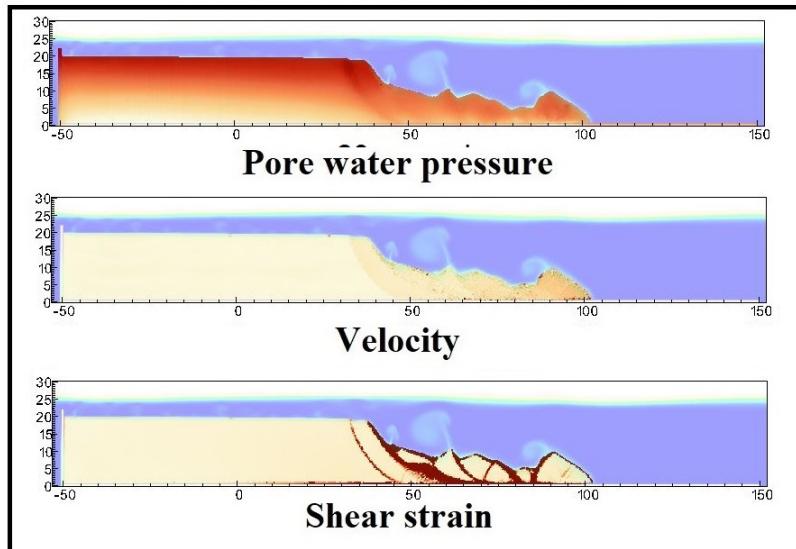


- 1 Graphical Abstract
- 2 MPMICE: A hybrid MPM-CFD model for simulating coupled problems in porous media. Application to earthquake-induced submarine landslides
- 3 Quoc Anh Tran, Gustav Grimstad, Seyed Ali Ghoreishian Amiri



**Application to earthquake-induced submarine landslide**

<sub>6</sub> Highlights

<sub>7</sub> **MPMICE: A hybrid MPM-CFD model for simulating coupled prob-**  
<sub>8</sub> **lems in porous media. Application to earthquake-induced subma-**  
<sub>9</sub> **rine landslides**

<sub>10</sub> Quoc Anh Tran, Gustav Grimstad, Seyed Ali Ghoreishian Amiri

- <sub>11</sub> • MPMICE is introduced for multiphase flow in porous media.
- <sub>12</sub> • Material Point method allows to model large deformation of non-isothermal
- <sub>13</sub> porous media.
- <sub>14</sub> • ICE (compressible multi-material CFD formulation) allows stabilizing
- <sub>15</sub> pore water pressure and turbulent flow.
- <sub>16</sub> • MPMICE is validated and apply to simulate the earthquake-induced
- <sub>17</sub> submarine landslide.

<sup>18</sup> MPMICE: A hybrid MPM-CFD model for simulating  
<sup>19</sup> coupled problems in porous media. Application to  
<sup>20</sup> earthquake-induced submarine landslides

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<sup>22</sup> **Abstract**

<sup>23</sup> In this paper, we describe a soil-fluid-structure interaction model that com-  
<sup>24</sup> bines soil mechanics (saturated sediments), fluid mechanics (seawater or air),  
<sup>25</sup> and solid mechanics (structures). The formulation combines the Material  
<sup>26</sup> Point Method, which models large deformation of the porous media and the  
<sup>27</sup> structure, with the Implicit Continuous-fluid Eulerian, which models com-  
<sup>28</sup> plex fluid flows. We validate the model and simulate the whole process of  
<sup>29</sup> earthquake-induced submarine landslides. We show that this model captures  
<sup>30</sup> complex interactions between saturated sediment, seawater, and structure,  
<sup>31</sup> so we can use the model to estimate the impact of potential submarine land-  
<sup>32</sup> slides on offshore structures.

<sup>33</sup> *Keywords:*

<sup>34</sup> Material Point Method, MPMICE, submarine landslide.

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<sup>78</sup> **Nomenclature**

**General variables**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$V$	$[L^3]$	Representative volume
$n$		Porosity
$\sigma$	$[ML^{-1}t^{-2}]$	Total stress tensor
$\Delta t$	$[t]$	Time increment
$\mathbf{b}$	$[ML^1t^{-2}]$	Body force
$c_v$	$[L^2t^{-2}T^{-1}]$	Constant volume specific heat
$f_d$	$[MLt^{-2}]$	Drag forces in momentum exchange term
$f^{int}$	$[MLt^{-2}]$	Internal forces
$f^{ext}$	$[MLt^{-2}]$	External forces
$q_{fs}$	$[MLt^{-2}]$	Heat exchange term
$S$		Weighting function
$\nabla S$		Gradient of weighting function

<sup>79</sup>

**Solid phase**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$m_s$	$[M]$	Solid mass
$\rho_s$	$[ML^{-3}]$	Solid density
$\phi_s$		Solid volume fraction
$\bar{\rho}_s$	$[ML^{-3}]$	Bulk Solid density
$\mathbf{x}_s$	$[L]$	Solid Position vector
$\mathbf{U}_s$	$[Lt^{-1}]$	Solid Velocity vector
$\mathbf{a}_s$	$[Lt^{-2}]$	Solid Acceleration vector
$\sigma'$	$[ML^{-1}t^{-2}]$	Effective Stress tensor
$\epsilon$		Strain tensor
$e_s$	$[L^2t^{-2}]$	Solid Internal energy per unit mass
$T_s$	$[T]$	Solid Temperature
$\mathbf{F}_s$		Solid Deformation gradient
$V_s$	$[L^3]$	Solid Volume

**Fluid phase**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$m_f$	[M]	Fluid mass
$\rho_f$	[ML <sup>-3</sup> ]	Fluid density
$\phi_f$		Fluid volume fraction
$\bar{\rho}_f$	[ML <sup>-3</sup> ]	Bulk Fluid density
$\mathbf{U}_f$	[Lt <sup>-1</sup> ]	Fluid Velocity vector
$\boldsymbol{\sigma}_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid stress tensor
$p_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid isotropic pressure
$\boldsymbol{\tau}_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid shear stress tensor
$e_f$	[L <sup>2</sup> t <sup>-2</sup> ]	Fluid Internal energy per unit mass
$T_f$	[T]	Fluid Temperature
$v_f$	[L <sup>3</sup> /M]	Fluid Specific volume $\frac{1}{\rho_f}$
$\alpha_f$	[1/T]	Thermal expansion
$\mu$	[ML <sup>-1</sup> t <sup>-1</sup> ]	Fluid viscosity
$V_f$	[L <sup>3</sup> ]	Fluid Volume

**Superscript**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$n$		Current time step
$L$		Lagrangian values
$n + 1$		Next time step

**Subscript**

$c$	Cell-centered quantity
$p$	Particle quantity
$i$	Node quantity
$FC$	Cell face quantity
$L, R$	Left and Right cell faces

81 **Introduction**

82 Many geological natural processes and their interactions with man-made  
83 structures are influenced by soil-fluid-structure interactions. The prediction  
84 of these processes requires a tool that can capture complex interactions  
85 between soil, fluid, and structure, such as the process of submarine land-  
86 slides. Indeed, The offshore infrastructure as well as coastal communities  
87 may be vulnerable to submarine landslides. Submarine landslides contain  
88 three stages: triggering, failure, and post-failure. Erosion or earthquakes can  
89 trigger slope failures in the first stage. Following the failure, sediments move  
90 quickly after the post-failure stage. In other words, solid-like sediments will  
91 behave like a fluid after failure. This transition, where the sediment trans-  
92 forms into fluid-like debris and then re-establishes a porous medium, poses  
93 a challenging task for simulating submarine landslides.

94 Due to this transition, submarine landslides can be modeled using either the  
95 Eulerian framework or the Lagrangian framework. The Eulerian framework  
96 involves the calculation of material response at specific time-space points.  
97 For instance, methods within Computational Fluid Dynamics, for example  
98 using Finite Volume Methods (FVM) are employed to simulate submarine  
99 landslides [1, 2, 3, 4] by solving governing equations in a full-Eulerian frame-  
100 work. While FVM is capable of handling complex flows, such as turbulent  
101 flows, it falls short of accounting for the triggering mechanisms of subma-  
102 rine landslides. This is due to the challenge of incorporating 'constitutive  
103 laws' of sediment materials within the Eulerian framework. This is particu-  
104 larly significant because converting material time derivatives into fixed space  
105 derivatives involves arduous mathematical tasks, especially for soil materials  
106 that rely on nonlinear tensor operations and history-dependent state/internal  
107 variables. In contrast, the Lagrangian framework, including various particle-  
108 based methods, provides a solution to this problem. In the Lagrangian frame-  
109 work, material "particles" are tracked individually through space, and ma-  
110 terial properties and internal variables are determined at and follow these  
111 particles. These methods have been extensively used to simulate landslides,  
112 like Material Point Method (MPM) [5], Smooth Particle Hydro Dynamics [6],  
113 Particle Finite Element Method [7], or Coupled Eulerian-Lagrangian Method  
114 [8]. For simplicity, these previous simulations have adopted a total stress  
115 analysis, neglecting the pore pressure development which is a key factor trig-  
116 gering slope failures.

117 Recent advancements in particle-based Lagrangian methods have allowed for

the modeling of fluid flows in porous media using sets of Lagrangian particles. Within the MPM family, there is a specific approach known as double-point MPM [9, 10, 11]. In this method, fluid particles and solid particles are overlaid within a single computational grid. However, it is important to note that particle-based methods encounter numerical instability when modeling fluid flows. To address this, various numerical techniques are employed, including the B-bar method [9], null-space filter [12], or least square approximation [13, 14]. These methods are necessary, especially when dealing with complex and turbulent fluid flows, such as those seen in submarine landslides. In such scenarios, Eulerian methods like FVM/CFD are preferred due to their computational efficiency, particularly when turbulence occurs at fine resolutions. CFD has even been employed in combination with the Discrete Element Method [15, 16] to study granular grain - fluid interactions, allowing for the examination of micro-scale behavior and realistic grain morphology. However, the computational demands of Discrete Element Methods can be quite challenging when applied to practical scenarios. Therefore, an ideal approach might involve the integration of CFD with particle-based continuum methods. Additionally, MPM can also be coupled with thermal effects [17, 18, 19], opening up the possibility of capturing hydro-thermal-mechanical coupling. Over the past two decades, more than 50 particle-based methods have been developed to address the simulation of large deformations in solids [20]. Among these, MPM emerges as a strong candidate for coupling with CFD. This is because MPM incorporates a stationary mesh during computation, just like CFD. As such, MPM and CFD can be seamlessly integrated within a unified computational mesh, offering a promising approach for tackling complex fluid-solid interactions.

A numerical method for simulating soil-fluid-structure interaction (Figure 1) involving large deformations, is presented in this work in order to simulate the interaction between sediment (soil), seawater (fluid) and offshore structures (structure) namely MPMICE (Figure 2). In the MPMICE, the Material Point Method (MPM) is coupled with the Implicit Continuous Eulerian (ICE) [21]. The MPM method is a particle method that allows the porous soil to undergo arbitrary distortions. The ICE method, on the other hand, is a conservative finite volume technique with all state variables located at the cell center (temperature, velocity, mass, pressure). The ICE method offers certain advantages in comparison to conventional FVM in the realm of flow computation encompassing all velocity ranges. An initial technical report [22] at Los Alamos National Laboratory provided the theoretical and

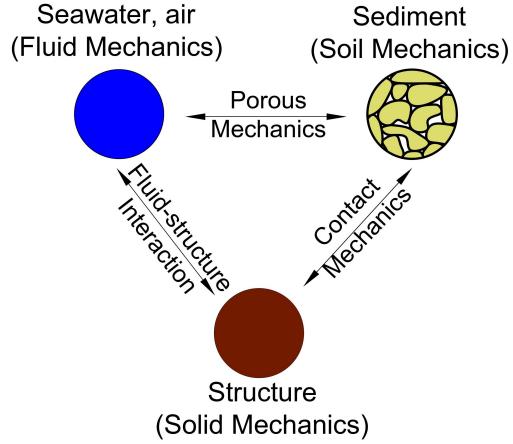


Figure 1: Interaction between soil-fluid-structure

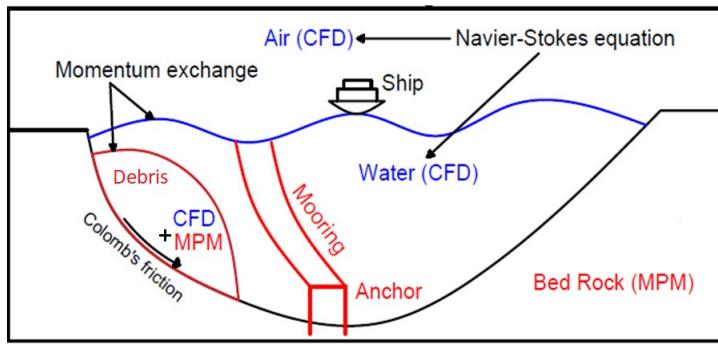


Figure 2: Coupling of soil-water-structure interaction using MPMICE

156 algorithmic foundation for the MPMICE, followed by the MPMICE development  
 157 and implementation in the high-performance Uintah computational  
 158 framework for simulating fluid-structure interactions [23]. This paper pri-  
 159 marily contributes further to the development of the MPMICE for analyzing  
 160 the **soil**-fluid-structure interaction, since sediment should be considered as a  
 161 porous media (soil) and not as a solid to capture the evolution of the pore  
 162 water pressure. Baumgarten et al. [24] made the first attempt at coupling  
 163 the FVM with the MPM for the simulation of soil-fluid interaction by us-  
 164 ing an explicit time integration for the single-phase flow. In contrast to the  
 165 mentioned work, we use implicit time integration for the multi phase flows.

166    **Theory and formulation**

167    This section lay out the theoretical framework for the MPMICE model.  
168    We use the common notation of the continuum mechanics with vector and  
169    tensor denoted simply by using bold font and scalar denoted by using normal  
170    font. The notation are shown in Nomenclature.

171    **Assumptions**

172    The following assumptions are made for the MPMICE model.

- 173    1. Solid phases (MPM) are described in a Lagrangian formulation while  
174    fluid phases (ICE) are described in an Eulerian formulation in the  
175    framework of continuum mechanics and mixture theory.  
176    2. Solid grains are incompressible while the fluid phases are compressible.  
177    3. There is no mass exchange between solid and fluid phases.  
178    4. Terzaghi's effective stress is valid.

179    **Governing equations**

180    A representative element volume  $\Omega$  is decomposed by two domains: solid  
181    domains  $\Omega_s$  and fluid domains  $\Omega_f$ . Then, all domains are homogenized  
182    into two overlapping continua. Considering the volume fraction of solid  
183     $\phi_s = \Omega_s/\Omega$  and fluid  $\phi_f = \Omega_f/\Omega$  with the true (or Eulerian) porosity  
184     $n = \sum \phi_f$  of the representative element volume, the average density of solid  
185    and fluid phases are defined as:

186

$$\bar{\rho}_s = \phi_s \rho_s, \quad \bar{\rho}_f = \phi_f \rho_f \quad (1)$$

187    The mass of solid and fluid phases are:

188

$$m_s = \int_{\Omega_s} \rho_s dV = \bar{\rho}_s V, \quad m_f = \int_{\Omega_f} \rho_f dV = \bar{\rho}_f V \quad (2)$$

189    Reviewing the Terzaghi's effective stress concept for the saturated porous  
190    media, the total stress  $\boldsymbol{\sigma}$  is calculated by:

191

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - p_f \mathbf{I} \quad (3)$$

192    The balance equations are derived based on the mixture theory. The rep-  
193    resentative thermodynamic state of the fluid phases are given by the vector  
194     $[m_f, \mathbf{U}_f, e_f, T_f, v_f]$  which are mass, velocity, internal energy, temperature,

195 specific volume. The representative state of the solid phases are given by the  
 196 vector  $[m_s, \mathbf{U}_s, e_s, T_s, \boldsymbol{\sigma}', p_f]$  which are mass, velocity, internal energy, temper-  
 197 ature, effective stress and pore water pressure. The derivation is presented  
 198 in detail in the Appendix.

