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# Contents

<b>1</b>	<b>Measurement report</b>	<b>1</b>
<b>2</b>	<b>Theory &amp; Preparation</b>	<b>3</b>
2.1	Compton scattering . . . . .	3
2.2	Cross section . . . . .	4
2.3	Compton spectrum . . . . .	5
<b>3</b>	<b>Experiment &amp; Evaluation</b>	<b>6</b>
	<b>Bibliography</b>	<b>7</b>

## 2. Theory & Preparation

### 2.1 Compton scattering

Consider the scenario of a high-energy photon interacting with an unbound electron as shown in Figure 2.1a. To describe this process we choose a coordinate frame where the electron is at rest with respect to us. In the experiments to be presented in this report such a coordinate frame conveniently is the lab frame anyways. Furthermore, we employ natural units,  $\epsilon_0 = \hbar = c = 1$ .

From the conservation of energy and impulse we can construct a theoretical description of this process based on the initial and final energies of both particles.

$$\begin{aligned} E_{\gamma,i} + \underbrace{E_{e,i}}_{=0} &= E_{\gamma,f} + E_{e,f} \\ p_{\gamma,i} + \underbrace{p_{e,i}}_{=0} &= p_{\gamma,f} + p_{e,f} \end{aligned}$$

From the above relations an expression for the energy of the photon after interacting with the electron can be obtained as lined out in [Sch17]. It reads

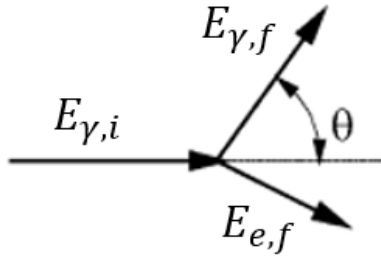
$$E_{\gamma,f} = \frac{E_{\gamma,i}}{1 + \frac{E_{\gamma,i}}{m_e}(1 - \cos \theta)}, \quad (2.1)$$

where  $\theta$  defines the angle spanned between the incident photon and its path post scattering. It follows that the electron gains energy from the interaction.

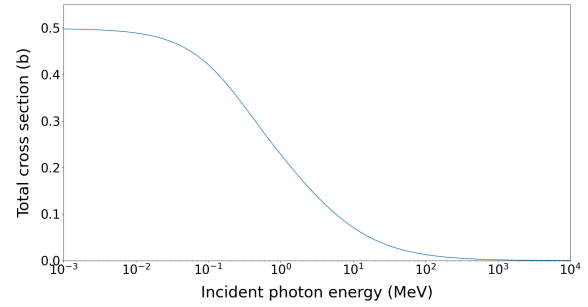
$$E_{e,f} = E_{\gamma,i} - E_{\gamma,f} = E_{\gamma,f} \cdot \frac{E_{\gamma,i}}{m_e} \cdot (1 - \cos \theta). \quad (2.2)$$

The measureable change in the photons wavelength  $\lambda = \frac{hc}{E_{\gamma}}$  due to the interaction is called the **Compton effect**. The underlying elastic scattering of photons and unbound electrons is consequently labelled **Compton scattering**. Alongside Photoionisation and Pair production it represents one of the important processes by which electromagnetic radiation interacts with matter.

The physical characteristics of Compton scattering, namely its cross section and the resulting distribution of electron energies will be discussed in the following section 2.2 and section 2.3.



(a) Scattering kinematics



(b) Total cross section

(a) A high energy photon scatters off a free electron at rest. The defining variables to describe this process are given by  $E_{\gamma,i}$  and  $\theta$ . Figure adapted with changes from [Sch17].  
 (b) The total cross section as a function of the incident photon energy. Roughly constant for low-energy photons, the total cross section drops off quickly for higher energies.

## 2.2 Cross section

Compton scattering is the dominating effect by which photons with an energy between 100 keV and 10 MeV interact with matter [DG70]. A theoretical description of the processes cross section is given by the **Klein-Nishina formula** (KN).

$$\frac{d\sigma}{d\Omega}_{\text{KN}} = \frac{\alpha^2}{2m_e} \left( \frac{E_{\gamma,f}}{E_{\gamma,i}} \right)^2 \left[ \frac{E_{\gamma,f}}{E_{\gamma,i}} + \frac{E_{\gamma,i}}{E_{\gamma,f}} - \sin^2 \theta \right] \quad (2.3)$$

Integrating over all solid angles and defining  $x = \frac{E_{\gamma,i}}{m_e}$ , one obtains the total cross section.

$$\sigma_{\text{tot.}} = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{\pi\alpha^2}{m_e^2} \frac{1}{x^3} \left( \frac{2x(2 + x(1+x)(8+x))}{(1+2x)^2} + ((x-2)x - 2\log(1+2x)) \right) \quad (2.4)$$

