Digital Communications and Networks 5 (2019) 84–93

Contents lists available at ScienceDirect

Digital Communications and Networks

journal homepage: www.keaipublishing.com/dcan

Percolation analysis of large-scale wireless balloon networks

M.D. Nashid Anjum, Honggang Wang *, Hua Fang

University of Massachusetts Dartmouth, 285 Old Westport Rd, Dartmouth, MA 02747, USA

A R T I C L E I N F O

Keywords:
Percolation
Connectivity
WBN
PPP
GDM

A B S T R A C T

Recent advancements in wireless technology have tested Wireless Balloon Networks (WBNs) as an ideal solution for the provision of internet facilities in deprived and challenging areas. A few high profile companies, such as Google, Space Data Inc., etc., have already made news by initiating projects based on high-altitude WBNs in order to provide internet facilities in remote areas. Unfortunately, the technical details have mainly been kept confidential so far. In this paper, we attempt to analyze the percolation properties of large-scale WBNs, considering both homogenous and heterogenous wireless nodes. In order to do so, we modeled a WBN as a large-scale random network where the path-loss models of homogenous and heterogenous WBNs were reduced to GDM (Gilbert's Disk Model) and RGDM (Random Gilbert's Disk Model), respectively. The bounds of the critical density regime were derived for both percolation models. Additionally, this paper implemented an experimental test bed for the WBN percolation model. Consequently, the findings of this research may prove crucial in estimating critical network properties.

1. Introduction

Balloons have been regularly used to gather weather information over the last few decades [1]. However, the concept of Wireless Balloon Networks (WBNs) was introduced only recently. A few highly regarded companies, such as Google, Space Data Inc., etc., recently commenced projects based on high-altitude wireless balloon networks in order to provide internet facilities in remote areas [1–3]. The balloon nodes of a high-altitude WBN are deployed over the stratosphere, which is in the range $6-31$ miles above ground level. However, controlling the movements of balloon nodes and network topology can be rather challenging for high-altitude WBNs. In order to tackle these difficulties, Facebook has offered a groundbreaking solution by replacing the WBN balloon nodes with solar drones [2,4].

Besides high-altitude WBNs, low-altitude WBNs have been envisioned as a rapid solution to provide temporary networking facilities over hazardous and affected areas during natural disasters [5–8]. In addition, pilot tests of WBNs have been successfully carried out on both high-altitude [2] and low-altitude platforms [7,8]. Furthermore, the simple infrastructure, low cost, easy portability, and rapid installation capability of WBNs have proved their immense potential for the application of wireless technology.

Unfortunately, the research progress on WBNs has so far been kept behind closed doors and very little has been disclosed on public research platforms. Although there are some studies on low-altitude WBNs ([5–10]) there are very few covering high-altitude WBNs [1–3]. Furthermore, these have mainly studied the application, infrastructure, and potential of WBN technology. To date, the analysis of network properties has largely been ignored.

In this paper, we aim to analyze the percolation properties of a WBN. Unlike ordinary ad-hoc networks, a WBN is three-dimensional in nature, and thus percolation modeling becomes more challenging. Moreover, a WBN can be homogenous or heterogenous in nature. In a homogenous WBN, all the nodes have equal transmission power and range. On the contrary, in a heterogenous WBN, the nodes may have different transmission powers and ranges. We reduced the percolation model of a homogenous WBN to a Gilbert's Disk Model (GDM), while the percolation model of a heterogenous WBN was reduced to a Random Gilbert's Disk Model (RGDM). We derived the upper and lower bounds of the critical node distribution density of both the homogenous and heterogenous WBNs. Determining this percolation property is crucial in order to estimate the network connectivity, cost, throughput, transmission capacity, and so on.

The rest of the paper is organized as follows: Section 2 describes the system model. Section 3 deals with the percolation model of the WBN. Section 4 and 5 derive the bounds of the critical density of the homogenous and heterogenous WBNs, respectively. Finally, the conclusion is provided in Section 7.

* Corresponding author.
E-mail addresses: manjum@umassd.edu (M.D.N. Anjum), hwang1@umassd.edu (H. Wang), hfang2@umassd.edu (H. Fang).

https://doi.org/10.1016/j.dcan.2018.03.001
Received 29 January 2018; Accepted 2 March 2018
Available online 27 March 2018

2. System model

A typical system model for a high-altitude WBN is depicted in Fig. 1 where the network nodes are deployed over the stratosphere. The network nodes are basically high altitude floating balloons. A network node may use two types of communication [1,11]. First, horizontal communication, by which the balloon nodes create a horizontal, ad-hoc skymesh. The second is vertical communication, by which a node creates a vertical network with ground stations. In this paper, our interest is to analyze the percolation and connectivity of a horizontal mesh network. The vertical network connectivity is beyond our interest. In the rest of this paper, WBN only refers to the ad-hoc skymesh of the balloon network.

The altitude of the WBN varies depending on the application. Low Altitude Platform (LAP) based WBNs are used to provide networking facilities in emergencies, such as searching and rescuing purposes for disaster management or providing temporary internet facilities in remote areas that are hard to reach. In general, typical low-altitude WBNs are deployed 40–100 m above ground level [1]. On the other hand, High-Altitude Platform (HAP) based WBNs are useful for gathering weather information and providing commercial internet facilities in remote areas. High-altitude WBNs are typically deployed over the stratosphere [1]. Hence, the altitude of a high-altitude WBN network may vary from 6 miles to 31 miles above ground level. Regardless of the applications, the purpose of the WBN is to fulfil network coverage along the horizontal plane (XY-plane). Thus, in the case of an ideal WBN, to ensure the maximum network coverage along the horizontal plane, all of the network nodes need to obtain equal altitude. However, in practice it can sometimes be challenging to achieve this ideal condition, especially for high altitude WBNs, due to air flow, cloud effects, etc. In this paper, we ignore the altitude differences of the network nodes if the nodes belong to a common WBN. This assumption significantly reduces the complexity of the connectivity analysis of WBNs using percolation theory.

3. Percolation model of a WBN

In a WBN, a number of balloon nodes are spatially distributed over a large area, hence a WBN can be described as a large-scale random network. Consequently, it can be modeled as a graph, where the balloons represent the nodes of the graph and the wireless links between the balloons represent the edges of the graph. In addition, percolation theory has been identified as a crucial mathematical tool to analyze the connectivity of a large-scale random graph, *i.e.*, large-scale random wireless networks [12].

Percolation is the event of creating a connected component in a graph [13] and connected component is a set of nodes that are connected to each other with open edges (*i.e.*, links). In a discrete percolation model, an edge (*i.e.*, a link) between two network nodes is considered to be open with a probability p. In other words, the edge can be assumed closed with the probability $(1-p)$. Moreover, p_c is the critical probability where if $p > p_c$, the network percolates with a probability $\theta(p) > 0$. Discrete percolation helps to understand the fundamental concept of percolation theory, but it does not properly reflect the path-loss model of the wireless network. In order to overcome this limitation, the Gilbert's Disk Model (GDM) was introduced in [14]. A GDM incorporates the path-loss property of the wireless communication into percolation theory. According to GDM, two nodes are connected if the distance between the nodes is smaller than a predefined critical distance.

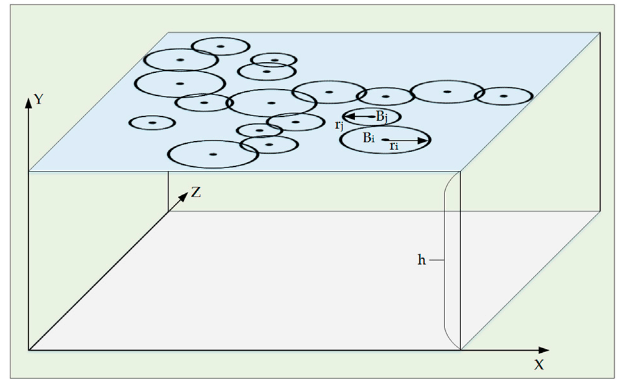


Fig. 2. Random Gilbert disk percolation model of a typical wireless balloon network (3D view).