199

---

## 200 Mass Conservation

201 The mass balance equations for both fluid (e.g., water, air) and solid phases  
 202 are:

203

$$\frac{1}{V} \frac{\partial m_f}{\partial t} + \nabla \cdot (m_f \mathbf{U}_f) = 0, \frac{1}{V} \frac{D_s m_s}{Dt} = 0 \quad (4)$$

204 Solving the mass balance equation of the solid phase leads to:

$$\frac{D_s n}{Dt} = \phi_s \nabla \cdot \mathbf{U}_s \quad (5)$$

205

---

## 206 Momentum Conservation

207 The momentum balance equations for each fluid phases (e.g., water, air) are:

$$\frac{1}{V} \left[ \frac{\partial(m_f \mathbf{U}_f)}{\partial t} + \nabla \cdot (m_f \mathbf{U}_f \mathbf{U}_f) \right] = -\phi_f \nabla p_f + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d \quad (6)$$

208 The momentum balance equations for each solid phases are:

$$\frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}') - \phi_s \nabla p_f + \bar{\rho}_s \mathbf{b} + \sum \mathbf{f}_{fric} - \sum \mathbf{f}_d \quad (7)$$

209

---

## 210 Energy Conservation

211 The internal energy balance equations for each fluid phases (e.g., water, air)  
 212 are:

$$\frac{1}{V} \left[ \frac{\partial(m_f e_f)}{\partial t} + \nabla \cdot (m_f e_f \mathbf{U}_f) \right] = -\bar{\rho}_f p_f \frac{D_f v_f}{Dt} + \boldsymbol{\tau}_f : \nabla \mathbf{U}_f + \nabla \cdot \mathbf{q}_f + \sum q_{sf} \quad (8)$$

213 The internal energy balance equations for each solid phases are:

$$\frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (9)$$

214 where  $c_v$  is the specific heat at constant volume of the solid materials.

215

---

216 Closing the systems of equations, the following additional models are needed:  
 217 (1) A constitutive equation to describe the stress - strain behaviour of solid  
 218 phase (computing effective stress  $\sigma'$ ).  
 219 (2) Optional turbulent model to compute the viscous shear stress  $\tau_f$ .  
 220 (3) Frictional forces  $\mathbf{f}_{fric}$  for the contact for soil-structure interaction be-  
 221 tween solid/porous materials with the friction coefficient  $\mu_{fric}$ .  
 222 (4) Exchange momentum models (computing drag force  $\mathbf{f}_d$ ) for interaction  
 223 between materials.  
 224 (5) Energy exchange models (computing temerature exhange term  $q_{sf}$ ) for  
 225 interaction between materials.  
 226 (6) An equation of state to establish relations between thermodynamics vari-  
 227 ables of each fluid materials  $[P_f, \bar{\rho}_f, v_f, T_f, e_f]$ .  
 228 (7) Thermal conduction model to compute thermal flux of solid phase  $\mathbf{q}_s$  and  
 229 liquid phase  $\mathbf{q}_s$ .  
 230 Four thermodynamic relations for the equation of states are:

$$\begin{aligned}
 e_f &= e_f(T_f, v_f) \\
 P_f &= P_f(T_f, v_f) \\
 \phi_f &= v_f \bar{\rho}_f \\
 0 &= n - \sum_{f=1}^{N_f} v_f \bar{\rho}_f
 \end{aligned} \tag{10}$$

231 *Constitutive soil model*

232 As a result of the explicit MPM formulation, we can derive the consti-  
 233 tutive law in the updated Lagrangian framework of "small strain - large  
 234 deformation". Therefore, the rotation of the particles (representative ele-  
 235 ment volume) is manipulated by rotating the Cauchy stress tensor. First,  
 236 the deformation gradient is decomposed into the polar rotation tensor  $\mathbf{R}_s^{n+1}$   
 237 and stretch tensor  $\mathbf{V}_s^{n+1}$  as:

$$\mathbf{F}_s^{n+1} = \mathbf{V}_s^{n+1} \mathbf{R}_s^{n+1} \tag{11}$$

238 Then, before calling the constitutive model, the stress and strain rate tensor  
 239 are rotated to the reference configuration as:

$$\boldsymbol{\sigma}'^{n*} = (\mathbf{R}_s^{n+1})^T \boldsymbol{\sigma}'^n \mathbf{R}_s^{n+1} \tag{12}$$

$$\delta\boldsymbol{\epsilon}^{n*} = (\mathbf{R}_s^{n+1})^T \delta\boldsymbol{\epsilon}^n \mathbf{R}_s^{n+1} \tag{13}$$

241 Using the constitutive model with the input tensors  $\sigma'^{n*}, \delta\epsilon^{n*}$  to compute  
 242 the Cauchy stress tensor at the advanced time step  $\sigma'^{n+1*}$  then rotating it  
 243 back to current configuration as:

$$\sigma'^{n+1} = R_s^{n+1} \sigma'^{n+1*} (R_s^{n+1})^T \quad (14)$$

244 In this paper, we adopt the hyper-elastic Neo Hookean model for the structure  
 245 materials and additionally Mohr-Coulomb failure criteria for the soil (porous  
 246 media) materials. The Cauchy stress of the hyper-elastic Neo Hookean model  
 247 can be written as:

$$\sigma' = \frac{\lambda \ln(J)}{J} + \frac{\mu}{J} (\mathbf{F}\mathbf{F}^T - \mathbf{J}) \quad (15)$$

248 where  $\lambda$  and  $\mu$  are bulk and shear modulus ad  $J$  is the determinant of the  
 249 deformation gradient  $\mathbf{F}$ . And the yield function  $f$  and flow potentials  $g$  of  
 250 the Mohr-Coulomb can be written as:

$$\begin{aligned} f &= -\sigma'_1 + \sigma'_3 + 2c' \cos(\phi') + (\sigma'_1 + \sigma'_3) \sin(\phi') \\ g &= -\sigma'_1 + \sigma'_3 + 2c' \cos(\psi') + (\sigma'_1 + \sigma'_3) \sin(\psi') \end{aligned} \quad (16)$$

251 In the equations,  $c'$ ,  $\phi'$ , and  $\psi'$  represent the cohesion, friction angle, and  
 252 dilation angle, respectively.  $\sigma'_1$  and  $\sigma'_3$  denote the maximum and minimum  
 253 principal stresses, with the condition  $\sigma'_1 < \sigma'_3 < 0$ . It is important to note  
 254 that in our assumptions, stress is considered positive during extension, which  
 255 means the signs of the stresses in these equations are opposite to those in  
 256 standard Soil Mechanic's textbooks. The numerical implementation follows  
 257 the approach described in Clausen et al. [25].

### 258 *Turbulent model*

259 The turbulent effect is modelled using a statistical approach namely large-  
 260 eddy simulation. In this approach, the micro-scale turbulent influence in the  
 261 dynamics of the macro-scale motion is computed through simple models like  
 262 Smagorinsky model [26]. In the Smagorinsky model, the residual stress tensor  
 263 is:

$$\tau_{ij} = 2\mu_{eff}(\bar{S}_{ij} - \frac{1}{3}\delta_{ij}\bar{S}_{kk}) + \frac{1}{3}\delta_{ij}\tau_{kk} \quad (17)$$

264 where the the strain rate tensor is given by:

$$\bar{S}_{ij} = \frac{1}{2}(\frac{\delta \bar{\mathbf{U}}_i}{\delta x_j} + \frac{\delta \bar{\mathbf{U}}_j}{\delta x_i}) \quad (18)$$

265 and the effective viscosity is sum of molecular viscosity and turbulent viscosity  
 266  $\mu_{eff} = \mu + \mu_t$  in which the turbulent viscosity  $\mu_t$  is calculated by:

$$\mu_t = (C_s \Delta)^2 \sqrt{2S_{ij} S_{ij}} \quad (19)$$

267 where  $C_s$  is the Smagorinsky constant with the value of 0.1 and  $\Delta = \sqrt[3]{dxdydz}$   
 268 is the grid size that defines the subgrid length scale.

269 *Frictional force for soil-structure interaction*

270 MPMICE includes a contact law for the interaction between soil and  
 271 structure using the first Coulomb friction contact for MPM presented by  
 272 Bardenhagen et al. [27]. The magnitude of the friction force at the contact  
 273 depends on the friction coefficient  $\mu_{fric}$  and the normal force  $\mathbf{f}_{norm}$  computed  
 274 from the projection of the contact force in the normal direction.

$$\mathbf{f}_{fric} = \mu_{fric} \mathbf{f}_{norm} \quad (20)$$

275 The contact determines whether the soil is sliding or sticking to the structure  
 276 by comparing the friction force with the sticking force  $\mathbf{f}_{stick}$  can be computed  
 277 from the projection of the contact force in the tangent direction as:

$$\begin{aligned} & \text{if } \mathbf{f}_{fric} \geq \mathbf{f}_{stick} \text{ no sliding} \\ & \text{if } \mathbf{f}_{fric} < \mathbf{f}_{stick} \text{ sliding occurs} \end{aligned} \quad (21)$$

278 Frictional sliding between solid materials also generates dissipation and the  
 279 work rate generated from the sliding can be calculated as:

$$\Delta W_{friction} = \mathbf{f}_{fric} d \quad (22)$$

280 where  $d$  is the sliding distance which can be computed based on the sliding  
 281 velocity between two materials.

282 *Momentum and Energy exchange model*

283 Currently, the energy exchange coefficient  $H_{sf}$  is assumed to be constant  
 284 for the sake of simplicity. Then the energy exchange can be written as:

$$q_{sf} = H_{sf} (T_f - T_s) \quad (23)$$

285 On the other hand, the drag force can be calculated as:

$$f_d = K(\mathbf{U}_s - \mathbf{U}_f) \quad (24)$$

286 For the momentum exchange between fluid flows and porous media, we as-  
 287 sume that the drag force  $\mathbf{f}_d$  depends on the average grain size of the grains  
 288  $D_p$ , the porosity  $n$ , the fluid viscosity  $\mu_f$ , and is proportional to the relative  
 289 velocities of soil grains and fluid  $(\mathbf{U}_s - \mathbf{U}_f)$ . Based on recent investigation  
 290 of CFD simulations of fluid flow around mono- and bi-disperse packing of  
 291 spheres for  $0.1 < \phi_s < 0.6$  and  $Re < 1000$  [28]. The drag force is given by:  
 292

$$\mathbf{f}_d = \frac{18\phi_s(1-\phi_s)\mu_f}{D_p^2} F(\phi_s, Re)(\mathbf{U}_s - \mathbf{U}_f) \quad (25)$$

293 where Reynolds number  $Re$  are computed as:

$$Re = \frac{n\rho_f D_p}{\mu_f} \|\mathbf{U}_s - \mathbf{U}_f\| \quad (26)$$

294 The function  $F(\phi_s, Re)$  can be calculated as:

$$F(\phi_s, Re) = F(\phi_s, 0) + \frac{0.413Re}{24(1-\phi_s)^2} \left( \frac{(1-\phi_s)^{-1} + 3\phi_s(1-\phi_s) + 8.4Re^{-0.343}}{1 + 10^{3\phi_s} Re^{-(1+4\phi_s)/2}} \right) \quad (27)$$

295 where the low Reynold coefficient  $F(\phi_s, Re \rightarrow 0)$  is:

$$F(\phi_s, 0) = \frac{10\phi_s}{(1-\phi_s)^2} + (1-\phi_s)^2(1 + 1.5\sqrt{\phi_s}) \quad (28)$$

296 When validating the model with analytical solution, it requires to know the  
 297 hydraulic conductivity  $K$ . In such case, we convert the equation (25) to  
 298 Kozeny-Carman formula by assuming  $F(\phi_s, Re) = 10\phi_s/(1-\phi_s)^2$ , leading to

$$\mathbf{f}_d = \frac{180\phi_s^2\mu_f}{D_p^2(1-\phi_s)} (\mathbf{U}_s - \mathbf{U}_f) \quad (29)$$

299 Then, the draging force following the Darcy law is given by:

$$\mathbf{f}_d = \frac{n^2\mu_f}{\kappa} (\mathbf{U}_s - \mathbf{U}_f) \quad (30)$$

300 where  $\kappa$  being intrinsic permeability of soil which can be written as:

$$\kappa = \frac{K\mu_f}{\rho_f g} \quad (31)$$

301 As such, the hydraulic conductivity will be expressed as:

$$K = \frac{D_p^2(1-\phi_s)^3}{180\phi_s^2\mu_f} \rho_f g \quad (32)$$

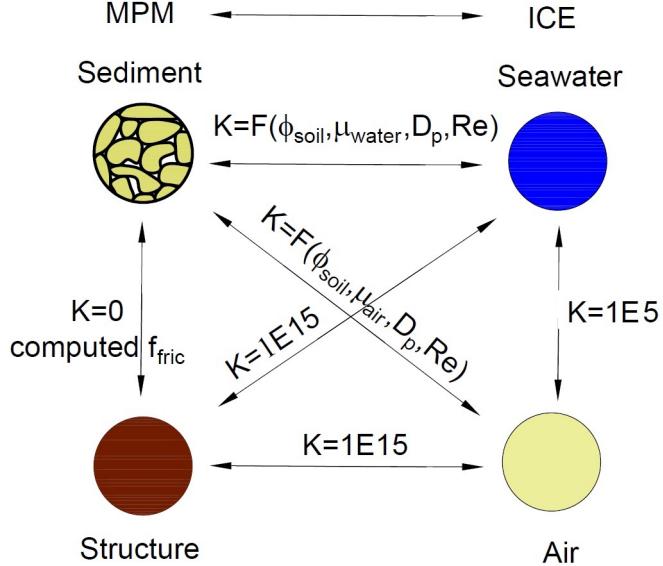


Figure 3: Momentum exchange coefficient between materials

302 Solving momentum and energy exchange with an implicit solver

303 The derivation of the implicit integration for the momentum exchange is  
 304 presented in the Appendix's section 'Momentum and energy exchange with  
 305 an implicit solver'. The linear equations for multi phases  $i,j=1:N$  has the  
 306 form as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_i \\ \Delta \mathbf{U}_j \end{vmatrix} = \begin{vmatrix} \beta_{ij} (\mathbf{U}_i^* - \mathbf{U}_j^*) \\ \beta_{ji} (\mathbf{U}_i^* - \mathbf{U}_j^*) \end{vmatrix}$$

307 where the intermediate velocity for fluid phases  $f=1:N_f$  and for solid/porous  
 308 phases  $s=1:N_s$  can be calculated by:

$$\begin{aligned} \mathbf{U}_f^* &= \mathbf{U}_f^n + \Delta t \left( -\frac{\nabla P_f^{n+1}}{\rho_f^n} + \frac{\nabla \cdot \boldsymbol{\tau}_f^n}{\bar{\rho}_f^n} + \mathbf{b} \right) \\ \mathbf{U}_s^* &= \mathbf{U}_s^n + \Delta t \left( \frac{\nabla \cdot \boldsymbol{\sigma}'^n}{\bar{\rho}_s^n} - \frac{\nabla P_f^{n+1}}{\rho_s} + \mathbf{b} \right) \end{aligned} \quad (33)$$

309 Also, the momentum exchange coefficient can be computed at every time  
 310 step as  $\beta_{12} = K/\bar{\rho}_f^n$  and  $\beta_{21} = K/\bar{\rho}_s^n$  with the coefficient depending on the  
 311 different type of interactions (see Figure 3) as for example:

312

- 313 1. The drag force is set to zero in soil-structure interactions, and instead  
 314 the frictional force is computed.  
 315 2. As a result of fluid-structure interaction, the momentum exchange coef-  
 316 ficient should be extremely high (1E15) when the solid material points  
 317 are considered to be zero-porosity/zero-permeability.  
 318 3. In the case of soil-fluid interaction, the drag force is calculated using  
 319 the equation (25). Considering that air has a much lower viscosity than  
 320 water, its drag force is much lower than the drag force of water in a  
 321 pore.  
 322 4. A momentum exchange coefficient of 1E5 is applied between multiphase  
 323 flows. This value is far higher than reality [29], but it is necessary to  
 324 have enough numerical stability to conduct simulations in the numerical  
 325 example.

