Lenguajes Formales y Compiladores

Syntax Analysis

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Motivation

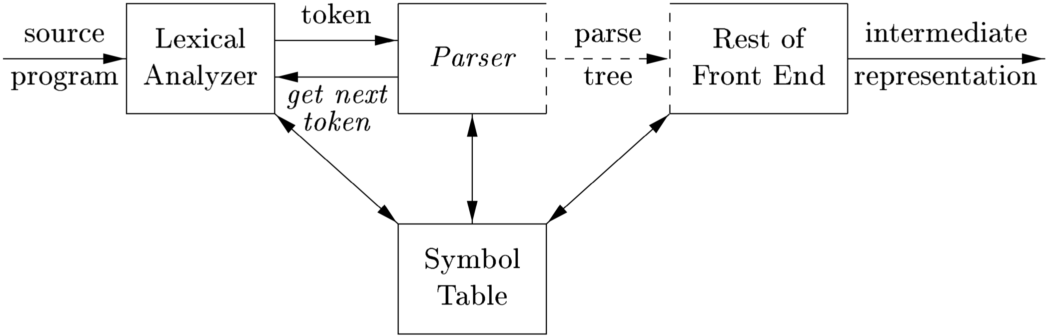

Figure 4.1 Aho et al. 2006.

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Recursive-Descent Parsing

Takeaway

This technique uses backtracking to find the correct production to be applied to derive the input string.

Recursive-Descent Parsing

Algorithm Recursive Procedure for Parsing

Read input left to right. Let a be the current input symbol
procedure $\textsc{Recursive-parser}(A)$ ▷ Starts with S
    Choose an A-production: $A \to x_1 x_2 \cdots x_k$ ▷ $x_i \in N \cup \Sigma$
    for $i := 1$ to k **do**
        if $x_i \in N$ **then**
            $\textsc{Recursive-parser}(x_i)$
        else if $x_i = a$ **then**
            advance to the next input symbol
        else
            `ERROR`

Example

Recursive-Descent Parsing

Consider the grammar

$$S \to cAd$$
$$A \to ab \mid a$$

Construct a parse tree top-down for the input string $w = cad$.

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Takeaway

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First and Follow are auxiliary sets that allow us to find the terminal symbols that start a string and the terminal symbols that follow a given nonterminal symbol, respectively.

Note: We assume that every string ends with the symbol \$.

First

Definition (First)

Let $G = (N, \Sigma, P, S)$ be a grammar.
Define $\textsc{First}(\alpha)$, where $\alpha \in (N \cup \Sigma)^*$, to be the set of terminals that begin strings derived from α.

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Remark: We allow ε to be in First sets under certain conditions.

First of a Symbol (I)

Compute $\textsc{First}(x)$

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    2.1 For each $1 \leq i \leq k$, if $\varepsilon \in \textsc{First}(y_j)$ for all $1 \leq j \leq i-1$, then $\textsc{First}(x) := \textsc{First}(x) \cup \textsc{First}(y_i) - \{\varepsilon\}$.

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   2.1 For each $1 \leq i \leq k$, if $\varepsilon \in \textsc{First}(y_j)$ for all $1 \leq j \leq i-1$, then $\textsc{First}(y_i) - \{\varepsilon\} \subseteq \textsc{First}(x)$.

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   - 2.2 If $\varepsilon \in \textsc{First}(y_j)$ for all $1 \leq j \leq k$, then $\varepsilon \in \textsc{First}(x)$.
3. If $x \to \varepsilon$ is a production, then $\varepsilon \in \textsc{First}(x)$.

First of a Symbol

$$
\textsc{First}(x) := \begin{cases} \{x\} & \text{if } x \in \Sigma \\ \textsc{First}(x) \cup \{\varepsilon\} & \text{if } (x \to y_1 y_2 \cdots y_k \wedge \varepsilon \in \bigcap_{j=1}^{k} \textsc{First}(y_j)) \vee x \to \varepsilon \\ \textsc{First}(x) \cup \textsc{First}(y_i) - \{\varepsilon\} & \text{if } x \to y_1 y_2 \cdots y_k \wedge \varepsilon \in \bigcap_{j=1}^{i-1} \textsc{First}(y_j) \end{cases}
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Continue on next slide

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4. and so on.
5. Add ε to $\textsc{First}(x_1 x_2 \cdots x_n)$ if, for all $1 \leq i \leq n$, $\varepsilon \in \textsc{First}(x_i)$.