We consider the nodes of a WBN as spatially distributed over the horizontal plane (X—Y) at altitude $z = h$ (Fig. 2, Fig. 3). Hence, $\boldsymbol{B} \subset \mathbb{R}^2$, where $\boldsymbol{B} = \{b_i, b_2, \dots, b_n\}$ represents the set of network nodes. The spatial distribution follows a **PPP** (Poisson Point Process) with a density ρ. The density ρ represents the mean number of nodes per unit area.

The network coverage area of node b_i is presented by a disk d_i. We consider the transmission range, *i.e.*, the radius of a disk d_i is presented by r_i, which is chosen randomly. The value of r_i is independent and identically distributed (*i.i.d*) for all of the network nodes. This implies that each network node may have a different transmission range, *i.e.*, coverage area, which means the network nodes are heterogenous in nature. We assume the random variable $\tilde{r}$ represents the disk radius. The location of node b_i is presented by $l_i \equiv (x_i, y_i, z_i)$. The Euclidian distance between two nodes b_i and b_j is given by:

$$\lambda_{ij} = \left\| l_i - l_j \right\| \tag{1}$$

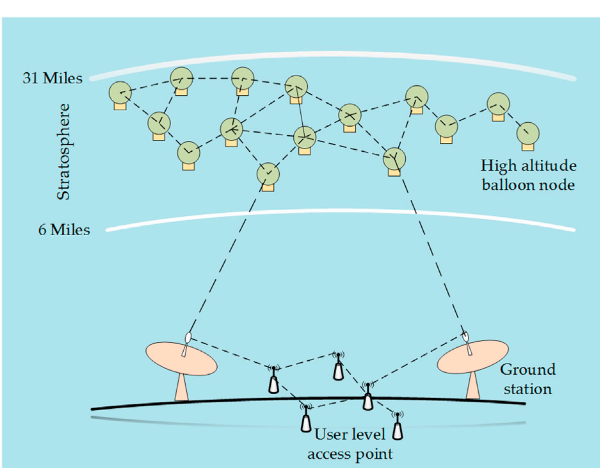


Fig. 1. System model of a typical high altitude wireless balloon network.

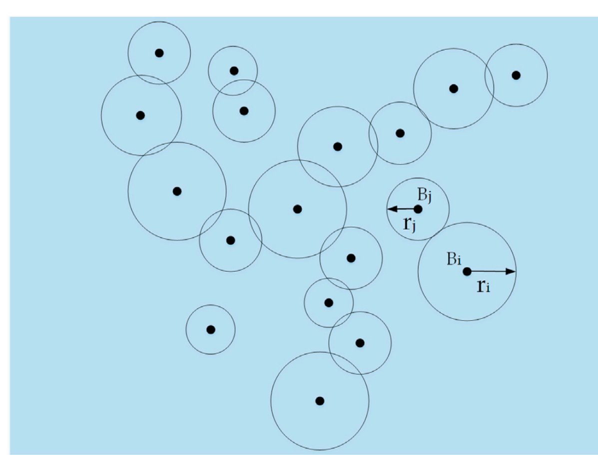


Fig. 3. Random Gilbert disk percolation model of a typical wireless balloon network (Bird's eye view).

$$= \left[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j) \right]^{\frac{1}{2}} \quad (2)$$

For the ideal case, $z_i = z_j = h$, where h is the altitude of the WBN network plane as depicted in Fig. 2. Hence, the node distance λ_{ij} is reduced to:

$$\lambda_{ij} = \left[(x_i - x_j)^2 + (y_i - y_j)^2 \right]^{\frac{1}{2}} \quad (3)$$

Two nodes are connected if and only if disk d_i and disk d_j overlap. This is known as the random Gilbert disk percolation model and is denoted by $G_R(\rho, r)$.

Definition 3.1. *Two nodes b_i and b_j of a WBN are connected iff $\lambda_{ij} < [|r_i| + |r_j|]$.*

Our aim is to analyze the percolation properties of a WBN in terms of the node density ρ and the random disk radius (*i.e.*, transmission range) $\tilde{r}$.

We are interested in determining the critical density ρ_c, which indicates the minimum node density. If the node density $\rho < \rho_c$, then the percolation of WBN does not occur.

Definition 3.2. The critical density ρ_c is the minimum density needed to obtain percolation in a WBN under the random Gilbert's disk model below which percolation does not exist. Mathematically,

$$\rho_c(\tilde{r}) = \inf\{\rho : |\mathscr{K}| = \infty\}; n = \infty \quad (4)$$

where $\mathscr{K}$ denotes the connected element of the origin [13] i.e., the set of the connected nodes. inf{} represents the infimum of the set.

4. Percolation of a homogenous WBN

Presumably, high altitude WBNs are homogenous in nature. This implies that each node has similar transmission characteristics, *i.e.*, an equal transmission power, rate, and range. This means that:

$$r_i = r; \forall i \quad (5)$$

Definition 4.1. For homogenous WBNs, two nodes b_i and b_j are connected if $\lambda_{ij} < [|r_i| + |r_j|] = 2r$.

Hence, two nodes are connected if the distance between the nodes is less than twice their transmission range. This is the condition of Gilbert's disk model, which is denoted by $G(\rho, r)$. A fundamental property of GDM is stated in theorem 4.1 [13].

Lemma 4.1. *The percolation properties of two WBNs $G(\rho_1, r_1)$ and $G(\rho_2, r_2)$ are identical i.e., $G(\rho_1, r_1) \equiv G(\rho_2, r_2)$ iff $\rho_1 r_1^2 = \rho_2 r_2^2$.*

Proof 4.1. Let $\boldsymbol{B}$ be distributed over a region $\mathscr{A}$. Thus, the expected number of nodes in a unit area is, $\mathbb{E}\{\boldsymbol{B}([0,1])\} = \rho$. Hence, the expected number of nodes over $\mathscr{A}$ is given by $\mathbb{E}\{\boldsymbol{B}(\mathscr{A})\} = |\mathscr{A}|\rho$. Thus, the expected number of neighbours in a unit disk of the percolation model $G(\rho_1, r_1)$ is $\mathbb{E}\{\boldsymbol{B}(\pi r_1^2)\} = \rho_1 \pi r_1^2 - 1$. Similarly, $\mathbb{E}\{\boldsymbol{B}(\pi r_2^2)\} = \rho_2 \pi r_2^2 - 1$. Thus:

$$\rho_1 r_1^2 = \rho_2 r_2^2 \quad (6)$$

$$\Rightarrow \rho_1 \pi r_1^2 - 1 = \rho_2 \pi r_2^2 - 1 \quad (7)$$

$$\Rightarrow \mathbb{E}\{\boldsymbol{B}(\pi r_1^2)\} = \mathbb{E}\{\boldsymbol{B}(\pi r_2^2)\} \quad (8)$$

And so the number of neighbours in a unit disk is equal for both models and $G(\rho_1, r_1) \equiv G(\rho_2, r_2)$.

Based on Theorem.4.1 , the following theorem can be deduced.

Lemma 4.2. *Any GDM of a WBN can be normalized into an equivalent GDM of the unit radius. Mathematically, $G(\rho, r) \equiv G(\rho r^2, 1)$.*

Let $\rho_0 = \rho r^2$, and thus, $G(\rho_0, 1) \equiv G(\rho r^2, 1)$, where ρ_0 is the normalized distribution density.

4.1. Upper bound of the critical density regime for homogenous WBN percolation

The upper bound of the critical density ρ_c can be derived by mapping GDM into a hexagonal lattice grid, as discussed in [13,15].

