Chapter Outline

Relational Algebra

- Unary Relational Operations
- Relational Algebra Operations From Set Theory
- Binary Relational Operations
- Additional Relational Operations
- Examples of Queries in Relational Algebra

Relational Calculus

- Tuple Relational Calculus
- Domain Relational Calculus

Example Database Application (COMPANY)

Overview of the QBE language (appendix D)

Relational Algebra Overview

Relational algebra is the basic set of operations for the relational model

These operations enable a user to specify **basic retrieval requests** (or **queries**)

The result of an operation is a *new relation*, which may have been formed from one or more *input* relations

- This property makes the algebra “closed” (all objects in relational algebra are relations)

Relational Algebra Overview (continued)

The **algebra operations** thus produce new relations

- These can be further manipulated using operations of the same algebra

A sequence of relational algebra operations forms a **relational algebra expression**

- The result of a relational algebra expression is also a relation that represents the result of a database query (or retrieval request)

Brief History of Origins of Algebra

Muhammad ibn Musa al-Khwarizmi (800-847 CE) wrote a book titled al-jabr about arithmetic of variables

- Book was translated into Latin.
- Its title (al-jabr) gave Algebra its name.

Al-Khwarizmi called variables “shay”

- “Shay” is Arabic for “thing”.
- Spanish transliterated “shay” as “xay” (“x” was “sh” in Spain).
- In time this word was abbreviated as x.

Where does the word Algorithm come from?

- Algorithm originates from “al-Khwarizmi"
- Reference: PBS (http://www.pbs.org/empires/islam/innoalgebra.html)

Relational Algebra Overview

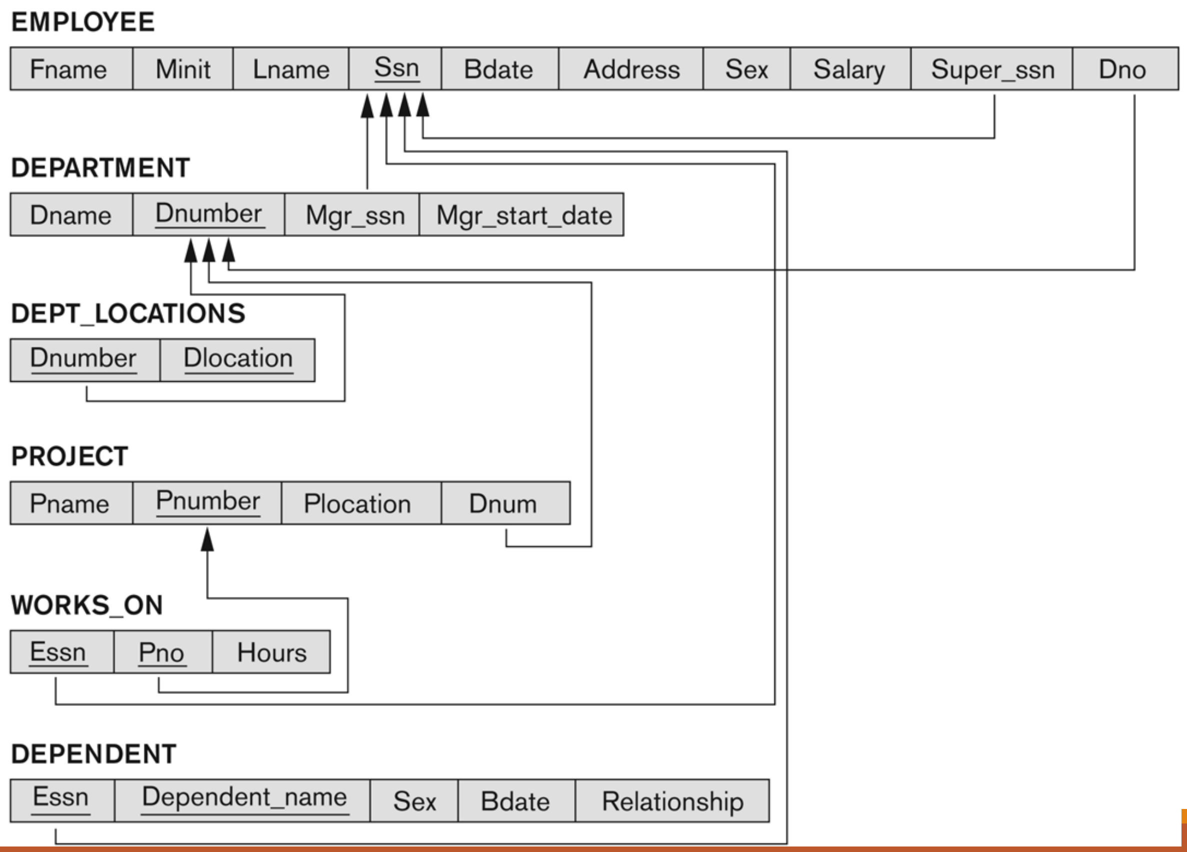
Relational Algebra consists of several groups of operations

