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2. Theory & Preparation

2.1 Compton scattering

Consider the scenario of a high-energy photon interacting with an unbound electron as shown in Figure 2.1a. To describe this process we choose a coordinate frame where the electron is at rest with respect to us. In the experiments to be presented in this report such a coordinate frame conveniently is the lab frame anyways. Furthermore, we employ natural units, $\epsilon_0 = \hbar = c = 1$.

From the conservation of energy and impulse we can construct a theoretical description of this process based on the initial and final energies of both particles.

$$\begin{aligned} E_{\gamma,i} + \underbrace{E_{e,i}}_{=0} &= E_{\gamma,f} + E_{e,f} \\ p_{\gamma,i} + \underbrace{p_{e,i}}_{=0} &= p_{\gamma,f} + p_{e,f} \end{aligned}$$

From the above relations an expression for the energy of the photon after interacting with the electron can be obtained as lined out in [Sch17]. It reads

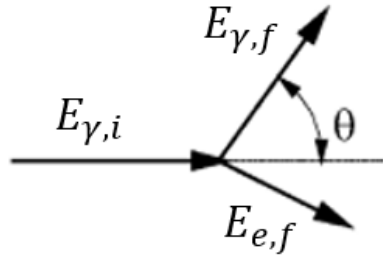
$$E_{\gamma,f} = \frac{E_{\gamma,i}}{1 + \frac{E_{\gamma,i}}{m_e}(1 - \cos \theta)}, \quad (2.1)$$

where θ defines the angle spanned between the incident photon and its path post scattering. It follows that the electron gains energy from the interaction.

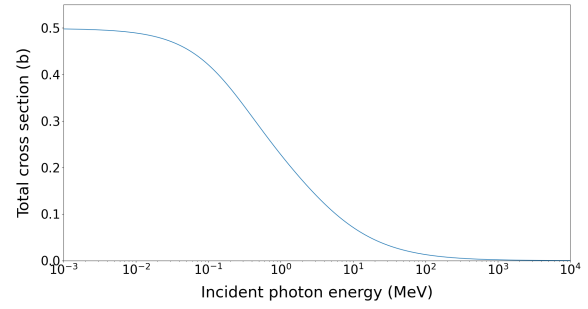
$$E_{e,f} = E_{\gamma,i} - E_{\gamma,f} = E_{\gamma,f} \cdot \frac{E_{\gamma,i}}{m_e} \cdot (1 - \cos \theta). \quad (2.2)$$

The measureable change in the photons wavelength $\lambda = \frac{hc}{E_{\gamma}}$ due to the interaction is called the **Compton effect**. The underlying elastic scattering of photons and unbound electrons is consequently labelled **Compton scattering**. Alongside Photoionisation and Pair production it represents one of the important processes by which electromagnetic radiation interacts with matter.

The physical characteristics of Compton scattering, namely its cross section and the resulting distribution of electron energies will be discussed in the following section 2.2 and section 2.3.



(a) Scattering kinematics



(b) Total cross section

(a) A high energy photon scatters off a free electron at rest. The defining variables to describe this process are given by $E_{\gamma,i}$ and θ . Figure adapted with changes from [Sch17].
 (b) The total cross section as a function of the incident photon energy. Roughly constant for low-energy photons, the total cross section drops off quickly for higher energies.

2.2 Cross section

Compton scattering is the dominating effect by which photons with an energy between 100 keV and 10 MeV interact with matter [DG70]. A theoretical description of the processes cross section is given by the **Klein-Nishina formula** (KN).

$$\frac{d\sigma}{d\Omega}_{\text{KN}} = \frac{\alpha^2}{2m_e} \left(\frac{E_{\gamma,f}}{E_{\gamma,i}} \right)^2 \left[\frac{E_{\gamma,f}}{E_{\gamma,i}} + \frac{E_{\gamma,i}}{E_{\gamma,f}} - \sin^2 \theta \right] \quad (2.3)$$

Integrating over all solid angles and defining $x = \frac{E_{\gamma,i}}{m_e}$, one obtains the total cross section.

$$\sigma_{\text{tot.}} = \int \frac{d\sigma}{d\Omega} d\Omega = \frac{\pi\alpha^2}{m_e^2} \frac{1}{x^3} \left(\frac{2x(2 + x(1+x)(8+x))}{(1+2x)^2} + ((x-2)x - 2\log(1+2x)) \right) \quad (2.4)$$

In the low-energy limit of $x \ll 1$ Equation 2.4 simplifies to a constant called the **Thomson cross section**, whereas in the high-energy limit $x \rightarrow \infty$ we expand in x to find that the cross section vanishes. This behaviour can also be seen in Figure 2.1b.