First of a String

This can be formalized as:

$$\textsc{First}(x_1 x_2 \cdots x_n) \coloneqq \bigcup_{i=1}^{n} \left\{ \textsc{First}(x_i) - \{\varepsilon\} \;\middle|\; \varepsilon \in \bigcap_{j=1}^{i-1} \textsc{First}(x_j) \right\}$$

Follow

Definition (Follow)

Let $G = (N, \Sigma, S, P)$ be a grammar.
Define $\textsc{Follow}(A)$, where $A \in N$, to be the set of terminals a that can appear immediately to the right of A in some sentential form, that is:

$$\{a \in \Sigma \mid S \xrightarrow{*} \alpha A a \beta \text{ for some } \alpha, \beta \in (N \cup \Sigma)^*\}$$

Computing Follow

Compute $\textsc{Follow}(A)$

Let $G = (N, \Sigma, S, P)$ be a grammar.
To compute $\textsc{Follow}(A)$ for all $A \in N$, apply the following rules until nothing can be added to any Follow set.

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3. If there is a production $A \to \alpha B$, or a production $A \to \alpha B \beta$ with $\varepsilon \in \text{First}(\beta)$, then $\text{Follow}(B) := \text{Follow}(B) \cup \text{Follow}(A)$.

Computing Follow (I)

1. $\textsc{Follow}(S) := \textsc{Follow}(S) \cup \{\$\}$.

2. If $A \to \alpha B \beta$, then $\textsc{Follow}(B) := \textsc{Follow}(B) \cup (\textsc{First}(\beta) - \{\varepsilon\})$.

3. If $A \to \alpha B$, or $A \to \alpha B \beta$ with $\varepsilon \in \textsc{First}(\beta)$, then
   $\textsc{Follow}(B) := \textsc{Follow}(B) \cup \textsc{Follow}(A)$.

Computing Follow (II)

1. $\textsc{Follow}(S) := \{\$\}$.
2. For every production $A \to \alpha B \beta$, then $\textsc{First}(\beta) - \{\varepsilon\} \subseteq \textsc{Follow}(B)$.
3. For every production $A \to \alpha B$, or $A \to \alpha B \beta$ with $\varepsilon \in \textsc{First}(\beta)$, then $\textsc{Follow}(A) \subseteq \textsc{Follow}(B)$.

Exercise

First and Follow

Compute the sets First and Follow for all nonteminal symbols of the following grammar:

$$S \rightarrow AB \qquad A \rightarrow aA \mid a \qquad B \rightarrow bBc \mid bc$$

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Motivation

Takeaway

- Top-Down parsing can be viewed as the problem of constructing a parse tree for a given input string.
- Equivalently, top-down parsing can be viewed as finding a leftmost derivation for an input string.

Predictive Parsing

- Predictive parsing chooses the correct production to use by looking ahead at the input a fixed number of symbols, typically we may look only at one (i.e., the next input symbol).

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- This technique does not require backtracking.
- This kind of parsers can be constructed for a class of grammars called LL(1).
- The first "L" in LL(1) stands for scanning the input from left to right, the second "L" for producing a leftmost derivation, and the "1" for using one input symbol of look ahead at each step to make parsing action decisions.

LL(1) Grammars

Definition (LL(1) Grammars)

A grammar G is LL(1) if and only if whenever $A \to \alpha \mid \beta$ with $\alpha \neq \beta$, the following conditions hold:

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2. At most one of α and β can derive the empty string.
3. If $\beta \xrightarrow{*} \varepsilon$, then α does not derive any string beginning with a terminal in $\textsc{Follow}(A)$. *Analogously*, if $\alpha \xrightarrow{*} \varepsilon$, then β does not derive any string beginning with a terminal in $\textsc{Follow}(A)$.