- Unary Relational Operations
  - SELECT (symbol: σ (sigma))
  - PROJECT (symbol: π (pi))
  - RENAME (symbol: ρ (rho))
- Relational Algebra Operations From Set Theory
  - UNION (∪), INTERSECTION (∩), DIFFERENCE (or MINUS, **–**)
  - CARTESIAN PRODUCT (**x**)
- Binary Relational Operations
  - JOIN (several variations of JOIN exist)
  - DIVISION
- Additional Relational Operations
  - OUTER JOINS, OUTER UNION
  - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

Database State for COMPANY

All examples discussed below refer to the COMPANY database shown here.

Figure 5.7
Referential integrity constraints displayed on the COMPANY relational database schema.



EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS

Dnumber	Dlocation

PROJECT

Pname	Pnumber	Plocation	Dnum

WORKS_ON

Essn	Pno	Hours

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship

Unary Relational Operations: SELECT

The SELECT operation (denoted by **σ** (sigma)) is used to select a *subset* of the tuples from a relation based on a **selection condition**.

- The selection condition acts as a **filter**
- Keeps only those tuples that satisfy the qualifying condition
- Tuples satisfying the condition are *selected* whereas the other tuples are discarded (*filtered out*)

Examples:

- Select the EMPLOYEE tuples whose department number is 4:

$$\sigma_{\text{DNO} = 4} (\text{EMPLOYEE})$$

- Select the employee tuples whose salary is greater than $30,000:

$$\sigma_{\text{SALARY} > 30{,}000} (\text{EMPLOYEE})$$

Unary Relational Operations: SELECT

- In general, the *select* operation is denoted by $\sigma_{\text{<selection condition>}}(R)$ where
  - the symbol σ (sigma) is used to denote the *select* operator
  - the selection condition is a Boolean (conditional) expression specified on the attributes of relation R
  - tuples that make the condition **true** are selected
    - appear in the result of the operation
  - tuples that make the condition **false** are filtered out
    - discarded from the result of the operation

Unary Relational Operations: SELECT (contd.)

SELECT Operation Properties

- The SELECT operation $\sigma_{\text{<selection condition>}}(R)$ produces a relation S that has the same schema (same attributes) as R
- SELECT σ is commutative:
  - $\sigma_{\text{<condition1>}}(\sigma_{\text{< condition2>}}(R)) = \sigma_{\text{<condition2>}}(\sigma_{\text{< condition1>}}(R))$
- Because of commutativity property, a cascade (sequence) of SELECT operations may be applied in any order:
  - $\sigma_{\text{<cond1>}}(\sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond2>}}(\sigma_{\text{<cond3>}}(\sigma_{\text{<cond1>}}(R)))$
- A cascade of SELECT operations may be replaced by a single selection with a conjunction of all the conditions:
  - $\sigma_{\text{<cond1>}}(\sigma_{\text{< cond2>}}(\sigma_{\text{<cond3>}}(R)) = \sigma_{\text{<cond1> AND < cond2> AND < cond3>}}(R)))$
- The number of tuples in the result of a SELECT is less than (or equal to) the number of tuples in the input relation R

