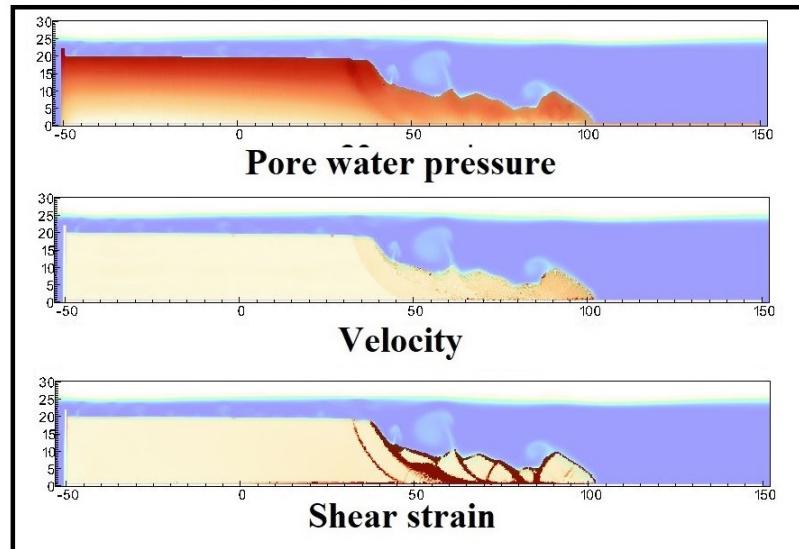


- 1 Graphical Abstract
- 2 MPMICE: A hybrid MPM-CFD model for simulating coupled problems in porous media. Application to earthquake-induced submarine landslides
- 3 Quoc Anh Tran, Gustav Grimstad, Seyed Ali Ghoreishian Amiri



**Application to earthquake-induced submarine landslide**

<sub>6</sub> Highlights

<sub>7</sub> **MPMICE: A hybrid MPM-CFD model for simulating coupled prob-**  
<sub>8</sub> **lems in porous media. Application to earthquake-induced subma-**  
<sub>9</sub> **rine landslides**

<sub>10</sub> Quoc Anh Tran, Gustav Grimstad, Seyed Ali Ghoreishian Amiri

- <sub>11</sub> • MPMICE is introduced for multiphase flow in porous media.
- <sub>12</sub> • Material Point method allows to model large deformation of non-isothermal
- <sub>13</sub> porous media.
- <sub>14</sub> • ICE (compressible multi-material CFD formulation) allows to stabilize
- <sub>15</sub> pore water pressure and turbulent flow.
- <sub>16</sub> • MPMICE is validated and apply to simulate the earthquake-induced
- <sub>17</sub> submarine landslide.

<sup>18</sup> MPMICE: A hybrid MPM-CFD model for simulating  
<sup>19</sup> coupled problems in porous media. Application to  
<sup>20</sup> earthquake-induced submarine landslides

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## <sup>22</sup> Abstract

<sup>23</sup> In this paper, we describe a soil-fluid-structure interaction model that com-  
<sup>24</sup> bines soil mechanics (saturated sediments), fluid mechanics (seawater or air),  
<sup>25</sup> and solid mechanics (structures). The formulation combines the Material  
<sup>26</sup> Point Method, which models large deformation of the porous media and the  
<sup>27</sup> structure, with the Implicit Continuous-fluid Eulerian, which models com-  
<sup>28</sup> plex fluid flows. We validate the model and simulate the whole process of  
<sup>29</sup> earthquake-induced submarine landslides. We show that this model captures  
<sup>30</sup> complex interactions between saturated sediment, seawater, and structure,  
<sup>31</sup> so we can use the model to estimate the impact of potential submarine land-  
<sup>32</sup> slides on offshore structures.

<sup>33</sup> *Keywords:*

<sup>34</sup> Material Point Method, MPMICE, submarine landslide.

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<sup>75</sup> **Nomenclature**

**General variables**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$V$	$[L^3]$	Representative volume
$n$		Porosity
$\sigma$	$[ML^{-1}t^{-2}]$	Total stress tensor
$\Delta t$	$[t]$	Time increment
$\mathbf{b}$	$[ML^1t^{-2}]$	Body force
$c_v$	$[L^2t^{-2}T^{-1}]$	Constant volume specific heat
$f_d$	$[MLt^{-2}]$	Drag forces in momentum exchange term
$f^{int}$	$[MLt^{-2}]$	Internal forces
$f^{ext}$	$[MLt^{-2}]$	External forces
$q_{fs}$	$[MLt^{-2}]$	Heat exchange term
$S$		Weighting function
$\nabla S$		Gradient of weighting function

<sup>76</sup>

**Solid phase**

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$m_s$	$[M]$	Solid mass
$\rho_s$	$[ML^{-3}]$	Solid density
$\phi_s$		Solid volume fraction
$\bar{\rho}_s$	$[ML^{-3}]$	Bulk Solid density
$\mathbf{x}_s$	$[L]$	Solid Position vector
$\mathbf{U}_s$	$[Lt^{-1}]$	Solid Velocity vector
$\mathbf{a}_s$	$[Lt^{-2}]$	Solid Acceleration vector
$\sigma'$	$[ML^{-1}t^{-2}]$	Effective Stress tensor
$\epsilon_s$		Strain tensor
$e_s$	$[L^2t^{-2}]$	Solid Internal energy per unit mass
$T_s$	$[T]$	Solid Temperature
$\mathbf{F}_s$		Solid Deformation gradient
$V_s$	$[L^3]$	Solid Volume

<sup>77</sup>

### Fluid phase

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$m_f$	[M]	Fluid mass
$\rho_f$	[ML <sup>-3</sup> ]	Fluid density
$\phi_f$		Fluid volume fraction
$\bar{\rho}_f$	[ML <sup>-3</sup> ]	Bulk Fluid density
$\mathbf{U}_f$	[Lt <sup>-1</sup> ]	Fluid Velocity vector
$\boldsymbol{\sigma}_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid stress tensor
$p_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid isotropic pressure
$\boldsymbol{\tau}_f$	[ML <sup>-1</sup> t <sup>-2</sup> ]	Fluid shear stress tensor
$e_f$	[L <sup>2</sup> t <sup>-2</sup> ]	Fluid Internal energy per unit mass
$T_f$	[T]	Fluid Temperature
$v_f$	[L <sup>3</sup> /M]	Fluid Specific volume $\frac{1}{\rho_f}$
$\alpha_f$	[1/T]	Thermal expansion
$\mu$	[ML <sup>-1</sup> t <sup>-1</sup> ]	Fluid viscosity
$V_f$	[L <sup>3</sup> ]	Fluid Volume

### Superscript

<u>Variable</u>	<u>Dimensions</u>	<u>Description</u>
$n$		Current time step
$L$		Lagrangian values
$n + 1$		Next time step

### Subscript

$c$	Cell-centered quantity
$p$	Particle quantity
$i$	Node quantity
$FC$	Face-centered quantity
$L, R$	Left and Right cell faces

78     **Introduction**

79     Many geological natural processes and their interactions with man-made  
80     structures are influenced by soil-fluid-structure interactions. The prediction  
81     of these processes requires a tool that can capture complex interactions  
82     between soil, fluid, and structure, such as the process of submarine land-  
83     slides. Indeed, The offshore infrastructure as well as coastal communities  
84     may be vulnerable to submarine landslides. Submarine landslides contain  
85     three stages: triggering, failure, and post-failure. Erosion or earthquakes can  
86     trigger slope failures in the first stage. Following the failure, sediments move  
87     quickly after the post-failure stage. In other words, solid-like sediments will  
88     behave like a fluid after failure. This phase transition makes the simulation  
89     of submarine landslides a challenging task.

90  
91     Due to this phase transition, submarine landslide can be modeled by ei-  
92     ther the Computational Fluid Dynamics (CFD) or the particle-based meth-  
93     ods. For simulating submarine slides, CFD methods solve governing equa-  
94     tions in a full-Eulerian framework [1, 2, 3, 4] with interface capturing tech-  
95     niques. While CFD can handle complex flows (such as turbulent flows), it  
96     cannot account for the triggering mechanism of submarine landslides because  
97     it is not straightforward to consider 'soil constitutive laws' of sediment ma-  
98     terials in the Eulerian framework. In contrast, particle-based methods can  
99     overcome this problem by using the Lagrangian framework. These meth-  
100   ods have been extensively used to simulate landslides, like Material Point  
101   Method (MPM) [5], Smooth Particle Hydro Dynamics [6], Particle Finite  
102   Element Method [7], or Coupled Eulerian Lagrangian Method [8]. For sim-  
103   plicity, these simulations adopt total stress analysis which neglects the pore  
104   pressure development which is key factor triggering slope failures.

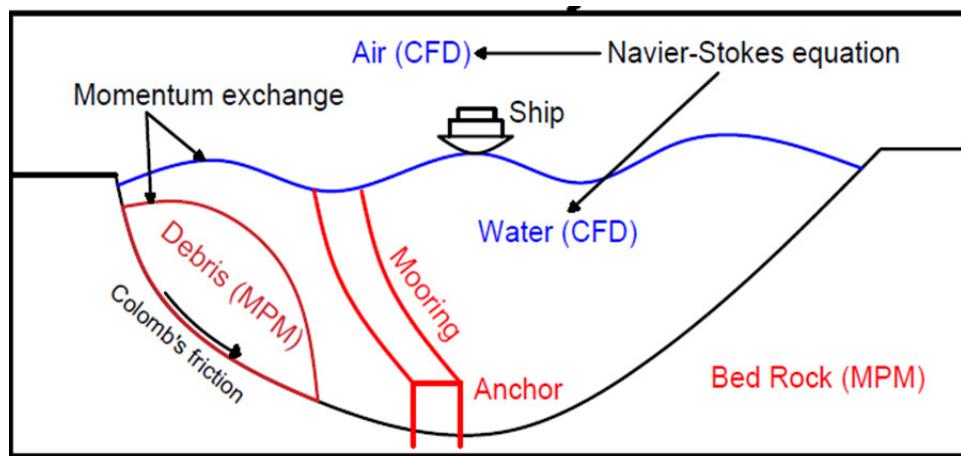
105  
106   Recent developments in particle-based methods model the coupling of  
107   fluid flows in porous media by sets of Lagrangian particles. For the MPM  
108   family, it is the double-point MPM ([9, 10, 11]) where fluid particles and  
109   solid particles are overlapped in a single computational grid. Even if fluid  
110   flows are considered, particle-based methods have numerical instability in  
111   modeling the fluid flow, which requires additional numerical treatments such  
112   as the B-bar method [9], null-space filter [12], or least square approximation  
113   [13, 14]. Indeed, CFD is a more optimal option for complex fluid flows  
114   especially dealing with large distortions of continuum fluid media. Therefore,

115 it could be ideal to combine the CFD with particle-based methods. More than  
 116 50 particle-based methods have been developed to solve large deformations  
 117 of solids over the last two decades [15], but the MPM appears to be the  
 118 best candidate for coupling with the CFD. Because MPM incorporates a  
 119 stationary mesh during computation, just like CFD. As such, both MPM  
 120 and CFD can be coupled naturally in a unified computational mesh.