Consider a hexagonal lattice grid, as shown in Fig. 4. A hexagon is open if it possesses at least one node. An open hexagon is marked "green". The probability of a hexagon face being open is p_0. Moreover, the distribution of the nodes follows *PPP,* hence $p_0 = 1 - \exp(-\rho|h|)$, where, $|h|$ represents the cell area. It is known that a hexagonal lattice percolates *if* $p_0 > \frac{1}{2}$ [13]. If the length of an edge of the hexagon is σ , then $|h| = \frac{3\sqrt{3}\sigma^2}{2}$. Thus, percolation happens if and only if:

$$1 - \exp\left(-\rho \frac{3\sqrt{3}\sigma^2}{2}\right) > \frac{1}{2} \quad (9)$$

$$\Rightarrow \sigma^2 > \frac{2 \ln 2}{3\sqrt{3}\rho} \quad (10)$$

From Fig. 4 we find that the maximum distance between two nodes within two neighboring open hexagons is:

$$ab = ac + bc = 2ac \quad (11)$$

$$= 2\sqrt{\left(\frac{\sigma}{2}\right)^2 + \left(2\sigma\cos\left(\frac{\pi}{6}\right)\right)^2} \quad (12)$$

$$= \sqrt{13}\sigma \quad (13)$$

According to GDM, percolation happens if:

$$\sqrt{13}\sigma < r \Rightarrow \sigma^2 < \frac{r^2}{13} \quad (14)$$

From 9 and 14, we derive, $\rho r^2 > \frac{26 \ln(2)}{3\sqrt{3}} \approx 5$, or, $\rho_0 > 5$

Theorem 4.3. *The probability of the percolation of the homogenous WBN is greater than* 0 if:

$$\rho > \frac{26 \ln(2)}{3\sqrt{3}r^2} \approx \frac{5}{r^2} \quad (15)$$

Theorem 4.4. *The upper bound of the critical density regime for the percolation of the homogeneous WBN is* $\rho_c^U = \frac{26 \ln(2)}{3\sqrt{3}r^2}$.

The normalized upper bound is $\rho_o^U c = \rho_c^U r^2$.

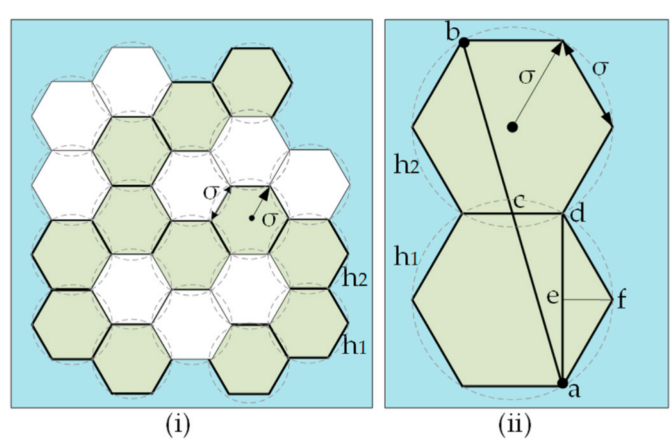


Fig. 4. (i) Hexagonal equivalent of Gilbert's disk percolation model of homogenous WBN. (ii) Distance between two possible farthest connected nodes in two connected closest hexagons.

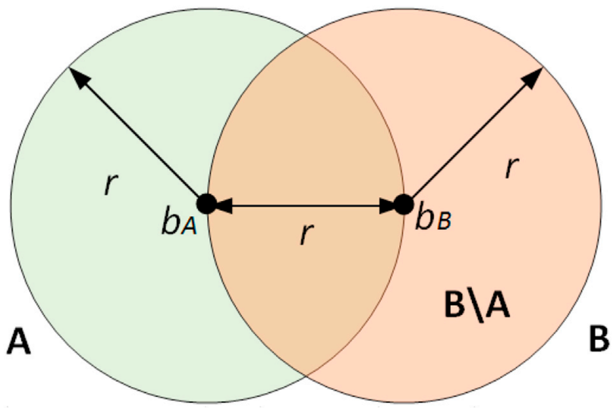


Fig. 5. Calculating lower bound of critical density of homogenous WBN.

4.2. Lower bound of the critical density regime for homogenous WBN percolation

Let **A** and **B** be the two circular regions with radius r, as shown in Fig. 5. If node b_A is located at the center of **A**, then the rest of the nodes that reside in **A** are the neighbor nodes of $b1$. The set of neighbor nodes of b_A is $\mathscr{N}_A$. Similarly, b_B is located at the center of **B** and the corresponding set of neighbor nodes is $\mathscr{N}_B$. Considering that $b_B \in \mathscr{N}_A$, the distance between b_A and b_B is $|l_{b_A} - l_{b_B}| \le r$ in order to connect to each other. The portion of **B** that does not overlap with **A** is presented by $\mathbf{B} \setminus \mathbf{A}$. Hence, $\mathscr{N}_{\mathscr{B}} \setminus \mathscr{N}_{\mathscr{A}}$ represents the set of nodes that are located in the region $\mathbf{B} \setminus \mathbf{A}$. $\#(\mathscr{N}_{\mathscr{B}} \setminus \mathscr{N}_{\mathscr{A}})$ is maximized when $\mathbf{B} \setminus \mathbf{A}$ is maximized, that is, when $|l_{b_A} - l_{b_B}| = r$.

Hence, the maximum expected number of neighbor nodes of b_B that are not the neighbor of b_A is [13,16]:

$$\max(\mathbb{E}\{\#(\mathscr{N}_{\mathscr{B}} \setminus \mathscr{N}_{\mathscr{A}})\}) = \max(|\mathbf{B} \setminus \mathbf{A}|)\rho \tag{16}$$

$$= \left(\pi r^2 - r^2 \cos^{-1}\left(\frac{1}{2}\right) + \frac{r}{2}\sqrt{3r^2}\right)\rho \tag{17}$$

$$= \left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right) r^2 \rho \tag{18}$$

Percolation is not possible if:

$$\left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right) r^2 \rho < 1 \tag{19}$$

$$\Rightarrow \rho < \frac{1}{r^2} \frac{6}{4\pi + 3\sqrt{3}}. \tag{20}$$

Theorem 4.5. *The homogenous WBN does not percolate* if

$$\rho < \frac{6}{(4\pi + 3\sqrt{3})r^2} \approx \frac{0.338}{r^2} \tag{21}$$

Theorem 4.6. *The lower bound of the critical density regime of the homogenous WBN is* $\rho_c^L = \frac{6}{(4\pi+3\sqrt{3})r^2}$.

The normalized lower bound is $\rho_{oc}^L = \rho_c^L r^2$.

Fig. 6 shows how the values of ρ_c^U and ρ_c^L vary with respect to the transmission range r. The difference between the values of ρ_c^U and ρ_c^L is known as the critical density regime. If the node distribution density ρ of **PPP** is less than ρ_c^L, then the percolation probability is zero, and thus connectivity between the nodes will be lost. On the other hand, if $\rho > \rho_c^U$, then percolation occurs, there is a high probability that the connectivity between the nodes will be maintained. If $\rho_c^L < \rho < \rho_c^U$, then connectivity is not impossible, but the probability is much lower.

Once the values of ρ_c^L and ρ_c^U are determined, it can be possible to estimate the required minimum number for the nodes to establish network connectivity. Fig. 7 shows the required minimum number of nodes, below which number percolation is impossible for a given area and transmission range. As might reasonably be expected, the required number of nodes increases as the area size increases. Furthermore, for the same area the number of required nodes decreases as the size of the transmission range increases.

However, if the deployed number of nodes is greater than the required minimum number of nodes, it does not necessarily ensure percolation, *i.e.*, network connectivity. To ensure network connectivity, the number of deployed nodes should be greater than the least required number of nodes. It is notable that the word "ensuring" here actually means "highly probable". For example, if $r = 1$ and $area = 30\,m^2$, then we can see from Fig. 7 that percolation is not possible if the number of deployed nodes is less than 10. Additionally, from Fig. 8 we can see that the number of deployed nodes should be more than 150 in order to maintain good connectivity.