LL(1) Grammars

The conditions in Definition 3 can be specified as:

Whenever $A \to \alpha \mid \beta$ with $\alpha \neq \beta$:

1. $\textsc{First}(\alpha) \cap \textsc{First}(\beta) = \emptyset$.
2. If $\varepsilon \in \textsc{First}(\beta)$, then $\textsc{First}(\alpha) \cap \textsc{Follow}(A) = \emptyset$.
   If $\varepsilon \in \textsc{First}(\alpha)$, then $\textsc{First}(\beta) \cap \textsc{Follow}(A) = \emptyset$.

Exercise

LL(1)

Verify that the following grammar is LL(1):

$$S \to AB \qquad A \to aA \mid a \qquad B \to bBc \mid bc$$

Observation

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- The class of LL(1) grammars is rich enough to cover most programming constructs, although care is needed in writing a suitable grammar for the source language.
- For instance, no *left-recursive* or *ambiguous* grammar can be LL(1).

Predictive Parsing Table

Let $G = (N, \Sigma, P, S)$ be a CFG. We shall construct a table $M \subseteq (N \times \Sigma \cup \{\$\}) \times \mathscr{P}(P)$.

Algorithm Construction of a predictive parsing table M

for each production $A \to \alpha \in P$ **do**
  for every $a \in \textsc{First}(\alpha)$ **do**
    $M[A,a] \leftarrow M[A,a] \cup \{A \to \alpha\}$
  if $\varepsilon \in \textsc{First}(\alpha)$ **then**
    for each terminal $b \in \textsc{Follow}(A)$ **do**
      $M[A,b] \leftarrow M[A,b] \cup \{A \to \alpha\}$
  if $\varepsilon \in \textsc{First}(\alpha) \land \$ \in \textsc{Follow}(A)$ **then**
    $M[A,\$] \leftarrow M[A,\$] \cup \{A \to \alpha\}$

If after performing the algorithm there is no production at all in some $M[A,a]$, then set $M[A,a]$ to `error` (normally represented by an empty entry in the table).

Predictive Parsing Table

Observation

- The algorithm can be applied to any grammar G to produce a parsing table M.
- However, for every LL(1) grammar, each parsing-table entry uniquely identifies a production or signals an error.
- For some grammars, M may have some entries that are multiply defined.

Exercise

Exercise

Construct the parsing table M for the grammar

$$S \to AB \qquad A \to aA \mid a \qquad B \to bBc \mid bc$$

Predictive Parsing Algorithm

Algorithm Predictive Parsing

Let T be a stack ▷ At the beginning it contains $S\$$
Let w be a string and a be its first symbol
Let X be the top stack symbol
while $X \neq \$$ **do**
    if $X = a$ **then**
        T.`pop()`
        Let a be the next symbol of w
    else if X is a terminal **then** ERROR()
    else if $M[X, a]$ is an `error` entry **then** ERROR()
    else if $M[X, a] = X \to Y_1 Y_2 \cdots Y_k$ **then**
        T.`pop()`
        T.`push`$(Y_1 Y_2 \cdots Y_k)$ ▷ Y_1 on top

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Motivation

Takeaway

Bottom-up parse corresponds to the construction of a parse tree for an input string beginning at the leaves (the bottom) and working up towards the root (the top).

Handle Pruning

Definition (Handle-Pruning)

Let G be a grammar. If $S \xrightarrow{*} \alpha A w \rightarrow \alpha\beta w$, we say that a production $A \rightarrow \beta$ is a **handle** of $\alpha\beta w$.

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Objective

By "handle pruning" we can obtain a rightmost derivation.

$$S = \gamma_0 \rightarrow_{rm} \gamma_1 \rightarrow_{rm} \cdots \rightarrow_{rm} \gamma_{n-1} \rightarrow_{rm} \gamma_n = w$$

for every γ_i we have a **handle** $A_i \rightarrow \beta_i$

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3. **Accept**. Announce successful completion of parsing.
4. **Error**. Discover a syntax error and call an error recovery routine.