*Figure 1: Interaction between soil-fluid-structure*

121



*Figure 2: Coupling of soil-water-structure interaction using MPMICE*

122 A numerical method for simulating soil-fluid-structure interaction (Figure  
123 1) involving large deformations, is presented in this work in order to simu-  
124 late the interaction between sediment (soil), seawater (fluid) and offshore  
125 structures (structure) namely MPMICE (Figure 2). In the MPMICE, the  
126 Material Point Method (MPM) is coupled with the Implicit Continuous Eu-  
127 lerian (ICE). The MPM method is a particle method that allows the porous  
128 soil to undergo arbitrary distortions. The ICE method, on the other hand,  
129 is a conservative finite volume technique with all state variables located at  
130 the cell center (temperature, velocity, mass, pressure). An initial technical  
131 report [16] at Los Alamos National Laboratory provided the theoretical and  
132 algorithmic foundation for the MPMICE, followed by the MPMICE devel-  
133 opment and implementation in the high-performance Uintah computational  
134 framework for simulating fluid-structure interactions [17]. This paper pri-  
135 marily contributes futher to the development of the MPMICE for analyzing  
136 the **soil**-fluid-structure interaction, since sediment should be considered as a  
137 porous media (soil) and not as a solid to capture the evolution of the pore  
138 water pressure. Baumgarten et al. [18] made the first attempt at coupling  
139 the Finite Volume Method with the MPM for the simulation of soil-fluid  
140 interaction. In contrast to the mentioned work, we use implicit time inte-  
141 gration for the multi phase flows instead of explicit time integration for the  
142 single-phase flow.

### 143 **Theory and formulation**

144 This section lay out the theoretical framework for the MPMICE model.  
145 We use the common notation of the continuum mechanics with vector and  
146 tensor denoted simply by using bold font and scalar denoted by using normal  
147 font. The notation are shown in Nomenclature.

### 148 *Assumptions*

149 The following assumptions are made for the MPMICE model.

- 150 1. Solid phases (MPM) are described in a Lagrangian formulation while  
151 fluid phases (ICE) are described in an Eulerian formulation in the  
152 framework of continuum mechanics and mixture theory.
- 153 2. Solid grains are incompressible while the fluid phases are compressible.  
154 Solid's thermal expansion is negligible.
- 155 3. There is no mass exchange between solid and fluid phases.
- 156 4. Terzaghi's effective stress is valid.

157 *Governing equations*

158 A representative element volume  $\Omega$  is decomposed by two domains: solid  
159 domains  $\Omega_s$  and fluid domains  $\Omega_f$ . Then, all domains are homogenized  
160 into two overlapping continua. Considering the volume fraction of solid  
161  $\phi_s = \Omega_s/\Omega$  and fluid  $\phi_f = \Omega_f/\Omega$  with the true (or Eulerian) porosity  
162  $n = \sum \phi_f$  of the representative element volume, the average density of solid  
163 and fluid phases are defined as:

164

$$\bar{\rho}_s = \phi_s \rho_s, \quad \bar{\rho}_f = \phi_f \rho_f \quad (1)$$

165 The mass of solid and fluid phases are:

166

$$m_s = \int_{\Omega_s} \rho_s dV = \bar{\rho}_s V, \quad m_f = \int_{\Omega_f} \rho_f dV = \bar{\rho}_f V \quad (2)$$

167 Reviewing the Terzaghi's effective stress concept for the saturated porous  
168 media, the total stress  $\boldsymbol{\sigma}$  is calculated by:

169

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - p_f \mathbf{I} \quad (3)$$

170 The balance equations are derived based on the mixture theory. The rep-  
171 resentative thermodynamic state of the fluid phases are given by the vector  
172  $[m_f, \mathbf{U}_f, e_f, T_f, v_f]$  which are mass, velocity, internal energy, temperature,  
173 specific volume. The representative state of the solid phases are given by the  
174 vector  $[m_s, \mathbf{U}_s, e_s, T_s, \boldsymbol{\sigma}', p_f]$  which are mass, velocity, internal energy, temper-  
175 ature, effective stress and pore water pressure. The derivation is presented  
176 in detail in the Appendix.

177

---

178 Mass Conservation

179 The mass balance equations for both fluid (e.g., water, air) and solid phases  
180 are:

181

$$\frac{1}{V} \frac{\partial m_f}{\partial t} + \nabla \cdot (m_f \mathbf{U}_f) = 0, \quad \frac{1}{V} \frac{D_s m_s}{Dt} = 0 \quad (4)$$

182 Solving the mass balance equation leads to:

$$\frac{D_s n}{Dt} = \phi_s \nabla \cdot \mathbf{U}_s \quad (5)$$

183

---

184 Momentum Conservation

<sup>185</sup> The momentum balance equation for the fluid phases (e.g., water, air) are:

<sup>186</sup>

$$\frac{1}{V} \left[ \frac{\partial(m_f \mathbf{U}_f)}{\partial t} + \nabla \cdot (m_f \mathbf{U}_f \mathbf{U}_f) \right] = -\phi_f \nabla p_f + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d \quad (6)$$

<sup>187</sup> The momentum balance equation for the solid phases are:

<sup>188</sup>

$$\frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}') - \phi_s \nabla p_f + \bar{\rho}_s \mathbf{b} + \sum \mathbf{f}_{fric} - \sum \mathbf{f}_d \quad (7)$$

---

<sup>189</sup> **Energy Conservation**

<sup>190</sup> The internal energy balance equation for the fluid phases (e.g., water, air)  
<sup>191</sup> are:

$$\frac{1}{V} \left[ \frac{\partial(m_f e_f)}{\partial t} + \nabla \cdot (m_f e_f \mathbf{U}_f) \right] = -\bar{\rho}_f p_f \frac{D_f v_f}{Dt} + \boldsymbol{\tau}_f : \nabla \mathbf{U}_f + \nabla \cdot \mathbf{q}_f + \sum q_{sf} \quad (8)$$

<sup>192</sup> The internal energy balance equation for the solid phase is:

$$\frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (9)$$

<sup>193</sup> where  $c_v$  is the specific heat at constant volume of the solid materials.

<sup>194</sup> Closing the systems of equations, the following additional models are needed:

<sup>195</sup> (1) A constitutive equation to describe the stress - strain behaviour of solid  
<sup>196</sup> phase (computing effective stress  $\boldsymbol{\sigma}'$ ).

<sup>197</sup> (2) Optional turbulent model to compute the viscous shear stress  $\boldsymbol{\tau}_f$ .

<sup>198</sup> (3) Frictional forces  $\mathbf{f}_{fric}$  for the contact for soil-structure interaction be-  
<sup>199</sup> tween solid/porous materials with the friction coefficient  $\mu_{fric}$ .

<sup>200</sup> (4) Exchange momentum models (computing drag force  $\mathbf{f}_d$ ) for interaction  
<sup>201</sup> between materials.

<sup>202</sup> (5) Energy exchange models (computing temerature exhange term  $q_{sf}$ ) for  
<sup>203</sup> interaction between materials.

<sup>204</sup> (6) An equation of state to establish relations between thermodynamics vari-  
<sup>205</sup> ables of each fluid materials  $[P_f, \bar{\rho}_f, v_f, T_f, e_f]$ .

208 Four thermodynamic relations for the equation of states are:

$$\begin{aligned} e_f &= e_f(T_f, v_f) \\ P_f &= P_f(T_f, v_f) \\ \phi_f &= v_f \bar{\rho}_f \\ 0 &= n - \sum_{f=1}^{N_f} v_f \bar{\rho}_f \end{aligned} \quad (10)$$

209 *Constitutive soil model*

210 As a result of the explicit MPM formulation, we can derive the consti-  
 211 tutive law in the updated Lagrangian framework of "small strain - large  
 212 deformation". Therefore, the rotation of the particles (representative ele-  
 213 ment volume) is manipulated by rotating the Cauchy stress tensor. First,  
 214 the deformation gradient is decomposed into the polar rotation tensor  $\mathbf{R}_s^{n+1}$   
 215 and sketch tensor  $\mathbf{V}_s^{n+1}$  as

$$\mathbf{F}_s^{n+1} = \mathbf{V}_s^{n+1} \mathbf{R}_s^{n+1} \quad (11)$$

216 Then, before calling the constitutive model, the stress and strain rate tensor  
 217 are rotated to the reference configuration as

$$\boldsymbol{\sigma}'^{n*} = (\mathbf{R}_s^{n+1})^T \boldsymbol{\sigma}'^{n*} \mathbf{R}_s^{n+1} \quad (12)$$

$$\delta\boldsymbol{\epsilon}^{n*} = (\mathbf{R}_s^{n+1})^T \delta\boldsymbol{\epsilon}_s^{n*} \mathbf{R}_s^{n+1} \quad (13)$$

218 Using the constitutive model with the input tensors  $\boldsymbol{\sigma}'^{n*}, \delta\boldsymbol{\epsilon}^{n*}$  to compute  
 219 the Cauchy stress tensor at the advanced time step  $\boldsymbol{\sigma}'^{n+1*}$  then rotating it  
 220 back to current configuration

$$\boldsymbol{\sigma}'^{n+1} = \mathbf{R}_s^{n+1} \boldsymbol{\sigma}'^{n+1*} (\mathbf{R}_s^{n+1})^T \quad (14)$$

221 In this paper, we adopt the hyper-elastic Neo Hookean model for the structure  
 222 materials and additionally Mohr-Coulomb failure criteria for the soil (porous  
 223 media) materials. The Cauchy stress of the hyper-elastic Neo Hookean model  
 224 can be written as:

$$\boldsymbol{\sigma}' = \frac{\lambda \ln(J)}{J} + \frac{\mu}{J} (\mathbf{F} \mathbf{F}^T - \mathbf{J}) \quad (15)$$

225 where  $\lambda$  and  $\mu$  are bulk and shear modulus ad  $J$  is the determinant of the  
 226 deformation gradient  $\mathbf{F}$ . And the yield function  $f$  and flow potentials  $g$  of

<sup>228</sup> the Mohr-Coulomb can be written as:

$$\begin{aligned} f &= \sigma'_1 - \sigma'_3 - 2c' \cos(\phi') - (\sigma'_1 + \sigma'_3) \sin(\phi') \\ g &= \sigma'_1 - \sigma'_3 - 2c' \cos(\psi') - (\sigma'_1 + \sigma'_3) \sin(\psi') \end{aligned} \quad (16)$$

<sup>229</sup> where the  $c'$ ,  $\phi'$  and  $\psi'$  are cohesion and friction angle and dilation angle.  $\sigma'_1$   
<sup>230</sup> and  $\sigma'_3$  are maximum and minimum principal stress.