The following query results refer to this database state

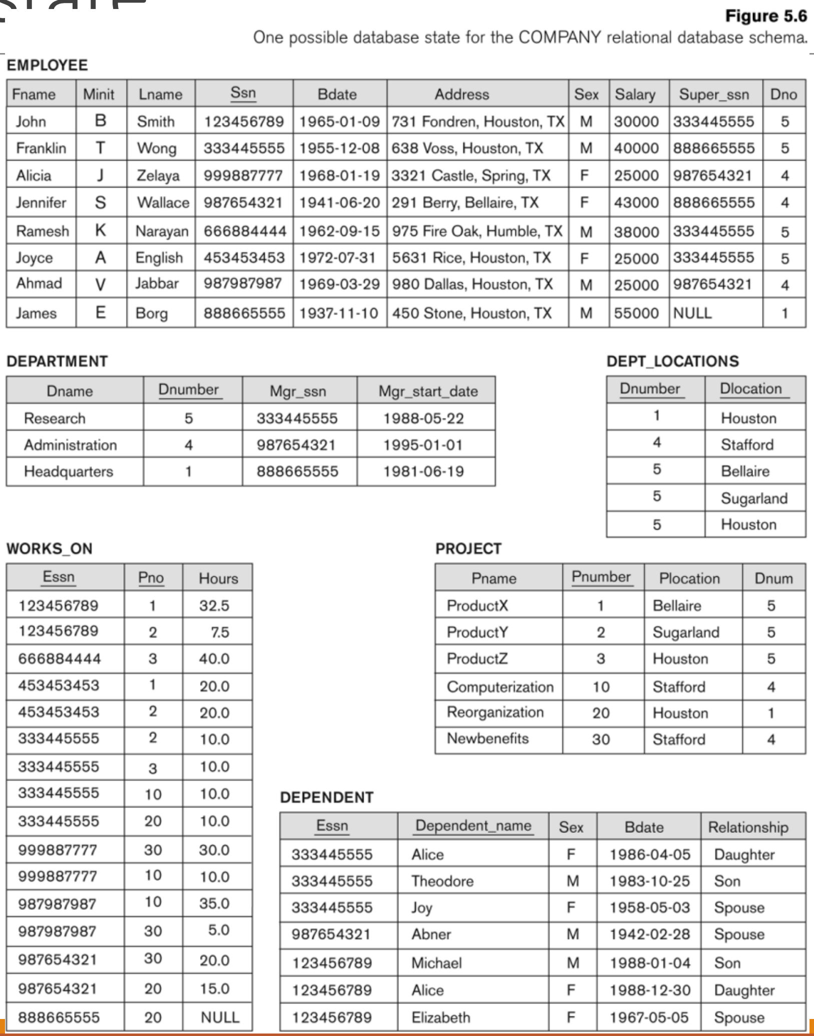


Figure 5.6
One possible database state for the COMPANY relational database schema.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Unary Relational Operations: PROJECT

PROJECT Operation is denoted by π (pi)

This operation keeps certain *columns* (attributes) from a relation and discards the other columns.

- PROJECT creates a vertical partitioning
  - The list of specified columns (attributes) is kept in each tuple
  - The other attributes in each tuple are discarded

Example: To list each employee's first and last name and salary, the following is used:

$$\pi_{\text{LNAME, FNAME,SALARY}}(\text{EMPLOYEE})$$

Unary Relational Operations: PROJECT (cont.)

The general form of the *project* operation is:

$$\pi_{\text{<attribute list>}}(R)$$

- π (pi) is the symbol used to represent the *project* operation
- <attribute list> is the desired list of attributes from relation R.

The project operation *removes any duplicate tuples*

- This is because the result of the *project* operation must be a *set of tuples*
  - Mathematical sets *do not allow* duplicate elements.

Unary Relational Operations: PROJECT (contd.)

PROJECT Operation Properties

- The number of tuples in the result of projection $\pi_{\text{<list>}}(R)$ is always less or equal to the number of tuples in R
  - If the list of attributes includes a *key* of R, then the number of tuples in the result of PROJECT is *equal* to the number of tuples in R
- PROJECT is *not* commutative
  - $\pi_{\text{<list1>}} (\pi_{\text{<list2>}} (R)) = \pi_{\text{<list1>}} (R)$ as long as <list2> contains the attributes in <list1>

