Orientation estimation using smartphone sensors

sensor fusion and nonlinear filtering

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Abstract

The task is to build a filter which can be used to estimate the orientation of a smartphone by using its sensors (accelerator, gyroscope and magnetometer)

Task 1 Discuss pros and cons regarding this choice

(1)Setting gyroscope data as input

Pros:
1)Real-time orientation estimation: Gyroscopes provide information about the rotational motion of the mobile phone, allowing for real-time estimation of its direction. The Kalman filter provides a framework to process and fuse the gyroscope data with other sensor measurements to estimate the orientation accurately.

2)Low latency: The Kalman filter, when implemented efficiently, can provide low-latency estimates of the phone's direction. This is essential for applications that require immediate or responsive feedback, such as augmented reality, motion tracking, or virtual reality.

3)Improved accuracy: Incorporating gyroscope data helps improve the accuracy of the estimation process. By continuously updating the orientation estimate based on gyroscope measurements, the Kalman filter can mitigate errors introduced by other sensors (such as accelerometers or magnetometers) or external disturbances.

Cons:
1)Gyroscope limitations: Gyroscopes have limitations that can impact the accuracy of the estimation. Gyroscopes suffer from drift over time, leading to errors in the estimated direction. Noise in gyroscope measurements can also introduce inaccuracies into the filtering process. These limitations need to be carefully accounted for and compensated to achieve reliable orientation estimates.

2)Complementary sensor integration: While gyroscope data is valuable for estimating short-term changes in direction, it may not be sufficient on its own for accurate long-term estimation. Combining gyroscope data with other sensors like accelerometers and magnetometers (sensor fusion) is often necessary to improve the accuracy and robustness of the direction estimation process.

(2)Inappropriate situations

In the following cases, gyroscope data is not suitable as an input alone, 1)The first situation is ***Gyroscope Drift***: Gyroscopes are prone to drift, meaning that over time, small errors in the gyroscope measurements can accumulate, leading to significant inaccuracies in the estimated direction. If the drift is not effectively compensated for, the estimated direction may become increasingly erroneous.

2)The second situation is ***Limited Gyroscope Range***:Gyroscopes have finite ranges and can saturate or become less accurate when subjected to high rotational rates or accelerations. In extreme cases, the gyroscope may provide unreliable measurements or even fail to capture certain motions accurately, leading to incorrect direction estimation.

3)The thirf situation is ***Environmental factors***:Gyroscopes can be affected by environmental factors such as temperature changes, magnetic interference, and vibrations. These external influences can introduce errors and uncertainties into the gyroscope measurements, adversely affecting the accuracy of the direction estimation.

(3)Include angular velocity

Including angular velocities in the state vector can be beneficial in certain scenarios where a more detailed estimation of the system's dynamics is required:
1)***High-speed or agile movements***: If the system undergoing estimation involves high-speed or agile movements, such as a fast-moving robotic arm or an aerial drone, including angular velocities in the state vector can help capture the system's dynamics more accurately. Angular velocities provide information about the rotational speed and direction of the system, enabling better tracking of fast and dynamic motions.

2)***Nonlinear system dynamics***: When dealing with nonlinear system dynamics, including angular velocities in the state vector allows for a more comprehensive modeling of the system. Nonlinear systems often exhibit complex and nonlinear relationships between states and inputs, and incorporating angular velocities can help capture these non-linearization and improve the estimation accuracy.

Task 2

Histograms of measurements for some sensors and axes

As is shown in the figure 1 3 5.

A plot of the signals overtime

As is shown in the figure 2 4 6.