<sup>231</sup> *Turbulent model*

<sup>232</sup> The turbulent effect is modelled using a statistical approach namely large-  
<sup>233</sup> eddy simulation. In this approach, the micro-scale turbulent influence in the  
<sup>234</sup> dynamics of the macro-scale motion is computed through simple models like  
<sup>235</sup> Smagorinsky model. In the Smagorinsky mode, the residual stress tensor is:

$$\tau_{ij} = 2\mu_{eff}(\bar{S}_{ij} - \frac{1}{3}\delta_{ij}\bar{S}_{kk}) + \frac{1}{3}\delta_{ij}\tau_{kk} \quad (17)$$

<sup>236</sup> where the strain rate tensor is given by

$$\bar{S}_{ij} = \frac{1}{2}(\frac{\delta \bar{U}_i}{\delta x_j} + \frac{\delta \bar{U}_j}{\delta x_i}) \quad (18)$$

<sup>237</sup> and the effective viscosity is sum of molecular viscosity and turbulent viscosity  
<sup>238</sup>  $\mu_{eff} = \mu + \mu_t$  in which the turbulent viscosity  $\mu_t$  is calculated by

$$\mu_t = (C_s \Delta)^2 \sqrt{2\bar{S}_{ij}\bar{S}_{ij}} \quad (19)$$

<sup>239</sup> where  $C_s$  is the Smagorinsky constant and  $\Delta = \sqrt[3]{dxdydz}$  is the grid size  
<sup>240</sup> that defines the subgrid length scale.

<sup>241</sup> *Frictional force for soil-structure interaction*

<sup>242</sup> MPMICE includes a contactlaw for the interaction between soil and struc-  
<sup>243</sup> ture using the first Coulomb friction contact for MPM presented by Barden-  
<sup>244</sup> hagen et al. ([19]). The magnitude of the friction force at the contact depends  
<sup>245</sup> on the friction coefficient  $\mu_{fric}$  and the normal force  $\mathbf{f}_{norm}$  computed from  
<sup>246</sup> the projection of the contact force in the normal direction.

$$\mathbf{f}_{fric} = \mu_{fric} \mathbf{f}_{norm} \quad (20)$$

247 The contact determines whether the soil is sliding or sticking to the structure  
 248 by comparing the friction force with the sticking force  $\mathbf{f}_{stick}$  computed from  
 249 the projection of the contact force in the tangent direction:

$$\begin{aligned} \text{if } \mathbf{f}_{fric} &\geq \mathbf{f}_{stick} \text{ no sliding} \\ \text{if } \mathbf{f}_{fric} &< \mathbf{f}_{stick} \text{ sliding occurs} \end{aligned} \quad (21)$$

250 Frictional sliding between solid materials also generates dissipation and the  
 251 work rate generated from the sliding can be calculated as:

$$\Delta W_{friction} = \mathbf{f}_{fric} d \quad (22)$$

252 where  $d$  is the sliding distance which can be computed based on the sliding  
 253 velocity between two materials.

254 *Momentum and Energy exchange model*

255 Currently, the energy exchange coefficient  $H_{sf}$  is assumed to be constant  
 256 for the sake of simplicity. Then the energy exchange can be written as:

$$q_{sf} = H_{sf}(T_f - T_s) \quad (23)$$

257 On the other hand, the drag force can be calculated as:

$$f_d = K(\mathbf{U}_s - \mathbf{U}_f) \quad (24)$$

258 For the momentum exchange between fluid flows and porous media, we as-  
 259 sume that the drag force  $\mathbf{f}_d$  depends on the average grain size of the grains  
 260  $D_p$ , the porosity  $n$ , the fluid viscosity  $\mu_f$ , and is proportional to the relative  
 261 velocities of soil grains and fluid  $(\mathbf{U}_s - \mathbf{U}_f)$ . Based on recent investigation  
 262 of CFD simulations of fluid flow around mono- and bi-disperse packing of  
 263 spheres for  $0.1 < \phi_s < 0.6$  and  $Re < 1000$  [20]. The drag force is given by:

$$\mathbf{f}_d = \frac{18\phi_s(1-\phi_s)\mu_f}{D_p^2} F(\phi_s, Re)(\mathbf{U}_s - \mathbf{U}_f) \quad (25)$$

264 where Reynolds number  $Re$  are computed as:

$$Re = \frac{n\rho_f D_p}{\mu_f} \|(\mathbf{U}_s - \mathbf{U}_f)\| \quad (26)$$

266 The function  $F(\phi_s, Re)$  can be calculated as:

$$F(\phi_s, Re) = F(\phi_s, 0) + \frac{0.413Re}{24(1 - \phi_s)^2} \left( \frac{(1 - \phi_s)^{-1} + 3\phi_s(1 - \phi_s) + 8.4Re^{-0.343}}{1 + 10^{3\phi_s} Re^{-(1+4\phi_s)/2}} \right) \quad (27)$$

267 where the low Reynold coefficient  $F(\phi_s, Re \rightarrow 0)$  is:

$$F(\phi_s, 0) = \frac{10\phi_s}{(1 - \phi_s)^2} + (1 - \phi_s)^2(1 + 1.5\sqrt{\phi_s}) \quad (28)$$

268 When validating the model with analytical solution, it requires to know the  
269 hydraulic conductivity. In such case, we convert the equation (25) to Kozeny-  
270 Carman formula by assuming  $F(\phi_s, Re) = 10\phi_s/(1 - \phi_s)^2$ , then the hydraulic  
271 conductivity will be expressed as  $D_p^2(1 - \phi_s)^3/180\mu\phi_s^2$ .

272 *Solving momentum and energy exchange with an implicit solver*

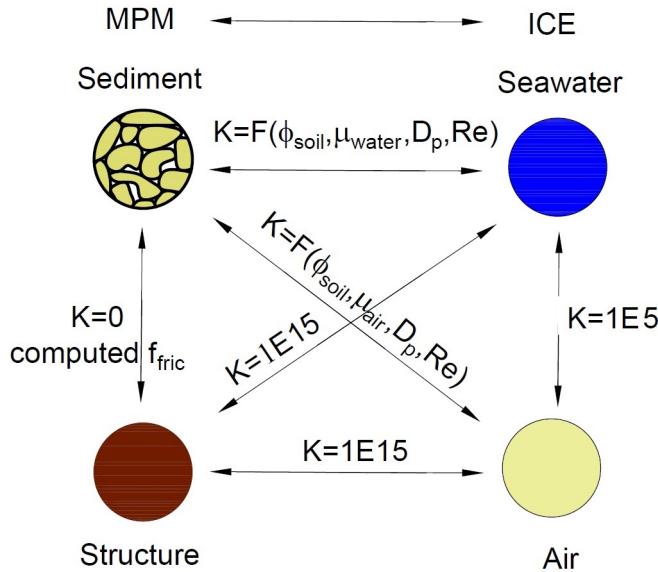


Figure 3: Momentum exchange coefficient between materials

273 The derivation of the implicit integration for the momentum exchange is  
274 presented in the Appendix's section 'Momentum and energy exchange with  
275 an implicit solver'. The linear equations for multi phases i,j=1:N has the

276 form:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_i \\ \Delta \mathbf{U}_j \end{vmatrix} = \begin{vmatrix} \beta_{ij} (\mathbf{U}_i^* - \mathbf{U}_j^*) \\ \beta_{ji} (\mathbf{U}_i^* - \mathbf{U}_j^*) \end{vmatrix}$$

277 where the intermediate velocity for fluid phases  $f=1:N_f$  and for solid/porous  
278 phases  $s=1:N_s$  can be calculated by

$$\begin{aligned} \mathbf{U}_f^* &= \mathbf{U}_f^n + \Delta t \left( -\frac{\nabla P_f^{n+1}}{\rho_f^n} + \frac{\nabla \cdot \boldsymbol{\tau}_f^n}{\bar{\rho}_f^n} + \mathbf{b} \right) \\ \mathbf{U}_s^* &= \mathbf{U}_s^n + \Delta t \left( \frac{\nabla \cdot \boldsymbol{\sigma}'^n}{\bar{\rho}_s^n} - \frac{\nabla P_f^{n+1}}{\rho_s} + \mathbf{b} \right) \end{aligned} \quad (29)$$

279 Also, the momentum exchange coefficient can be computed at every time  
280 step as  $\beta_{12} = K/\bar{\rho}_f^n$  and  $\beta_{21} = K/\bar{\rho}_s^n$  with the coefficient depending on the  
281 different type of interactions (see Figure 3) as for example:

282

- 283 1. The drag force is set to zero in soil-structure interactions, and instead  
284 the frictional force is computed.
- 285 2. As a result of fluid-structure interaction, the momentum exchange co-  
286 efficient should be extremely high when the solid material points are  
287 considered to be zero-porosity/zero-permeability.
- 288 3. In the case of soil-fluid interaction, the drag force is calculated using  
289 the equation (25). Considering that air has a much lower viscosity than  
290 water, its drag force is much lower than the drag force of water in a  
291 pore.
- 292 4. A momentum exchange coefficient of 1E5 is applied between multiphase  
293 flows. This value is far higher than reality [21], but it is necessary to  
294 have enough numerical stability to conduct simulations in the numerical  
295 example.