Examples of applying SELECT and PROJECT operations

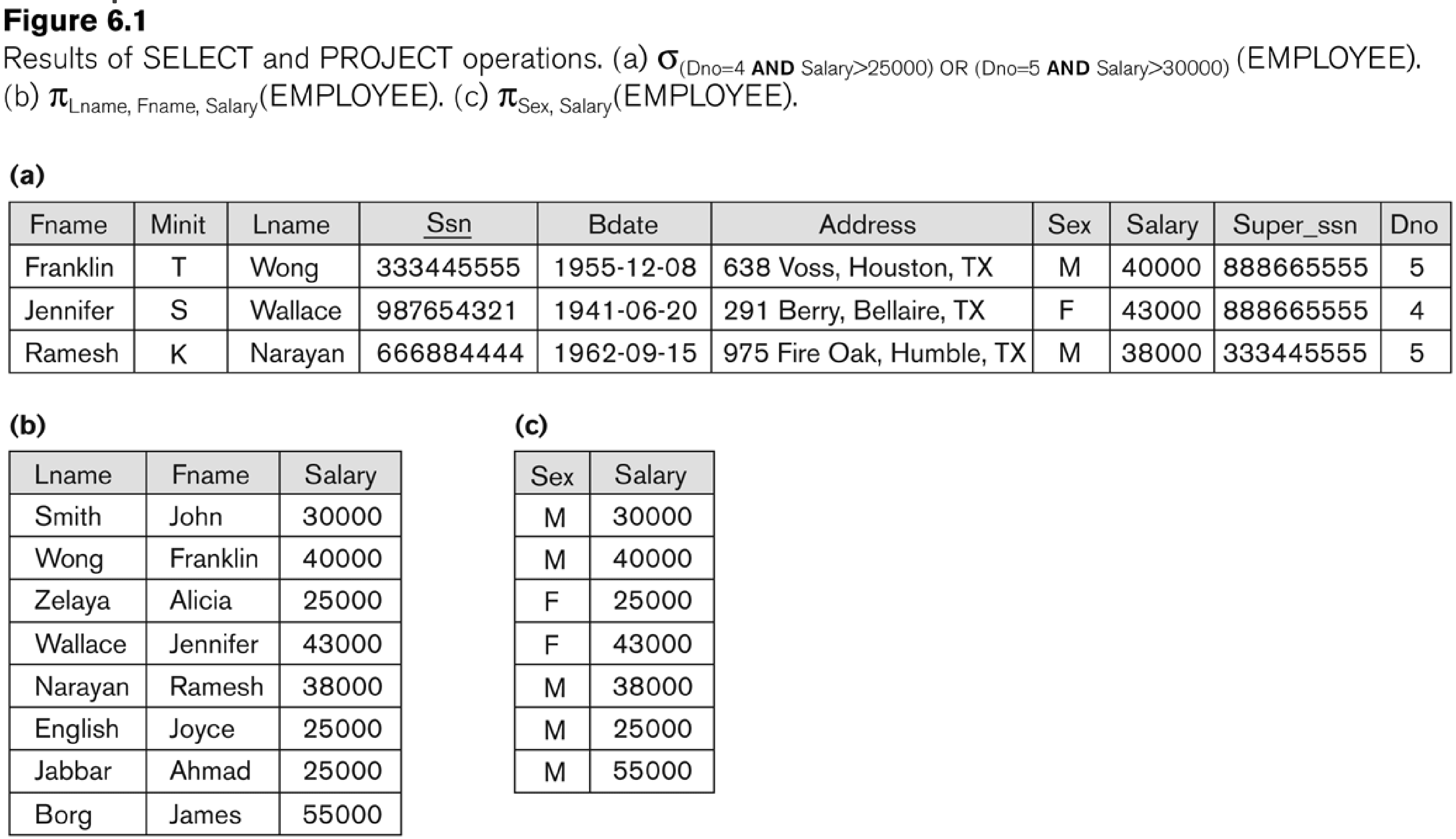


Figure 6.1

Results of SELECT and PROJECT operations. (a) $\sigma_{(\text{Dno}=4\ \textbf{AND}\ \text{Salary}>25000)\ \text{OR}\ (\text{Dno}=5\ \textbf{AND}\ \text{Salary}>30000)}$(EMPLOYEE). (b) $\pi_{\text{Lname, Fname, Salary}}$(EMPLOYEE). (c) $\pi_{\text{Sex, Salary}}$(EMPLOYEE).

(a)

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5

(b)

Lname	Fname	Salary
Smith	John	30000
Wong	Franklin	40000
Zelaya	Alicia	25000
Wallace	Jennifer	43000
Narayan	Ramesh	38000
English	Joyce	25000
Jabbar	Ahmad	25000
Borg	James	55000

(c)

Sex	Salary
M	30000
M	40000
F	25000
F	43000
M	38000
M	25000
M	55000

Relational Algebra Expressions

We may want to apply several relational algebra operations one after the other

- Either we can write the operations as a single **relational algebra expression** by nesting the operations, or
- We can apply one operation at a time and create **intermediate result relations**.

In the latter case, we must give names to the relations that hold the intermediate results.

