Seminar for Interdisciplinary and Applied Research

Quirijn Meijer & Ruben Meeuwese

Introducing Ourselves

- **Quirijn**: background in mathematics and computer science, future PhD candidate in either physics or mathematics.
- **Ruben**:
- Yourselves.

Specifics for this Seminar

- **Vision**: promoting interdisciplinary and applied research.
- **Participation**: no obligation, and can take different forms.
- **Shared knowledge**: quickly reviewed in this presentation.

Shared knowledge

- Determining what can safely be assumed is difficult.
- The fewer assumptions, the more talks will have to rely on intuition.
- The review in this presentation is meant to jog everyone’s memory.

Proofs

Proof Methods

- **Induction**: assuming one case, induce the rest.
- **Contraposition**: $P \Rightarrow Q$ if and only if $\neg Q \Rightarrow \neg P$.
- **Contradiction**: deriving a contradiction.
- **Construction**: constructing an example.

Calculus

Differentiation

- Differentiation finds the rate of change of a function f.
- For a function f of one variable, the **derivative** is denoted $\frac{\mathrm{d}f}{\mathrm{d}x}$.
- A **partial derivative** of a multi-variable function f is denoted $\frac{\partial f}{\partial x_i}$.

Integration

- Integration is the reverse of differentiation.
- A **definite integral** of $f(x)$ from a to b is written $\int_a^b f(x)\mathrm{d}x$.
- The **fundamental theorem of calculus** states that:

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a).$$

Limits

- If it exists, the **limit** of $f(x)$ as x approaches c is L, or $\lim_{x \to c} f(x) = L$.
- Convergence and divergence.
- Used for the definitions of derivatives and integrals.

Linear Algebra

- **Vector spaces** are collections of objects that can be put together.
- **Vectors** can be all kinds of objects, including functions.
- **Basis vectors** give a natural understanding of the dimension of a space.

Matrices

- A **matrix** $M : V \to W$ is a linear map between vector spaces.
- Its dimensions are determined by the domain and co-domain.
- Matrices can be composed by summation and multiplication.

Determinants

- The **determinant** $\det(M)$ is a value containing important information.
- Intuitively, it gives the scaling factor of the matrix.
- The value can tell us if a matrix can be inverted.

Inner Products

- **Inner products** are maps that take two vectors, and return a scalar value.
- As an example, take the dot product.
- An **inner product space** is a pair of a space and an inner product.

Eigenvectors, Eigenvalues and Eigenspaces

- An **eigenvector** x of M is a vector for which $Mx = \lambda x$.
- The factor λ is the **eigenvalue** of the eigenvector x.
- Eigenvectors span **eigenspaces**.

Probability & Statistics

Random Variables

- The value of a **random variable** X depends on a random event.
- Examples are coin tosses or die throws.
- There is a distinction between **discrete** and **continuous** variables.

Probability Distributions

- **Distributions** assign **probabilities** to random events.
- They therefore also give probabilities for the values of a random variable.

Expectation and Variance

- The **expectation** $\mathbb{E}[X]$ is the expected value X will take on average.
- The **variance** $\mathrm{Var}(X)$ measures the spread of X around its expected value.
- Calculated differently for discrete and continuous random variables.
- Fun fact: if we use measure theory, we don't need to separate cases!

Statistical Tests

- **Statistical tests** try to infer the truth of a **hypothesis**.
- We are not going into detail, but we will remind you of their existence.
- Examples include the χ^2 test, ANOVA and Student's t-test.

Room for Questions