296 Similar approach applied for the energy exchange term leading to:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_i \\ \Delta T_j \end{vmatrix} = \begin{vmatrix} \eta_{ij} (T_i^n - T_j^n) \\ \eta_{ji} (T_j^n - T_i^n) \end{vmatrix}$$

297 with  $\eta$  is the energy exchange coefficient.

298

299    *Equation of state for fluid phases*

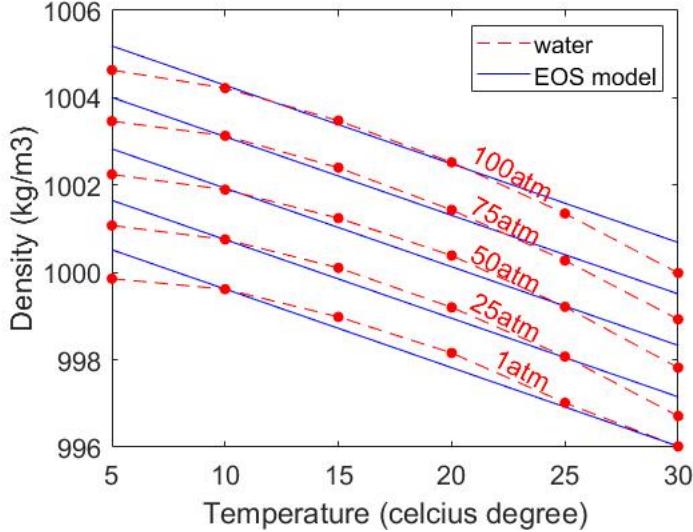


Figure 4: Equation of state of water

300    The equation of state establishes relations between thermodynamics vari-  
 301    ables  $[P_f, \rho_f, T_f]$ . The choice of the equation of state depends on the types  
 302    of the fluid materials. For example, for the air, it is possible to assume the  
 303    equation of state for the perfect gas which obeys:

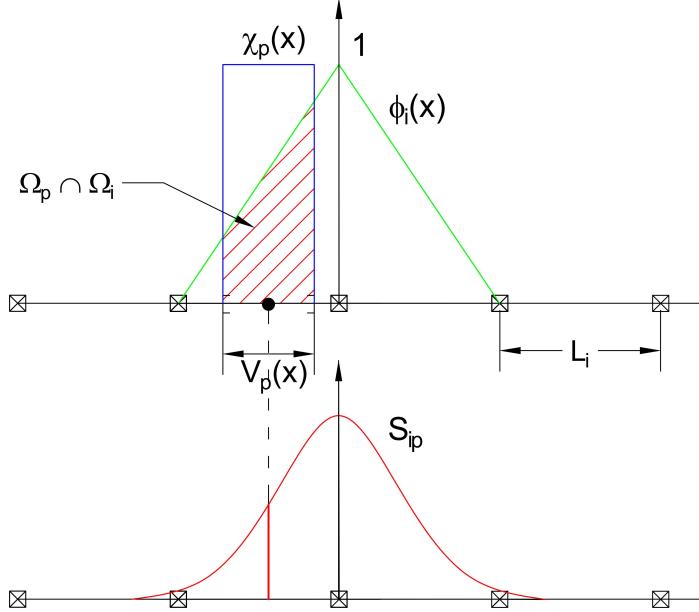
$$P_f = \rho_f R T_f \quad (30)$$

304    where  $R$  is the gas constant. For the water, a simple linear equation of state  
 305    is in the following form:

$$P_f = P_{ref} + K_f(\rho_f - \rho_{ref} - \alpha_f(T_f - T_{ref})) \quad (31)$$

306    where reference pressure  $P_{ref} = 1 \text{ atm} = 101325 \text{ Pa}$ , reference temperature  
 307     $T_{ref} = 10^\circ\text{C}$ , reference density  $\rho_{ref} = 999.8 \text{ kg/m}^3$ , the bulk modulus of water  
 308     $K_f = 2 \text{ GPa}$ , and the water thermal expansion  $\alpha_f = 0.18 \text{ }^\circ\text{C}^{-1}$ . Equation  
 309    (31) matches well with the state of the water (see Figure 4).

310 **Numerical implementation**



*Figure 5: GIMP weighting function (red as a convolution of the linear basis shape function (green) and the characteristic function (blue))*

311 The fluid phases are discretized in the grid with the state variables stored  
312 at the centroid of the cells  $[\rho_{fc}, \mathbf{U}_{fc}, T_{fc}, v_{fc}]$  while the solid phase is dis-  
313 cretized in the particles with the state variables  $[m_{sp}, \mathbf{U}_{sp}, T_{sp}, \boldsymbol{\sigma}'_{sp}]$ . In the  
314 Material Point Method, we use the generalized interpolation technique [22]  
315 using the weight function as a convolution of a grid shape function  $N_i(\mathbf{x})$  in  
316 a nodal domain  $\Omega_i$  and a characteristic function  $\chi_p(\mathbf{x})$  in a particle domain  
317  $\Omega_p$  with the volume  $V_p(\mathbf{x})$  as follows:

$$S_{ip} = \frac{1}{V_p} \int_{\Omega_i \cap \Omega_p} N_i(\mathbf{x}) \chi_p(\mathbf{x}) d\mathbf{x} \quad (32)$$

318 where the volume  $V_p(\mathbf{x})$  of the material point p can be calculated as:

$$V_p = \int_{\Omega_p} \chi_p(\mathbf{x}) d\mathbf{x} \quad (33)$$

319 The characteristic function is the Heaviside function as  $\chi_p = 1$  if  $\mathbf{x} \in \Omega_p$ ,  
 320 otherwise 0 (see Figure 5). For the interpolation of the centroid of the cell,  
 321 the linear basis function is used as:

$$S_{ci} = N_i(\mathbf{x}_c) \quad (34)$$

322 The time discretization are solved using the following steps:

323 *Interpolation from Solid Particle to Grid*

324 The nodal values of the solid state (mass, velocity, temperature, volume)  
 325 are:

$$\begin{aligned} m_{si}^n &= \sum S_{ip} m_{sp} \\ \mathbf{U}_{si}^n &= \frac{\sum S_{ip} (m\mathbf{U})_{sp}^n}{m_{si}^n} \\ T_{si}^n &= \frac{\sum S_{ip} (mT)_{sp}^n}{m_{si}^n} \\ V_{si}^n &= \frac{\sum S_{ip} (mV)_{sp}^n}{m_{si}^n} \\ \boldsymbol{\sigma}_{si}^n &= \frac{\sum S_{ip} (\boldsymbol{\sigma}V)_{sp}^n}{V_{si}^n} \end{aligned} \quad (35)$$

326 The nodal internal forces is calculated by

$$\mathbf{f}_{si}^{int,n} = - \sum \nabla S_{ip} (\boldsymbol{\sigma}'_{sp})^n V_{sp}^n \quad (36)$$

327 The nodal external forces  $f_{si}^{ext,n}$  and extra momentum from contact forces  
 328 are computed here. The nodal velocity and nodal temperature are applied  
 329 boundary conditions.

330 Then we compute the solid cell variables as:

$$\begin{aligned} m_{sc}^n &= \sum S_{ci} m_{si} \\ \rho_{sc}^n &= \frac{m_{sc}^n}{V} \\ \mathbf{U}_{sc}^n &= \sum S_{ci} \mathbf{U}_{si}^n \\ T_{sc}^n &= \sum S_{ci} T_{si}^n \\ V_{sc}^n &= \sum S_{ci} V_{si}^n \\ \boldsymbol{\sigma}_{sc}^n &= \sum S_{ci} \boldsymbol{\sigma}_{si}^n \end{aligned} \quad (37)$$

331    *Compute equation of state for fluid phase*

332    Considering the total fluid material volume of a cell is:

$$V_{total} = \sum_{f=1}^{N_f} M_f v_f \quad (38)$$

333    Then we need to find  $P_{eq}$  which allows each fluid materials obey their equation  
 334    of states  $[P_f, \rho_f, v_f, T_f, e_f]$  but also allow mass of all fluid materials to fill the  
 335    entire the pore volume without ongoing compression or expansion following  
 336    the condition:

$$0 = n - \sum_{f=1}^{N_f} v_f \bar{\rho}_f \quad (39)$$

337    Then, we can use he Newton-Raphson interation to find the value of  $P_{eq}$   
 338    which satisfies the equation (38, 39) and each equation of states of each fluid  
 339    materials.

340    *Compute faced-centered velocity*

341    Following the derivation in the Appendix: Advanced Fluid Pressure, we  
 342    first compute the fluid face-centered velocity as

$$\mathbf{U}_{f,FC}^* = \frac{(\bar{\rho}\mathbf{U})_{f,FC}^n}{\bar{\rho}_{f,FC}^n} + \Delta t \left( -\frac{\nabla^{FC} P_{eq}}{\rho_{f,FC}^n} + \frac{\nabla^{FC} \cdot \boldsymbol{\tau}^n}{\bar{\rho}_{s,FC}} + \mathbf{b} \right) \quad (40)$$

344    The equation (40) is discretized in three dimension (noted that  $\nabla^{FC} \cdot \boldsymbol{\tau} = 0$ ),  
 345    for example the discretized equation in the x direction is

$$U_{fx}^* = \frac{(\bar{\rho}U)_{fx,R}^n + (\bar{\rho}U)_{fx,L}^n}{\bar{\rho}_{fx,L}^n + \bar{\rho}_{fx,R}^n} + \Delta t \left( -\frac{2(v_{fx,L}^n v_{fx,R}^n)}{v_{fx,L}^n + v_{fx,R}^n} \frac{P_{eqx,R} - P_{eqx,L}}{\Delta x} + b_x \right) \quad (41)$$

346    The face-centered solid velocity can be calculated as

$$\mathbf{U}_{s,FC}^* = \frac{(\bar{\rho}\mathbf{U})_{s,FC}^n}{\bar{\rho}_{s,FC}^n} + \Delta t \left( \frac{\nabla^{FC} \cdot \boldsymbol{\sigma}_c'^n}{\bar{\rho}_{s,FC}} - \frac{\nabla^{FC} P_{eq}}{\rho_s} + \mathbf{b} \right) \quad (42)$$