Single expression versus sequence of relational operations (Example)

To retrieve the first name, last name, and salary of all employees who work in department number 5, we must apply a select and a project operation

We can write a *single relational algebra expression* as follows:

- $\pi_{\text{FNAME, LNAME, SALARY}}(\sigma_{\text{DNO=5}}(\text{EMPLOYEE}))$

OR We can explicitly show the *sequence of operations*, giving a name to each intermediate relation:

- $\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO=5}}(\text{EMPLOYEE})$
- $\text{RESULT} \leftarrow \pi_{\text{FNAME, LNAME, SALARY}}(\text{DEP5\_EMPS})$

Unary Relational Operations: RENAME

The RENAME operator is denoted by ρ (rho)

In some cases, we may want to *rename* the attributes of a relation or the relation name or both

- Useful when a query requires multiple operations
- Necessary in some cases (see JOIN operation later)

Unary Relational Operations: RENAME (contd.)

The general RENAME operation ρ can be expressed by any of the following forms:

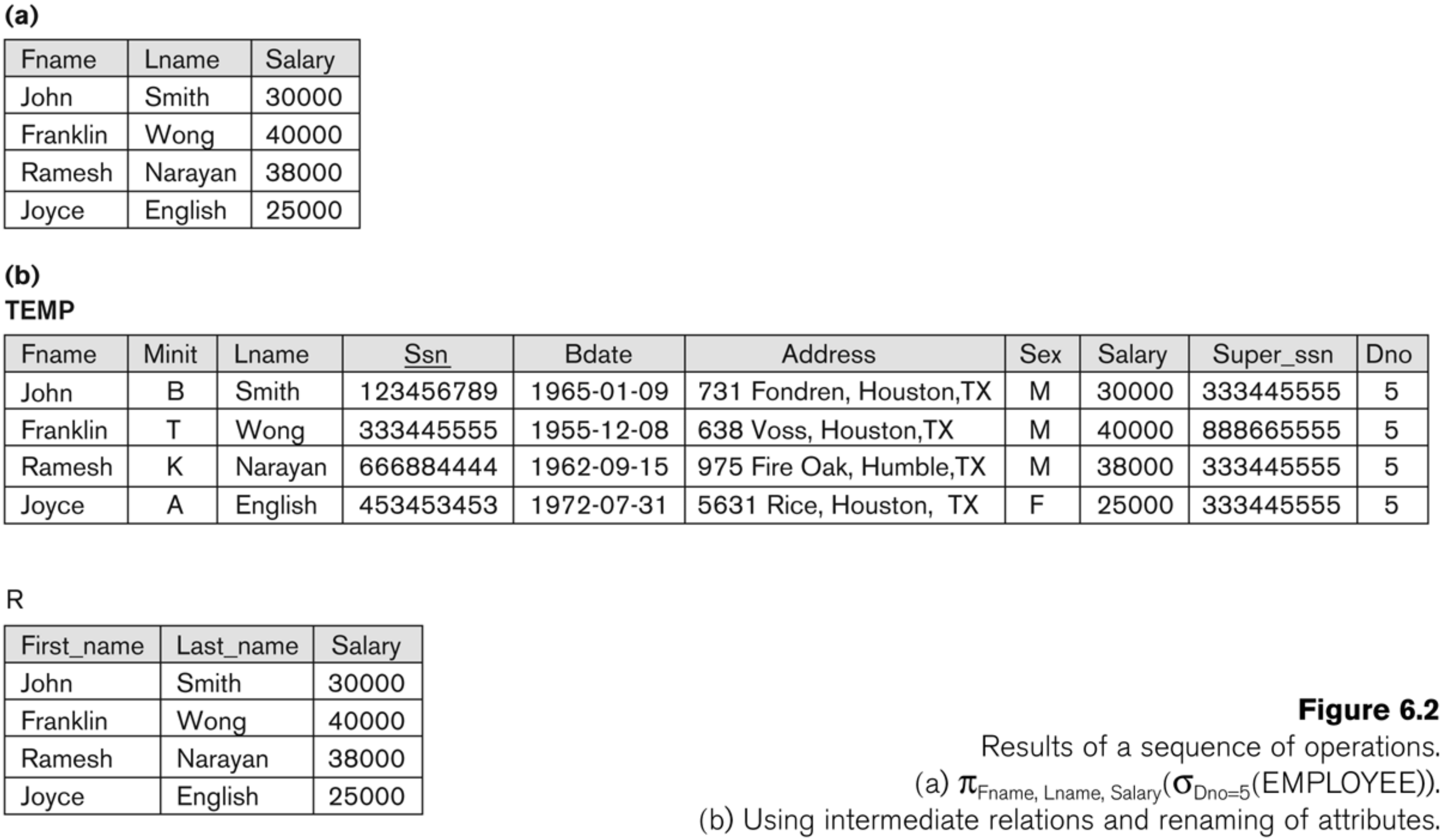
- $\rho_{S\ (B1,\ B2,\ \ldots,\ Bn\ )}(R)$ changes both:
  - the relation name to S, *and*
  - the column (attribute) names to B1, B1, …..Bn
- $\rho_S(R)$ changes:
  - the *relation name* only to S
- $\rho_{(B1,\ B2,\ \ldots,\ Bn\ )}(R)$ changes:
  - the *column (attribute) names* only to B1, B1, …..Bn