326 Similar approach applied for the energy exchange term leading to:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_i \\ \Delta T_j \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_i^n - T_j^n) \\ \eta_{ji}(T_j^n - T_i^n) \end{vmatrix}$$

327 with  $\eta$  being the energy exchange coefficient.

328 *Equation of state for fluid phases*

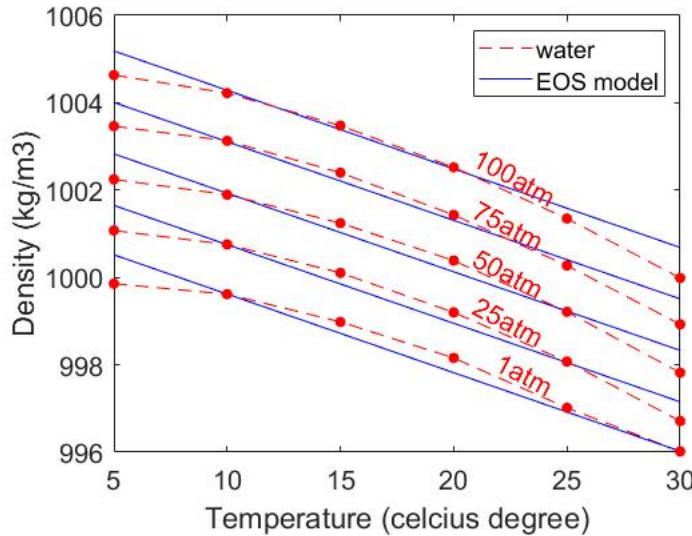


Figure 4: Equation of state of water

329 The equation of state establishes relations between thermodynamics vari-  
 330 ables  $[P_f, \rho_f, T_f]$ . The choice of the equation of state depends on the types  
 331 of the fluid materials. For example, for the air, it is possible to assume the  
 332 equation of state for the perfect gas which obeys:

$$P_f = \rho_f R T_f \quad (34)$$

333 where  $R$  is the gas constant. For the water, a simple linear equation of state  
 334 is in the following form:

$$P_f = P_{ref} + K_f(\rho_f - \rho_{ref} + \alpha_f(T_f - T_{ref})) \quad (35)$$

335 where reference pressure  $P_{ref} = 1 \text{ atm} = 101325 \text{ Pa}$ , reference temperature  
 336  $T_{ref} = 10^\circ\text{C}$ , reference density  $\rho_{ref} = 999.8 \text{ kg/m}^3$ , the bulk modulus of water  
 337  $K_f = 2 \text{ GPa}$ , and the water thermal expansion  $\alpha_f = 0.18 \text{ }^\circ\text{C}^{-1}$ . Equation  
 338 (35) matches well with the state of the water (see Figure 4).

### 339 Numerical implementation

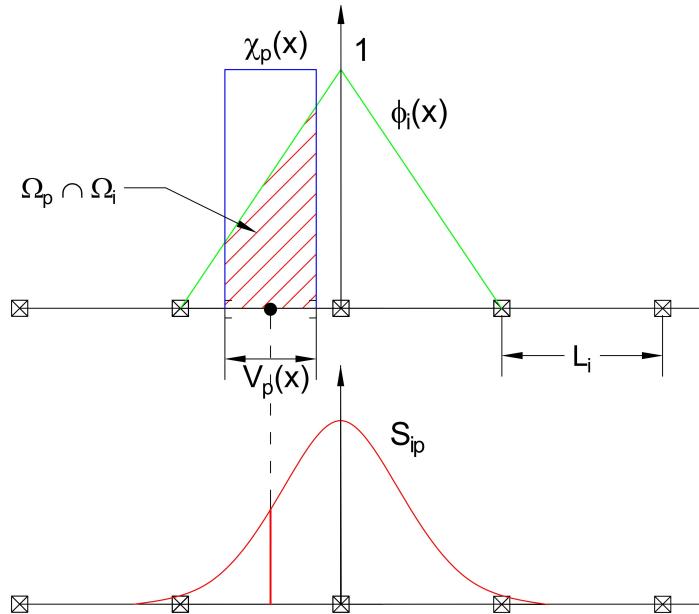


Figure 5: GIMP weighting function (red as a convolution of the linear basis shape function (green) and the characteristic function (blue))

340 The fluid phases are discretized in the grid with the state variables stored  
 341 at the centroid of the cells  $[\rho_{f,c}, \mathbf{U}_{f,c}, T_{f,c}, v_{f,c}]$  while the solid phase is dis-  
 342 cretized in the particles with the state variables  $[m_{sp}, \mathbf{U}_{sp}, T_{sp}, \boldsymbol{\sigma}'_{sp}]$ . In the  
 343 Material Point Method, we use the generalized interpolation technique [30]  
 344 using the weight function as a convolution of a grid shape function  $N_i(\mathbf{x})$  in  
 345 a nodal domain  $\Omega_i$  and a characteristic function  $\chi_p(\mathbf{x})$  in a particle domain  
 346  $\Omega_p$  with the volume  $V_p(\mathbf{x})$  as follows:

$$S_{ip} = \frac{1}{V_p} \int_{\Omega_i \cap \Omega_p} N_i(\mathbf{x}) \chi_p(\mathbf{x}) d\mathbf{x} \quad (36)$$

347 where the volume  $V_p(\mathbf{x})$  of the material point p can be calculated as:

$$V_p = \int_{\Omega_p} \chi_p(\mathbf{x}) d\mathbf{x} \quad (37)$$

348 The characteristic function is the Heaviside function as  $\chi_p = 1$  if  $\mathbf{x} \in \Omega_p$ ,  
 349 otherwise 0 (see Figure 5). For the interpolation of the centroid of the cell,  
 350 the linear basis function is used as:

$$S_{ci} = N_i(\mathbf{x}_c) \quad (38)$$

351 The time discretization are solved using the following steps.

352 *Interpolation from Solid Particle to Grid*

353 The nodal values of the solid state (mass, velocity, temperature, volume)  
 354 are:

$$\begin{aligned} m_{si}^n &= \sum S_{ip} m_{sp} \\ \mathbf{U}_{si}^n &= \frac{\sum S_{ip} (m\mathbf{U})_{sp}^n}{m_{si}^n} \\ T_{si}^n &= \frac{\sum S_{ip} (mT)_{sp}^n}{m_{si}^n} \\ V_{si}^n &= \frac{\sum S_{ip} (mV)_{sp}^n}{m_{si}^n} \\ \boldsymbol{\sigma}_{si}^n &= \frac{\sum S_{ip} (\boldsymbol{\sigma}V)_{sp}^n}{V_{si}^n} \end{aligned} \quad (39)$$

355 The nodal internal forces is calculated by:

$$\mathbf{f}_{si}^{int,n} = - \sum \nabla S_{ip} (\boldsymbol{\sigma}'_{sp})^n V_{sp}^n \quad (40)$$

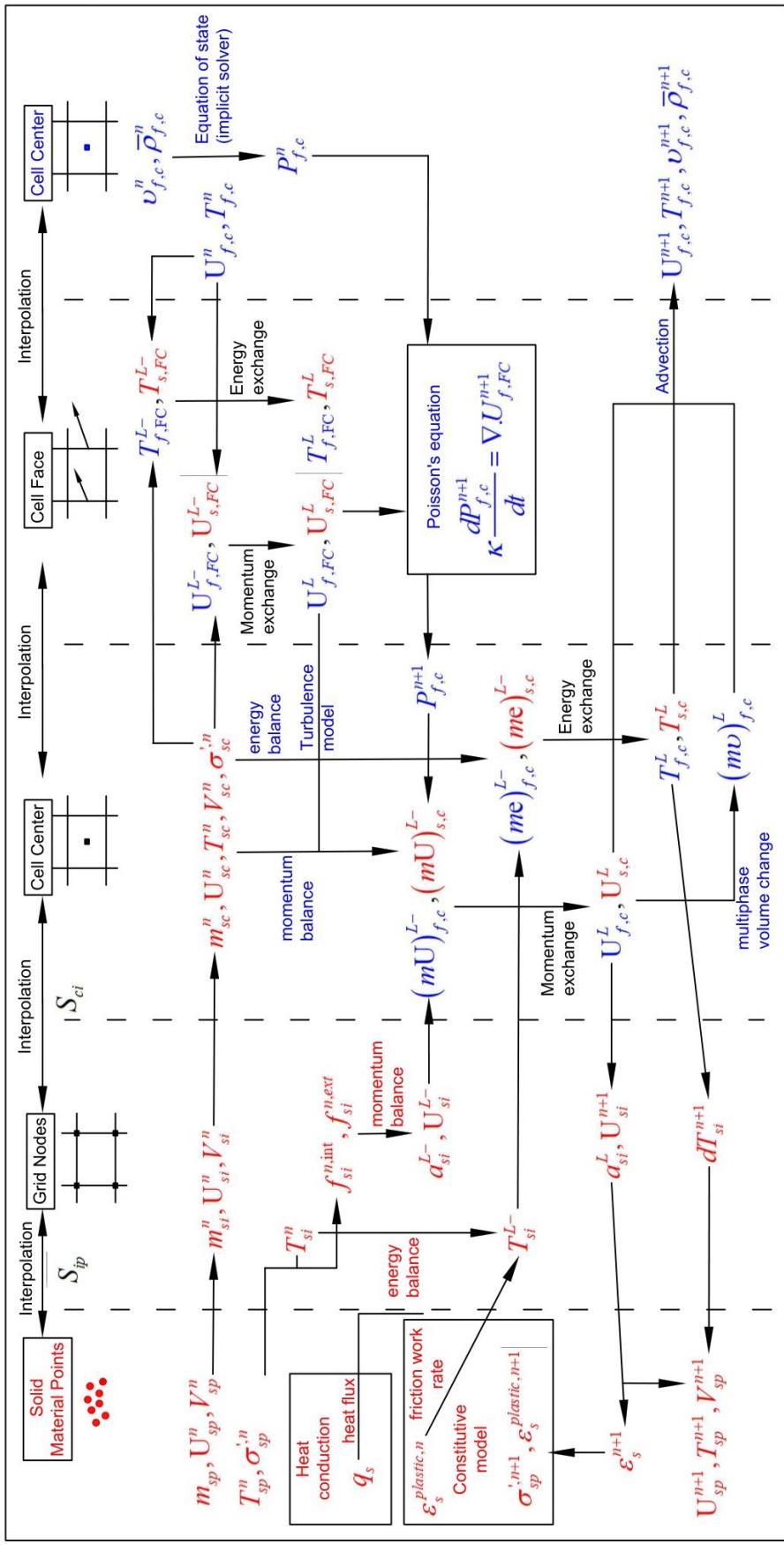


Figure 6: Numerical implementation of MPMICE

356 The nodal external forces  $f_{si}^{ext,n}$  and the frictional forces from soil-structure  
 357 interaction  $f_{fric,si}^n$  from contact between materials are computed here.  
 358 Then we compute the solid cell variables as:

$$\begin{aligned} m_{sc}^n &= \sum S_{ci} m_{si} \\ \rho_{sc}^n &= \frac{m_{sc}^n}{V} \\ \mathbf{U}_{sc}^n &= \sum S_{ci} \mathbf{U}_{si}^n \\ T_{sc}^n &= \sum S_{ci} T_{si}^n \\ V_{sc}^n &= \sum S_{ci} V_{si}^n \\ \boldsymbol{\sigma}_{sc}^n &= \sum S_{ci} \boldsymbol{\sigma}_{si}^n \end{aligned} \tag{41}$$

359 *Compute equation of state for fluid phase*

360 The total fluid material volume of a cell is:

$$V_{total} = \sum_{f=1}^{N_f} M_f v_f \tag{42}$$

361 We need to find  $P_{f,c}^n$  which allows each fluid materials obey their equation of  
 362 states  $[P_f, \rho_f, v_f, T_f, e_f]$  but also allow mass of all fluid materials to fill the  
 363 entire the pore volume without ongoing compression or expansion following  
 364 the condition as follows:

$$0 = n - \sum_{f=1}^{N_f} v_f \bar{\rho}_f \tag{43}$$

365 Then, we can use the Newton-Raphson interation to find the value of  $P_{f,c}^n$   
 366 which satisfies the equation (42, 43) and each equation of states of each fluid  
 367 materials.

368 *Compute cell face velocity*

369 Following the derivation in the Appendix: Advanced Fluid Pressure, we  
 370 first compute the fluid cell face velocity as:

$$\mathbf{U}_{f,FC}^{L-} = \frac{(\bar{\rho} \mathbf{U})_{f,FC}^n}{\bar{\rho}_{f,FC}^n} + \Delta t \left( -\frac{\nabla^{FC} P_{f,c}^n}{\rho_{f,FC}^n} + \frac{\nabla^{FC} \cdot \boldsymbol{\tau}^n}{\bar{\rho}_{s,FC}} + \mathbf{b} \right) \tag{44}$$

<sup>371</sup> The equation (44) is discretized in three dimension (noted that  $\nabla^{FC} \cdot \boldsymbol{\tau} = 0$ ),  
<sup>372</sup> for example the discretized equation in the x direction is:

$$U_{fx}^{L-} = \frac{(\bar{\rho}U)_{fx,R}^n + (\bar{\rho}U)_{fx,L}^n}{\bar{\rho}_{fx,L}^n + \bar{\rho}_{fx,R}^n} + \Delta t \left( -\frac{2(v_{fx,L}^n v_{fx,R}^n)}{v_{fx,L}^n + v_{fx,R}^n} \frac{P_{f,cx,R}^n - P_{f,cx,L}^n}{\Delta x} + b_x \right) \quad (45)$$

<sup>373</sup> The cell face solid velocity can be calculated as:

$$\mathbf{U}_{s,FC}^{L-} = \frac{(\bar{\rho}\mathbf{U})_{s,FC}^n}{\bar{\rho}_{s,FC}^n} + \Delta t \left( \frac{\nabla^{FC} \cdot \boldsymbol{\sigma}_c'^n}{\bar{\rho}_{s,FC}} - \frac{\nabla^{FC} P_{f,c}^n}{\rho_s} + \mathbf{b} \right) \quad (46)$$

<sup>374</sup> The equation (46) is discretized in three dimension (noted that  $\nabla^{FC} \cdot \boldsymbol{\sigma}_{ij} = 0$   
<sup>375</sup> with  $i \neq j$ ), for example the discretized equation in the x direction is:

$$U_{sx}^{L-} = \frac{(\bar{\rho}U)_{sx,R}^n + (\bar{\rho}U)_{sx,L}^n}{\bar{\rho}_{sx,L}^n + \bar{\rho}_{sx,R}^n} + \Delta t \left( \frac{2(\sigma_{xx,R} - \sigma_{xx,L})}{(\bar{\rho}_{sx,L}^n + \bar{\rho}_{sx,R}^n) \Delta x} - \frac{P_{f,cx,R}^n - P_{f,cx,L}^n}{\rho_s \Delta x} + b_x \right) \quad (47)$$