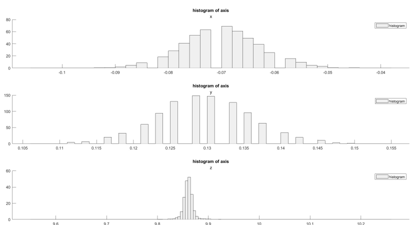

Figure 1: histogram of acc

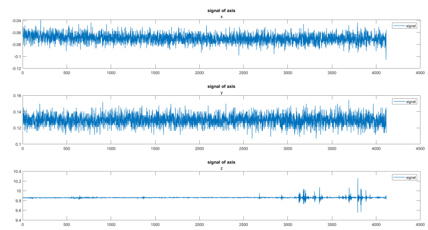

Figure 2: signal of acc

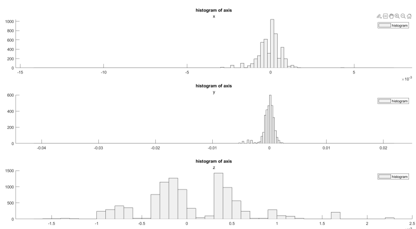

Figure 3: histogram of gyro

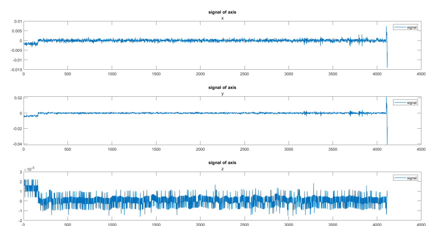

Figure 4: signal of gyro

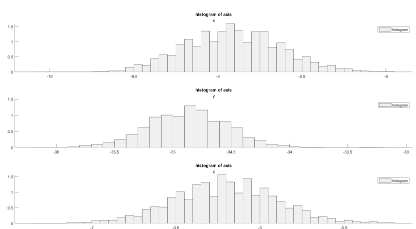

Figure 5: histogram of mag

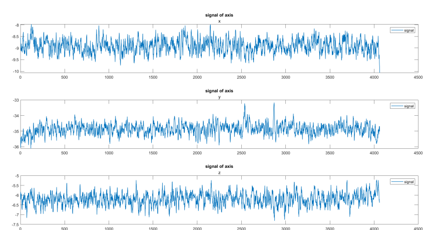

Figure 6: signal of mag

Compute mean and covariances

As is shown in table 1 and table 2, they are computed Compute mean and covariances.

Table 1: Calculated mean values of calibration data

	Accelerometer	Gyroscope	Magnetometer
μ_x	-0.0698	$-5.963 \cdot 10^{-5}$	-8.9078
μ_y	0.1296	$-1.483 \cdot 10^{-4}$	-34.8681
μ_z	9.8607	$5.45 \cdot 10^{-5}$	-6.2037

Table 2: Calculated covariance of calibration data

	Accelerometer	Gyroscope	Magnetometer
σ_x^2	$5.3764 \cdot 10^{-5}$	$5.2583 \cdot 10^{-7}$	0.0812
σ_y^2	$4.162 \cdot 10^{-5}$	$2.0019 \cdot 10^{-6}$	0.1222
σ_z^2	$3.6202 \cdot 10^{-4}$	$3.0836 \cdot 10^{-7}$	0.0961

summary

In this task, Redmi k30s was putting on the table and away from the disturbance. As is shown in the figure 1-6, all of those data are norm distribution.
For the Gyroscope and Accelerometer, if they are placed on a table without movement or external forces, the data from these sensors would not exhibit a significant distribution pattern. The data would be centered around a stationary value with minimal variance, resulting in a narrow distribution, as is shown in figure 1 and figure 3. However, both of them have some trends.

- For the Accelerometer, due to it affected by gravity, its mean tend to be equal to 9.86, which is equal to earth gravity. Due to the imbalance of the table, there is some acceleration in both x and y directions, but small and acceptable
- For the Gyroscopebecause the mobile phone is placed statically, it presents a standard and narrow normal distribution.

For magnetometer, in general, the data from a magnetometer measuring the Earth's magnetic field or nearby magnetic objects would not be expected to follow a normal distribution. However, we put the phone in a stable magnetic environment (away from metal things)-In such cases, the variations in the magnetic field became small and follow a normal distribution due to the stability of the environment.