Motivation

LR(k) Parsers

- The “L” is for left-to-right scanning of the input, the “R” for constructing a rightmost derivation (in reverse), and the “k” for the number of input symbols of lookahead that are used in making parsing decisions.
- LR parsers are table-driven. If we can construct a parsing table for a grammar, it is said to be and *LR grammar*.

Model

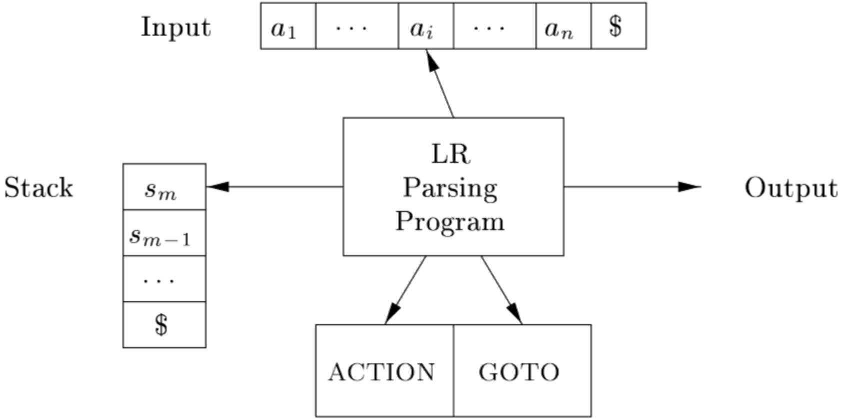

Figure 4.35 Aho et al.

LR(0) Automaton

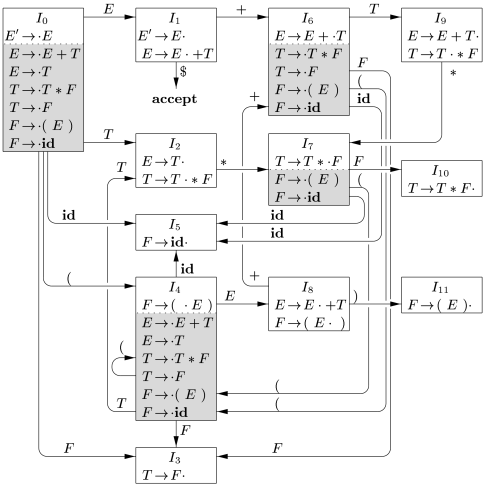

Figure 4.31 Aho et al.

Preliminaries

We need to define:

Preliminaries

- Items and closure of item sets
- Function GoTo
- Action
- Construction of parsing table
- Behavior of LR(0) automaton

Items

Definition (Items)

- Given a production $A \to \alpha$ we define an **item** to be this production with a dot "$\bullet$" somewhere in α, even at the beginning or the end.

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- If $A \to \varepsilon$, it produces the item $A \to \bullet$.
- The set of items for a grammar G is the set of all items that can be constructed for the productions of G.

Preliminaries

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It is the collection of sets of items that provides the basis to construct the parser.

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Definition (Augmented Grammar)

From now on, given a grammar G we work with the **augmented grammar** for G, denoted G'. It is G with a new start symbol S' and a new production $S' \to S$.

Closure of Item Sets

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1. $I \subseteq \textsc{Closure}(I)$
2. If $A \rightarrow \alpha \bullet B\beta \in \textsc{Closure}(I)$ and $B \rightarrow \gamma$ is a production, then add $B \rightarrow \bullet\gamma$ to $\textsc{Closure}(I)$, if it is not already there.

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If I is an item set for a grammar G, then $\textsc{Closure}(I)$ is the set of items constructed from I by the following rules:

1. $I \subseteq \textsc{Closure}(I)$
2. If $A \to \alpha \bullet B \beta \in \textsc{Closure}(I)$ and $B \to \gamma$ is a production, then add $B \to \bullet \gamma$ to $\textsc{Closure}(I)$, if it is not already there.
   Apply this rule until no more new items can be added to $\textsc{Closure}(I)$.

