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FAKULTÄT FÜR PHYSIK
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Name: Filip Vorname: Paul

Name: Janic Vorname: Beck

Emailadresse(n): ____________

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2. Theory & Preparation

2.1 Compton scattering

Consider the scenario of a high-energy photon interacting with an unbound electron as shown in Figure 2.1a. To describe this process we choose a coordinate frame where the electron is at rest with respect to us. In the experiments to be presented in this report such a coordinate frame conveniently is the lab frame anyways. Furthermore, we employ natural units, $\epsilon_0 = \hbar = c = 1$.
From the conservation of energy and impluse we can construct a theoretical description of this process based on the inital and final energies of both particles.

$$\begin{aligned} E_{\gamma,i} + \underbrace{E_{e,i}}_{=0} &= E_{\gamma,f} + E_{e,f} \\ p_{\gamma,i} + \underbrace{p_{e,i}}_{=0} &= p_{\gamma,f} + p_{e,f} \end{aligned}$$

From the above relations an expression for the energy of the photon after interacting with the electron can be obtained and reads

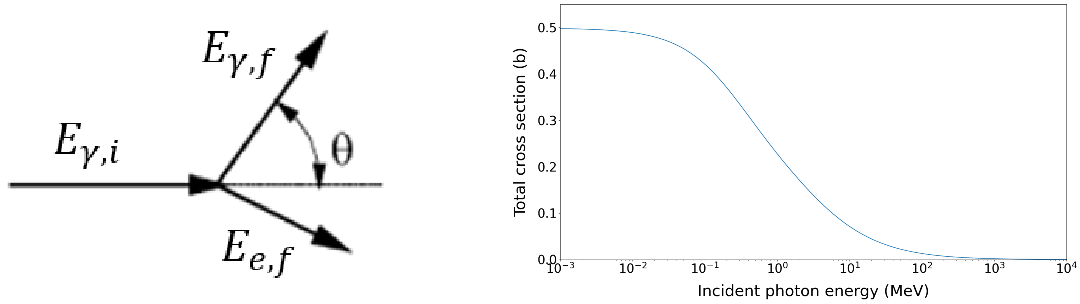
$$E_{\gamma,f} = \frac{E_{\gamma,i}}{1 + \frac{E_{\gamma,i}}{m_e}(1 - \cos\theta)}, \tag{2.1}$$

where θ defines the angle spanned between the incident photon and its path post scattering. It follows that the electron gains energy from the interaction.

$$E_{e,f} = E_{\gamma,i} - E_{\gamma,f} = E_{\gamma,f} \cdot \frac{E_{\gamma,i}}{m_e} \cdot (1 - \cos\theta). \tag{2.2}$$

The measureable change in the photons wavelength $\lambda = \frac{hc}{E_\gamma}$ due to the interaction is called the **Compton effect**. The underlaying elastic scattering of photons and unbound electrons is consequently labelled **Compton scattering**. Alongside Photoionisation and Pair production it represents one of the important processes by which electromagnetic radiation interacts with matter.

The physical characteristics of Compton scattering, namely its cross section and the resulting distribution of electron energies will be discused in the following section 2.2 and section 2.3.



(a) **Scattering kinematics** (b) **Total cross section**

(a) A high energy photon scatters off a free electron at rest. The defining variables to describe this process are given by $E_{\gamma,i}$ and θ. Figure adapted with changes from [**?**] (b) The total cross section as a function of the incident photon energy. The cross section decreases for large energies due to the increased likeliness of pair production.

2.2 Cross section

Compton scattering is the dominating effect by which photons with an energy between 100 keV and 10 MeV interact with matter [**?**]. A theoretical description of the processes cross section is given by the **Klein-Nishina formul** (KN).