Task 3

For the equation we already have:

$$\dot{q}(t) = \underbrace{\frac{1}{2}S(w_{k-1} + v_{k-1})}_{A}\, q(t)$$

Then we need to discrete it:

$$\begin{aligned} q(k) &= e^{AT}q(k-1) \\ &= (I + AT)q(k-1) \\ &= \big(I + \frac{T}{2}S(w_{k-1} + v_{k-1})\big)q(k-1) \end{aligned}$$

Then we should use the equation given in the task:

$$S(w_{k-1} + v_{k-1}) = S(w_{k-1}) + S(v_{k-1})$$

Also we should use the approximation:$G(q_{k-1})v_{k-1} \approx G(\hat{q}_{k-1})v_{k-1}$,Then we get the equation below:

$$q(k) = \underbrace{\big(I + \frac{T}{2}S(w_{k-1})\big)}_{F}\, q(k-1) + \underbrace{\frac{T}{2}S(q_{k-1})}_{G}\, v_{k-1}$$

Task 4

Since data ($\omega \sim N(x, P)$) and noise ($noise \sim N(0, R_w)$) follow normal distribution, then the x, P could be expressed below:

$$x = F * x$$
$$P = F * P * F^T + G * R_w * G^T$$

As is shown in figure 7, which is the code of tu_qw: If there is no angular rate

```
function [x,P] =tu_qw(x, P, omega, T, Rw)
%here is x=Fx+GVk
G=T/2*Sq(x);
F=eye(size(x,1))+T/2*Somega(omega);
x=F*x;
P=F*P*F'+G*Rw*G';
end
```

Figure 7: code of task 4

measurement available, we just let do repeat the last time x and P:

$$x_k = x_{k-1}$$
$$P_k = P_{k-1}$$

Task 5

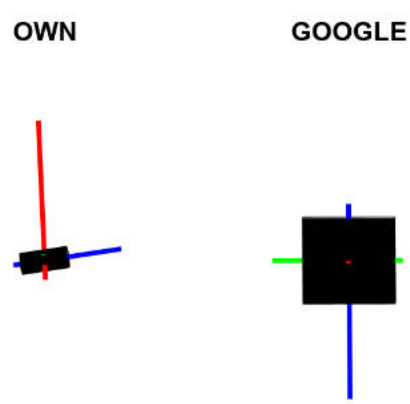

Figure 8: filter with only gyro

- Since the data from gyro could only give the information of angle velocity, the filter can not deal with wrong prior, it will perform very bad, and the estimated orientation would be sensitive to drift and errors over time.
- As is shown in figure 5, it is the result of filter with wrong prior(start the filter with the phone on the side instead of laying face up on the desk).
- When relying solely on the gyroscope, without Magnetometer and accelerometer, the initial orientation of the device is unknown. Without an absolute reference or prior position information, the gyroscope can only provide relative angular velocity measurements. This means that the estimated orientation will be relative to the device's starting position, but the actual orientation in the global frame of reference cannot be determined.

Task 6

Since we assume that $f_k^a = 0$, then the y_k^a could be expressed as below:

$$y_k^a = Q\left(q_k\right)^T g^0 + e_k^a$$

$$y_k^a = h\left(q_k\right) + e_k^a$$

Thus the updated $\hat{q}_{k|k}$ is shown below:

$$\begin{aligned}
\hat{q}_{k|k} &= \hat{q}_{k|k-1} + K_k\left(y_k^a - h\left(\hat{q}_{k|k-1}\right)\right) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T \\
S_k &= h'\left(\hat{q}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T + R_a \\
K_k &= P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T S_k^{-1}
\end{aligned}$$

The function mu_g is shown below:

```
function[x, P] = mu_g(x, P, yacc, Ra, g0)
    hx=Qq(x)'*g0;
    [Q0, Q1, Q2, Q3]=dQqdq(x);
    Hx=[Q0'*g0 Q1'*g0 Q2'*g0 Q3'*g0];
    S=Hx*P*Hx'+Ra;
    KK=P*Hx'*inv(S);
    x=x+KK*(yacc-hx);
    P=P-KK*S*KK';
end
```