5. Percolation of heterogenous WBNs

Unlike homogenous WBNs, the nodes of a heterogenous WBN do not

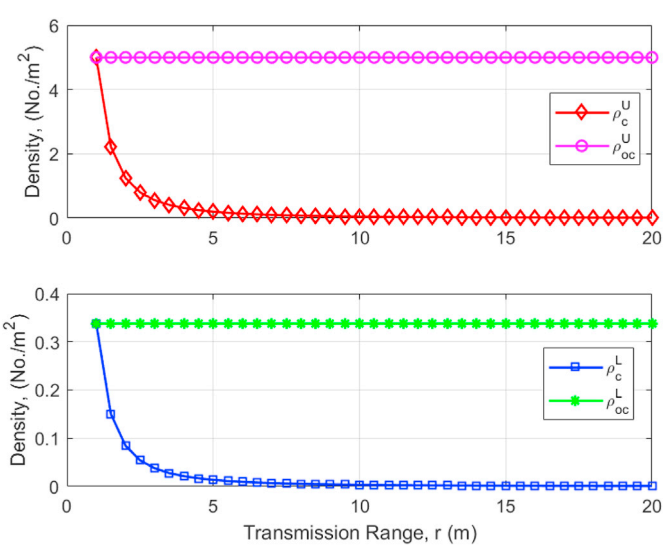


Fig. 6. Values of ρ_c^U and ρ_c^L with respect to transmission range r.

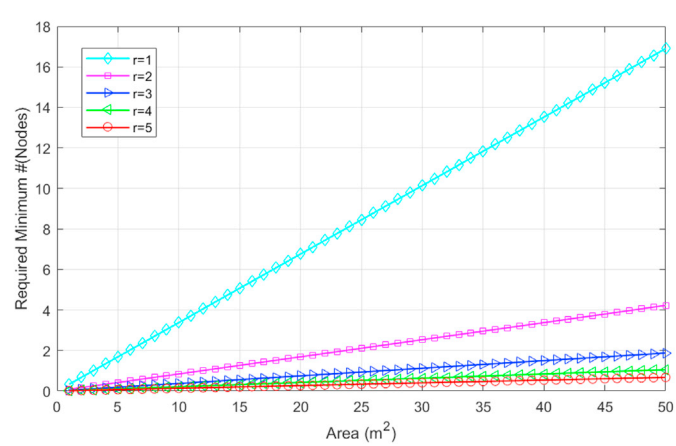


Fig. 7. Required minimum number of nodes under which percolation is impossible in a homogenous WBN.

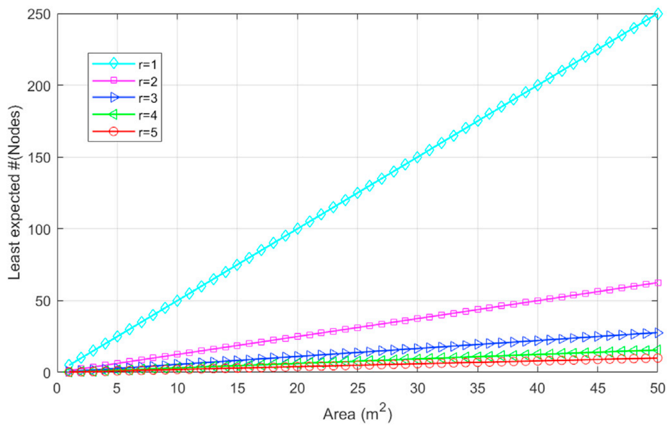


Fig. 8. Required least expected number of nodes to trigger percolation in a homogenous WBN.

possess an equal transmission range, but rather different nodes have different transmission ranges. Hence, the disk radius of the percolation model should be a random variable $\tilde{r}$ as we defined in Section 3. Naturally, a random variable $\tilde{r}$ should follow a uniform distribution. Finding the bounds of the critical density regime of heterogenous WBNs can be achieved based on RGDM. In the first part of this section, we find the upper bound of the critical density regime. The latter part deals with finding the lower bound followed by the approach introduced in reference [13].

5.1. Upper bound of the critical density regime of heterogenous WBN percolation

The upper bound of the RGDM can be derived by reducing the problem to a GDM [13] for a particular minimum transmission range. In order to do so, we chose a minimum radius r_{min}. Let $B_{r_{min}} \subset B$, where $B_{r_{min}}$ is the set of the nodes with a transmission range less than r_{min}. We are only interested in the nodes that have a transmission range greater than r_{min}. Thus, we are only interested in the nodes of set $B^c_{r_{min}}$. Hence, $B^c_{r_{min}} \subset B$, so $B^c_{r_{min}}$ also follows **PPP**. Thus, the effective node distribution is smaller than the actual distribution ρ, that is $\rho Pr\{\tilde{r} \geq r_{min}\}$. For the sake of calculating the upper bound, we assume the worst condition, that is $\tilde{r} = r_{min}$. A smaller range of transmission requires a greater number of nodes in an area for percolation to take place. Once we set $\tilde{r} = r_{min}$, then the problem is reduced to a GDM, as discussed in Section 4. Comparing the result found in Section 4, we can derive the following two theorems: 5.1 and 5.2.

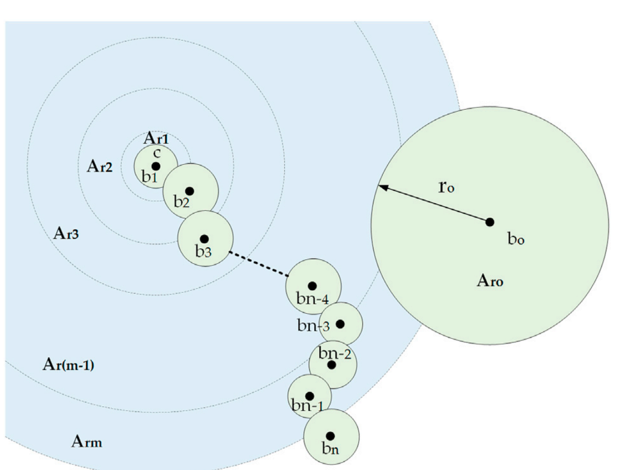


Fig. 9. Scenario setup for deriving the upper bound of critical density regime of a heterogenous WBN.

Theorem 5.1. *The probability of the percolation of the heterogenous WBN is greater than 0 for a chosen range r_0 if:*

$$\rho\left(\tilde{r} \geq r_0\right) \rangle \frac{26\ln(2)}{3\sqrt{3}r_0^2} \approx \frac{5}{r_0^2} \tag{22}$$

Theorem 5.2. *The upper bound of the critical density regime of the heterogenous WBN is:* $\dot{\rho}^U_c(\tilde{r} \geq r_0) = \frac{26\ln(2)}{3\sqrt{3}r_0^2} \approx \frac{5}{r_0^2}$.

5.2. Lower bound of the critical density regime of heterogenous WBN percolation

Let node b_1 reside at the center (c) of the circular region $\mathbf{A_{r_i}}$ with the radius r_i (Fig. 9), where $i \in \{1, 2, \ldots, m\}$. Moreover, $b_n \in \mathscr{K}$ is the farthest connected node of b_1, where $\mathscr{K}$ is the connected component.