$$\begin{aligned} x \ll 1 : \quad \sigma_{\text{tot.}} &= \frac{8\pi\alpha^2}{m_e^2} \approx 0.6652 \text{ b} \\ x \rightarrow \infty : \quad \sigma_{\text{tot.}} &= \frac{\pi\alpha^2}{xm_e^2} \left(\frac{1}{2} + \log 2x \right) \end{aligned}$$

Naively, one would therefore expect Compton scattering to be an important process for low to intermediate energy ranges of the electromagnetic spectrum, where the cross section $\sigma_{\text{Tot.}}$ is non-negligible. This is however not the case. Various other processes such as the photoelectric effect or Rayleigh scattering dominate the low energy regime and render Compton scattering only a fringe case. With higher photon energies, the cross section for the aforementioned processes drop off, and increase the importance of Compton scattering for gammas carrying an energy of 100 keV up to 10 MeV .

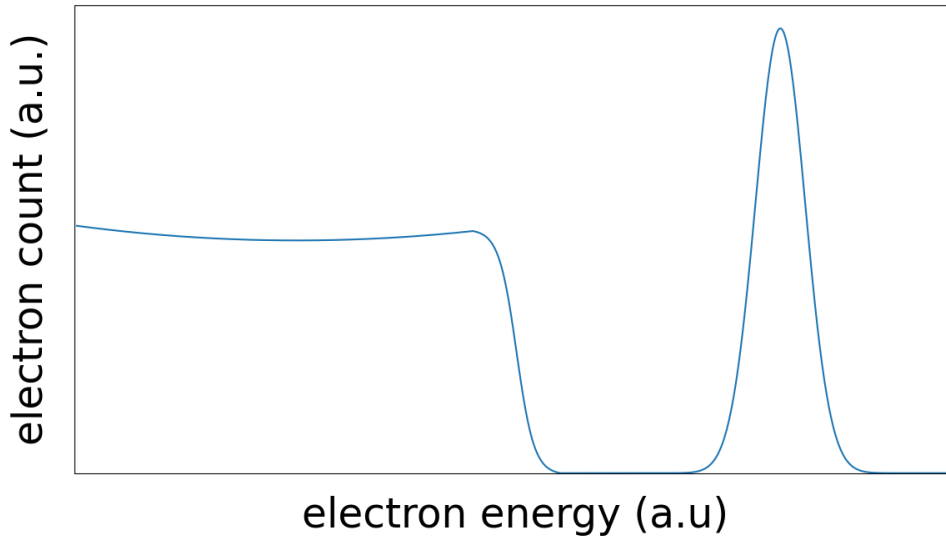


Figure 2.2: An idealised Compton spectrum. A relatively constant flux of electrons is measured up to an energy of E_{\max} , where the spectrum sharply drops off. At the high-energy end of the spectrum a gaussian shaped photopeak is visible.

2.3 Compton spectrum

Without knowing anything about the distribution of energies or deflection angles of the scattered electrons, by examining Equation 2.2 it can already be established that the energy the electron gains from the interaction is directly proportional to $(1 - \cos \theta)$, or in other words, by how much the photon is scattered away from its original path. It follows that for $\theta = 180^\circ$ the electron gains a maximum energy of

$$E_{\max} = \frac{E_\gamma}{1 + \frac{m_e c^2}{E_\gamma}}. \quad (2.5)$$

Since the photon physically cannot dump more energy by this process, a sharp drop in the Compton spectrum at E_{\max} is expected. This characteristic drop-off is commonly called the **Compton edge**. Energetically lower than this cutoff lays the **Compton continuum**, or main part of the spectrum. Over a wide range of energies that correspond to scattering angles $\theta \in [0^\circ, 180^\circ]$ the flux of electrons remains approximately constant. This follows directly from Equation 2.3.

Lastly, an idealised Compton spectrum as depicted in Figure 2.2 will also display a characteristic **photopeak**. This photopeak is caused by photons directly interacting with detector material via the photoelectric effect. In this case, the entire energy of the photon is dumped inside the detector. It is therefore a helpful reference point for calibrations, albeit not being part of the Compton spectrum itself.

The measured Compton spectrum discussed in chapter 3 may display several other characteristics such as a prominent X-ray line or a backscatter peak. These properties are dependant on the experimental setup. As such they will - if needed - be discussed in the appropriate sections of chapter 3.

3. Experiment & Evaluation

3.1 Experimental setup

The multi channel analyser (MCA) consists of a 20.43 cm^2 large NaJ scintillating crystal as well as a photo-multiplier (PMT) useful for signal amplification. A readout chip allows for the digital treatment of measurement data. The detector is placed on metallic guiding rails that allow the scintillator to be placed at various positions on a circle around a plastic pedestal. A small $1 \text{ cm} \times 1 \text{ cm}$ target can be placed on the pedestal such that it is located in the beam path of a high-energy γ -source. The angle enclosed by the beam path and a straight line from scintillator detector to the target pedestal defines the scattering angle θ .