347    The equation (42) is discretized in three dimension(noted that  $\nabla^{FC} \cdot \sigma_{ij} = 0$   
 348    with  $i \neq j$ ), for example the discretized equation in the x direction is

$$U_{sx}^* = \frac{(\bar{\rho}U)_{sx,R}^n + (\bar{\rho}U)_{sx,L}^n}{\bar{\rho}_{sx,L}^n + \bar{\rho}_{sx,R}^n} + \Delta t \left( \frac{2(\sigma_{xx,R} - \sigma_{xx,L})}{(\bar{\rho}_{sx,L}^n + \bar{\rho}_{sx,R}^n)\Delta x} - \frac{P_{eqx,R} - P_{eqx,L}}{\rho_s \Delta x} + b_x \right) \quad (43)$$

349 Computing the modified faced-centered velocity  $\mathbf{U}_{f,FC}^L$  considering the mo-  
 350 mentum exchange (see the Appendix: Momentum exchange with an implicit  
 351 solve)

$$\begin{aligned}\mathbf{U}_{f,FC}^L &= \mathbf{U}_{f,FC}^* + \Delta\mathbf{U}_{f,FC} \\ \mathbf{U}_{s,FC}^L &= \mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC}\end{aligned}\quad (44)$$

352 Solving the linear equation below to obtain the increment of velocity with i,j  
 353 = 1 : N as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta\mathbf{U}_{i,FC} \\ \Delta\mathbf{U}_{j,FC} \end{vmatrix} = \begin{vmatrix} \beta_{ij}(\mathbf{U}_{i,FC}^* - \mathbf{U}_{j,FC}^*) \\ \beta_{ji}(\mathbf{U}_{j,FC}^* - \mathbf{U}_{i,FC}^*) \end{vmatrix}$$

354 *Compute faced-centered temperature*

355 Similar to the velocity, the faced temperature is computed as:

$$T_{fx}^n = \frac{(\bar{\rho}T)_{fx,R}^n + (\bar{\rho}T)_{fx,L}^n}{\bar{\rho}_{fx,L}^n + \bar{\rho}_{fx,R}^n} \quad (45)$$

356 Computing the modified faced-centered temperature  $T_{f,FC}^L$  considering the en-  
 357 ergy exchange (see the Appendix: Momentum and energy exchange with an  
 358 implicit solver)

$$\begin{aligned}T_{f,FC}^L &= T_{f,FC}^n + \Delta T_{f,FC} \\ T_{s,FC}^L &= T_{s,FC}^n + \Delta T_{s,FC}\end{aligned}\quad (46)$$

359 Solving the linear equation below to obtain the increment of velocity with i,j  
 360 = 1 : N as:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_{i,FC} \\ \Delta T_{j,FC} \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_{i,FC}^n - T_{j,FC}^n) \\ \eta_{ji}(T_{j,FC}^n - T_{i,FC}^n) \end{vmatrix}$$

361 *Compute fluid pressure (implicit scheme)*

362 For single phase flow, the increment of the fluid pressure can be computed  
 363 as:

$$\kappa_f \frac{\Delta P}{dt} = \nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} \quad (47)$$

364 For multi-phase flows, the increment of the fluid pressure of the mixture can  
 365 be computed as:

$$\kappa \frac{\Delta P}{dt} = \sum_{f=1}^{N_f} \nabla^c \cdot (\phi_{f,FC} \mathbf{U}_{f,FC})^{n+1} \quad (48)$$

<sup>366</sup> where  $\kappa = \sum_{f=1}^{N_f} (\phi_{f,FC} \kappa_f) / \sum_{f=1}^{N_f} (\phi_{f,FC})$ . Then, the fluid pressure at cell  
<sup>367</sup> center is:

$$P_c^{n+1} = P_{eq} + \Delta P_c^n \quad (49)$$

<sup>368</sup> Finally, the faced-centered advanced fluid pressure is

$$P_{FC}^{n+1} = \left( \frac{P_{c,L}^{n+1}}{\bar{\rho}_{f,L}^n} + \frac{P_{c,R}^{n+1}}{\bar{\rho}_{f,R}^n} \right) / \left( \frac{1}{\bar{\rho}_{f,L}^n} + \frac{1}{\bar{\rho}_{f,R}^n} \right) = \left( \frac{P_{c,L}^{n+1} \bar{\rho}_{f,R}^n + P_{c,R}^{n+1} \bar{\rho}_{f,L}^n}{\bar{\rho}_{f,L}^n \bar{\rho}_{f,R}^n} \right) \quad (50)$$

<sup>369</sup> **Compute viscous shear stress term of the fluid phase**

<sup>370</sup> This part compute the viscous shear stress  $\Delta(m\mathbf{U})_{fc,\tau}$  for a single viscous  
<sup>371</sup> compressible Newtonian fluid and optionally shear stress induced by the tur-  
<sup>372</sup> bulent model.

<sup>373</sup> **Compute nodal internal temperature of the solid phase**

<sup>374</sup> The nodal internal temperature rate is computed based on the heat con-  
<sup>375</sup>duction model

$$dT_{si}^L = \frac{(\Delta W_{si}^n + \Delta W_{fric,i}^n + \nabla^i \cdot \mathbf{q}_{si}^n)}{m_{si}^n c_v} \quad (51)$$

<sup>376</sup> where  $\Delta W_{si}^n = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^p)}{Dt}$  is the mechanical work rate computed from the  
<sup>377</sup> constitutive model with  $\boldsymbol{\epsilon}_s^p$  is the plastic strain,  $\Delta W_{fric,i}^n$  is the work rate  
<sup>378</sup> compted from the contact law due to the frictional sliding between solid ma-  
<sup>379</sup>terials. The heat flux is  $\mathbf{q}_s = \bar{\rho}_s \beta_s \nabla T_s$  with  $\beta_s$  being the thermal conductivity  
<sup>380</sup> of the solid materials.

$$T_{si}^L = T_{si}^n + dT_{si}^L \quad (52)$$

<sup>381</sup> **Compute and integrate acceleration of the solid phase**

<sup>382</sup> After interpolating from material points to the nodes, the nodal acceler-  
<sup>383</sup>ation and velocity are calculate by

$$\mathbf{a}_{si}^{L-} = \frac{\mathbf{f}_{si}^{int,n} + \mathbf{f}_{si}^{ext,n}}{m_{si}^n} + \mathbf{g} \quad (53)$$

<sup>384</sup>  $\mathbf{U}_{si}^{L-} = \mathbf{U}_{si}^n + \mathbf{a}_{si}^{L-} \Delta t \quad (54)$

385 Compute Lagrangian value (mass, momentum and energy)

386 For the fluid phase, the linear momentum rate, the energy rate are

$$\Delta(m\mathbf{U})_{fc} = Vn_c^n \nabla^c P_c^{n+1} + \Delta(m\mathbf{U})_{fc,\tau} + V\bar{\rho}_{fc}^n g \quad (55)$$

387

$$\Delta(me)_{fc} = Vn_c^n P_c^{n+1} \nabla^c \cdot \mathbf{U}_{f,FC}^* + \nabla^c \cdot \mathbf{q}_{fc}^n \quad (56)$$

388 The Lagrangian value of the mass, linear momentum and energy of fluid  
389 phases without momentum exchange are

$$m_{fc}^L = V\bar{\rho}_{fc}^n \quad (57)$$

390

$$(m\mathbf{U})_{fc}^{L-} = V\bar{\rho}_{fc}^n \mathbf{U}_{fc}^n + \Delta(m\mathbf{U})_{fc} \quad (58)$$

391

$$(me)_{fc}^{L-} = V\bar{\rho}_{fc}^n T_{fc}^n c_v + \Delta(me)_{fc} \quad (59)$$

392 For the solid phase, the Lagrangian value of the linear momentum and energy  
393 of solid phase are

$$m_{sc}^L = m_{sc}^n \quad (60)$$

394

$$(m\mathbf{U})_{sc}^{L-} = \sum S_{ci} m_{si}^n \mathbf{U}_{si}^{L-} + V(1 - n_c^n) \nabla^c P_{fc}^{n+1} \quad (61)$$

395

$$(me)_{sc}^{L-} = \sum S_{ci} m_{si}^n T_{si}^L c_v \quad (62)$$

396 To consider the momentum exchange, the Lagrangian velocity is modified as

$$\begin{aligned} \mathbf{U}_{fc}^L &= \mathbf{U}_{fc}^{L-} + \Delta\mathbf{U}_{fc} \\ \mathbf{U}_{sc}^L &= \mathbf{U}_{sc}^{L-} + \Delta\mathbf{U}_{sc} \end{aligned} \quad (63)$$

397 where the cell-centered intermediate velocity can be calculated by

$$\begin{aligned} \mathbf{U}_{fc}^{L-} &= \frac{(m\mathbf{U})_{fc}^{L-}}{m_{fc}^L} \\ \mathbf{U}_{sc}^{L-} &= \frac{(m\mathbf{U})_{sc}^{L-}}{m_{sc}^L} \end{aligned} \quad (64)$$

398 And the increment of the velocity  $\mathbf{U}_{fc}$ ,  $\Delta\mathbf{U}_{sc}$  can be computed by solving the  
399 linear equation with  $i,j = 1 : N$  as:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta\mathbf{U}_{i,c} \\ \Delta\mathbf{U}_{j,c} \end{vmatrix} = \begin{vmatrix} \beta_{ij}(\mathbf{U}_{i,c}^* - \mathbf{U}_{j,c}^*) \\ \beta_{ji}(\mathbf{U}_{j,c}^* - \mathbf{U}_{i,c}^*) \end{vmatrix}$$

<sup>400</sup> To consider the energy exchange, the Lagrangian temperature is modified as

$$\begin{aligned} T_{fc}^L &= T_{fc}^{L-} + \Delta T_{fc} \\ T_{sc}^L &= T_{sc}^{L-} + \Delta T_{sc} \end{aligned} \quad (65)$$

<sup>401</sup> where the cell-centered intermediate temperature can be calculated by

$$\begin{aligned} T_{fc}^{L-} &= \frac{(mT)_{fc}^{L-}}{m_{fc}^L c_v} \\ T_{sc}^{L-} &= \frac{(mT)_{sc}^{L-}}{m_{sc}^L c_v} \end{aligned} \quad (66)$$

<sup>402</sup> And the increment of the velocity can be computed by solving the linear  
<sup>403</sup> equation with  $i,j = 1 : N$  as:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_{i,c} \\ \Delta T_{j,c} \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_{i,c}^n - T_{j,c}^n) \\ \eta_{ji}(T_{j,c}^n - T_{i,c}^n) \end{vmatrix}$$