Definition 5.1. *$E(0,r)$ is the event of having a connected path between a node $b_i \in \boldsymbol{B} \cap \boldsymbol{A}_{r_1}$ and $b_j \in \boldsymbol{B} \cap \boldsymbol{A}_{r_{m-1}} \setminus c + \boldsymbol{A}_{r_{m-2}}$, considering all the nodes on the path from b_i to b_j located inside $\mathbf{A}_{m_r} + c$.*

Definition 5.2. *$E_o(r)$ is the event of having a node $b_o \in \boldsymbol{B}$ outside the boundary of $\mathbf{A_{m_r}}$ for which there is overlap between the regions $\boldsymbol{A}_{\mathbf{r_o}}$ and $\boldsymbol{A}_{r_{m-1}}$.*

As shown in Fig. 9, b_n is located in $\boldsymbol{A}^{\mathbf{c}}_{\mathbf{r_m}}$ and the range of b_n does not overlap with $\boldsymbol{A}_{\mathbf{r}_{(m-1)}}$. Now, if $E^c_o(r)$ also occurs, then the event $E(0,r)$ happens. Thus:

$$Pr\left\{b_n \in \boldsymbol{A}^{\mathbf{c}}_{\mathbf{r_m}} \cap E^c_o(r)\right\} \leq Pr\{E(0,r)\} \tag{23}$$

$$\Rightarrow Pr\left\{b_n \in \boldsymbol{A}^{\mathbf{c}}_{\mathbf{r_m}}\right\} \leq Pr\{E(0,r)\} + Pr\{E_o(r)\} \tag{24}$$

The percolation occurs when b_n is located inside $\boldsymbol{A}_{\mathbf{r_m}}$, so that

$$Pr\{|\mathscr{K}| = \infty\} \leq \lim_{r\to\infty} Pr\{b_n \in \boldsymbol{A}_{\mathbf{r_m}}\} \tag{25}$$

$$\leq \lim_{r\to\infty} Pr\{E(0,r)\} + \lim_{r\to\infty} Pr\{E_o(r)\} \tag{26}$$

When $r \to \infty$, the probability of b_o residing outside $\boldsymbol{A}_{\mathbf{r_m}}$ is zero; hence,

$$\lim_{r\to\infty} Pr\{E_o(r)\} = 0. \tag{27}$$

Lemma 5.3. *If $\rho \leq \frac{1}{4k^2\mathbb{E}\{\tilde{r}^2\}}$, then $\lim_{r\to\infty} Pr\{E(0,r)\}$. Here, k is a constant.*[1]

Substituting the value of Equation (25) by Equation (28) and Lemma 5.3, we get that if $\rho \leq \frac{1}{4k^2\mathbb{E}\{\tilde{r}^2\}}$ then

$$Pr\{|\mathscr{K}| = \infty\} = 0 \tag{28}$$

Theorem 5.4. *If the expected number of neighbor nodes $\rho\mathbb{E}\{\tilde{r}^2\}$ is less than a constant value k, then the probability of percolation for a RGDM is equal to zero.*

Theorem 5.5. *The lower bound of the critical density regime for a heterogenous WBN is:* $\dot{\rho}^L_c(\tilde{r}) \geq \frac{1}{4k^2\mathbb{E}\{\tilde{r}^2\}}$.

5.3. Integrating fading with RGDM

We considered more general fading plus a path-loss wireless communication model for GRDM. In this case, we represented the fading coefficient of the wireless communication between node B_i and B_j by η_{ij}. The path-loss coefficient is α. Nodes B_i and B_j are connected if the

[1] See Ref. [13] for detail proof.

received signal strength is greater than the threshold (*i.e.*, the minimum required signal strength) τ. Mathematically,

$$\left|\eta_{ij}\right|^2 l_i - l_j^{-\alpha} > \tau \tag{29}$$

$$\Rightarrow \left\|l_i - l_j\right\| \le \left(\frac{\tau}{\left|\eta_{ij}\right|^2}\right)^{-\frac{1}{\alpha}} \tag{30}$$

$$\Rightarrow \lambda_{ij} \le \left(\frac{\tau}{\left|\eta_{ij}\right|^2}\right)^{-\frac{1}{\alpha}} \tag{31}$$

Thus, the random connection distance to maintain connectivity between $\boldsymbol{B_i}$ and $\boldsymbol{B_j}$ is $\tilde{\lambda}_{ij} = \left(\frac{\tau}{|\eta_{ij}|^2}\right)^{-\frac{1}{\alpha}}$. $\tilde{\lambda}$ resembles the random radius $\tilde{r}$ of RGDM, but $\tilde{\lambda}_{ij}$ depends on both i and j. In the rest of this paper, we denote $\tilde{\lambda}_{ij}$ as $\tilde{\lambda}$ to avoid notation complexity. In this case, the critical density of the path-loss and fading-based RGDM is given by $\rho_c(\tilde{\lambda})$.

In this case, the upper bound is the same as in the general RGDM model, as we show in Sub-Section 5.1. The lower bound of the critical density can be found by comparing it with Theorem 5.5:

$$\rho_c(\tilde{\lambda}) \ge \frac{1}{k' \mathbb{E}\{\tilde{\lambda}^2\}} \tag{32}$$

where k' is a constant.

However, in this case, the distribution of the random variable $\tilde{\lambda}$ does not necessarily follow the uniform distribution, but rather the distribution follows the same distribution as the channel coefficient η.

6. Network test bed

This section deals with an experimental test bed setup of a WBN percolation model. To conduct the test bed, we arbitrarily deployed N wireless nodes over a predefined bounded area A on a common two dimensional XY plane with a constant height h. As for network nodes, we used $GL - AR300M - Lite$ smart routers. The routers operate at a 2.4 GHz Wi-Fi frequency with a maximum transmission rate of 300 Mbps and a maximum transmission power of 20 dBm. The hardware of the network node is based on the *Atheros* 9631 WiSoC (Wi-Fi System-On-Chip) with a processing speed of 650 MHz, 128 MB DDR2 RAM, and 16 MB NOR Flash storage. The firmware of the network node was developed on the open source OpenWrt operating system, which provides a wide range of flexibility to configure the router nodes. Table 1 provides the detailed technical specifications of the test bed network nodes.

We set up three test bed scenarios and compared the results. In *Scenario I*, we chose a $13 \times 10\,m$ bounded area ($A = 130\,m^2$), where an Access Point (AP) is connected with an End Node (EN) or client through five network nodes. The locations of these seven network elements were chosen arbitrarily. As for the access point, we used a *NETGEAR N*300 *DOCSIS* 3.0 Wi-Fi cable modem router with maximum bit transmission rate of 300 Mbps. As for the end node/client, we used a personal computer. The technical details of the connecting nodes are described in Table 1, as mentioned above. We set the node transmission power to 1 dBm using the LuCI-OpenWrt configuration interface. We assumed the SNR threshold to be 25 dB. This means that a wireless link is sufficiently connected if the $SNR \ge 25$ dB. The measured network coverage range of a node was approximately $r = 5m$.

The network topology for *Scenario I* is depicted in Fig. 10. The dotted lines in the figure show the wireless links between the network nodes. Table 2 describes the Service Set IDentifiers (SSIDs) of the nodes, node IPs, corresponding connected nodes, and types of connection. For example, from Table 2 we can see that Node_2 is connected to Node 1 and Node 3, where the connection type between Node 1 and Node 2 is a WLAN-bridge. On the other hand, Node 3 acts as the WLAN STA of Node 2.