<sup>376</sup> Then, we compute the modified cell face velocity  $\mathbf{U}_{FC}^L$  considering the mo-  
<sup>377</sup> mentum exchange (see the Appendix: Momentum exchange with an implicit  
<sup>378</sup> solve) as follows:

$$\begin{aligned} \mathbf{U}_{f,FC}^L &= \mathbf{U}_{f,FC}^{L-} + \Delta \mathbf{U}_{f,FC} \\ \mathbf{U}_{s,FC}^L &= \mathbf{U}_{s,FC}^{L-} + \Delta \mathbf{U}_{s,FC} \end{aligned} \quad (48)$$

<sup>379</sup> The linear equation below is solved to obtain the increment of velocity with  
<sup>380</sup> i,j = 1 : N as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_{i,FC} \\ \Delta \mathbf{U}_{j,FC} \end{vmatrix} = \begin{vmatrix} \beta_{ij} (\mathbf{U}_{i,FC}^{L-} - \mathbf{U}_{j,FC}^{L-}) \\ \beta_{ji} (\mathbf{U}_{j,FC}^{L-} - \mathbf{U}_{i,FC}^{L-}) \end{vmatrix}$$

### <sup>381</sup> Compute cell face temperature

<sup>382</sup> Similar to the velocity, the faced temperature is computed, for example  
<sup>383</sup> in x direction, as:

$$\begin{aligned} T_{fx}^{L-} &= \frac{(\bar{\rho}T)_{fx,R}^n + (\bar{\rho}T)_{fx,L}^n}{\bar{\rho}_{fx,L}^n + \bar{\rho}_{fx,R}^n} \\ T_{sx}^{L-} &= \frac{(\bar{\rho}T)_{sx,R}^n + (\bar{\rho}T)_{sx,L}^n}{\bar{\rho}_{sx,L}^n + \bar{\rho}_{sx,R}^n} \end{aligned} \quad (49)$$

<sup>384</sup> Then, we compute the modified cell face temperature  $T_{FC}^L$  considering the  
<sup>385</sup> energy exchange (see the Appendix: Momentum and energy exchange with

<sup>386</sup> an implicit solver) as follows:

$$\begin{aligned} T_{f,FC}^L &= T_{f,FC}^{L-} + \Delta T_{f,FC} \\ T_{s,FC}^L &= T_{s,FC}^{L-} + \Delta T_{s,FC} \end{aligned} \quad (50)$$

<sup>387</sup> The linear equation below is solved to determine the increment of tempera-  
<sup>388</sup> ture due to energy exchange with  $i,j = 1 : N$  as:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_{i,FC} \\ \Delta T_{j,FC} \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_{i,FC}^{L-} - T_{j,FC}^{L-}) \\ \eta_{ji}(T_{j,FC}^{L-} - T_{i,FC}^{L-}) \end{vmatrix}$$

<sup>389</sup> *Compute fluid pressure (implicit scheme)*

<sup>390</sup> For single phase flow, the increment of the fluid pressure can be computed  
<sup>391</sup> as:

$$\kappa_f \frac{\Delta P}{dt} = \nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} \quad (51)$$

<sup>392</sup> For multi-phase flows, the increment of the fluid pressure of the mixture can  
<sup>393</sup> be computed as:

$$\kappa \frac{\Delta P}{dt} = \sum_{f=1}^{N_f} \nabla^c \cdot (\phi_{f,FC} \mathbf{U}_{f,FC})^{n+1} \quad (52)$$

<sup>394</sup> where  $\kappa = \sum_{f=1}^{N_f} (\phi_{f,FC} \kappa_f) / \sum_{f=1}^{N_f} (\phi_{f,FC})$ . Then, the fluid pressure at cell  
<sup>395</sup> center is:

$$P_c^{n+1} = P_c^n + \Delta P_c^n \quad (53)$$

<sup>396</sup> Finally, the cell face advanced fluid pressure is:

$$P_{FC}^{n+1} = \left( \frac{P_{c,L}^{n+1}}{\bar{\rho}_{f,L}^n} + \frac{P_{c,R}^{n+1}}{\bar{\rho}_{f,R}^n} \right) / \left( \frac{1}{\bar{\rho}_{f,L}^n} + \frac{1}{\bar{\rho}_{f,R}^n} \right) = \left( \frac{P_{c,L}^{n+1} \bar{\rho}_{f,R}^n + P_{c,R}^{n+1} \bar{\rho}_{f,L}^n}{\bar{\rho}_{f,L}^n \bar{\rho}_{f,R}^n} \right) \quad (54)$$

<sup>397</sup> *Compute viscous shear stress term of the fluid phase*

<sup>398</sup> This part compute the viscous shear stress  $\Delta(m\mathbf{U})_{f,c,\tau}$  for a single vis-  
<sup>399</sup> cous compressible Newtonian fluid and optionally shear stress induced by the  
<sup>400</sup> turbulent model.

401    *Compute nodal internal temperature of the solid phase*

402    The nodal internal temperature rate is computed based on the heat con-  
403    duction model as below:

$$dT_{si}^{L-} = \frac{(\Delta W_{si}^n + \Delta W_{fric,i}^n + \nabla^i \cdot \mathbf{q}_{si}^n)}{m_{si}^n c_v} \quad (55)$$

404    where  $\Delta W_{si}^n = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt}$  is the mechanical work rate computed from the  
405    constitutive model with  $\boldsymbol{\epsilon}_s^p$  is the plastic strain,  $\Delta W_{fric,i}^n$  is the work rate  
406    computed from the contact law due to the frictional sliding between solid  
407    materials. The heat flux is  $\mathbf{q}_s = \bar{\rho}_s \beta_s \nabla T_s$  with  $\beta_s$  being the thermal conduc-  
408    tivity of the solid materials.

$$T_{si}^{L-} = T_{si}^n + dT_{si}^{L-} \quad (56)$$

409    *Compute and integrate acceleration of the solid phase*

410    After interpolating from material points to the nodes, the nodal acceler-  
411    ation and velocity are calculated by:

$$\mathbf{a}_{si}^{L-} = \frac{\mathbf{f}_{si}^{int,n} + \mathbf{f}_{si}^{ext,n}}{m_{si}^n} + \mathbf{g} \quad (57)$$

$$\mathbf{U}_{si}^{L-} = \mathbf{U}_{si}^n + \mathbf{a}_{si}^{L-} \Delta t \quad (58)$$

413    *Compute Lagrangian value (mass, momentum and energy)*

414    For the fluid phase, the linear momentum rate, the energy rate are:

$$\Delta(m\mathbf{U})_{f,c} = V n_c^n \nabla^c P_c^{n+1} + \Delta(m\mathbf{U})_{f,c,\tau} + V \bar{\rho}_{f,c}^n g \quad (59)$$

$$\Delta(me)_{f,c} = V n_c^n P_c^{n+1} \nabla^c \cdot \mathbf{U}_{f,FC}^* + \nabla^c \cdot \mathbf{q}_{f,c}^n \quad (60)$$

416    The heat flux is  $\mathbf{q}_f = \bar{\rho}_f \beta_f \nabla T_f$  with  $\beta_f$  being the thermal conductivity of the  
417    fluid materials. The Lagrangian value of the mass, linear momentum and  
418    energy of fluid phases without momentum exchange are:

$$m_{f,c}^L = V \bar{\rho}_{f,c}^n \quad (61)$$

$$(m\mathbf{U})_{f,c}^{L-} = V \bar{\rho}_{f,c}^n \mathbf{U}_{f,c}^n + \Delta(m\mathbf{U})_{f,c} \quad (62)$$

$$(me)_{f,c}^{L-} = V \bar{\rho}_{f,c}^n T_{f,c}^n c_v + \Delta(me)_{f,c} \quad (63)$$

<sup>421</sup> For the solid phase, the Lagrangian value of the linear momentum and energy  
<sup>422</sup> of solid phase are:

$$m_{sc}^L = m_{sc}^n \quad (64)$$

$$(m\mathbf{U})_{sc}^{L-} = \sum S_{ci} m_{si}^n \mathbf{U}_{si}^{L-} + V(1 - n_c^n) \nabla^c P_{f,c}^{n+1} \quad (65)$$

$$(me)_{sc}^{L-} = \sum S_{ci} m_{si}^n T_{si}^L c_v \quad (66)$$

<sup>425</sup> To consider the momentum exchange, the Lagrangian velocity is modified as:

$$\begin{aligned} \mathbf{U}_{f,c}^L &= \mathbf{U}_{f,c}^{L-} + \Delta \mathbf{U}_{f,c} \\ \mathbf{U}_{sc}^L &= \mathbf{U}_{sc}^{L-} + \Delta \mathbf{U}_{sc} \end{aligned} \quad (67)$$

<sup>426</sup> where the cell-centered intermediate velocity can be calculated by:

$$\begin{aligned} \mathbf{U}_{f,c}^{L-} &= \frac{(m\mathbf{U})_{f,c}^{L-}}{m_{f,c}^L} \\ \mathbf{U}_{sc}^{L-} &= \frac{(m\mathbf{U})_{sc}^{L-}}{m_{sc}^L} \end{aligned} \quad (68)$$

<sup>427</sup> And the increment of the velocity  $\mathbf{U}_{f,c}$ ,  $\Delta \mathbf{U}_{sc}$  can be computed by solving  
<sup>428</sup> the linear equation with  $i,j = 1:N$  as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_{i,c} \\ \Delta \mathbf{U}_{j,c} \end{vmatrix} = \begin{vmatrix} \beta_{ij} (\mathbf{U}_{i,c}^{L-} - \mathbf{U}_{j,c}^{L-}) \\ \beta_{ji} (\mathbf{U}_{j,c}^{L-} - \mathbf{U}_{i,c}^{L-}) \end{vmatrix}$$

<sup>429</sup> To consider the energy exchange, the Lagrangian temperature is modified as:

$$\begin{aligned} T_{f,c}^L &= T_{f,c}^{L-} + \Delta T_{f,c} \\ T_{sc}^L &= T_{sc}^{L-} + \Delta T_{sc} \end{aligned} \quad (69)$$

<sup>430</sup> where the cell-centered intermediate temperature can be calculated by:

$$\begin{aligned} T_{f,c}^{L-} &= \frac{(mT)_{f,c}^{L-}}{m_{f,c}^L c_v} \\ T_{sc}^{L-} &= \frac{(mT)_{sc}^{L-}}{m_{sc}^L c_v} \end{aligned} \quad (70)$$

<sup>431</sup> And the increment of the temperature due to energy exchange can be com-  
<sup>432</sup> puted by solving the linear equation with  $i,j = 1:N$  as:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_{i,c} \\ \Delta T_{j,c} \end{vmatrix} = \begin{vmatrix} \eta_{ij} (T_{i,c}^{L-} - T_{j,c}^{L-}) \\ \eta_{ji} (T_{j,c}^{L-} - T_{i,c}^{L-}) \end{vmatrix}$$

<sup>433</sup> Finally, we obtain the cell-centered solid acceleration and temperature rate  
<sup>434</sup> as:

$$d\mathbf{U}_{sc}^L = \frac{(m\mathbf{U})_{sc}^L - (m\mathbf{U})_{sc}^n}{m_{sc}^L \Delta t} \quad (71)$$

<sup>435</sup>

$$dT_{sc}^L = \frac{(me)_{sc}^L - (me)_{sc}^n}{m_{sc}^L c_v \Delta t} \quad (72)$$

<sup>436</sup> **Compute Lagrangian specific volume of the fluid phase**

<sup>437</sup> To compute the Lagrangian value of the specific volume of the fluid phase,  
<sup>438</sup> we need to compute the Lagrangian temperature rate as below:

$$T_{f,c}^{n+1} = \frac{(me)_{f,c}^L}{m_{f,c}^L c_v} \quad (73)$$

<sup>439</sup>

$$\frac{D_f T_{f,c}}{Dt} = \frac{T_{f,c}^{n+1} - T_{f,c}^n}{\Delta t} \quad (74)$$

<sup>440</sup> As such, the Lagrangian specific volume rate is:

$$\Delta(mv)_{f,c} = V f_{f,c}^\phi \nabla \cdot \mathbf{U} + (\phi_{f,c} \alpha_{f,c} \frac{D_f T_{f,c}}{Dt} - f_{f,c}^\phi \sum_{n=1}^N \phi_{nc} \alpha_{nc} \frac{D_n T_{n,c}}{Dt}) \quad (75)$$

<sup>441</sup> where  $f_f^\phi = (\phi_f \kappa_f) / (\sum_{n=1}^N \phi_n \kappa_n)$  and  $\mathbf{U} = \nabla \cdot (\sum_{s=1}^{N_s} \phi_{sc} \mathbf{U}_{sc} + \sum_{f=1}^{N_f} \phi_{fc} \mathbf{U}_{f,c})$ .  
<sup>442</sup> Finally, the Lagrangian specific volume is:

$$(mv)_{f,c}^L = V \bar{\rho}_{f,c}^n v_{f,c}^n + \Delta(mv)_{f,c} \quad (76)$$

<sup>443</sup> **Compute advection term and advance in time**

<sup>444</sup> The mass, linear momentum, energy and specific volume with advection  
<sup>445</sup> are:

$$m_{f,c}^{n+1} = m_{f,c}^L - \Delta t \nabla \cdot (\bar{\rho}_{f,c}^L, \mathbf{U}_{f,FC}^L) \quad (77)$$

<sup>446</sup>

$$(m\mathbf{U})_{f,c}^{n+1} = (m\mathbf{U})_{f,c}^L - \Delta t \nabla \cdot ((\bar{\rho}\mathbf{U})_{f,c}^L, \mathbf{U}_{f,FC}^L) \quad (78)$$

<sup>447</sup>

$$(me)_{f,c}^{n+1} = (me)_{f,c}^L - \Delta t \nabla \cdot ((\bar{\rho}c_v T)_{f,c}^L, \mathbf{U}_{f,FC}^L) \quad (79)$$

<sup>448</sup>

$$(mv)_{f,c}^{n+1} = (mv)_{f,c}^L - \Delta t \nabla \cdot ((\bar{\rho}v)_{f,c}^L, \mathbf{U}_{f,FC}^L) \quad (80)$$

<sup>449</sup> Finally, the state variables of the fluid phases of the next time step are:

$$\bar{\rho}_{f,c}^{n+1} = \frac{m_{f,c}^{n+1}}{V} \quad (81)$$

450

$$\mathbf{U}_{f,c}^{n+1} = \frac{(m\mathbf{U})_{f,c}^{n+1}}{m_{f,c}^{n+1}} \quad (82)$$

451

$$T_{f,c}^{n+1} = \frac{(me)_{f,c}^{n+1}}{m_{f,c}^{n+1}} \quad (83)$$

452

$$v_{f,c}^{n+1} = \frac{(mv)_{f,c}^{n+1}}{m_{f,c}^{n+1}} \quad (84)$$

453 *Interpolate from cell to node of the solid phase*

454 First we interpolate the acceleration, velocity and temperature rate to  
455 the node as below:

456

$$\mathbf{a}_{si}^n = \sum S_{ci} d\mathbf{U}_{sc}^L \quad (85)$$

457

$$\mathbf{U}_{si}^{n+1} = \sum S_{ci} d\mathbf{U}_{sc}^L \Delta t \quad (86)$$

458

$$dT_{si}^n = \sum S_{ci} dT_{sc}^L \quad (87)$$

459 Then the boundary condition and contact forces  $f_{si}^{fric}$  are applied to the nodal  
velocity, and then accelerations are modified by:

$$\mathbf{a}_{si}^n = \frac{\mathbf{v}_{si}^{n+1} - \mathbf{v}_{si}^n}{\Delta t} \quad (88)$$

