

Simplification of problem (a)



$$AB + A(B+C) + B(B+C)$$

$$= AB + AB + AC + B + BC$$

[Distributivity]

$$= AB + AC + B + BC$$

[Idempotency]

$$= AC + AB + B + BC$$

[Commutativity]

$$= AC + B(A + 1 + C)$$

[Distributivity]

$$= AC + B$$

[Idempotency]

Simplification of problem (B)



$$A'B + BC' + BC + AB'C'$$

$$= A'B + BC + BC' + AB'C' \quad [\text{Commutativity}]$$

$$= A'B + BC + C'(B + AB') \quad [\text{Distributivity}]$$

$$= A'B + BC + C'((B + A)(B + B')) \quad [\text{Distributivity}]$$

$$= A'B + BC + C'(A + B)$$

$$= A'B + BC + C'A + BC' \quad [\text{Distributivity}]$$

$$= A'B + BC + BC' + C'A \quad [\text{Commutativity}]$$

$$= BA' + BC + BC' + C'A \quad [\text{Commutativity}]$$

$$= B(A' + C + C') + C'A \quad [\text{Distributivity}]$$

$$= B + C'A \quad [\text{Complement}]$$

$$= B + AC' \quad [\text{Commutativity}]$$

Simplification of problem (c)



$$(X + Y' + Z')(X + Y' + Z)(X + Y + Z')$$

$$= XY'Z' + XY'Z + XYZ' \quad [\text{Duality}]$$

$$= XY'(Z' + Z) + XYZ' \quad [\text{Distributivity}]$$

$$= XY' + XYZ' \quad [\text{Complement}]$$

$$= X(Y' + YZ') \quad [\text{Distributivity}]$$

$$= X(Y' + Y)(Y' + Z') \quad [\text{Distributivity}]$$

$$= X(Y' + Z') \quad [\text{Complement}]$$

$$= X + Y'Z' \quad [\text{DUALITY}]$$