In the low-energy limit of  $x \ll 1$  Equation 2.4 simplifies to a constant called the **Thomson cross section**, whereas in the high-energy limit  $x \rightarrow \infty$  we expand in  $x$  to find that the cross section vanishes. This behaviour can also be seen in Figure 2.1b.

$$\begin{aligned} x \ll 1 : \quad \sigma_{\text{tot.}} &= \frac{8\pi\alpha^2}{m_e^2} \approx 0.6652 \text{ b} \\ x \rightarrow \infty : \quad \sigma_{\text{tot.}} &= \frac{\pi\alpha^2}{xm_e^2} \left( \frac{1}{2} + \log 2x \right) \end{aligned}$$

Naively, one would therefore expect Compton scattering to be an important process for low to intermediate energy ranges of the electromagnetic spectrum, where the cross section  $\sigma_{\text{Tot.}}$  is non-negligible. This is however not the case. Various other processes such as the photoelectric effect or Rayleigh scattering dominate the low energy regime and render Compton scattering only a fringe case. With higher photon energies, the cross section for the aforementioned processes drop off, and increase the importance of Compton scattering for gammas carrying an energy of 100 keV up to 10 MeV .

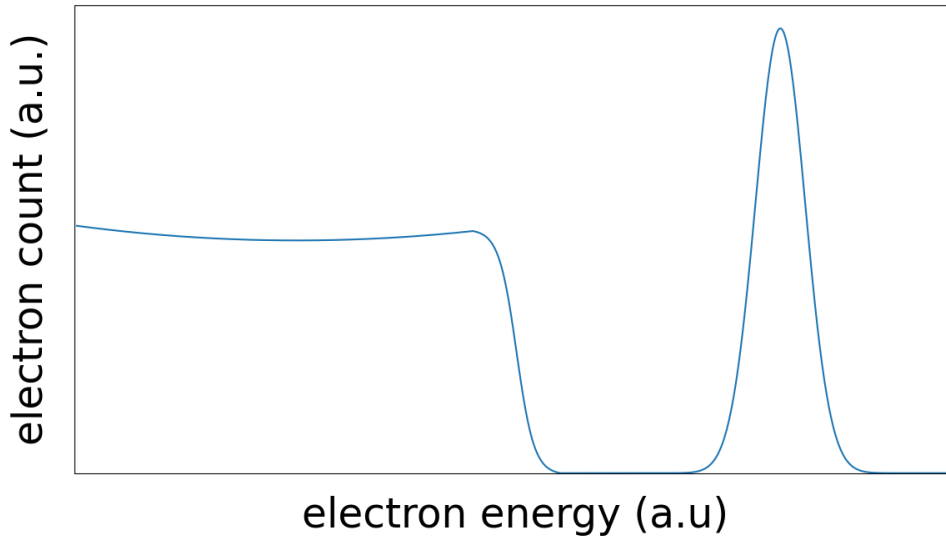


Figure 2.2: An idealised Compton spectrum. A relatively constant flux of electrons is measured up to an energy of  $E_{\max}$ , where the spectrum sharply drops off. At the high-energy end of the spectrum a gaussian shaped photopeak is visible.

### 2.3 Compton spectrum

Without knowing anything about the distribution of energies or deflection angles of the scattered electrons, by examining Equation 2.2 it can already be established that the energy the electron gains from the interaction is directly proportional to  $(1 - \cos \theta)$ , or in other words, by how much the photon is scattered away from its original path. It follows that for  $\theta = 180^\circ$  the electron gains a maximum energy of

$$E_{\max} = \frac{E_\gamma}{1 + \frac{m_e c^2}{E_\gamma}}. \quad (2.5)$$

Since the photon physically cannot dump more energy by this process, a sharp drop in the Compton spectrum at  $E_{\max}$  is expected. This characteristic drop-off is commonly called the **Compton edge**. Energetically lower than this cutoff lays the **Compton continuum**, or main part of the spectrum. Over a wide range of energies that correspond to scattering angles  $\theta \in [0^\circ, 180^\circ]$  the flux of electrons remains approximately constant. This follows directly from Equation 2.3.

Lastly, an idealised Compton spectrum as depicted in Figure 2.2 will also display a characteristic **photopeak**. This photopeak is caused by photons directly interacting with detector material via the photoelectric effect. In this case, the entire energy of the photon is dumped inside the detector. It is therefore a helpful reference point for calibrations, albeit not being part of the Compton spectrum itself.

The measured Compton spectrum discussed in chapter 3 will display several other characteristics such as a prominent X-ray line or a backscatter peak. These properties are dependant on the experimental setup. As such they will be discussed in the appropriate sections of chapter 3.

### **3. Experiment & Evaluation**

# Bibliography

- [DG70] Damashek, Marc und Frederick J Gilman: *Forward compton scattering*. Physical Review D, 1(5):1319, 1970.
- [Sch17] Schmidt, F.K.: *Einführung in das Kernphysikalische Praktikum*. Institut für Experimentelle Kernphysik, Karlsruhe, Ausgabe Oktober 2017.