460 *Update the particle variables*

461 The state variables of the solid phase  $[\mathbf{U}_{sp}^{n+1}, \mathbf{x}_{sp}^{n+1}, \nabla \mathbf{U}_{sp}^{n+1}, T_{sp}^{n+1}, \nabla T_{sp}^{n+1}, \mathbf{F}_{sp}^{n+1}, V_{sp}^{n+1}]$   
462 (velocity, position, velocity gradient, temperature, temperature gradient, de-  
463 formation gradient, volume) are updated as:

464

$$\mathbf{U}_{sp}^{n+1} = \mathbf{U}_{sp}^n + \sum S_{sp} \mathbf{a}_{si}^n \Delta t \quad (89)$$

465

$$\mathbf{x}_{sp}^{n+1} = \mathbf{x}_{sp}^n + \sum S_{sp} \mathbf{U}_{si}^{n+1} \Delta t \quad (90)$$

466

$$\nabla \mathbf{U}_{sp}^{n+1} = \sum \nabla S_{sp} \mathbf{U}_{si}^{n+1} \quad (91)$$

467

$$T_{sp}^{n+1} = T_{sp}^n + \sum S_{sp} dT_{si}^n \Delta t \quad (92)$$

$$\nabla T_{sp}^{n+1} = \sum \nabla S_{sp} T_{sp}^n \Delta t \quad (93)$$

$$\mathbf{F}_{sp}^{n+1} = (\mathbf{I} + \nabla \mathbf{U}_{sp}^{n+1} \Delta t) \mathbf{F}_{sp}^n \quad (94)$$

$$V_{sp}^{n+1} = \det(\mathbf{F}_{sp}^{n+1}) V_{sp}^o \quad (95)$$

Finally, the effective stress ( $\sigma'$ )<sup>n+1</sup> is updated from the constitutive model and the pore water pressure is interpolated from the cell as:

$$p_f^{n+1} = \sum S_{si} P_c^{n+1} \quad (96)$$

## 472 Numerical validation

For all simulations, water is characterized by a bulk modulus of 2 GPa, a density of 998 kg/m<sup>3</sup> at a reference temperature of 5 degrees Celsius and a reference pressure of 10325 Pa (1 atm), and a dynamic viscosity denoted as  $\mu_f$  of 1 mPa s. The air is treated as an ideal gas with a density of 1.17 kg/m<sup>3</sup> at a reference temperature of 5 degrees Celsius and a reference pressure of 10325 Pa (1 atm), and it possesses a dynamic viscosity  $\mu_f$  of  $18.45 \times 10^{-3}$  mPa s.”

480 *Fluid Flow through isothermal porous media*

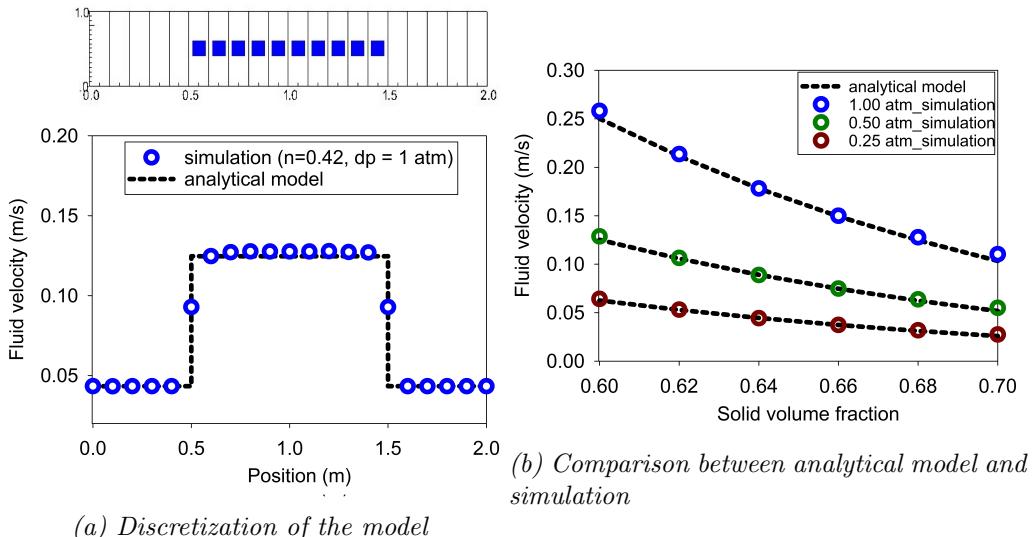


Figure 7: Numerical results of the fluid flow through isothermal porous media

481 Fluid flow through porous media is important in many engineering disciplines,  
 482 like predicting water flow in soil. Fluid flow velocity in one dimension  
 483 can be calculated from the porous media's hydraulic conductivity  $K$  as:

$$484 \quad U_f = K \frac{\Delta p_f}{L} \quad (97)$$

485 If the Carman-Kozeny formula is adopted  $F = 10\phi_s/(1 - \phi_s)^2$ , the hydraulic  
 486 conductivity will be expressed as  $K = D_p^2(1 - \phi_s)^3\rho_f g/180\phi_s^2\mu_f$ . Then, the  
 487 analytical formula of average velocity in one dimension through the porous  
 488 media is:

$$489 \quad U_f = \frac{1}{n} \frac{D_p^2(1 - \phi_s)^3}{180\phi_s^2\mu_f} \frac{\Delta p_f}{L} \quad (98)$$

490 Our numerical model's validity is confirmed through the simulation of  
 491 fluid flow in a 1m long porous medium. This porous medium is represented  
 492 by an elastic material with the following properties: Young's modulus of  
 493 10 MPa, Poisson's ratio of 0.3, and a density of 2650 kg/m<sup>3</sup>. The volume  
 494 fraction of the porous medium, denoted as  $\phi_s$ , is varied as [0.6, 0.62, 0.66,  
 495 0.68, 0.7], while the average grain diameter  $d$  is set at 1mm. The model is  
 496 discretized into 20 finite elements, with the porous medium represented by  
 497 10 finite elements, each containing one material point per element. We apply  
 498 pressure gradients with three different values: [0.25, 0.5, 1] atm. As depicted  
 499 in Figure 7, our model demonstrates excellent agreement with theoretical  
 500 predictions in simulating fluid flow.

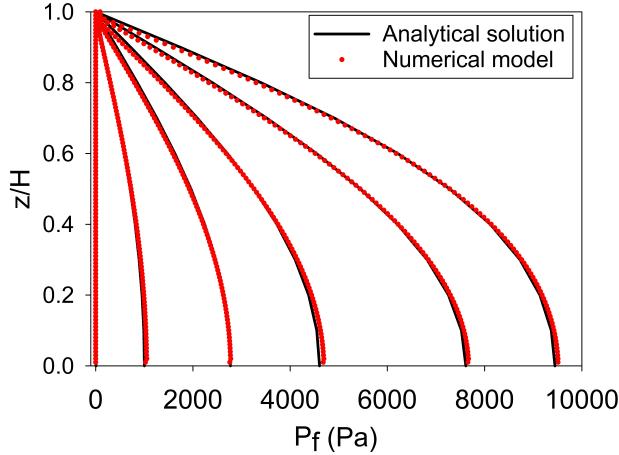
### 501 *Isothermal consolidation*

502 A common benchmark for fully saturated porous media is the simulation  
 503 of one-dimensional consolidation. Using the Carman-Kozeny formula, the  
 504 time-dependent pressure can be calculated as:

$$505 \quad p_f = \sum_{m=1}^{\infty} \frac{2F_{ext}}{M} \sin\left(\frac{Mz}{H}\right) e^{-M^2 T_V} \text{ with } M = \frac{\pi}{2}(2m + 1) \quad (99)$$

506 Here, the consolidation rate is defined as  $T_v = C_v t / H^2$ , the consolidation  
 507 coefficient as  $C_v = E_v n^3 d^2 / (180(1 - n)^2 \mu)$ , and the Oedometer modulus as  
 508  $E_v = E(1 - v) / (1 + v) / (1 - 2v)$ .

509 To validate our numerical model, we simulated the consolidation of a  
 510 1m column of porous media. The porous media is modeled as an elastic



*Figure 8: Comparison between analytical solution and numerical solution*

material with a Young's modulus of 10 MPa, a Poisson's ratio of 0.3, and a density of  $2650 \text{ kg/m}^3$ . The volume fraction of porous media  $\phi_s$  is set to 0.7, equivalent to a porosity of 0.3, and the average grain diameter  $d$  is 1 mm. The model is discretized into 100 finite elements, each with 1 material point per element. An external pressure of 10 kPa is applied to the top of the column. Figure 8 demonstrates a strong agreement between the predicted fluid flow and theoretical results.

#### 517 *Thermal induced cavity flow*

Another benchmark involves the study of thermally-induced cavity flow in porous media. This simulation calculates temperature and velocity distributions within a square, non-deformable, saturated porous medium. The top and bottom walls are insulated, while the left and right walls maintain a fixed temperature gradient of 1 degree, leading to fluid motion in the form of cavity flow due to temperature-induced density variation. Our numerical model is validated by comparing it with the numerical solution obtained using the finite element method.

The porous medium in this simulation is modeled as a non-deformable material with a density of  $2500 \text{ kg/m}^3$ . The specific heat capacity of the water and porous skeleton is  $4181 \text{ J/kg}\cdot\text{K}$  and  $835 \text{ J/kg}\cdot\text{K}$ , respectively. Thermal conductivity values are  $0.598 \text{ W/m}\cdot\text{K}$  for water and  $0.4 \text{ W/m}\cdot\text{K}$  for the porous skeleton. The volume fraction of porous media  $\phi_s$  is set at 0.6, equivalent

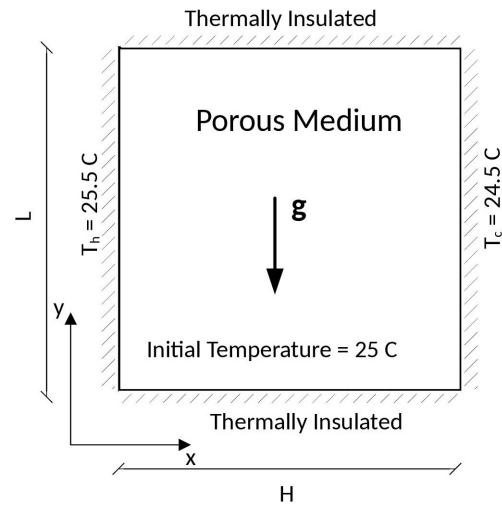


Figure 9: Model schematic [31]

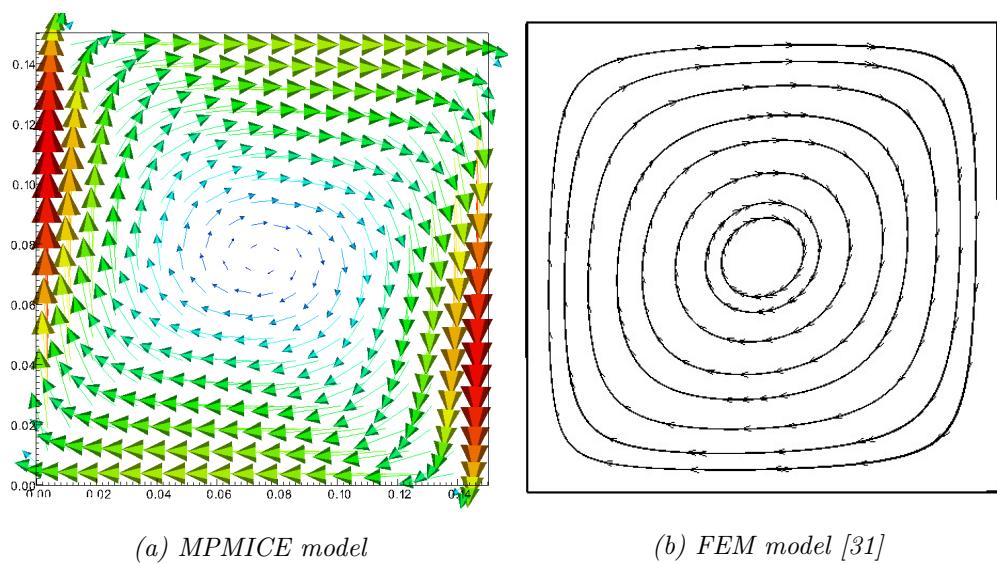


Figure 10: Comparison between MPMICE model and FEM model

531 to a porosity of 0.4, and the average grain diameter  $d$  is 1mm. The model  
 532 is discretized into a 20x20 grid of finite elements, with 4 material points per  
 533 element. Figure 10 demonstrates that our numerical results align well with  
 534 the numerical solution obtained using the finite element method.

535 **Numerical examples**

536 *Underwater debris flow*

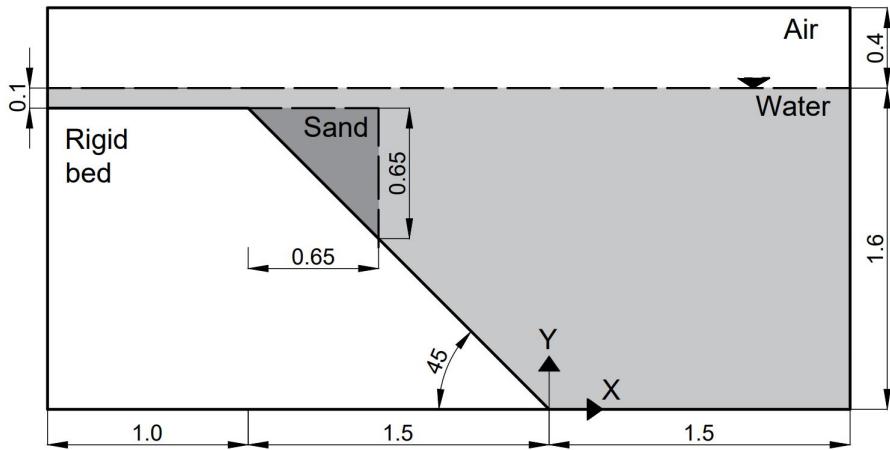


Figure 11: Model schematic

537 The numerical example is compared to the experimental work of Rzad-  
 538 kiewicz et al. on submarine debris flow [32]. In their experiment, sand within  
 539 a triangular box is released and slides along a rigid bed inclined at 45 de-  
 540 grees underwater (see Figure 11). The material properties in the numerical  
 541 model are chosen based on Rzadkiewicz et al.'s experiment [32]. The sand  
 542 has a saturated density of  $1985 \text{ kg/m}^3$  and a friction angle of 10 degrees.  
 543 Young's modulus, despite its negligible effect on debris flow run-out due to  
 544 extreme deformation, is set at 50 MPa with a Poisson's ratio of 0.25. The  
 545 rigid bed, significantly stiffer, possesses bulk modulus and shear modulus  
 546 values of  $117E^7 \text{ Pa}$  and  $43.8E^7 \text{ Pa}$ , respectively. The numerical parameters  
 547 used in this example are outlined in Table 1.  
 548 The boundary conditions applied in the numerical model are as follows: all  
 549 boundary faces have zero velocity ( $U = 0 \text{ m/s}$ ) and a temperature of 5 degrees  
 550 Celsius ( $T = 5^\circ\text{C}$ ). At the top boundary, pressure has a Neumann boundary