Figure 9: task 6

Task 7

By adding the accelerometer to the sensor fusion filter with the gyroscope, the orientation estimation becomes more accurate, stable, and resistant to drift. The accelerometer helps compensate for the gyroscope's drift, enhances stability, and provides an absolute reference for the device's orientation. This fusion improves performance in applications like rotating slowly.
However, if the f_k^a is too large, it violate our assumption before-an outlier can distort the fusion process and lead to incorrect orientation estimates. Thus, outlier detection and rejection mechanisms is necessary.

Task 8

The outlier detection and rejection mechanisms are shown below:

```
if ~any(isnan(acc))  % Acc measurements are available.
    if norm(acc)>0.7*norm(g0) && norm(acc)<1.3*norm(g0)
      [x, P] = mu_g(x, P, acc, Sigma_acc, g0);
      [x, P] = mu_normalizeQ(x, P);
      ownView.setAccDist(false);
    else
      ownView.setAccDist(true);
     end

end
```

Figure 10: task 8

As is shown below, if the acceleration is out of the tolerance, it will not be accept to update the filter. After adding such a outlier into filter, it result into a better performance in rotation (slide the device quickly back and forth on the horizontal surface of a table).

Figure 11: introduce a acc disturbance

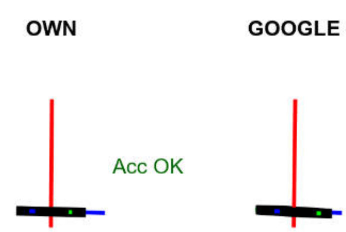

Figure 12: without acc disturbance

Task9

Since we assume that $f_k^m = 0$, then the y_k^a could be expressed as below:

$$y_k^a = Q\left(q_k\right)^T m^0 + e_k^m$$

$$y_k^a = h\left(q_k\right) + e_k^a$$

Thus the updated $\hat{q}_{k|k}$ is shown below:

$$\begin{aligned}
\hat{q}_{k|k} &= \hat{q}_{k|k-1} + K_k\left(y_k^m - h\left(\hat{q}_{k|k-1}\right)\right) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T \\
S_k &= h'\left(\hat{q}_{k|k-1}\right) P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T + R_m \\
K_k &= P_{k|k-1} h'\left(\hat{q}_{k|k-1}\right)^T S_k^{-1}
\end{aligned}$$

According to the equation, the mu_m is shown below:

```
function [x, P] = mu_m(x, P, mag, m0,Rm)
    hx=Qq(x)'*m0;
    [Q0, Q1, Q2, Q3]=dQqdq(x);
    Hx=[Q0'*m0 Q1'*m0 Q2'*m0 Q3'*m0];
    S=Hx*P*Hx'+Rm;
    KK=P*Hx'*inv(S);
    x=x+KK*(mag-hx);
    P=P-KK*S*KK';
end
```

Figure 13: task 9

Task10

However, When introducing a magnetic disturbance into filter with magnetometer update, several effects can be observed(as is shown in figure 10):

- Orientation Deviation: The estimated orientation can deviate from the true orientation due to the altered magnetometer readings caused by the disturbance.

- Drift and Error Amplification: The disturbance can introduce errors that accumulate over time, leading to drift and potentially amplifying estimation errors.