The Euclidian distance between two nodes was calculated based on the corresponding node location coordinates as depicted in Table 3. The real-time signal strength and Additive White Gaussian Noise (AWGN) was measured using the GL-iNet-provided LuCI-OpenWrt Configuration (LOC) interface. Based on the peak signal strength, average signal strength, and AWGN, the average and peak SNR were measured for each wireless link. We also measured the PHY layer bit rate for each link, such as that shown in Fig. 11 for the Node 1-EN link. The result is summarized in Table 3. Moreover, from Equation (21) we found that the network does not percolate if the node density $\rho_c < \frac{0.338}{5^2} = 0.014$. For *Scenario I*, $\rho =$

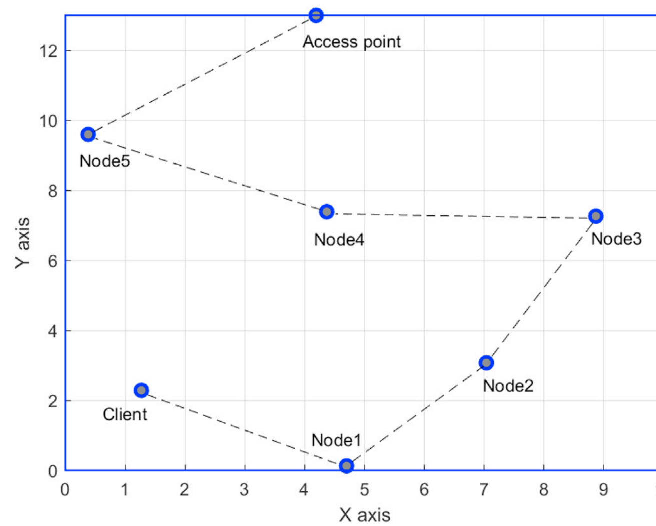


Fig. 10. Network topology of the WBN test bed *Scenario – I*.

Table 1
Network node technical specifications.

Parameter	Specification
CPU	Atheros 9531, @650 MHz
MemoryStorage	DDR2 128 MB NOR-Flash 16 MB
Frequency	2.4 GHz
Transmission Rate	300 Mbps
Max. TX Power	20 dBm
Used TX Power	1 dBm
Protocol	802.11 b/g/n
WAN	10/100 Mbps
Power Input	5 v/1 A
Power Consumption	< 2 W
Operating System	OpentWrt

Table 2
Test bed specifications.

Node	SSID	IP	Connected Device: BR-LAN	Connected Device: WLAN-STA
Node_1	GL-AR300M-495-NOR-Node4	192.168.8.4	EN	Node_2
Node_2	GL-AR300M-495-NOR-Node5	192.168.8.5	Node_1	Node_3
Node_3	GL-AR300M-495-NOR-Node16	192.168.8.16	Node_2	Node_4
Node_4	GL-AR300M-495-NOR-Node1	192.168.8.1	Node_3	Node_5
Node_5	GL-AR300M-495-NOR-Node2	192.168.8.2	Node_4	AP
EN	NASHID	192.168.8.232	–	Node_1
AP	NETGEAR96	192.168.0.22	Node_5	–

Table 3
Test bed results.

Link	ED/m	TX Power/dBm	Avg. Received Power/dBm	Received Max. Power/dBm	AWGN/dBm	Avg. SNR/dB	Max. SNR/dB	Max. Phy Bit Rate/Mbps
$Node_1$-EN	4.0521	1	– 64	– 62	– 94	29	32	65
$Node_2$-$Node_1$	3.7606	1	– 62	– 56	– 94	31	38	65
$Node_3$-$Node_2$	4.5726	1	– 65	– 63	– 94	29	31	65
$Node_4$-$Node_3$	4.4976	1	– 62	– 65	– 94	31	34	65
$Node_5$-$Node_4$	4.5592	1	– 68	– 65	– 94	25	29	65

$\frac{N}{A} = 0.0385 > \rho_c$. Hence, the network should percolate and there should be a sufficient connection between the *AP* and *EN*. This means that the SNR of the links between the two end nodes should be greater than or equal to 25 dB. From Table 3, it can be seen that the average SNR of each link was greater than or equal to 25 dB. Using the LOC interface, we were also able to analyze the real-time network load, connections, and bit rates of each network link, as shown in Fig. 12, Fig. 13, and Fig. 14 for the wireless Node 1-EN link, for instance.

In test bed *Scenario II* and *Scenario III*, we used a lower number of nodes to connect the *AP* and *EN,* as depicted in Fig. 15(a) and Fig. 15(b), respectively. In the case of *Scenario III*, we found that $\rho << \rho_c$, hence the network should not percolate. The experiment results provided in Fig. 16(b) show that the average SNR at *EN* over the Node 5-EN link was 22 dB, which was less than the required threshold of 25 dB, hence the network was not sufficiently connected and we can say it does not percolate.

On the contrary, the *Scenario II* results show interesting consequences that provide important practical insight. In this case, $\rho = 0.015$, which is slightly greater than the critical density (0.014), hence theoretically the network should percolate. However, from Fig. 16(a) we can see that the measured SNR was 24 dB, which is 1 dB less than the required threshold. This indicates that the network does not percolate. Interestingly, we saw that the peak SNR was much higher (38 dB) than the required threshold. Moreover, by observing the real-time trend of the SNR graph we can see that the received SNR fluctuates around the required threshold. Additionally, based on the 3 min window in Fig. 16(a), we can see that the average SNR improved over the time period. Possible reasons for the slightly lower average SNR in *Scenario II* are as follows: (i) imperfect hardware and (ii) unexpected fading. It is plausible that in practice, hardware is not perfect and thus the network nodes/devices that were used in this test bed do not guarantee 100% accuracy in practice. This fact suggests that ρ should be chosen in such a way that $\rho >> \rho_c$, so as to ensure network connectivity.

7. Conclusion

Wireless Balloon Networks (WBNs) are a recently introduced technology that has been envisioned as a rapid solution for the provision of internet access to remote places and hazardous areas. A few companies