Closure of Item Sets

Observation

There are two classes of items:

- *Kernel items*: the initial item, $S' \to \bullet S$, and all items whose dots are not at the left end.
- *Nonkernel items*: all items with their dots at the left end, except for $S' \to \bullet S$

Computing a Closure Set

Let $G = (N, \Sigma, S, P)$ be a grammar and I a set of items.

Algorithm $\textsc{Closure}(I)$

```
J ← I
repeat
    for each item A → α•Bβ ∈ J do
        for each production B → γ ∈ P do
            if B → •γ ∉ J then
                J ← J ∪ {B → •γ}
until no more items are added to J
```

The Function GoTo

Definition (GoTo)

Let I be a set of items and X a grammar symbol. $\textsc{GoTo}(I, X)$ is defined to be the closure of the set of all items $A \to \alpha X \bullet \beta$ such that $A \to \alpha \bullet X \beta \in I$.

States of the LR(0) Automaton

Given an augmented grammar G' we are going to construct the LR(O) automaton by using $\textsc{Closure}(\_)$ and $\textsc{GoTo}(\_,\_)$.

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Given an augmented grammar G' we are going to construct the LR(O) automaton by using $\textsc{Closure}(\_)$ and $\textsc{GoTo}(\_,\_)$.

- The initial state, I_0, of the parser is the closure of the set of items containing $S' \to \bullet S$.
- By using $\textsc{GoTo}(\_,\_)$, we create both the other states and the transitions of the automaton.

Constructing Canonical LR(0)

Algorithm Computes the set of items of an extended grammar G'

procedure $\textsc{SetsOfItems}(G')$
  $C \leftarrow \{\textsc{Closure}(\{S' \rightarrow \bullet S\})\}$
  repeat
    for each set of items $I \in C$ **do**
      for each grammar symbol X **do**
        if $\textsc{GoTo}(I, X) \neq \emptyset \wedge \textsc{GoTo}(I, X) \notin C$ **then**
          $C \leftarrow C \cup \{\textsc{GoTo}(I, X)\}$
  until no more sets of items are added to C

The Function Action

Definition (Action)

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3. **Accept**. It accepts the input and finishes parsing.
4. **Error**. The parser discovers an error in its input and takes some corrective action.

Use of the LR(0) Automaton

SLR

We now construct a *simple LR* (SLR) parser based on the LR(0) automaton.
We begin by constructing the SLR-parsing table.

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   2.2 If $A \to \alpha \bullet \in I_i$ with $A \neq S'$, then $\textsc{Action}(i, a) \leftarrow$ "reduce $A \to \alpha$" for all $a \in \textsc{Follow}(A)$

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   If any conflicting actions result from the above rules, we say the grammar is not SLR(1)
3. If $\textsc{GoTo}(I_i, A) = I_j$, then $\textsc{GoTo}(i, A) \leftarrow j$

All entries not defined by (2) and (3) are made "error"

LR-parsing Algoritm

Let G be a grammar and w an input string.

Algorithm LR-parsing. Returns a reduction for w, otherwise an error.

```
Let T be a stack                                  ▷ At the beginning T contains 0$
Let a be the first symbol of w$
while true do
    Let s be the state on top of the stack
    if ACTION(s,a) = shift t then
        T.push(t)
        Let a be the next input symbol
    else if ACTION(s,a) = reduce A → β then
        pop |β| symbols of the stack
        T.push(GOTO(t,A))                         ▷ t is the temporary top of the stack
        output the production A → β
    else if ACTION(s,a) = accept then break       ▷ Parsing is done
    else error
```

Example

Example

Construct an LR-parser for the grammar

1. $E \to E + T$	3. $T \to T * F$	5. $F \to (E)$
2. $E \to T$	4. $T \to F$	6. $F \to \mathbf{i}$

Example

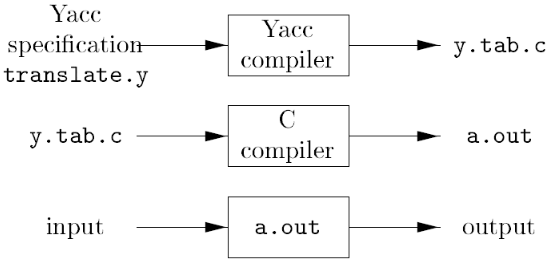
STATE	ACTION						GOTO		
	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

Table 4.37, Aho et al.