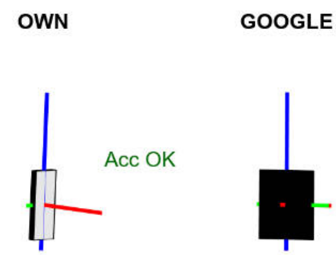

Figure 14: introduce a magnetic disturbance

Task11

$\hat{L}_k = (1-\alpha)\hat{L}_{k-1} + \alpha \|m_k\|$ allows for adaptive tracking of the local magnetic field strength while attenuating the impact of outliers or disturbances in the magnetometer measurements. In this equation we choose $\alpha = 0.02$, the function is shown below:

```
if ~any(isnan(mag))  % Mag measurements are available.
    L=0.98*L+0.02*norm(mag);
    % disp(L)
    if L>20 && L<40
        [x, P] = mu_m(x, P, mag, m0, Sigma_mag);
        [x, P] = mu_normalizeQ(x, P);
        ownView.setMagDist(false);
    else
        ownView.setMagDist(true);
    end
  % Do something
end
```

Figure 15: task 11

- Assumption: The assumption is that outliers in magnetometer measurements are distinct from normal variations and can be identified as anomalous data points.
- When is reasonable: The assumptions are reasonable in environments where the magnetic field is relatively stable and consistent.
- What will happen and why: Introducing outlier rejection on the magnetometer improves accuracy, reduces error propagation, and enhances the robustness of magnetic field estimation by identifying and excluding outlier readings from the estimation process.

After adding all sensors into the filter, the Euler angles of both your orientation filter and the built in filter in the phone are shown below:

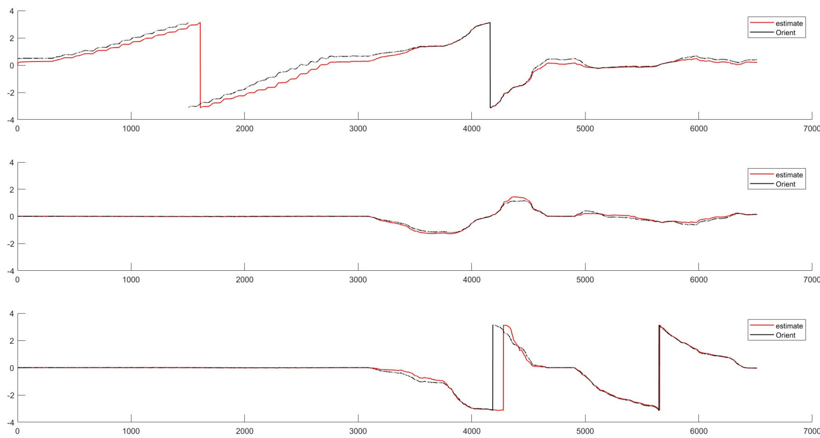

Figure 16: comparison of filter, yaw, roll, pitch

Task12

Accelerometer-Only Configuration:

- Pros: Enables estimation of tilt or inclination based on linear acceleration information.
- Cons: Prone to drift and unable to accurately estimate rotation or angular velocity without gyroscopic data. Limited in tracking dynamic movements and changes in orientation.

Magnetometer-Only Configuration:

- Pros: Provides orientation estimation with respect to magnetic north based on Earth's magnetic field information.
- Cons: Lacks accelerometer and gyroscope data, limiting the ability to estimate linear acceleration, dynamic movements, or changes in angular velocity.

However, Combining all three sensors (accelerometer, gyroscope, and magnetometer) in sensor fusion provides:

- Drift compensation for the gyroscope's tendency to accumulate errors over time.
- Comprehensive understanding of tilt, angular velocity, and orientation relative to the Earth's magnetic field.

- Improved accuracy, robustness, and responsiveness in estimating orientation, motion, and position.

In comparison to the built-in filter in smartphones, our orient filter demonstrates superior performance, as depicted in Figure 16. The built-in filter's limitations become apparent in scenarios involving data loss, particularly during yaw rotations, leading to noticeable gaps and lag in the estimation. Additionally, due to the absence of explicit outlier rejection mechanisms in most smartphone built-in sensor fusion filters, their performance is compromised when faced with disturbances in the accelerometer or magnetometer readings.

In contrast, our orient filter incorporates outlier rejection mechanisms, resulting in enhanced accuracy and robustness. By explicitly addressing outliers in the sensor data, our filter mitigates the adverse effects of disturbances, ensuring more reliable and precise estimation outcomes.