Nome: Carlos Augusto Fernandes Leitão RA: 211270628

Trabalho 3

Lista 7

2.1) B.10.3) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$; K para os polos: $S = -2 \pm j4$ e $S = 10$

Por Ackermann, temos que $K = [0 \quad 0 \quad 1] . \mathbb{C}^{-1} . \varphi_{(A)}$

Calculando $\mathbb{C} \rightarrow \mathbb{C} = [B \quad AB \quad A^2B]$

(Pela calculadora) $\begin{bmatrix} 1 \\ 1 \\ -11 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -11 \\ 60 \end{bmatrix}$ (Pela calculadora)

$$\mathbb{C} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & -11 \\ 1 & -11 & 60 \end{bmatrix} \rightarrow \mathbb{C}^{-1} = \frac{adj(\mathbb{C})}{det(\mathbb{C})} = -1/83 \begin{bmatrix} 61 & 71 & 12 \\ 71 & 1 & -1 \\ 12 & -1 & 1 \end{bmatrix}$$

$$\mathbb{C}_{11} = (-1)^2 . \begin{vmatrix} 1 & -11 \\ -11 & 60 \end{vmatrix} = 60 - 121 = -61$$

$$\mathbb{C}_{12} = (-1)^3 . \begin{vmatrix} 1 & -11 \\ 1 & 60 \end{vmatrix} = -(60 + 11) = -71$$

$$\mathbb{C}_{13} = (-1)^4 . \begin{vmatrix} 1 & 1 \\ 1 & -11 \end{vmatrix} = -11 - 1 = -12$$

$$\mathbb{C}_{21} = (-1)^3 . \begin{vmatrix} 1 & 1 \\ -11 & 60 \end{vmatrix} = -(60 + 11) = -71$$

$$\mathbb{C}_{22} = (-1)^4 . \begin{vmatrix} 0 & 1 \\ 1 & 60 \end{vmatrix} = 1$$

$$C_{23} = (-1)^5 \cdot \begin{vmatrix} 0 & 1 \\ 1 & -11 \end{vmatrix} = -(-1) = 1$$

$$C_{31} = (-1)^4 \cdot \begin{vmatrix} 1 & 1 \\ 1 & -11 \end{vmatrix} = -11 - 1 = -12$$

$$C_{32} = (-1)^5 \cdot \begin{vmatrix} 0 & 1 \\ 1 & -11 \end{vmatrix} = -(-1) = 1$$

$$C_{33} = (-1)^6 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = -1$$

Calculando $\varphi_{(A)}$:

$$\Delta_{(S)} = (S\,i - A + BK) = (S + 2 + j4)(S + 2 - j4)(S + 10)$$

$$\Delta_{(S)} = (S^2 + 2S - \cancel{j4S} + 2S + 4 - \cancel{j8} + \cancel{j4S} + \cancel{j8} + 16)(S + 10) = (S^2 + 4S + 20)(S + 10)$$

$$\Delta_{(S)} = S^3 + 4S^2 + 20S + 10S^2 + 40S + 200 = S^3 + 14S^2 + 60S + 200$$

$$\Delta_{(A)} = A^3 + 4A^2 + 60A + 200i \quad //$$

$$A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -5 & -6 \\ 6 & 24 & 31 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 1 \\ -1 & -5 & -6 \\ 6 & 24 & 31 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} = \begin{bmatrix} -1 & -5 & -6 \\ 6 & 24 & 31 \\ -31 & -144 & -157 \end{bmatrix}$$

$$\varphi_{(A)} = \begin{bmatrix} -1 & -5 & -6 \\ 6 & 24 & 31 \\ -31 & -144 & -157 \end{bmatrix} + 14 \begin{bmatrix} 0 & 0 & 1 \\ -1 & -5 & -6 \\ 6 & 24 & 31 \end{bmatrix} + 60 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 200 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 200 \end{bmatrix}$$

$$\varphi_{(A)} = \begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix} \quad //$$

Com as matrizes encontradas, podemos calcular a fórmula de Ackermann.

$$K = [0 \quad 0 \quad 1] \cdot 1/83 \cdot \begin{bmatrix} 61 & 71 & 12 \\ 71 & 1 & -1 \\ 12 & -1 & 1 \end{bmatrix} \begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix}$$

$$K = 1/83 \cdot [12 \quad -1 \quad 1] \begin{bmatrix} 199 & 55 & 8 \\ -8 & 154 & 7 \\ -7 & -43 & 117 \end{bmatrix}$$

$$K = \frac{1}{83} \cdot [2389 \quad 458 \quad 206] = [28{,}7831 \quad 5{,}5181 \quad 2{,}4819] \;//$$