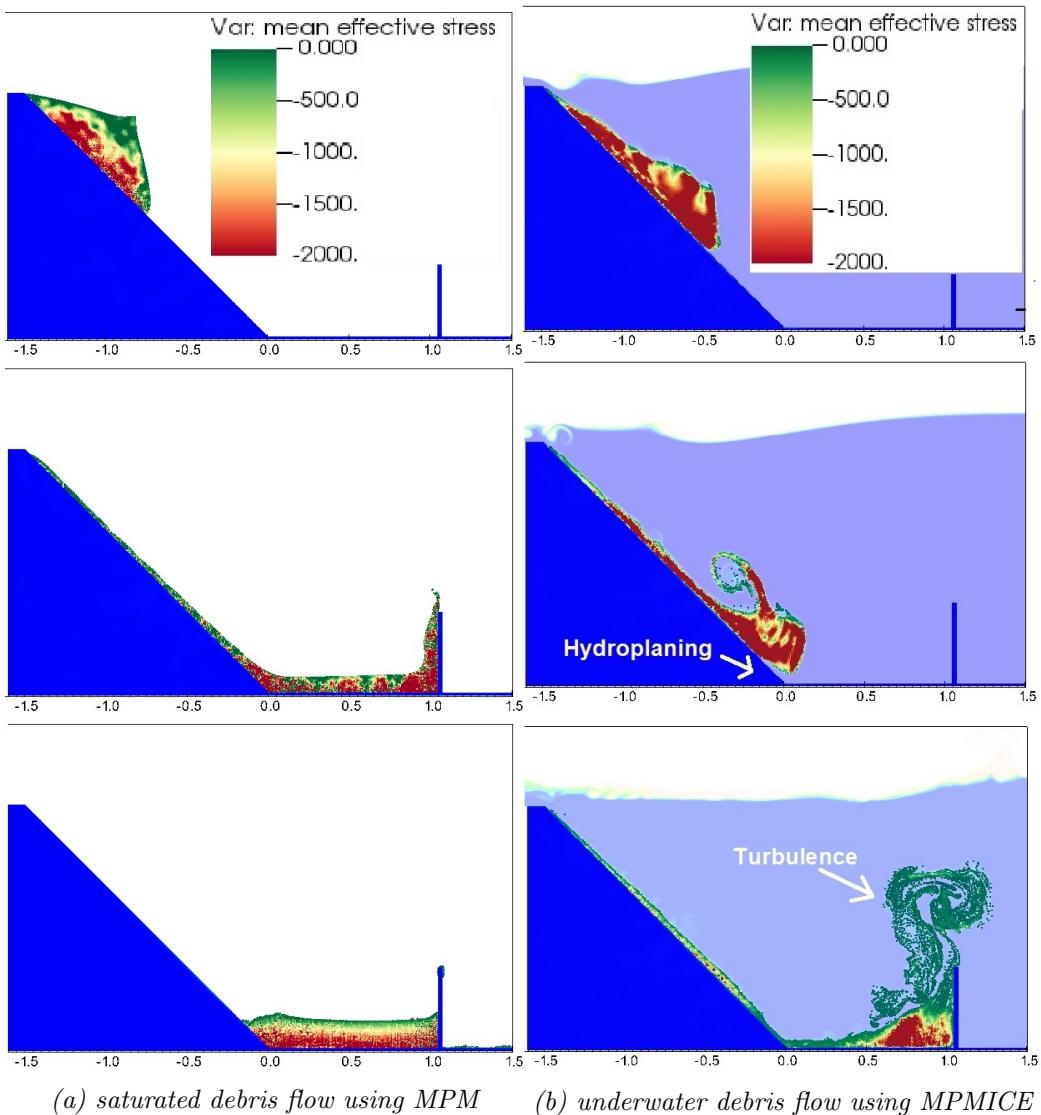
551 condition of  $dp/dx = 0$  kPa, and density has a Neumann boundary condition  
 552 of  $d\rho/dx = 0$   $kg/m^3$ . The background mesh comprises 700 x 400 cells,  
 553 resulting in a total of 280,000 cells. Each cell within the debris flow and rigid  
 554 bed contains 2 x 2 material points.

555 Figure 12b illustrates snapshots of underwater debris flow sliding, effectively  
 556 capturing the typical hydroplaning mechanism of the debris flow. Hydroplaning  
 557 refers to the lifting of the debris flow, causing it to lose contact with the  
 558 bottom layer. In addition, Figure 13 compares the elevation of the free  
 559 surface at 0.4s and 0.8s between our proposed method and other methods,  
 560 demonstrating the alignment of our computed results with experimental re-  
 561 sults [7].

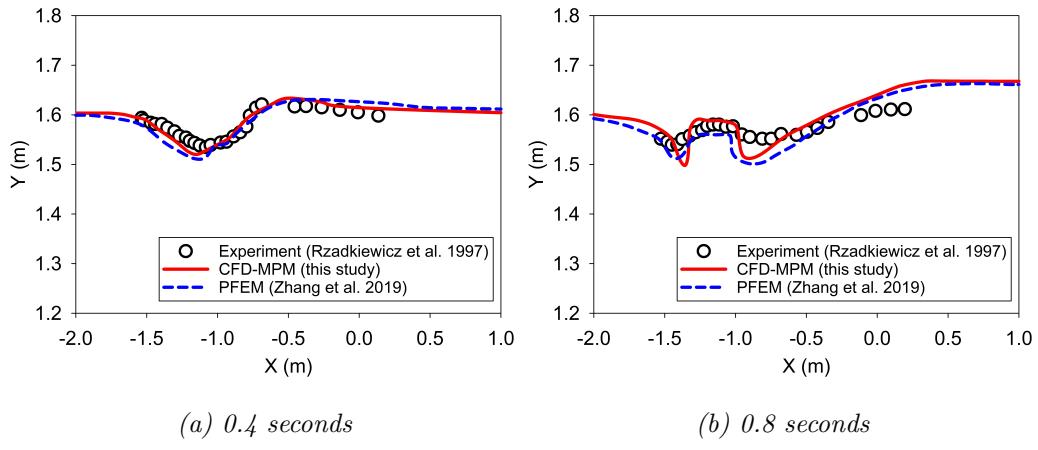
562 What sets our model apart is its utilization of effective stress analysis in-  
 563 stead of total stress analysis. This allows for the analysis of water pressure  
 564 and temperature within the debris flow. Furthermore, we investigate the dif-  
 565 ferences between underwater debris flow and saturated debris flow in terms  
 566 of their interaction with obstacles. Figure 12 presents snapshots of simula-  
 567 tions of both underwater and saturated debris flow. The saturated debris  
 568 flow (Figure 12a) exhibits behavior similar to frictional flow, with grains in  
 569 contact with each other. Conversely, underwater debris flow (Figure 12b)  
 570 behaves like turbulent flow, with grains separated and showing no contact  
 571 forces, as reflected by the near-zero effective stress in the turbulence domain.

Materials	Bulk modul (Pa)	Shear modul (Pa)	Density (kg/m3)	Temp (C)	Dynamic viscosity (Pa s)	Friction angle (degrees)
Water (at surface)	2.15e9	-	999.8	5	855e-6	-
Air (at top boundary)	-	-	1.177	5	18.45e-6	-
Sand (porous media)	8.33e6	20e6	1985	5	-	10
Rigid bed (solid)	117e7	43.8e7	8900	5	-	-

Table 1: Numerical parameters for the underwater submarine debris flow

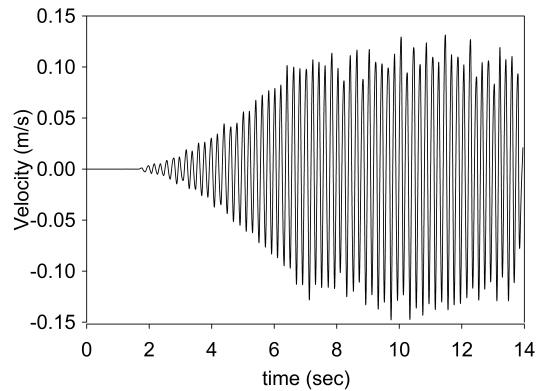


*Figure 12: Simulation of Debris Flow: Mean Effective Stress Distribution (Green Color Indicates Near-Zero Effective Stress)*



*Figure 13: Evolution of water level in the simulation of underwater debris flow*

572 Validation of soil response to the seismic loading



*Figure 14: Seismic loading*

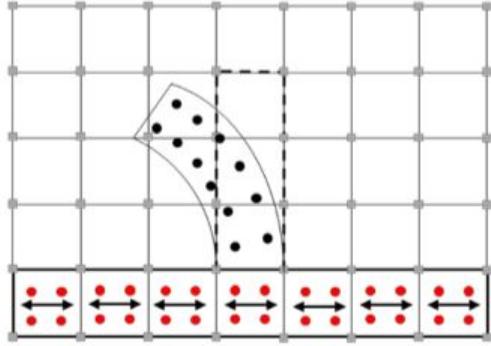


Figure 15: Material points prescribed velocity as kinematic boundary condition [33]

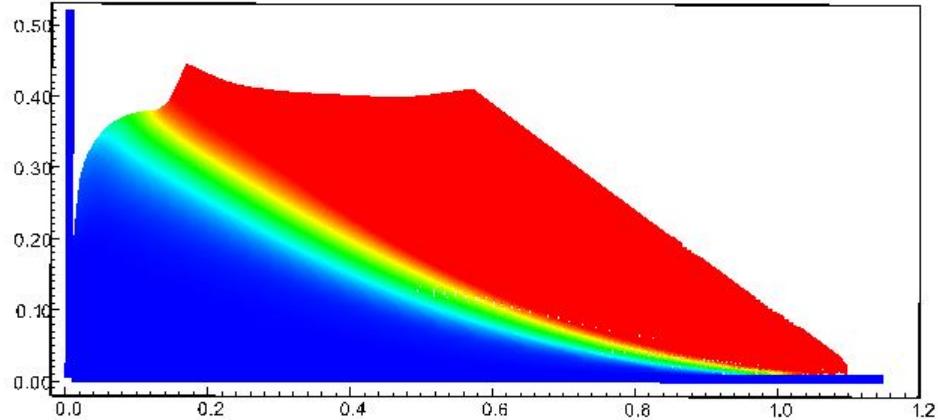


Figure 16: Numerical model of the seismic-induced slope failure with displacement color

573 An experimental study conducted by Hiraoka et al. [34] aimed to invest-  
 574 igate the influence of seismic shaking on the deformation of a 0.5 m-high  
 575 sand slope. The sand used in the experiment was partially saturated, with a  
 576 moisture content of 10 percent. The provided soil parameters for the Mohr  
 577 Coulomb model include the effective friction angle of 23 degrees, apparent  
 578 cohesion of 0.78 kPa, Young's modulus of 2.57 MPa, Poisson's ratio of 0.33,  
 579 and moist unit weight of 16.5 kN/m<sup>3</sup>. The soil's dilatancy angle was assumed  
 580 to be 0 [34]. The experimental setup consisted of a shaking table box with a  
 581 steel horizontal base and smooth glass vertical sidewalls. Laser sensors were  
 582 used to monitor the displacement of the slope's toe and crest. Figure 14

583 displays the velocity-time history employed in the experiment.  
 584 To simulate the seismic loading in our numerical model, we adopted a method  
 585 presented by Alsardi et al. [33], which involves specifying the velocity at the  
 586 corresponding material points representing either the shaking table or the  
 587 bedrock at the site (see Figure 15). In our simulation, we considered the  
 588 horizontal base to be fully rough and the vertical contact to be fully smooth.  
 589 The initial stress condition was initiated using gravity, and seismic loading  
 590 induced the slope failure (see Figure 16).  
 591 Previous studies by Bhandari et al. [35], Alsardi et al. [33], and Hiraoka  
 592 et al. [34] attempted to model this experiment using MPM and SPH mod-  
 593 els. In this study, we compared our results with those obtained from other  
 594 particle-based methods (Figure 17). The main difference is that we did not  
 595 apply 5 percent numerical damping in our model, unlike the other methods.  
 596 We found that the final displacement of the slope toe in our MPM model was  
 597 higher than that observed in the experiment. Nevertheless, the validation of  
 598 the Mohr-Coulomb model under seismic response demonstrated reasonable  
 599 soil behavior in terms of displacement.

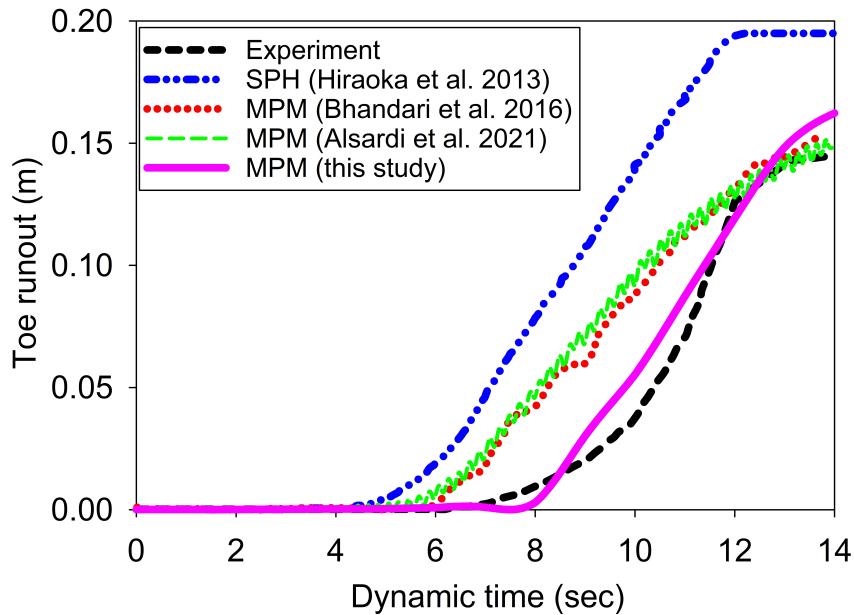


Figure 17: Displacement of the toe of the slope

600 *Earthquake-induced submarine landslides*

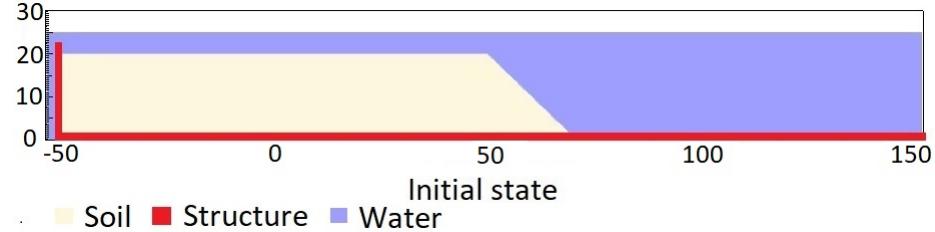


Figure 18: Numerical simulation of the earthquake-induced submarine landslide

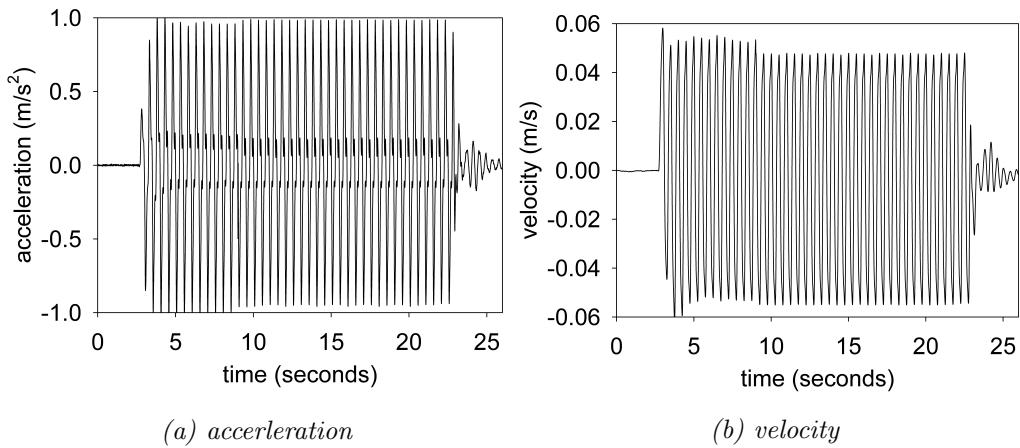


Figure 19: Ground acceleration profile, frequency of 2Hz and magnitude of 1g

601 In the final example, we conduct a numerical analysis of earthquake-  
 602 induced submarine landslides. We utilize a plane strain model featuring an  
 603 underwater slope, as depicted in Figure 18. This model consists of a 20m high  
 604 slope with a gradient of 45 degrees, placed within a horizontal and vertical  
 605 structure formerly used as a shaking table to apply earthquake loading. To  
 606 simplify the earthquake loading, we simulate ground shaking for 20 seconds,  
 607 maintaining a constant ground acceleration of 1g and a consistent frequency  
 608 of 2 Hz (Figure 19a). This magnitude of earthquake is plausible; for instance,  
 609 during the 2023 Turkey-Syria Earthquake, significant ground shaking with  
 610 peak ground acceleration exceeding 1g was recorded at numerous locations.  
 611 This real-world example demonstrates the practical occurrence of such high  
 612 levels of ground acceleration during seismic events. To generate the seismic

613 loading, we employ the same method as presented in the previous numerical  
 614 example.

