

Conjugate Gradient with Preconditioning

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Convergence behavior of Block Jacobi

Compare the convergence behaviour for the different block sizes and matrices. Which block size performs better? For which matrix does the method work better?

- The Block Jacobi Method for Nos5 matrix performs almost similar in both block size executions (Blocksize 1 and 5). It starts at 10^{-1} but stays relatively the same.
- The Convergence of NOS6 on the other hand looks a little different. Both executions seem to converge, signified that both residuals drop significantly and even start at the low value of 10^{-7} for blocksize 1 and $9 \cdot 10^{-8}$ for blocksize 5. So blocksize 5 performs better.
- In direct comparison NOS6 matrix performs better with our algorithm. indicated by the convergence indicated by the small and dropping residual.

Convergence of CG vs PCG

Compare the convergence behaviour of unpreconditioned CG and of PCG with the different preconditioners for the two test matrices. Which preconditioner performs best for the respective matrix? What could be the reason for that? Do you see a relation to the respective results shown in Part 1? If yes, explain this relation. If no, explain what you would have expected from the results in Part 1.

unpreconditioned CG:

- Nos5 converges slowly with small oscillations in 400 iterations.
- Nos6 oscillates even more extreme and needs more than 600 iterations to converge.

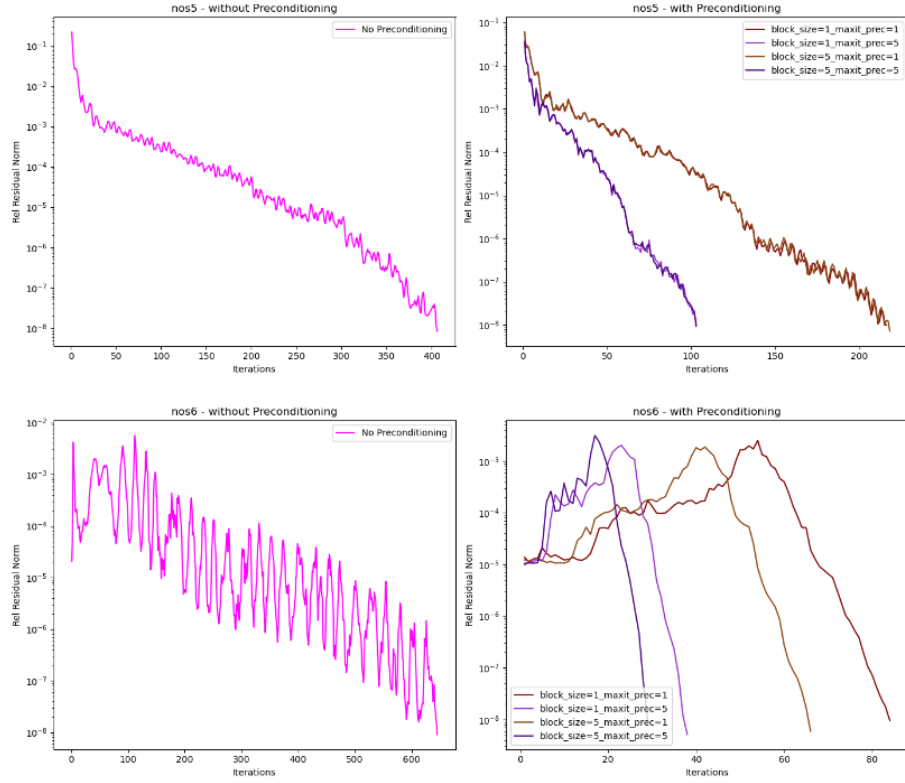


Figure 1: Convergence CG vs PCG

preconditioned CG:

Nos5

- "blocksize1 and prec_maxiterations 5" and "blocksize5 and prec_maxiterations 5" converged the best with about 100 iterations.
- "blocksize1 and prec_maxiterations 1" and "blocksize5 and prec_maxiterations 1" performed not as good as the other two but are still a lot better than the unpreconditioned variante, both converge at about 220 iterations.

Nos6

Similarly to before "blocksize 5 and prec_maxiterations 5" and "blocksize 1 and prec_maxiterations 5" perform optimal with a convergence at about 30 and 38 iterations (really fast). Whereas "blocksize 5 and prec_maxiterations 1" and "blocksize 1 and prec_maxiterations 1" need about double the time of their previously mentioned counterparts with convergence seemingly kicking in at about 64 and 85 iterations.

Matrix Type	Execution Type	Runtime (sec)
nos5	no-prec	0.009997
nos5	block_size=1, maxit_prec=1	0.084496
nos5	block_size=1, maxit_prec=5	0.137000
nos5	block_size=5, maxit_prec=1	0.066000
nos5	block_size=5, maxit_prec=5	0.183002
nos6	no-prec	0.029001
nos6	block_size=1, maxit_prec=1	0.074996
nos6	block_size=1, maxit_prec=5	0.084984
nos6	block_size=5, maxit_prec=1	0.031017
nos6	block_size=5, maxit_prec=5	0.050965

Interesting observation is that all variants first diverge in their initial iterations before plummeting to a really good solution.

Potential Reasons and Reflexion to Part 1:

First of all it seems that many iterations of preconditioning help both matrices in the speed of their overall convergence. With Matrix Nos6 explicitly it also seems to help to have bigger block sizes, because the executions with 5 as blocksize outperform their counterparts with only 1 as block size significantly. Seems that computational problems that the matrix has can be mitigated more with larger blocks.

We can also see some parallels to task 1. where with matrix nos5 block size did not seem to help a lot in convergence, but with nos6 we can clearly see that a larger blocksize gives us the better residual.

Performance Table of CG vs PCG

- No preconditioned runs are by far the fastest, but we also saw from task 1 and 2 that they need longer to converge and we don't reach as good of a residual norm as with the conditioned variants.
- Nos5:
 - Block_size=1, maxit_prec=5 and block_size=5, maxit_prec=5 perform the best in terms of convergence but they also need the longest to compute with 0.137 and 0.183 seconds.
 - Block_size=1, maxit_prec=1 and block_size=5, maxit_prec=1 run the fastest but also perform less well as the runs with maxit_prec=5.
- Nos6:
 - All executions run faster on average than with Nos5 matrix.

- Again `block_size=1`, `maxit_prec=5` and `block_size=5`, `maxit_prec=5` perform the best in terms of convergence, but they also lie at the lower end of runtime - with the former being the slowest of the variants with 0.085 seconds.
- `Block_size=5`, `maxit_prec=1` performs better than `block_size=1`, `maxit_prec=1` in terms of convergence, and in terms of speed it looks no different.

For both matrix types we observe that a larger block size helps with convergence. Precondition the problem also helps with convergence but it comes at the cost that the algorithm now needs to run additional "iterations" to condition the matrix which costs more time of course.