Unary Relational Operations: RENAME (contd.)

For convenience, we also use a *shorthand* for renaming attributes in an intermediate relation:

- If we write:
  - RESULT ← $\pi_{\text{FNAME, LNAME, SALARY}}$ (DEP5_EMPS)
  - RESULT will have the *same attribute names* as DEP5_EMPS (same attributes as EMPLOYEE)
- If we write:
  - RESULT (F, M, L, S, B, A, SX, SAL, SU, DNO)← $\pi_{\text{FNAME, LNAME, SALARY}}$ (DEP5_EMPS)
  - The 10 attributes of DEP5_EMPS are *renamed* to F, M, L, S, B, A, SX, SAL, SU, DNO, respectively

Example of applying multiple operations and RENAME



(a)

Fname	Lname	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

(b)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Figure 6.2

Results of a sequence of operations.

(a) $\pi_{\text{Fname, Lname, Salary}}(\sigma_{\text{Dno=5}}(\text{EMPLOYEE}))$.

(b) Using intermediate relations and renaming of attributes.

Relational Algebra Operations from Set Theory: UNION

UNION Operation

- Binary operation, denoted by ∪
- The result of R ∪ S, is a relation that includes all tuples that are either in R or in S or in both R and S
- Duplicate tuples are eliminated
- The two operand relations R and S must be "type compatible" (or UNION compatible)
  - R and S must have same number of attributes
  - Each pair of corresponding attributes must be type compatible (have same or compatible domains)

Relational Algebra Operations from Set Theory: UNION

Example:

- To retrieve the social security numbers of all employees who either *work in department 5* (RESULT1 below) or *directly supervise an employee who works in department 5* (RESULT2 below)
- We can use the UNION operation as follows:

$$\text{DEP5\_EMPS} \leftarrow \sigma_{\text{DNO=5}} \text{(EMPLOYEE)}$$

$$\text{RESULT1} \leftarrow \pi_{\text{SSN}}\text{(DEP5\_EMPS)}$$

$$\text{RESULT2(SSN)} \leftarrow \pi_{\text{SUPERSSN}}\text{(DEP5\_EMPS)}$$

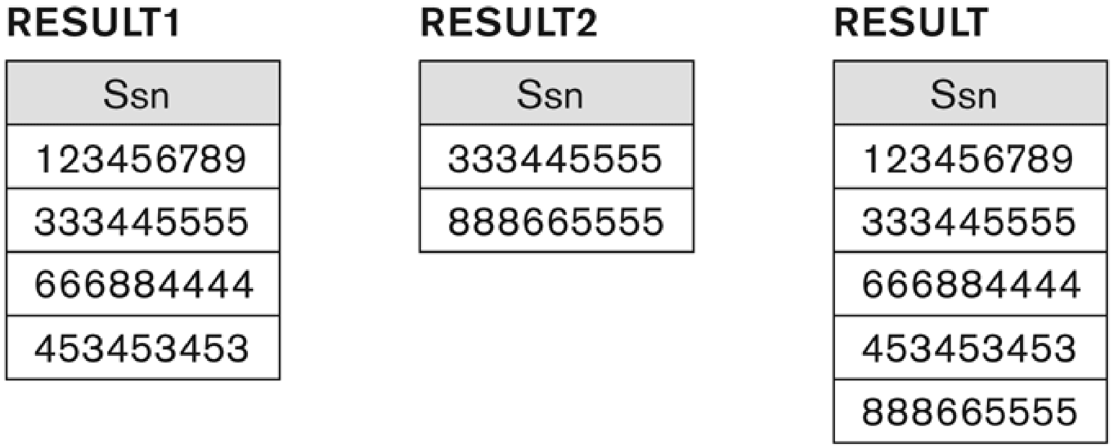
$$\text{RESULT} \leftarrow \text{RESULT1} \cup \text{RESULT2}$$

- The union operation produces the tuples that are in either RESULT1 or RESULT2 or both

Example of the result of a UNION operation

UNION Example

Figure 6.3
Result of the UNION operation RESULT ← RESULT1 ∪ RESULT2.