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}_{\mathrm{KN}} = \frac{\alpha^2}{2m_e}\left(\frac{E_{\gamma,f}}{E_{\gamma,i}}\right)^2\left[\frac{E_{\gamma,f}}{E_{\gamma,i}} + \frac{E_{\gamma,i}}{E_{\gamma,f}} - \sin^2\theta\right] \tag{2.3}$$

Integrating over all solid angles and defining $x = \frac{E_{\gamma,i}}{m_e}$, one obtains the total cross section.

$$\sigma_{\mathrm{tot.}} = \int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,\mathrm{d}\Omega = \frac{\pi\alpha^2}{m_e^2}\frac{1}{x^3}\left(\frac{2x(2+x(1+x)(8+x))}{(1+2x)^2} + ((x-2)x - 2\log(1+2x)\right) \tag{2.4}$$

In the low-energy limit of $x \ll 1$ Equation 2.4 simplifies to the **Thomson cross section**, whereas in the high-energy limit $x \to \infty$ we expand in x to find the following.

$$\begin{aligned} x \ll 1: &\qquad \sigma_{\mathrm{tot.}} = \frac{8\pi\alpha^2}{m_e^2} \\ x \longrightarrow \infty: &\qquad \sigma_{\mathrm{tot.}} = \frac{\pi\alpha^2}{xm_e^2}\left(\frac{1}{2} + \log 2x\right) \end{aligned}$$

As it turns out, the cross section is constant for low-energy photons, where ionisation is more probable. Furthermore the likeliness of an interaction actually decreases for high-energy photons and eventually drops to zero. This behaviour can also be seen in Figure 2.1b and explains the importance of Compton scattering over the intermediate ranges of energy from 100 keV to 10 MeV.

2.3 Compton spectrum

Without knowing anything about the distribution of energies or deflection angles of the scattered electrons, by examining Equation 2.2 it can already be established that the energy the electron gains from the interaction is directly proportional to $(1 - \cos\theta)$, or in other

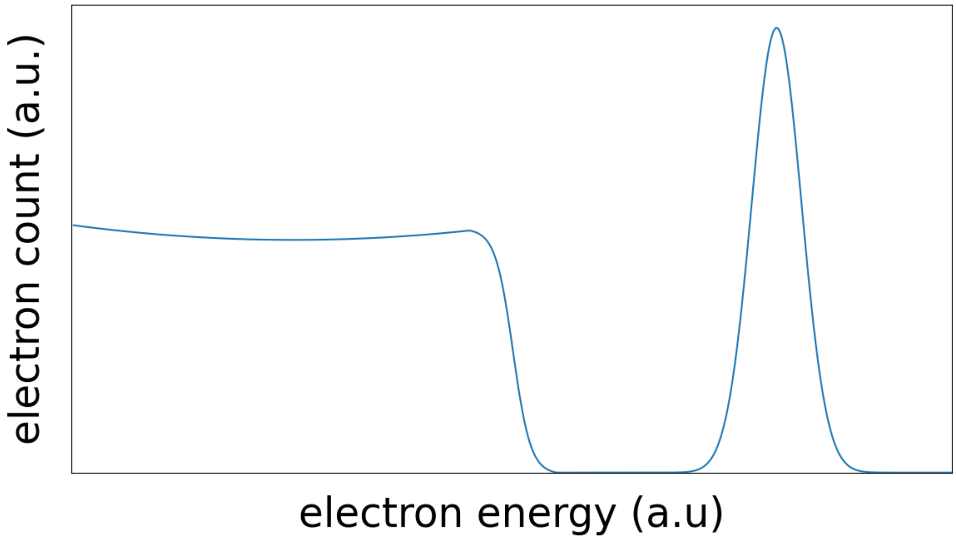


Figure 2.2: An idealised Compton spectrum. A relatively constant flux of electrons is measured up to an energy of E_{max}, where the spectrum sharply drops off. At the high-energy end of the spectrum a gaussian shaped photopeak is visible.

words, by how much the photon is scattered away from its original path. It follows that for $\theta = 180°$ the electron gains a maximum energy of

$$E_{\text{max}} = \frac{E_\gamma}{1 + \frac{m_e c^2}{E_\gamma}}. \tag{2.5}$$

Since the photon physically cannot dump more energy by this process, a sharp drop in the Compton spectrum at E_{max} is expected. This characteristic drop-off is commonly called the **Compton edge**. Energetically lower than this cutoff lays the **Compton continuum**, or main part of the spectrum. Over a wide range of energies that correspond to scattering angles $\theta \in [0°, 180°]$ the flux of electrons remains approximately constant. This follows directly from Equation 2.3.

Lastly, an idealised Compton spectrum as depicted in Figure 2.2 will also display a characteristic **photopeak**. This photopeak is caused by photons directly interacting with detector material via the photoelectric effect. In this case, the entire energy of the photon is dumped inside the detector. It is therefore a helpful reference point for calibrations, albeit not being part of the Compton spectrum itself.

3. Experiment & Evaluation