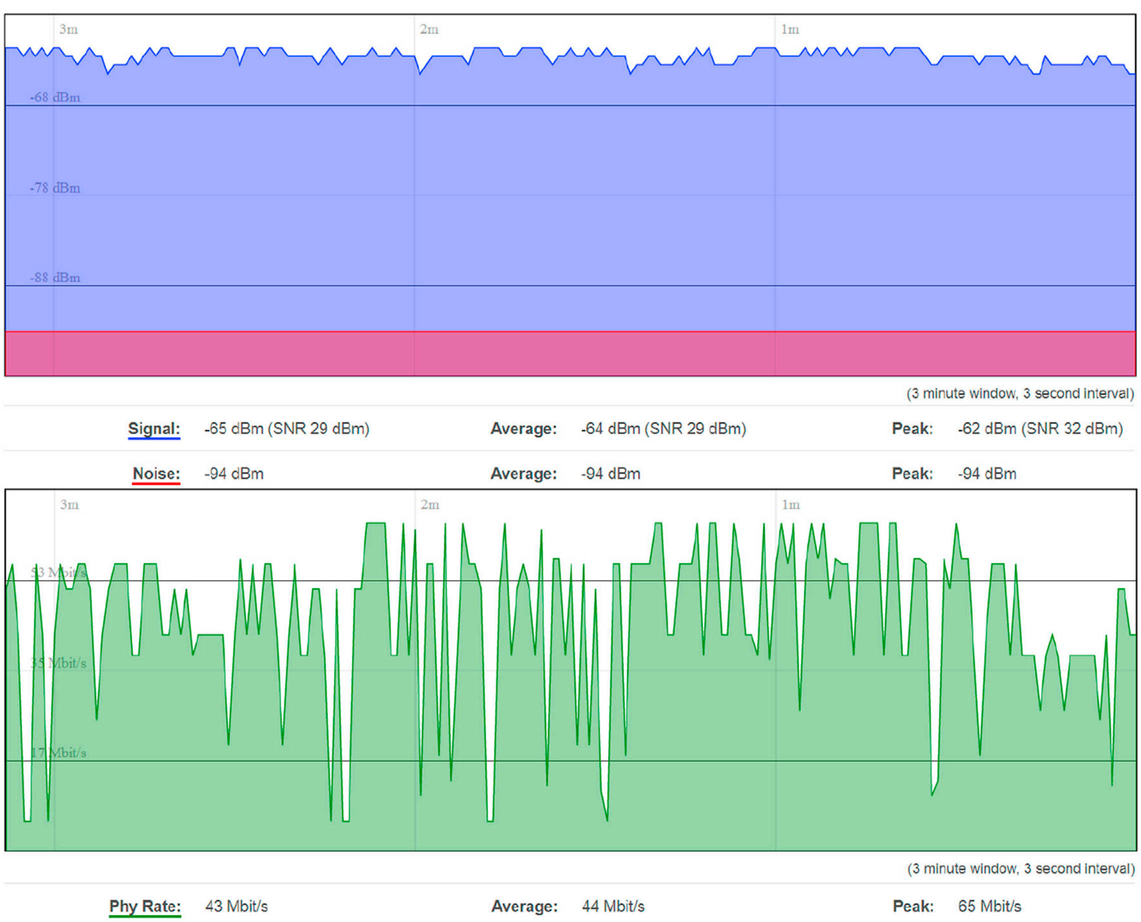
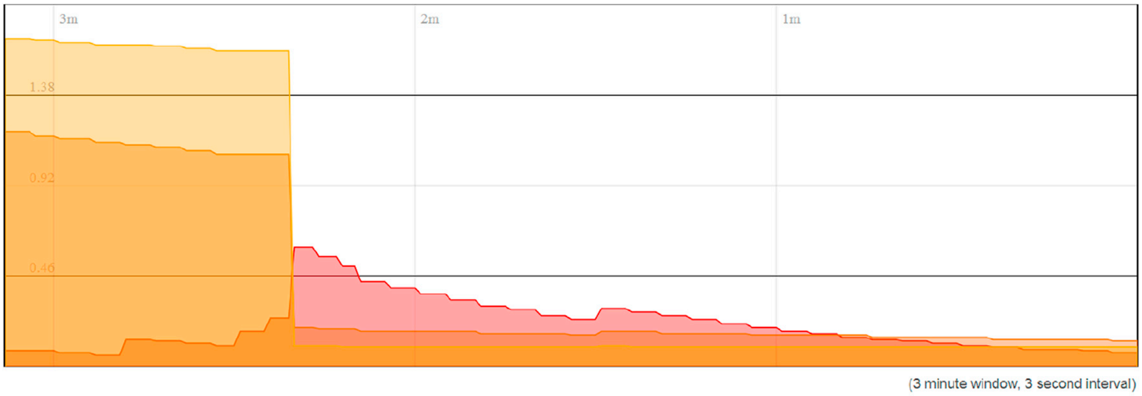
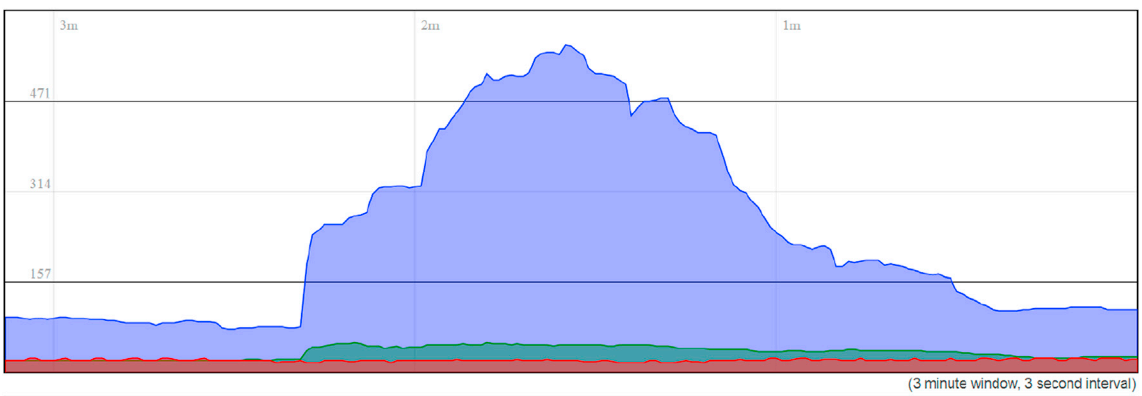


Fig. 11. Realtime signal strength, noise, SNR, and physical layer bitrate over 3 min window at $Node_1$-EN link.



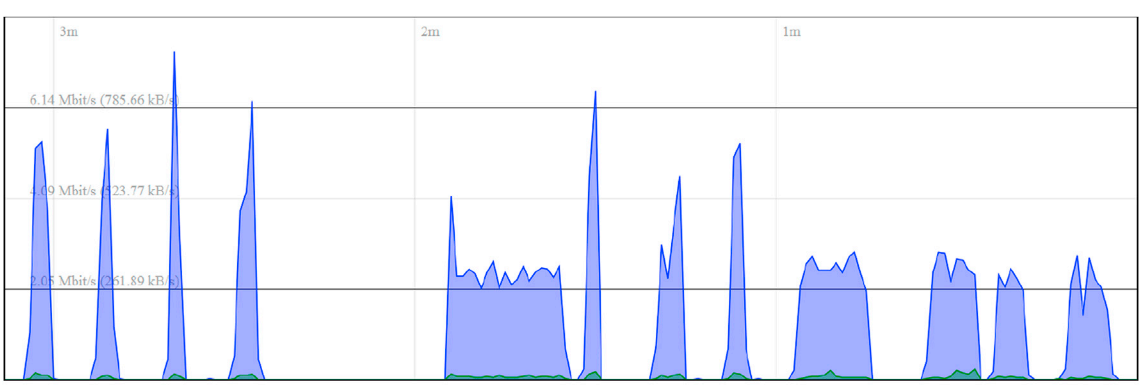
1 Minute Load:	0.07	Average:	0.07	Peak:	0.61
5 Minute Load:	0.13	Average:	0.13	Peak:	1.20
15 Minute Load:	0.10	Average:	0.10	Peak:	1.67

Fig. 12. Realtime network load at Node$_1$-EN link.



UDP:	110	Average:	110	Peak:	572
TCP:	29	Average:	28	Peak:	53
Other:	24	Average:	23	Peak:	26

Fig. 13. Realtime active connections over 3 min window at Node$_1$-EN link.



Inbound:	27.76 kbit/s (3.47 kB/s)	Average:	147.1 kbit/s (18.39 kB/s)	Peak:	7.44 Mbit/s (952.31 kB/s)
Outbound:	5.97 kbit/s (0.75 kB/s)	Average:	11.08 kbit/s (1.39 kB/s)	Peak:	275.53 kbit/s (34.44 kB/s)

Fig. 14. Realtime bit rate between WLAN-STA Node$_2$ and Node$_1$.

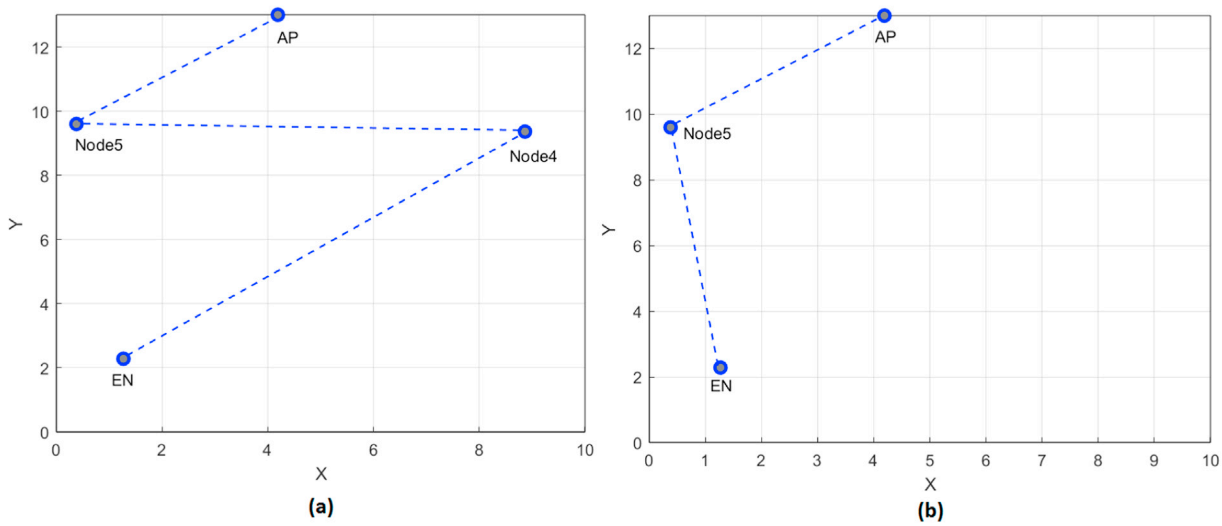


Fig. 15. Test bed topology for Scenario-II (a) and Scenario-III (b).