RESULT1

Ssn
123456789
333445555
666884444
453453453

RESULT2

Ssn
333445555
888665555

RESULT

Ssn
123456789
333445555
666884444
453453453
888665555

Relational Algebra Operations from Set Theory

Type Compatibility of operands is required for the binary set operation UNION $\cup$, (also for INTERSECTION $\cap$, and SET DIFFERENCE –, see next slides)

R1(A1, A2, ..., An) and R2(B1, B2, ..., Bn) are type compatible if:

- they have the same number of attributes, and
- the domains of corresponding attributes are type compatible (i.e. dom(Ai)=dom(Bi) for i=1, 2, ..., n).

The resulting relation for R1$\cup$R2 (also for R1$\cap$R2, or R1–R2, see next slides) has the same attribute names as the *first* operand relation R1 (by convention)

Relational Algebra Operations from Set Theory: INTERSECTION

INTERSECTION is denoted by ∩

The result of the operation R ∩ S, is a relation that includes all tuples that are in both R and S

- The attribute names in the result will be the same as the attribute names in R

The two operand relations R and S must be “type compatible”

Relational Algebra Operations from Set Theory: SET DIFFERENCE (cont.)

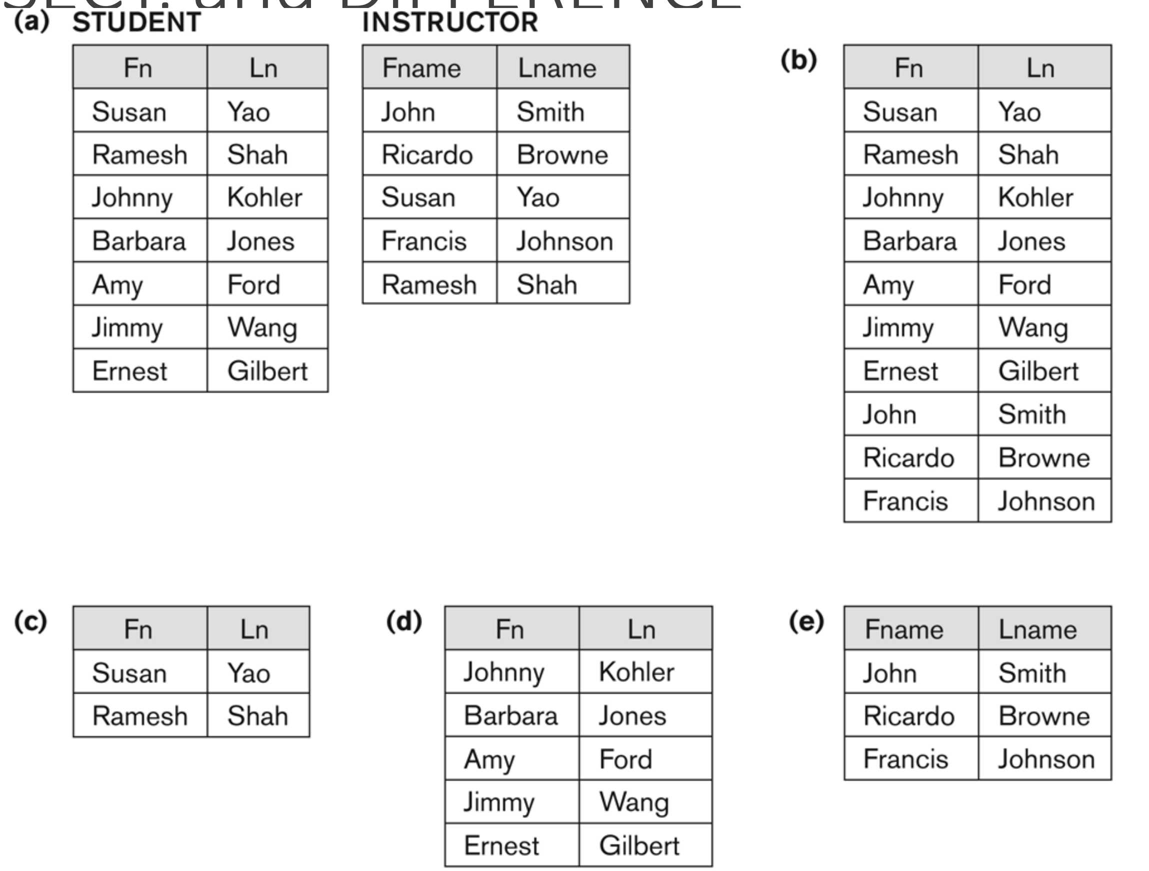
SET DIFFERENCE (also called MINUS or EXCEPT) is denoted by –