<sup>404</sup> Finally, we obtain the cell-centered solid acceleration and temperature rate  
<sup>405</sup> as

$$d\mathbf{U}_{sc}^L = \frac{(m\mathbf{U})_{sc}^L - (m\mathbf{U})_{sc}^n}{m_{sc}^L \Delta t} \quad (67)$$

$$dT_{sc}^L = \frac{(me)_{sc}^L - (me)_{sc}^n}{m_{sc}^L c_v \Delta t} \quad (68)$$

#### <sup>407</sup> Compute Lagrangian specific volume of the fluid phase

<sup>408</sup> To compute the Lagrangian value of the specific volume of the fluid phase,  
<sup>409</sup> we need to compute the Lagrangian temperature rate as below

$$T_{fc}^{n+1} = \frac{(me)_{fc}^L}{m_{fc}^L c_v} \quad (69)$$

$$\frac{D_f T_{fc}}{Dt} = \frac{T_{fc}^{n+1} - T_{fc}^n}{\Delta t} \quad (70)$$

<sup>410</sup> As such, the Lagrangian specific volume rate is:

$$\Delta(mv)_{fc} = V f_{fc}^\phi \nabla \cdot \mathbf{U} + (\phi_{fc} \alpha_{fc} \frac{D_f T_{fc}}{Dt} - f_{fc}^\phi \sum_{n=1}^N \phi_{nc} \alpha_{nc} \frac{D_n T_{nc}}{Dt}) \quad (71)$$

<sup>412</sup> where  $f_f^\phi = (\phi_f \kappa_f) / (\sum_{n=1}^N \phi_n \kappa_n)$  and  $\mathbf{U} = \nabla \cdot (\sum_{s=1}^{N_s} \phi_{sc} \mathbf{U}_{sc} + \sum_{f=1}^{N_f} \phi_{fc} \mathbf{U}_{fc})$ .

<sup>413</sup> Finally, the Lagrangian specific volume is

$$(mv)_{fc}^L = V \bar{\rho}_{f,c}^n v_{fc}^n + \Delta(mv)_{fc} \quad (72)$$

414    *Compute advection term and advance in time*

415    The time advanced mass, linear momentum, energy and specific volume  
416    are:

$$417 \quad m_{fc}^{n+1} = m_{fc}^L - \Delta t \nabla \cdot (\bar{\rho}_{fc}^L, \mathbf{U}_{f,FC}^L) \quad (73)$$

$$418 \quad (m\mathbf{U})_{fc}^{n+1} = (m\mathbf{U})_{fc}^L - \Delta t \nabla \cdot ((\bar{\rho}\mathbf{U})_{fc}^L, \mathbf{U}_{f,FC}^L) \quad (74)$$

$$419 \quad (me)_{fc}^{n+1} = (me)_{fc}^L - \Delta t \nabla \cdot ((\bar{\rho}c_v T)_{fc}^L, \mathbf{U}_{f,FC}^L) \quad (75)$$

$$420 \quad (mv)_{fc}^{n+1} = (mv)_{fc}^L - \Delta t \nabla \cdot ((\bar{\rho}v)_{fc}^L, \mathbf{U}_{f,FC}^L) \quad (76)$$

420    Finally, the state variables of the fluid phases of the next time step are

$$421 \quad \bar{\rho}_{fc}^{n+1} = \frac{m_{fc}^{n+1}}{V} \quad (77)$$

$$422 \quad \mathbf{U}_{fc}^{n+1} = \frac{(m\mathbf{U})_{fc}^{n+1}}{m_{fc}^{n+1}} \quad (78)$$

$$423 \quad T_{fc}^{n+1} = \frac{(me)_{fc}^{n+1}}{m_{fc}^{n+1}} \quad (79)$$

$$424 \quad v_{fc}^{n+1} = \frac{(mv)_{fc}^{n+1}}{m_{fc}^{n+1}} \quad (80)$$

424    *Interpolate from cell to node of the solid phase*

425    First we interpolate the acceleration, velocity and temperature rate to  
426    the node

$$427 \quad \mathbf{a}_{si}^n = \sum S_{ci} d\mathbf{U}_{sc}^L \quad (81)$$

$$428 \quad \mathbf{U}_{si}^{n+1} = \sum S_{ci} d\mathbf{U}_{sc}^L \Delta t \quad (82)$$

$$429 \quad dT_{si}^n = \sum S_{ci} dT_{sc}^L \quad (83)$$

429    Then the boundary condition and contact forces are applied to the nodal  
430    velocity and the acceleration is modified by

$$430 \quad \mathbf{a}_{si}^n = \frac{\mathbf{v}_{si}^{n+1} - \mathbf{v}_{si}^n}{\Delta t} \quad (84)$$

431    *Update the particle variables*

432    The state variables of the solid phase [ $\mathbf{U}_{sp}^{n+1}, \mathbf{x}_{sp}^{n+1}, \nabla \mathbf{U}_{sp}^{n+1}, T_{sp}^{n+1}, \nabla T_{sp}^{n+1}, \mathbf{F}_{sp}^{n+1}, V_{sp}^{n+1}$ ]  
 433    (velocity, position, velocity gradient, temperature, temperature gradient, de-  
 434    formation gradient, volume) are updated here

$$\mathbf{U}_{sp}^{n+1} = \mathbf{U}_{sp}^n + \sum S_{sp} \mathbf{a}_{si}^n \Delta t \quad (85)$$

$$\mathbf{x}_{sp}^{n+1} = \mathbf{x}_{sp}^n + \sum S_{sp} \mathbf{U}_{si}^{n+1} \Delta t \quad (86)$$

$$\nabla \mathbf{U}_{sp}^{n+1} = \sum \nabla S_{sp} \mathbf{U}_{si}^{n+1} \quad (87)$$

$$T_{sp}^{n+1} = T_{sp}^n + \sum S_{sp} dT_{si}^n \Delta t \quad (88)$$

$$\nabla T_{sp}^{n+1} = \sum \nabla S_{sp} T_{sp}^n \Delta t \quad (89)$$

$$\mathbf{F}_{sp}^{n+1} = (\mathbf{I} + \nabla \mathbf{U}_{sp}^{n+1} \Delta t) \mathbf{F}_{sp}^n \quad (90)$$

$$V_{sp}^{n+1} = \det(\mathbf{F}_{sp}^{n+1}) V_{sp}^o \quad (91)$$

441    Finally, the effective stress ( $\sigma'$ )<sup>n+1</sup> is updated from the constitutive model  
 442    and the pore water pressure is interpolated from the cell as:

$$p_f^{n+1} = \sum S_{si} P_c^{n+1} \quad (92)$$

443    **Numerical examples**

444    All input files and the analytical calculations in this section are provided  
 445    in the Github repository for the reproduction of the numerical results.

446

447    *Fluid Flow through isothermal porous media*

448    Fluid flow through porous media is important in many engineering disci-  
 449    plines, like predicting water flow in soil. Fluid flow velocity in one dimension  
 450    can be calculated from the porous media's hydraulic conductivity  $K$  as:

451

$$U_f = K \frac{\Delta p_f}{L} \quad (93)$$

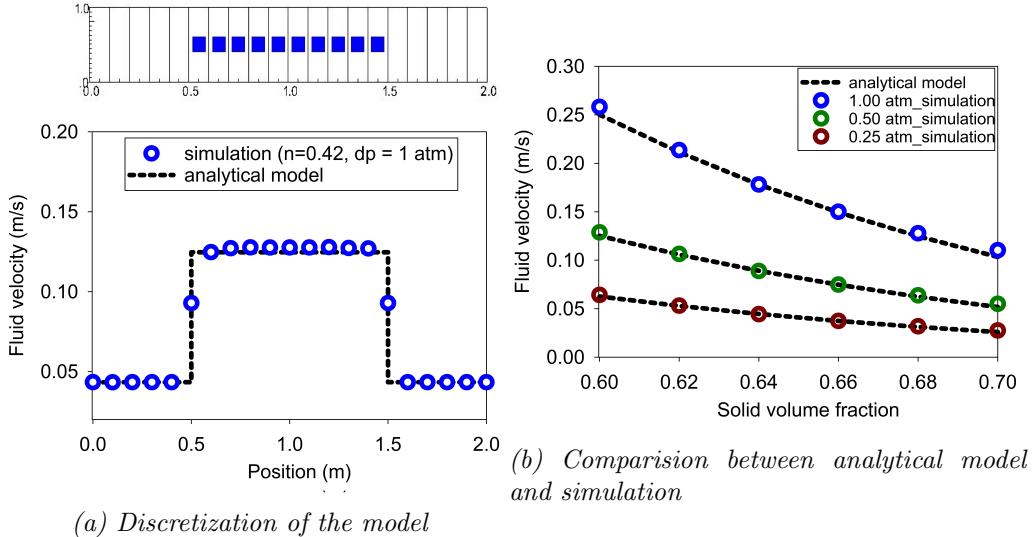


Figure 6: Numerical results of the fluid flow through isothermal porous media

If the Carman-Kozeny formula is adopted  $F = 10\phi_s/(1-\phi_s)^2$ , the hydraulic conductivity will be expressed as  $K = d^2(1-\phi_s)^3/180\mu\phi_s^2$ . Then, the analytical formula of average velocity in one dimension through the porous media is:

$$U_f = \frac{1}{n} \frac{d^2(1-\phi_s)^3}{180\mu\phi_s^2} \frac{\Delta p_f}{L} \quad (94)$$

Our numerical model is validated by modeling fluid flow through a 1m long porous media. This fluid has water properties (bulk modulus is 2GPa, density is 998 kg/m<sup>3</sup> at 5 degrees Celsius and 10325 Pa (1atm) pressure, dynamic viscosity  $\mu$  is 1mPa s). The porous media is modeled by elastic material with Young's modulus is 10 MPa, Poisson's ratio is 0.3, and density is 2650 kg/m<sup>3</sup>. The volume fraction of porous media  $\phi_s$  is [0.6, 0.62, 0.66, 0.68, 0.7] and the average grain diameter  $d$  is 1mm. The model is discretized in 20 finite element and the porous media in 10 finite element with 1 material point per element. The pressure gradient is applied with three different value [0.25, 0.5, 1] atm. Figure 6 shows a good agreement of fluid flow prediction between the theory and the model.