Table of Contents

Parser Generators

`Yacc`



- Declarations
- Translation rules
- Supporting C routines

Example: Grammar Specification

$$E \to E + T \mid T$$
$$T \to T * F \mid F$$
$$F \to (E) \mid \textbf{digit}$$

```
line    : expr '\n'          { printf("%d\n", $1); }
        ;
expr    : expr '+' term      { $$ = $1 + $3; }
        | term
        ;
term    : term '*' factor    { $$ = $1 * $3; }
        | factor
        ;
factor  : '(' expr ')'       { $$ = $2; }
        | DIGIT
        ;
```

Example: Complete Specification

```
%{
#include <ctype.h>
%}

%token DIGIT

%%
line   : expr '\n'        { printf("%d\n", $1); }
       ;
expr   : expr '+' term    { $$ = $1 + $3; }
       | term
       ;
term   : term '*' factor  { $$ = $1 * $3; }
       | factor
       ;
factor : '(' expr ')'     { $$ = $2; }
       | DIGIT
       ;
%%
yylex() {
    int c;
    c = getchar();
    if (isdigit(c)) {
        yylval = c-'0';
        return DIGIT;
    }
    return c;
}
```

Example: Grammar Specification

$$
\begin{aligned}
E \to\ & E + E \mid E - E \mid E * E \\
& \mid E/E \mid -E \mid (E) \mid \textbf{number}
\end{aligned}
$$

```
lines : lines expr '\n'   { printf("%g\n", $2); }
      | lines '\n'
      | /* empty */
      ;
expr  : expr '+' expr     { $$ = $1 + $3; }
      | expr '-' expr     { $$ = $1 - $3; }
      | expr '*' expr     { $$ = $1 * $3; }
      | expr '/' expr     { $$ = $1 / $3; }
      | '(' expr ')'      { $$ = $2; }
      | '-' expr  %prec UMINUS { $$ = - $2; }
      | NUMBER
      ;
```

Example: Complete Specification

```
%{
#include <ctype.h>
#include <stdio.h>
#define YYSTYPE double  /* double type for Yacc stack */
%}
%token NUMBER

%left '+' '-'
%left '*' '/'
%right UMINUS
%%

lines : lines expr '\n'   { printf("%g\n", $2); }
      | lines '\n'
      | /* empty */
      ;
expr  : expr '+' expr     { $$ = $1 + $3; }
      | expr '-' expr     { $$ = $1 - $3; }
      | expr '*' expr     { $$ = $1 * $3; }
      | expr '/' expr     { $$ = $1 / $3; }
      | '(' expr ')'      { $$ = $2; }
      | '-' expr  %prec UMINUS { $$ = - $2; }
      | NUMBER
      ;
%%
yylex() {
    int c;
    while ( ( c = getchar() ) == ' ' );
    if ( (c == '.') || (isdigit(c)) ) {
        ungetc(c, stdin);
        scanf("%lf", &yylval);
        return NUMBER;
    }
    return c;
}
```

Referencias

Aho, Alfred V. et al. (2006). *Compilers: Principles, Techniques, and Tools (2nd Edition)*. USA: Addison-Wesley Longman Publishing Co., Inc. ISBN: 0321486811.

Kozen, Dexter C. (1997). *Automata and Computability*. en. New York, NY: Springer New York. ISBN: 978-1-4612-7309-7 978-1-4612-1844-9. DOI: 10.1007/978-1-4612-1844-9. URL: http://link.springer.com/10.1007/978-1-4612-1844-9 (visited on 01/23/2024).

Milner, R. (1999). *Communicating and Mobile Systems: The Pi-Calculus*. Cambridge University Press, Cambridge, UK.

Reghizzi, Stefano Crespi, Luca Breveglieri, and Angelo Morzenti (2019). *Formal Languages and Compilation*. 3rd. Springer Publishing Company, Incorporated. ISBN: 3030048780.