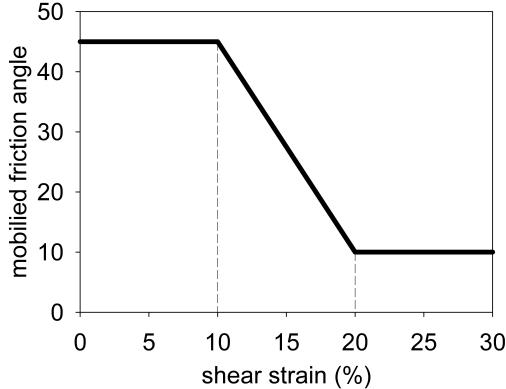


Figure 20: Mobilized friction angle in Mohr Coulomb model

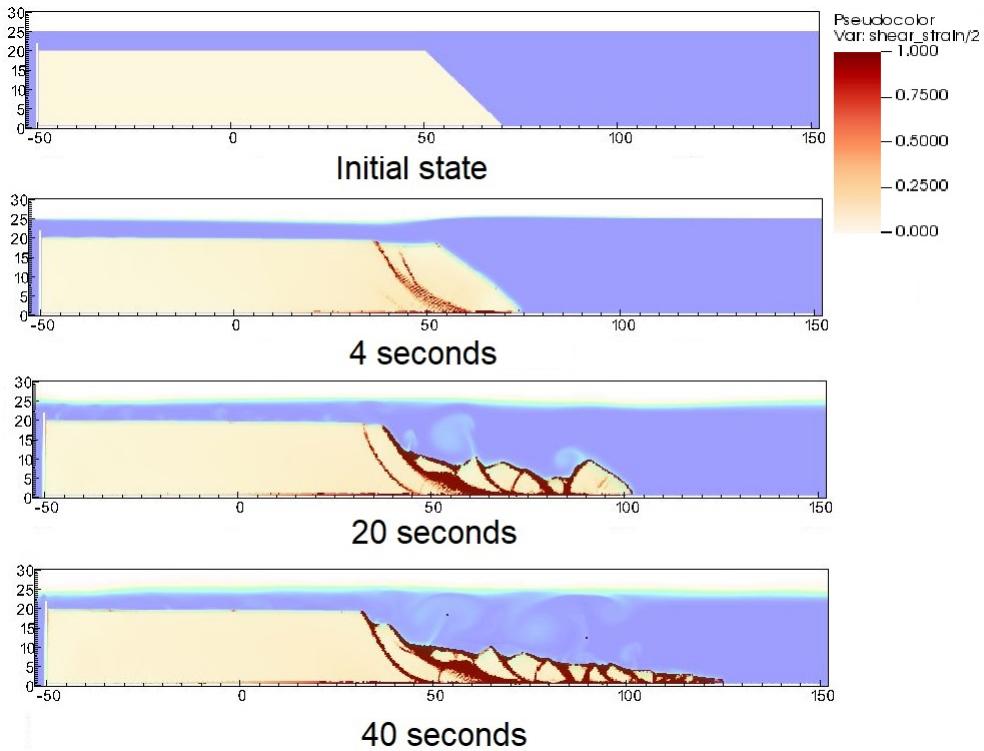
615 A non-associated Mohr-Coulomb model is employed to represent the soil  
 616 in our simulation. The soil grains have a density of  $2650 \text{ kg/m}^3$ , a Young's  
 617 modulus of 10 kPa, a Poisson's ratio of 0.3, and zero cohesion. The mobilized  
 618 friction angle  $\phi'_m$  is determined based on the softening curve (as depicted in  
 619 Figure 20), with a peak friction angle  $\phi'_p$  of 45 degrees and a residual friction  
 620 angle  $\phi'_r$  of 10 degrees. The porosity is set to 0.3, and the average grain  
 621 size of the soil is approximately  $0.1 \mu\text{m}$  to mimic undrained behavior. The  
 622 mobilized dilatancy angle is calculated using Rowe's stress dilatancy theory  
 623 [36] as follows:

$$\sin \psi'm = \frac{\sin \phi'm - \sin \phi'r}{1 - (\sin \phi'r \sin \phi'_m)} \quad (100)$$

624 The solid plane is modeled as a rigid body, acting as a shaking table. Frictional  
 625 contact with a friction coefficient of 0.1 is considered between the  
 626 horizontal plane and the sand. No artificial damping is applied in the sim-  
 627 ulation. The contact between the vertical plane and the sand is treated as  
 628 smooth, with a zero friction coefficient.

629 Symmetric boundary conditions are imposed on all boundary faces, while  
 630 Neumann boundary conditions are applied at the top boundary for pressure  
 631 ( $d\rho/dx = 0 \text{ kPa}$ ) and density ( $d\rho/dx = 0 \text{ kg/m}^3$ ). In the context of the  
 632 simulation, a symmetric boundary condition means that the normal compo-  
 633 nent of the velocity at the boundary face is set to zero, and the tangential

634 component matches the tangential component of the neighboring cells.  
 635 The mesh size is set to 0.25 m x 0.25 m, resulting in 300,852 element cells  
 636 and 142,316 material points. The simulation takes several hours to complete  
 637 60 seconds of simulation time, utilizing 4,096 CPUs.



*Figure 21: Shear strain during the earthquake-induced submarine landslides*

638 We have illustrated the entire process and mechanism of earthquake-  
 639 induced submarine landslides by presenting the shear strain (Figure 21),  
 640 pore water pressure in atm (Figure 22), and velocity (Figure 23). The failure  
 641 mechanism can be characterized as a progressive failure mechanism, and here  
 642 are some key numerical observations:

- 643 1. At the onset of the seismic event, the seismic loading triggers the initial  
 644 slide at 3 seconds. By 4 seconds, the debris starts moving at a maximum  
 645 speed of around 2-3 m/s, with multiple shear bands developing in the  
 646 slope. A wave is generated from the submarine slide, propagating with  
 647 approximately 2-3 m high in the direction of the slide.

648     2. When the onset of the shear band occurs in the slope (e.g., at 4 seconds  
649       and 20 seconds), negative excess pore water pressure develops along  
650       this shear band, with pore water pressure dropping below 1 atm. This  
651       behavior is typical of dilatancy when the soil undergoes rapid shearing  
652       in an undrained state.

653     3. As the seismic loading ends at 23 seconds, the last shear band is mobi-  
654       lized, and the slope quickly reaches its final deposition. There are no  
655       further progressive failures in the slope at this stage. A turbulent flow  
656       develops due to the interaction between the debris flow and seawater.

657     In summary, we have presented a comprehensive view of the earthquake-  
658       induced submarine landslides, covering (1) the earthquake-triggering mech-  
659       anism, (2) the initiation of shear bands with the development of negative  
660       excess pore water pressure, (3) the progressive failure mechanism, and (4)  
661       the generation of submarine landslide-induced waves leading to the final de-  
662       position of debris.

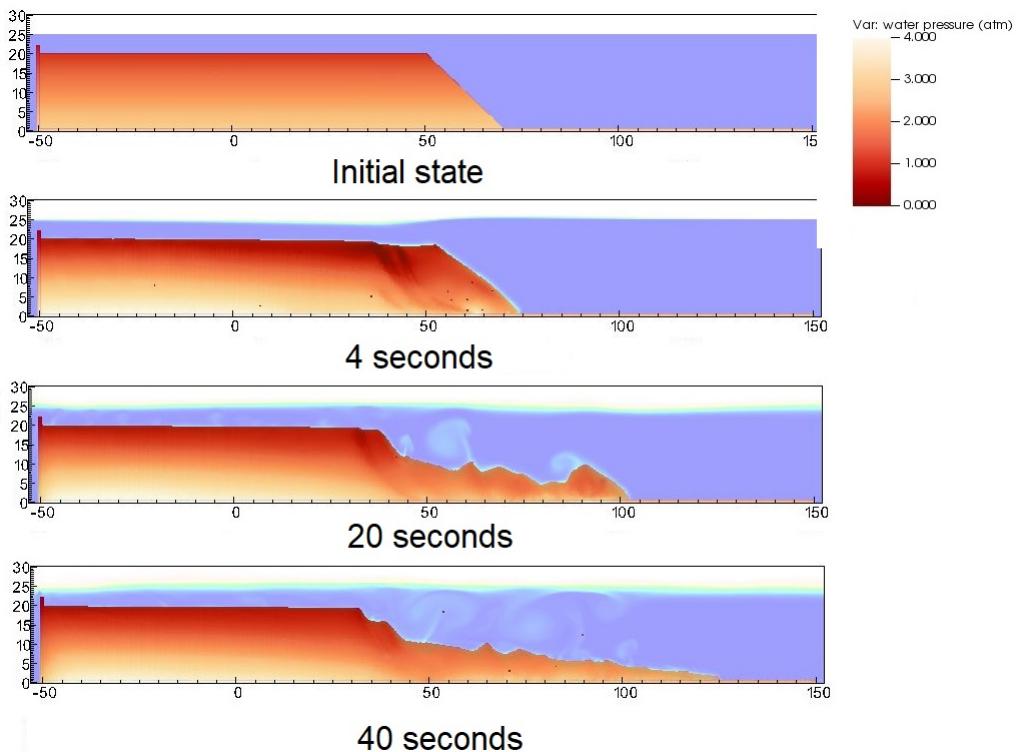
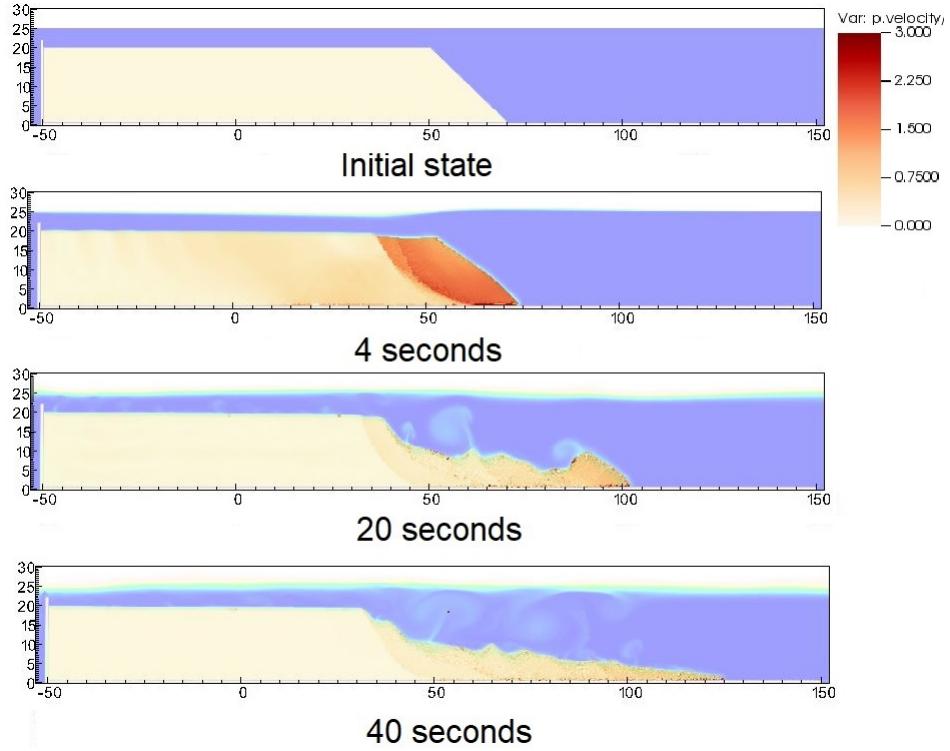


Figure 22: pore water pressure during the earthquake-induced submarine landslides



*Figure 23: Velocity during the earthquake-induced submarine landslides*

### 663      **Conclusions**

664      We have introduced a numerical approach called MPMICE for simulating  
 665      large deformation soil-fluid-structure interactions, with a specific focus  
 666      on earthquake-induced submarine landslides. This model leverages two key  
 667      components: Material Point Method (MPM): MPM is employed to accu-  
 668      rately capture the large deformations occurring in iso-thermal porous me-  
 669      dia and solid structures. Implicit Continuous Eulerian (CFD Formulation):  
 670      This component is used for modeling the intricate fluid flow, including tur-  
 671      bulence, within the system. It adopts a compressible, conservative multi-  
 672      material CFD formulation. Our model has been implemented within the  
 673      high-performance Uintah computational framework and rigorously validated  
 674      through comparisons with analytical solutions and experimental data. Sub-  
 675      sequently, we have demonstrated the model's capabilities in simulating the  
 676      complete process of earthquake-induced submarine landslides.

677 **Acknowledgements**

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679 Harman from the University of Utah for sharing the insight on the theoretical  
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687 Porelab. The computations were performed on High Performance Comput-  
688 ing resources provided by UNINETT Sigma2 - the National Infrastructure  
689 for High Performance Computing and Data Storage in Norway.

690 **Data Availability Statement**

691 The authors confirm that the data supporting the findings of this study  
692 are available within the article. All input files and the analytical calculations  
693 in this section are provided in the Github repository  
694 ([https://github.com/QuocAnh90/Uintah\\_NTNU](https://github.com/QuocAnh90/Uintah_NTNU)) for the reproduction of the  
695 numerical results.