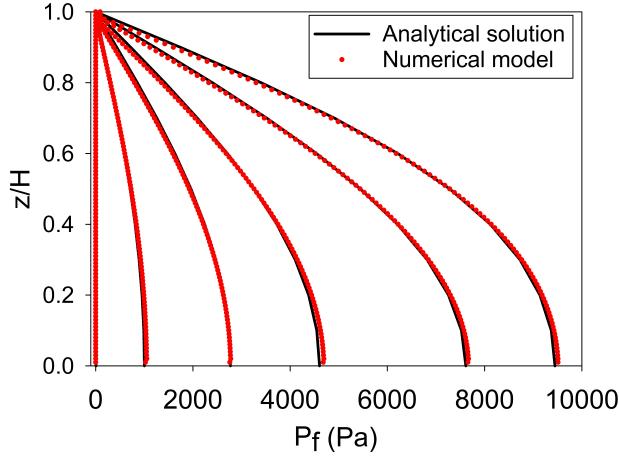


Figure 7: Comparison between analytical solution and numerical solution

468     *Isothermal consolidation*

469     A common benchmark for a fully saturated porous media is the simulation  
 470     of one-dimensional consolidation. Using the Carman-Kozeny formula, the  
 471     time-dependent pressure can be calculated as:

$$p_f = \sum_{m=1}^{\infty} \frac{2F_{ext}}{M} \sin\left(\frac{Mz}{H}\right) e^{-M^2 T_v} \text{ with } M = \frac{\pi}{2}(2m+1) \quad (95)$$

472     where the consolidation rate  $T_v = C_v t / H^2$ , the consolidation coefficient  
 473      $C_v = E_v n^3 d^2 / (180(1-n)^2 \mu)$  and the Oedometer modulus  $E_v = E(1-\nu)/(1+\nu)/(1-2\nu)$ . Our numerical model is validated by modeling the consolidation  
 474     of a 1m column. This fluid has water properties (bulk modulus is 2GPa,  
 475     density is 998 kg/m<sup>3</sup> at 5 degrees Celsius and 101325 Pa (1atm) pressure,  
 476     dynamic viscosity  $\mu$  is 1mPa s). The porous media is modeled by elastic  
 477     material with Young's modulus is 10 MPa, Poisson's ratio is 0.3, and density  
 478     is 2650 kg/m<sup>3</sup>. The volume fraction of porous media  $\phi_s$  is 0.7 which is equiv-  
 479     alent to the porosity of 0.3 and the average grain diameter  $d$  is 1mm. The  
 480     model is discretized in 100 finite element with 1 material point per element.  
 481     The external pressure applies to the top of the column is 10 kPa. Figure 7  
 482     shows a good agreement of fluid flow prediction between the theory and the  
 483     model.

484

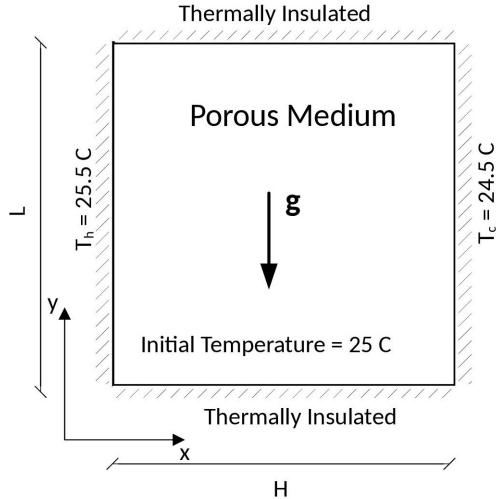


Figure 8: Model schematic [23]

486    *Thermal induced cavity flow*

487    Another benchmark is the thermal induced cavity flow in porous me-  
 488    dia. Temperature and velocity distributions are calculated for a square non-  
 489    deformable saturated porous media. The top and bottom walls are insulated,  
 490    and the left and right walls are at fixed temperatures differing by 1 degree.  
 491    The fluid motion at steady state are cavity flow due to the temperature in-  
 492    duced density variation.

493    The numerical is validated by comparing with the numerical solution of the fi-  
 494    nite element method. The fluid has water properties (bulk modulus is 2GPa,  
 495    density is 998 kg/m<sup>3</sup> at 5 degrees Celsius and 10325 Pa (1atm) pressure,  
 496    dynamic viscosity  $\mu$  is 1 mPa s). The porous media is modeled by non de-  
 497    formable material, and density is 2500 kg/m<sup>3</sup>. The specific heat capacity  
 498    of the water and porous skeleton are 4181 J/kg.K and 835 J/kg.K respec-  
 499    tively. The thermal conductivity of the water and porous skeleton are 0.598  
 500    W/m.K and 0.4 W/m.K. The volume fraction of porous media  $\phi_s$  is 0.6  
 501    which is equivalent to the porosity of 0.4 and the average grain diameter  $d$   
 502    is 1mm. The model is discretized in 20 x 20 finite element with 4 material  
 503    point per element. Figure 9 shows a good agreement of numerical results of  
 504    the model compared with the numerical solution of the finite element method.

505

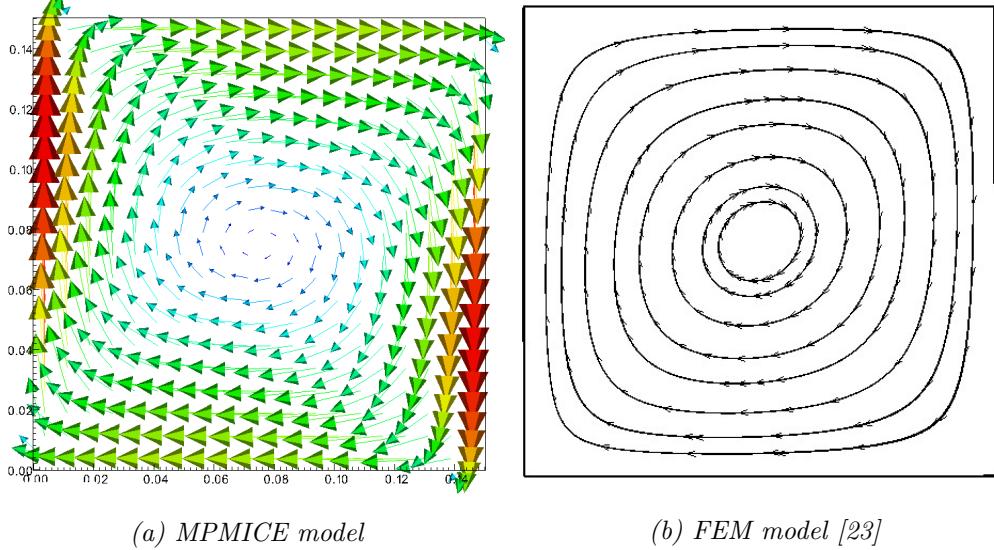


Figure 9: Comparision between MPMICE model and FEM model

#### 506 Underwater debris flow

507 The numerical example is validated by Rzadkiewicz et al.'s experiment  
 508 on submarine debris flow [24]. During the experiment, sand in a triangular  
 509 box is released and then slides along a rigid bed inclined 45 degrees under  
 510 water, see Figure 10.

511 In the numerical model, the material properties are selected based on the  
 512 experiment by Rzadkiewicz et al [24]. Sand has a saturated density of 1985  
 513  $kg/m^3$  and yield stress of 200 Pa. Young's modulus has little effect on debris  
 514 flow run-out because of the extreme large deformation of the debris. There-  
 515 fore, we select 50 MPa Young's modulus with 0.25 Poisson's ratio. The rigid  
 516 bed is much stiffer with bulk modulus and shear modulus of  $117E^7$  Pa and  
 517  $43.8E^7$  Pa. Under gravity, the density of the water at the surface is 999.8  
 518  $kg/m^3$  at the pressure of 1 atm. At the top boundary, the air has a density  
 519 of  $1.17 kg/m^3$  at the atmospheric pressure of 1 atm. At 5 Celcius degrees,  
 520 air and water have viscosity of  $18.45E^{-3}$  mPa s and 1 mPa s respectively.  
 521 The numerical parameters used in this example are presented in Table 1.  
 522 On all boundary faces, the Dirichlet boundary condition is imposed for veloc-  
 523 ity ( $u = 0$  m/s) and temperature ( $T = 5$  Celcius degrees), while the Neuman  
 524 boundary condition is imposed at the top boundary for pressure ( $dp/dx = 0$   
 525 kPa) and density ( $d\rho/dx = 0 kg/m^3$ ). For the background mesh, there are

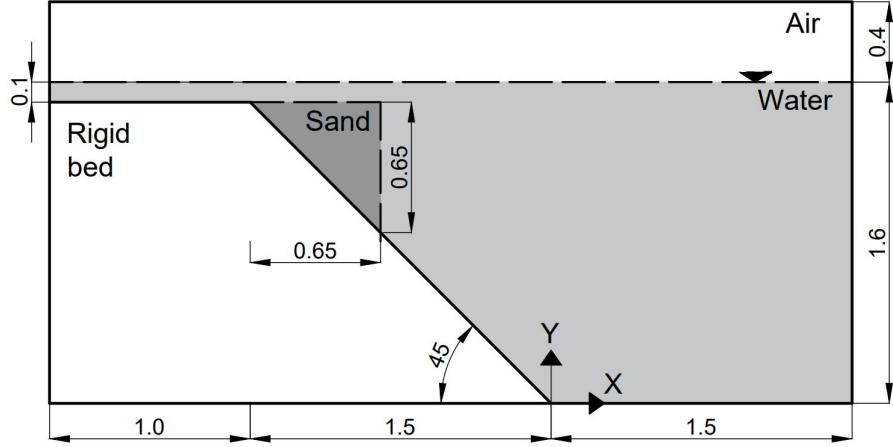


Figure 10: Model schematic

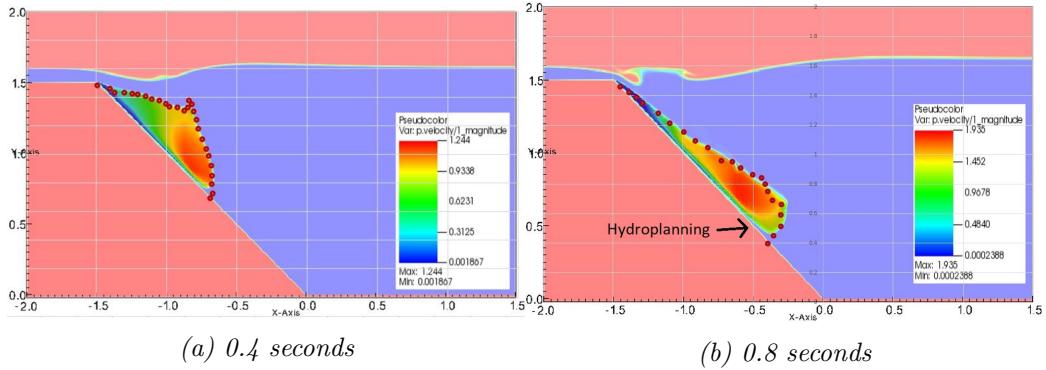


Figure 11: Simulation of underwater debris flow

526  $700 \times 400 = 280,000$  cells. In each cell of the debris flow and rigid bed, there  
 527 are  $2 \times 2$  material points.