3.2 Calibration of a multichannel analyser

As a first task the MCA needs to be calibrated so that measured channel numbers of an event can be related to the energy of the particles that caused it. The presented calibration is fourfold. In separate measurements the radionuclides Cobalt-57, and -60, as well as Sodium-22, and Caesium-137 are placed in front of the NaJ-scintillator. Their γ -spectrum is measured for 300 s. The resulting distribution of observed events across the different measuring channels is depicted in Figure 3.1. The interesting parts of the spectra are the gaussian shaped sections located at the tail ends. They represent the photopeaks discussed briefly in section 2.3.

Equipped with theoretical knowledge of the radioactive processes, one can compare the measured channel with the expected energy of a nuclear transition to establish a linear connection between the two. For a more solid result this calibration is done simultaneously for all measured radionuclides. A linear regression over all reference points then allows a quantitative analysis of measurement results. Below listed in ?? are the different observed radioactive transitions, their characteristic transition energies, as well as the channel they were observed in by the MCA, the last column holds information about the location of a gaussian fit applied to the signal. In the bottom plot of Figure 3.1 these results are furthermore visualised.

Table 3.1: MCA energy calibration parameters

Transition	Energy (keV)	mes. channel	fit channel
$^{57}\text{Co} \rightarrow ^{57}\text{Fe}$	122.061 [REA70]	16	15.3(1)
$^{22}\text{Na} \rightarrow ^{22}\text{Ne}$	546.544 [SM54]	91	90.8(4)
$^{137}\text{Cs} \rightarrow ^{137m}\text{Ba}$	661.659 [Wag51]	108	106.0(3)
$^{60}\text{Co} \rightarrow ^{60}\text{Ni}^*$	1173.200 [BKK ⁺ 01]	-	see text
$^{60}\text{Ni}^* \rightarrow ^{60}\text{Ni}$	1332.000 [BKK ⁺ 01]	-	see text

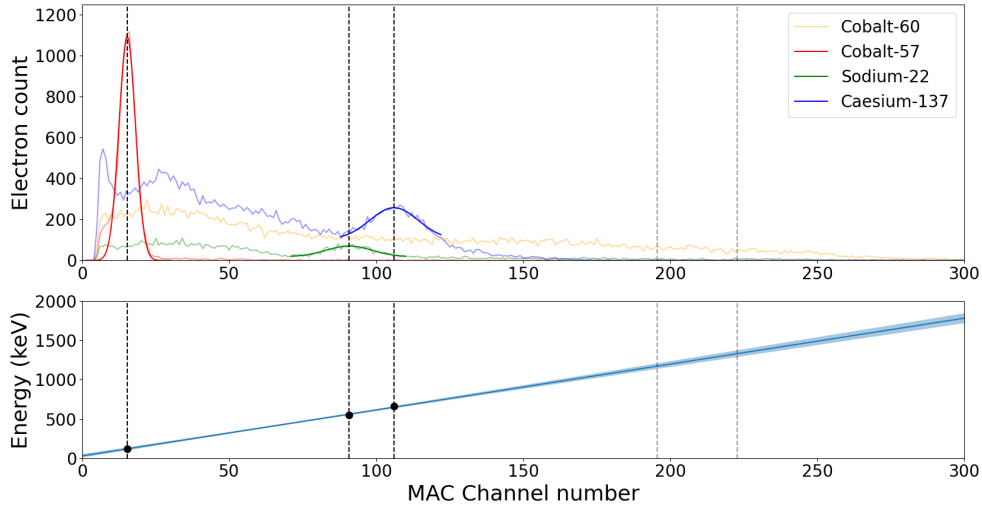


Figure 3.1

The gamma spectra of various radionuclides as measured by the MCA is depicted in the upper plot. A Gaussian fit highlights the characteristic photopeaks for each (except Co-60, see text) measurement. The lower plot sets the measured channel numbers into context with the energies of the observed transitions. A linear model that connects MCA channel number with an energy is established.

As can be seen both in the plot as well as the table, a calibration for the photopeaks of ^{60}Co are not conducted. From the location of other photopeaks the characteristic points of the ^{60}Co γ spectrum should be located around channels #195 and #223. This can however not be experimentally verified. A reasonable analysis cannot be completed, as the photopeaks are - provided the presented methodology is sound - far too faint. This can perhaps be prevented in future lab assignments by gathering more data of the ^{60}Co spectrum.

