

1 Deadline

The deadline date and time are available in the Interactiva mailbox.

2 Preliminaries

For this assignment some definitions and notions are required. Let $M = (Q, \Sigma, \delta, s, F)$ be a deterministic finite automaton (DFA).

1. **Inaccessible states.** A state $q \in Q$ is said to be **inaccessible** if there is no string $x \in \Sigma^*$ such that $\hat{\delta}(s, x) = q$.
2. **Equivalent states.** A pair of states $p, q \in Q$ is said to be **equivalent** if and only if

$$(\forall x \in \Sigma^*)(\hat{\delta}(p, x) \in F = \hat{\delta}(q, x) \in F).$$

That is, any string x that from p allows us to access a final state, must also allow us to reach a final state from q , and vice versa.

3. We say that two states can be **collapsed** if they are equivalent.

3 Assignment

The assignment is to implement the minimization algorithm presented in Kozen 1997, Lecture 14. Given a DFA with no inaccessible states, the algorithm returns the states that are equivalent. Therefore, such states can be collapsed and we shall obtain a minimized automaton.

4 Input/output

Your program should fulfill the following specifications.

Input

A case is a DFA M with no inaccessible states.

You may assume states are denoted by natural numbers and the initial state s is always 0 (zero). Alphabets are formed with letters of the latin alphabet (with 26 letters). In ASCII code, characters from 97 to 122.

The input of the program is as follows

1. A line with a number $c > 0$ indicating how many cases you will receive.
2. For each case, in a single line the number $n > 0$ of states of M .
3. Then, a single line with the alphabet of M . Symbols are separated by a single blank space.
4. Then, the final states of M separated by blank spaces.
5. Finally, n lines, one for each state. Each line contains a row of the table that represents M . Assume that the symbols of the alphabet appear in the table in the same order as they were given in step 3. If the automaton is

	a	b
$\rightarrow 0$	1	2
1F	3	4
2	4	5
3	5	5
4F	5	5
5F	5	5

the n lines are

```
0 1 2
1 3 4
2 4 3
3 5 5
4 5 5
5 5 5
```

Output

For each case, print the pairs of states that are equivalent in lexicographical order. All the pairs in a single line separated by blank spaces.

5 Delivery

1. The homework solution is in groups of 2, and the defense is individual.
2. You must deliver the implementation compressed in a zip file on Interactiva by the deadline.
3. Additionally, provide a link to a Replit workspace with the solution, enabling editing permissions.
4. Deliver a document in English containing the following information:
 - Your full name.
 - Versions used of the operating system, programming language, and tools in your implementation to run the code without using Replit.
 - Detailed instructions for running your implementation without using Replit.
 - Do not include unnecessary files or directories in the delivery in zip.

References

- [1] Kozen, Dexter C. *Automata and Computability*. Springer, Third printing, 1997 [2012]. Undergraduate Texts in Computer Science. <https://doi.org/10.1007/978-1-4612-1844-9>.

6 Example I/O

Input

4

6

a b

1 2 5

0 1 2

1 3 4

2 4 3

3 5 5

4 5 5

5 5 5

6

a b

3 4 5

0 1 2

1 3 4

2 4 3

3 5 5

4 5 5

5 5 5

6

a

1 4

0 1

1 2

2 3

3 4

4 5

5 0

4

a b

0 1

0 1 2

1 1 2

2 3 1

3 3 3

Output

(1, 2) (3, 4)

(1, 2) (3, 4) (3, 5) (4, 5)

(0, 3) (1, 4) (2, 5)

(0, 1)