696 **Appendix: Equation derivation**

697 Before deriving the governing equation, we define the Lagrangian deriva-  
698 tive for a state variable  $f$  as:

$$\frac{D_f f}{Dt} = \frac{\partial f}{\partial t} + \mathbf{U}_f \cdot \nabla f \quad (101)$$

$$\frac{D_s f}{Dt} = \frac{\partial f}{\partial t} + \mathbf{U}_s \cdot \nabla f \quad (102)$$

700 We adopt the following definitions as per [22]:

$$-\frac{1}{V} \left[ \frac{\partial V_f}{\partial p} \right] \equiv \kappa_f \quad \text{isothermal compressibility of fluid} \quad (103)$$

$$\frac{1}{V} \left[ \frac{\partial V_f}{\partial T} \right] \equiv \alpha_f \quad \text{constant pressure thermal expansivity of fluid} \quad (104)$$

702 Then, we calculate the rate of volume within incompressible solid grains as  
 703 follows:

$$\frac{1}{V} \frac{D_f V_f}{Dt} = \frac{1}{V} \left( \left[ \frac{\partial V_f}{\partial p} \right] \frac{D_f P_{eq}}{Dt} + \left[ \frac{\partial V_f}{\partial T_f} \right] \frac{D_f T}{Dt} \right) = \frac{1}{V} \left( -\kappa_f \frac{D_f P_{eq}}{Dt} + \alpha_f \frac{D_f T_f}{Dt} \right) \quad (105)$$

704 *Evolution of porosity*

705 Solving the solid mass balance equation (4) with the definition of solid  
 706 mass in equation (2), we obtain the rate of porosity as:

$$\frac{D_s m_s}{Dt} = \frac{D_s (\phi_s \rho_s V)}{Dt} = \rho_s V \frac{D_s \phi_s}{Dt} + \phi_s V \frac{D_s \rho_s}{Dt} + \phi_s \rho_s \frac{D_s V}{Dt} = 0 \quad (106)$$

707 Since soil grains are assumed to be incompressible, term 2 on the right-hand  
 708 side is zero, resulting in:

$$V \frac{D_s \phi_s}{Dt} + \phi_s \frac{D_s V}{Dt} = 0 \quad (107)$$

709 Dividing all terms by "V" and using the equation  $\frac{1}{V} \frac{D_s V}{Dt} = \nabla \cdot \mathbf{U}_s$ , we get:

$$\frac{D_s n}{Dt} = \frac{\partial n}{\partial t} + \mathbf{U}_s \cdot \nabla n = \phi_s \nabla \cdot \mathbf{U}_s \quad (108)$$

710 *Momentum conservation*

711 The linear momentum balance equations for the fluid phases based on  
 712 mixture theory are given by:

$$\frac{1}{V} \frac{D_f (m_f \mathbf{U}_f)}{Dt} = \nabla \cdot (-\phi_f p_f \mathbf{I}) + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d + \mathbf{f}_b \quad (109)$$

713 On the right hand side, the terms include the divergence of partial fluid phase  
 714 stress, body force, drag force (momentum exchange) and buoyant force as  
 715 described in [37] for immiscible mixtures, which takes the form:  
 716

$$\mathbf{f}_b = \boldsymbol{\sigma}_f \nabla(n) \quad (110)$$

717 Hence, the linear momentum balance equations for the fluid phases become:

$$\frac{1}{V} \frac{D_f (m_f \mathbf{U}_f)}{Dt} = -\phi_f \nabla p_f + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d \quad (111)$$

718 The Reynolds stress component can be included in the term  $\boldsymbol{\tau}_f$  to consider the  
 719 turbulent effects if needed. To derive the linear momentum balance equation  
 720 for the solid phase, we begin with the linear momentum balance equation for  
 721 the mixture as:

$$\frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} + \frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}) + \bar{\rho}_f \mathbf{b} + \bar{\rho}_s \mathbf{b} \quad (112)$$

722 Combining Terzaghi's equation (3) and subtracting both sides with equation  
 723 (111), we obtain the linear momentum balance equations for the solid phase  
 724 as:

$$\frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}') - \phi_s \nabla p_f + \bar{\rho}_s \mathbf{b} - \sum \mathbf{f}_d + \sum \mathbf{f}_{fric} \quad (113)$$

725 Here the  $\mathbf{f}_{fric}$  stems from the soil-structure interaction following the contact  
 726 law between the soil/structure interfaces.

727 *Energy conservation*

728 We utilize the general form of the total energy balance equation for porous  
 729 media from [38]. The total energy balance equations for the fluid phases take  
 730 the following form:

$$\frac{1}{V} \frac{D_f(m_f(e_f + 0.5 \mathbf{U}_f^2))}{Dt} = \nabla \cdot (-np_f \mathbf{I}) \cdot \mathbf{U}_f + \nabla \cdot \mathbf{q}_f + (\bar{\rho}_f \mathbf{b}) \cdot \mathbf{U}_f + \sum \mathbf{f}_d \cdot \mathbf{U}_f + \sum q_{sf} \quad (114)$$

731 By applying the product rule  $D(m\mathbf{U}^2) = D(m\mathbf{U} \cdot \mathbf{U}) = 2\mathbf{U} \cdot D(m\mathbf{U})$ , we can  
 732 express the left-hand side of equation (114) as:

$$\frac{1}{V} \frac{D_f(m_f(e_f + 0.5 \mathbf{U}_f^2))}{Dt} = \frac{1}{V} \frac{D_f(m_f e_f)}{Dt} + \frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} \cdot \mathbf{U}_f \quad (115)$$

733 Combining equations (111), (114), and (115), we derive the final form of the  
 734 internal energy balance equation for the fluid phases as:

$$\frac{1}{V} \frac{D_f(m_f e_f)}{Dt} = \frac{1}{V} \left[ \frac{\partial(m_f e_f)}{\partial t} + \nabla \cdot (m_f e_f \mathbf{U}_f) \right] = \bar{\rho}_f p_f \frac{D_f v_f}{Dt} + \nabla \cdot \mathbf{q}_f + \sum q_{sf} \quad (116)$$

735 On the right hand side, the terms include the average pressure-volume work,  
 736 the average viscous dissipation, the thermal transport and the energy ex-  
 737 change between solid and fluid respectively. The heat flux is  $\mathbf{q}_f = \bar{\rho}_f \beta_f \nabla T_f$

738 with  $\beta_f$  being the thermal conductivity coefficient. To derive the internal  
 739 energy balance equation for the solid phase, we introduce the rate of the  
 740 internal energy for the thermoelastic materials as a function of elastic strain  
 741 tensor  $\boldsymbol{\epsilon}_s^e$  and temperature  $T_s$  as belows:

$$\frac{m_s}{V} \frac{D_s(e_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + \frac{D_s(e_s)}{D_s(T_s)} \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + c_v \frac{D_s(T_s)}{Dt} \quad (117)$$

742  $c_v$  is the specific heat at the constant volume of the solid materials. The total  
 743 energy balance equation for the mixture based on [38] can be written as:

$$\begin{aligned} & \frac{1}{V} \frac{D_f(m_f(e_f + 0.5\mathbf{U}_f^2))}{Dt} + \frac{1}{V} \frac{D_s(m_s(e_s + 0.5\mathbf{U}_s^2))}{Dt} = \nabla \cdot (-\phi_f p_f \mathbf{I}) \cdot \mathbf{U}_f \\ & + \nabla \cdot (\boldsymbol{\sigma}' - \phi_s p_f \mathbf{I}) \cdot \mathbf{U}_s + (-\phi_f p_f \mathbf{I}) : \nabla \mathbf{U}_f + \boldsymbol{\sigma}' : \nabla \mathbf{U}_s \\ & + (\bar{\rho}_f \mathbf{b}) \cdot \mathbf{U}_f + (\bar{\rho}_s \mathbf{b}) \cdot \mathbf{U}_s + \nabla \cdot \mathbf{q}_f + \nabla \cdot \mathbf{q}_s + \sum \mathbf{f}_d \cdot (\mathbf{U}_f - \mathbf{U}_s) \end{aligned} \quad (118)$$

745 Subtracting equation (118), (117) to equations (114) and (113), we obtained  
 746 the internal energy balance equation for solid phase as:

$$\boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + \frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \Delta W_s + \Delta W_{friction} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (119)$$

747 On the right hand side, the terms include the work rate from frictional sliding  
 748 between solid materials  $\Delta W_{friction}$ , thermal transport and energy exchange  
 749 between solid and fluid respectively. The heat flux is  $\mathbf{q}_s = \bar{\rho}_s \beta_s \nabla T_s$  with  $\beta_s$   
 750 being the thermal conductivity of the solid materials, the mechanical work  
 751 rate  $\Delta W_s = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s)}{Dt} = \boldsymbol{\sigma}' : \left( \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt} \right)$  computed from the constitutive  
 752 model with  $\boldsymbol{\epsilon}_s^p$  is the plastic strain tensor, . By subtracting the term  $\boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt}$ ,  
 753 we get the final form of the energy balance equation as:

$$\frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt} + \Delta W_{friction} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (120)$$

### 754 Advanced Fluid Pressure

755 The discretization of the pressure equation begins with the Lagrangian  
 756 cell face velocity and the equation for the pressure as:

$$\bar{\rho}_{f,FC} \frac{\mathbf{U}_{f,FC}^{n+1} - \mathbf{U}_{f,FC}^n}{dt} = n \nabla^{FC} P_{f,c}^{n+1} + \bar{\rho}_{f,FC} \mathbf{b} \quad (121)$$

757

$$\kappa \frac{dP}{dt} = \nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} \quad (122)$$

758 The divergence of the equation (121) with  $\nabla \cdot \mathbf{b} = 0$  is:

$$\nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} - \nabla^c \cdot \mathbf{U}_{f,FC}^n = \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} (P_{f,c}^n + \Delta P_{f,c}^n) \quad (123)$$

759 To solve this equation, we define the cell face intermediate velocity  $\mathbf{U}_{f,FC}^*$  as:

$$\bar{\rho}_{f,FC} \frac{\mathbf{U}_{f,FC}^* - \mathbf{U}_{f,FC}^n}{\Delta t} = n \nabla^{FC} P_{f,c}^n + \bar{\rho}_{f,FC} \mathbf{b} \quad (124)$$

760 The divergence of the equation (124) is:

$$\nabla^c \cdot \mathbf{U}_{f,FC}^* - \nabla^c \cdot \mathbf{U}_{f,FC}^n = \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} P_{f,c}^n \quad (125)$$

761 Combining equations (122, 123, 125), it leads to:

$$\left( \kappa - \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} \right) \Delta P_{fc}^n = -\nabla^c \cdot \mathbf{U}_{f,FC}^* \quad (126)$$

762 When the fluid is incompressible,  $\kappa$  approaches to zero and the equation  
763 (126) becomes the Poisson's equation for the incompressible fluid flow.

#### 764 *Momentum and Energy exchange with an implicit solver*

765 Considering the fluid momentum balance equation as:

$$(m\mathbf{U})_{f,FC}^{n+1} = (m\mathbf{U})_{f,FC}^n - \Delta t (Vn \nabla^{FC} P_{f,c}^n + m_f \mathbf{b}) + VK \Delta t (\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (127)$$

766 And assuming  $m_{f,FC}^{n+1} = m_{f,FC}^n$ , we get:

$$\mathbf{U}_{f,FC}^{n+1} = \mathbf{U}_{f,FC}^n - \Delta t \left( \frac{\nabla^{FC} P_{fc}^n}{\rho_{f,FC}^n} + \mathbf{b} \right) + \frac{\Delta t K}{\rho_{f,FC}^n} (\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (128)$$

767 As defined in the section 'Advanced Fluid Pressure', the cell face intermediate  
768 fluid velocity  $\mathbf{U}_{f,FC}^* = \Delta t (\nabla^{FC} P_{fc}^n / \rho_{f,FC}^n + \mathbf{b})$  is computed as:

$$\mathbf{U}_{f,FC}^{n+1} = \mathbf{U}_{f,FC}^* + \frac{\Delta t K}{\rho_{f,FC}^n} (\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (129)$$

769 Considering the solid momentum balance equation as:

$$(m\mathbf{U})_{s,FC}^{n+1} = (m\mathbf{U})_{s,FC}^n - \Delta t(V\nabla^{FC} \cdot \boldsymbol{\sigma}'^n - V(1-n)\nabla^{FC}P_{f,c}^n + m_s\mathbf{b}) - VK\Delta t(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (130)$$

770 We define the cell face intermediate solid velocity as  $\mathbf{U}_{s,FC}^* = \Delta t(\nabla^{FC} \cdot \boldsymbol{\sigma}_c'^n / \bar{\rho}_{s,FC} - \nabla^{FC}P_{f,c}^n / \rho_s + \mathbf{b})$  leading to:

$$\mathbf{U}_{s,FC}^{n+1} = \mathbf{U}_{s,FC}^* - \frac{\Delta t K}{\bar{\rho}_{s,FC}^n} (\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (131)$$

772 Combining equation (129) and (131) we get:

$$\begin{aligned} \mathbf{U}_{f,FC}^* + \Delta\mathbf{U}_{f,FC} &= \mathbf{U}_{f,FC}^* + \frac{\Delta t K}{\bar{\rho}_{f,FC}^n} (\mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} - \mathbf{U}_{f,FC}^* - \Delta\mathbf{U}_{f,FC}) \\ \mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} &= \mathbf{U}_{s,FC}^* - \frac{\Delta t K}{\bar{\rho}_{s,FC}^n} (\mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} - \mathbf{U}_{f,FC}^* - \Delta\mathbf{U}_{f,FC}) \end{aligned} \quad (132)$$

773 Rearranging the equation (132), it leads to the linear system of equations as  
774 belows:

$$\begin{vmatrix} (1 + \beta_{12,FC}) & -\beta_{12,FC} \\ -\beta_{21,FC} & (1 + \beta_{21,FC}) \end{vmatrix} \begin{vmatrix} \Delta\mathbf{U}_{f,FC} \\ \Delta\mathbf{U}_{s,FC} \end{vmatrix} = \begin{vmatrix} \beta_{12,FC}(\mathbf{U}_{s,FC}^* - \mathbf{U}_{f,FC}^*) \\ \beta_{21,FC}(\mathbf{U}_{f,FC}^* - \mathbf{U}_{s,FC}^*) \end{vmatrix}$$

775 Solving this linear equations with  $\beta_{12,FC} = (\Delta t K) / \bar{\rho}_{f,FC}^n$  and  $\beta_{21,FC} =$   
776  $(\Delta t K) / \bar{\rho}_{s,FC}^n$  with K is the momentum exchange coefficient. Similar derivation  
777 can be performed to computed the cell-center velocity increment leading  
778 to:

$$\begin{vmatrix} (1 + \beta_{12c}) & -\beta_{12c} \\ -\beta_{21c} & (1 + \beta_{21c}) \end{vmatrix} \begin{vmatrix} \Delta\mathbf{U}_{f,c} \\ \Delta\mathbf{U}_{sc} \end{vmatrix} = \begin{vmatrix} \beta_{12c}(\mathbf{U}_{sc}^* - \mathbf{U}_{f,c}^*) \\ \beta_{21c}(\mathbf{U}_{f,c}^* - \mathbf{U}_{sc}^*) \end{vmatrix}$$

779 with  $\beta_{12c} = (\Delta t K) / \bar{\rho}_{f,c}^n$  and  $\beta_{21c} = (\Delta t K) / \bar{\rho}_{sc}^n$  and the cell-centered interme-  
780 diate velocity can be calculated by:

$$\begin{aligned} \mathbf{U}_{f,c}^* &= \mathbf{U}_{f,c}^n + \Delta t \left( -\frac{\nabla P_{f,c}^{n+1}}{\bar{\rho}_{f,c}^n} + \frac{\nabla \cdot \boldsymbol{\tau}_{f,c}^n}{\bar{\rho}_{f,c}^n} + \mathbf{b} \right) \\ \mathbf{U}_{sc}^* &= \mathbf{U}_{sc}^n + \Delta t \left( \frac{\nabla \cdot \boldsymbol{\sigma}_c'^n}{\bar{\rho}_{sc}^n} - \frac{\nabla P_{f,c}^{n+1}}{\rho_s} + \mathbf{b} \right) \end{aligned} \quad (133)$$

781 For generalize multi materials i,j = 1:N, the linear equations is in the form  
782 as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta\mathbf{U}_i \\ \Delta\mathbf{U}_j \end{vmatrix} = \begin{vmatrix} \beta_{ij}(\mathbf{U}_i^* - \mathbf{U}_j^*) \\ \beta_{ji}(\mathbf{U}_j^* - \mathbf{U}_i^*) \end{vmatrix}$$

783 Similar approach applied for the energy exchange term leading to:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_i \\ \Delta T_j \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_i^n - T_j^n) \\ \eta_{ji}(T_j^n - T_i^n) \end{vmatrix}$$

784 with  $\eta$  is the energy exchange coefficient.

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