In any case, a linear model that connects channel number \mathcal{C} to a particle energy E is obtained. They can be related to each other via Equation 3.1 that is presented below.

$$E(\mathcal{C}) = \underbrace{(5.8 \pm 0.3) \text{ keV}}_a \cdot \mathcal{C} + \underbrace{(30.2 \pm 22.0) \text{ keV}}_b \quad (3.1)$$

Albeit only three of the four radionuclides can be used effectively for a calibration, the results nevertheless seem promising. A quick check regarding the goodness of fit finds a reduced $\chi^2_\nu \approx 0.97$, indicating that the model conforms with observations to reasonable accuracy. This accuracy $\Delta E(\mathcal{C})$ can then be quantified using the covariance matrix $\text{COV}(a, b)$, estimated by the fitting algorithm, as well as the gradient ∇E , which can easily be calculated by hand.

$$\text{COV}(a, b) = \begin{bmatrix} 0.0736 & -5.2014 \\ -5.2014 & 483.5527 \end{bmatrix}, \quad \nabla E = \begin{pmatrix} \mathcal{C} \\ 1 \end{pmatrix}. \quad (3.2)$$

$$\Delta E(\mathcal{C}) = \sqrt{(\nabla E)^T \text{COV}(a, b) \nabla E}. \quad (3.3)$$

It is expected that the uncertainties in $E(\mathcal{C})$ grow large for energies located far away from the calibration points. This is especially the case for high scattering angles.

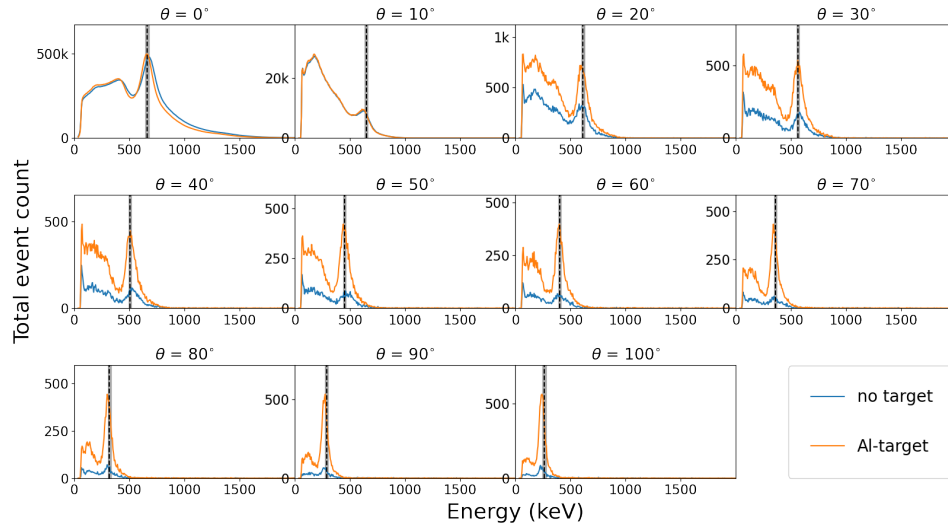


Figure 3.2

Raw number of events counted in every channel for each measured angle. The dashed black lines indicate the theoretical energy a photon has after undergoing Compton scattering at an angle θ . The distance between signal peaks and this energy grows with increasing scattering angles. This is expected due to the imperfect calibration.

3.3 Differential cross section

As a second task, the validity of the Klein-Nishina formula presented in Equation 2.3 is tested. For this purpose, the spectrometer is placed at various angles relative to the beam path. For every angle two measurements of 300s each are taken. One measurement with a cylindrical aluminum target in the beam path that allows for Compton scattering. The second measurement is conducted without any target and will help quantify the background noise measured by the spectrometer. The resulting distribution of electron energies is shown in Figure 3.2 for all measurements.

3.4 Energy shift & electron rest mass

3.5 Z dependence

Table 3.2: Calculated energy shift

θ (°)	$1 - \cos \theta$	Energy (keV)	inv. (keV ⁻¹)
20	0.060	601.07 ± 2.76	0.0017 ± 0.0001
30	0.134	556.07 ± 2.29	0.0018 ± 0.0001
40	0.234	503.18 ± 1.68	0.0020 ± 0.0001
50	0.357	445.76 ± 1.44	0.0022 ± 0.0001
60	0.500	398.55 ± 1.21	0.0025 ± 0.0001
70	0.658	346.06 ± 0.97	0.0029 ± 0.0001
80	0.826	305.15 ± 0.69	0.0033 ± 0.0001
90	1.000	270.31 ± 0.68	0.0037 ± 0.0001
100	1.174	240.19 ± 0.53	0.0042 ± 0.0001

Table 3.3: Z dependance of R

Element	Z/A	R	$\frac{R \cdot A}{\rho Z}$
Al	13/26.982	49.00±0.95	37.66±0.73
Fe	26/55.845	155.23±1.12	42.34±0.31
Cu	29/63.546	166.09±1.14	40.62±0.28
Pb	82/207.200	132.46±1.09	29.51±0.24

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