The result of R – S, is a relation that includes all tuples that are in R but not in S

- The attribute names in the result will be the same as the attribute names in R

The two operand relations R and S must be “type compatible”

Example to illustrate the result of UNION, INTERSECT, and DIFFERENCE



(a) STUDENT

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

INSTRUCTOR

Fname	Lname
John	Smith
Ricardo	Browne
Susan	Yao
Francis	Johnson
Ramesh	Shah

(b)

Fn	Ln
Susan	Yao
Ramesh	Shah
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert
John	Smith
Ricardo	Browne
Francis	Johnson

(c)

Fn	Ln
Susan	Yao
Ramesh	Shah

(d)

Fn	Ln
Johnny	Kohler
Barbara	Jones
Amy	Ford
Jimmy	Wang
Ernest	Gilbert

(e)

Fname	Lname
John	Smith
Ricardo	Browne
Francis	Johnson

Figure 6.4
The set operations UNION, INTERSECTION, and MINUS. (a) Two union-compatible relations. (b) STUDENT ∪ INSTRUCTOR. (c) STUDENT ∩ INSTRUCTOR. (d) STUDENT – INSTRUCTOR. (e) INSTRUCTOR – STUDENT.

Some properties of UNION, INTERSECT, and DIFFERENCE

Notice that both union and intersection are *commutative* operations; that is

- $R \cup S = S \cup R$, and $R \cap S = S \cap R$

Both union and intersection can be treated as n-ary operations applicable to any number of relations as both are *associative* operations; that is

- $R \cup (S \cup T) = (R \cup S) \cup T$
- $(R \cap S) \cap T = R \cap (S \cap T)$

The minus operation is not commutative; that is, in general

- $R - S \neq S - R$

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT

CARTESIAN (or CROSS) PRODUCT Operation

- This operation is used to combine tuples from two relations in a combinatorial fashion.
- Denoted by R(A1, A2, . . ., An) x S(B1, B2, . . ., Bm)
- Result is a relation Q with degree n + m attributes:
  - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.
- The resulting relation state has one tuple for each combination of tuples—one from R and one from S.
- Hence, if R has n_R tuples (denoted as |R| = n_R), and S has n_S tuples, then R x S will have n_R * n_S tuples.
- The two operands do NOT have to be "type compatible”

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

Generally, CROSS PRODUCT is not a meaningful operation

- Can become meaningful when followed by other operations

Example (not meaningful):

- FEMALE_EMPS ← $\sigma_{\text{SEX='F'}}$(EMPLOYEE)
- EMPNAMES ← $\pi_{\text{FNAME, LNAME, SSN}}$ (FEMALE_EMPS)
- EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

EMP_DEPENDENTS will contain every combination of EMPNAMES and DEPENDENT

- whether or not they are actually related

Relational Algebra Operations from Set Theory: CARTESIAN PRODUCT (cont.)

To keep only combinations where the DEPENDENT is related to the EMPLOYEE, we add a SELECT operation as follows

Example (meaningful):

- FEMALE_EMPS $\leftarrow \sigma_{\text{SEX='F'}}$(EMPLOYEE)
- EMPNAMES $\leftarrow \pi_{\text{FNAME, LNAME, SSN}}$ (FEMALE_EMPS)
- EMP_DEPENDENTS $\leftarrow$ EMPNAMES x DEPENDENT
- ACTUAL_DEPS $\leftarrow \sigma_{\text{SSN=ESSN}}$(EMP_DEPENDENTS)
- RESULT $\leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT\_NAME}}$ (ACTUAL_DEPS)

RESULT will now contain the name of female employees and their dependents