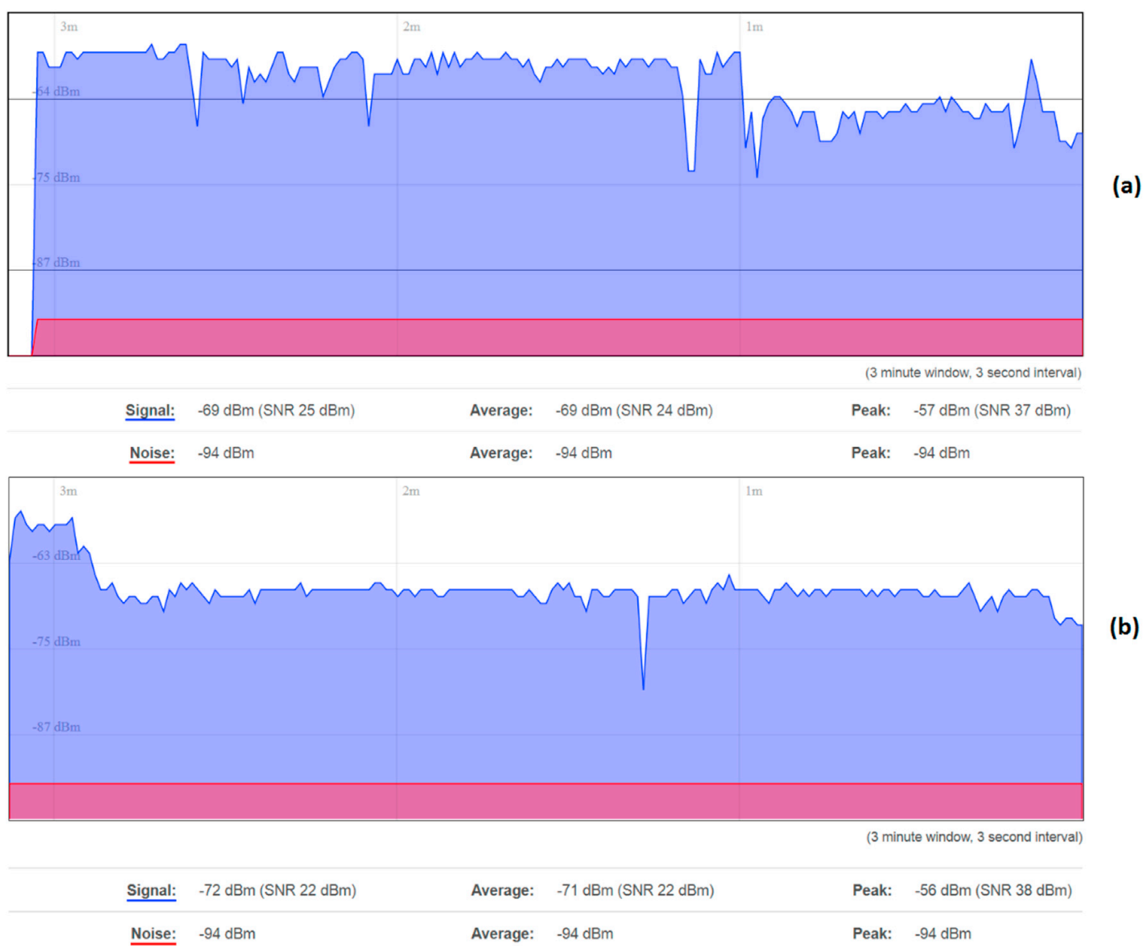


Fig. 16. Received signal strength, AWGN, and SNR for test bed Scenario-II (a) and Scenario-III (b).

have already commenced the development of WBNs in order to launch them commercially over the stratosphere. However, the research progress is kept behind closed doors and very little work has been published in the public arena. To the best of our knowledge, this is the first published attempt to carry out a network property analysis of a WBN. We analyzed the percolation of a WBN and derived the bounds for node distribution density both for homogenous and heterogenous WBNs. We modeled a WBN as a random network and showed that a path-loss model of homogenous WBNs can be achieved by reducing them to a Gilbert's Disk Model (GDM). Moreover, the path-loss model of a heterogenous WBN can be reduced to a Random Gilbert's Disk Model (RGDM). Our findings can be used to estimate the required minimum number of nodes in a target area in order to maintain the network connectivity of a WBN. This work could potentially play a larger role in the research field of WBNs to estimate their cost, connectivity, throughput, and transmission capacity.

Acknowledgement

This work is supported in part by National Science Foundation 1451629, 1401711, 1429120 and 1407882.

References

[1] D. Kim, A Survey of Balloon Networking Applications and Technologies, 2013.
[2] S. Katikala, Google¢ project loon, Insight: Rivier Academic Journal 10 (2) (2014) 1–6.
[3] S. Rushing, Analyzing" Global Access to the Internet for All" Projects, Network 9.
[4] T. Simonite, Facebooks Internet Drone Team Is Collaborating with Googles Stratospheric Balloons Project, MIT Technology Review.
[5] Y. Shibata, Y. Sato, N. Ogasawara, G. Chiba, A disaster information system by ballooned wireless adhoc network, in: Complex, Intelligent and Software Intensive Systems, 2009. CISIS'09. International Conference on, IEEE, 2009, pp. 299–304.
[6] Y. Shibata, N. Uchida, N. Shiratori, A resilient network and information management system for large scale disaster, in: Advanced Information Networking and Applications Workshops (WAINA), 2016 30th International Conference on, IEEE, 2016, pp. 655–660.
[7] Y. Shibata, Y. Sato, N. Ogasawara, G. Chiba, Ballooned wireless mesh network for emergency information system, in: Advanced Information Networking and Applications-workshops, 2008. AINAW 2008. 22nd International Conference on, IEEE, 2008, pp. 1118–1122.
[8] S. Arimura, Y. Shibata, Y. Ohashi, N. Uchida, A new wireless micro-balloon network for disaster use, in: Advanced Information Networking and Applications Workshops (WAINA), 2014 28th International Conference on, IEEE, 2014, pp. 617–622.
[9] J.-M. Kelif, Coverage and performance of stratospheric balloons wireless networks, in: Personal, Indoor, and Mobile Radio Communications (PIMRC), 2016 IEEE 27th Annual International Symposium on, IEEE, 2016, pp. 1–6.
[10] F. Ahdi, S. Subramaniam, Using unmanned aerial vehicles as relays in wireless balloon networks, in: Communications (ICC), 2015 IEEE International Conference on, IEEE, 2015, pp. 3795–3800.
[11] R. Premkumar, Wireless Networks for Disaster Relief, Department of Computer Science and Engineering, Washington University in St. Louis.
[12] M. Franceschetti, R. Meester, Random Networks for Communication: from Statistical Physics to Information Systems, vol. 24, Cambridge University Press, 2008.
[13] R. Vaze, Random Wireless Networks, Cambridge University Press, 2015.
[14] E.N. Gilbert, Random plane networks, J. Soc. Ind. Appl. Math. 9 (4) (1961) 533–543.
[15] B. Bollobás, O. Riordan, Percolation, Cambridge University Press, 2006.
[16] E. W. Weisstein, Circle-Circle Intersection, a Wolfram Web Resource.