528 Figure 11a and 11b show snapshots of the debris flow sliding in the plane  
 529 at 0.4 s and 0.8 s. Our simulations match the computed results from Rzad-  
 530 kiewicz et al. [24]. The model also captures typical hydroplaning mechanism  
 531 of the underwater debris flow (hydroplaning means the debris flow is lifted  
 532 up and no longer in contact with the bottom layer). The elevation of the free  
 533 surface at 0.4s and 0.8s is compared between our proposed method and other  
 534 methods in Figure 12. Once again, our computed results were consistent with  
 535 both the experiment and others computational results [7]. Unlike other com-  
 536 putational models based on total stress analysis, the proposed model based

Materials	Bulk modul (Pa)	Shear modul (Pa)	Density (kg/m3)	Temp (C)	Dynamic viscosity (Pa s)	Yield stress (Pa)
Water (at surface)	2.15e9	-	999.8	5	855e-6	-
Air (at top boundary)	-	-	1.177	5	18.45e-6	-
Sand (porous media)	8.33e6	20e6	1985	5	-	200
Rigid bed (solid)	117e7	43.8e7	8900	5	-	-

Table 1: Numerical parameters for the underwater submarine debris

537 on the effective stress analysis which allows to analyze the water pressure  
 538 and temperature in the debris flow.

539 We also explore the difference between underwater debris flow and satu-  
 540 rated debris flow in terms of interacting with obstacle. Figure 13 shows the  
 541 snapshot of the simulations of underwater and saturated debris flow. The  
 542 saturated debris flow (see Figure 13a) behaves like frictional flow as grain  
 543 have contact forces with each other. On the other hand, the underwater de-  
 544 bris flow (see Figure 13b) behaves like turbulent flow as grains are separated  
 545 from each other and exhibit no contact forces between grains.

546

547 *Earthquake-induced submarine landslides*

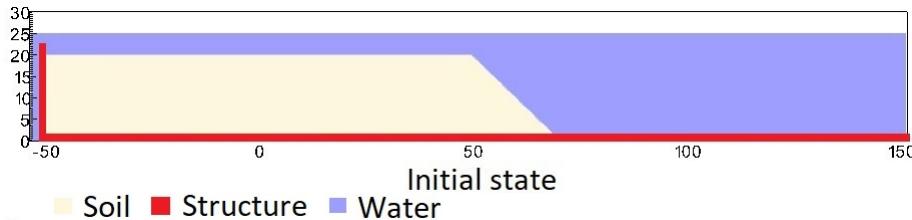
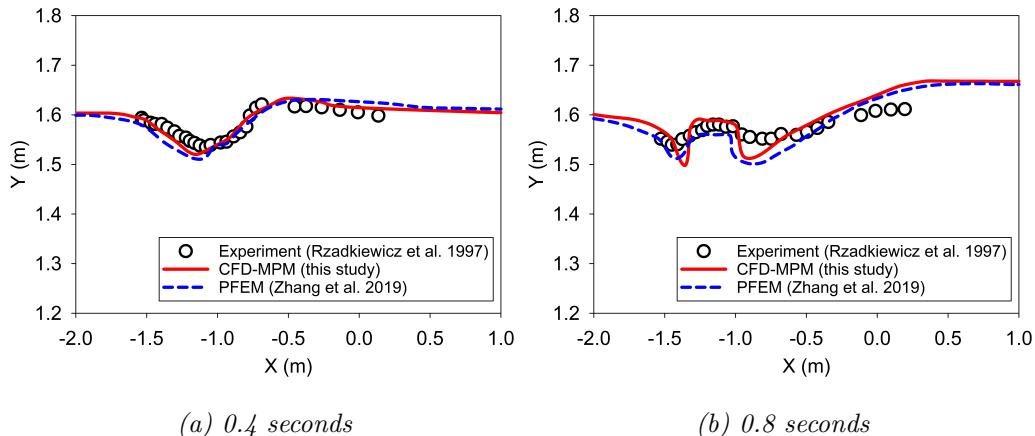
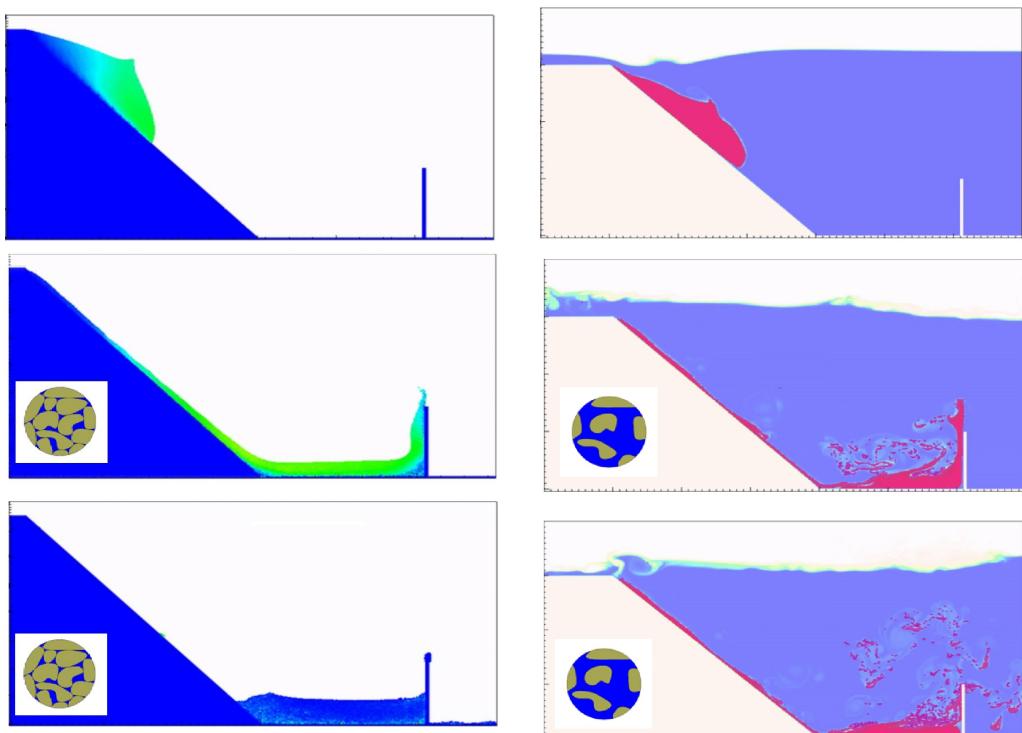


Figure 14: Numerical model of the earthquake-induced submarine landslide



*Figure 12: Simulation of underwater debris flow*



(a) saturated debris flow using MPM      (b) underwater debris flow using MPMICE

*Figure 13: Simulation of underwater debris flow*

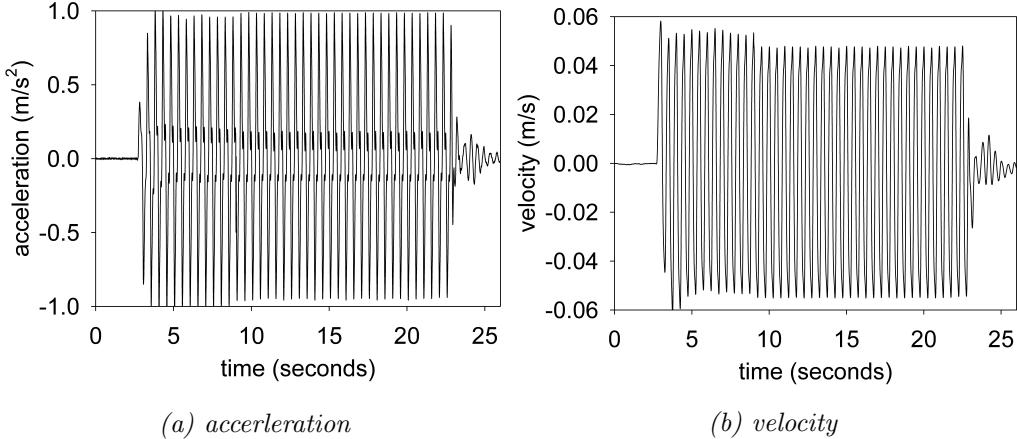


Figure 15: Ground acceleration profile, frequency of 2Hz and magnitude of 1g

In the final example, we perform numerical analysis of the earthquake induced submarine landslides. A plane strain model with the slope under water is shown in Figure 14. A 20m high slope with slope gradient of 45 degrees is placed in a horizontal and vertical structure which was used to be a shaking table to apply earthquake loading. We simplify the earthquake loading by simulating the ground shaking for 20 seconds with the peak ground acceleration of 1g and the frequency of 2Hz (Figure 15a). The ground motion is applied in terms of velocity (Figure 15b). The earthquake of this magnitude can occur typically for the earthquake of magnitude of more than 6.

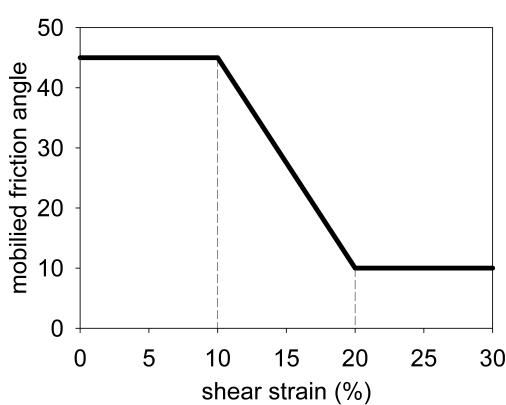


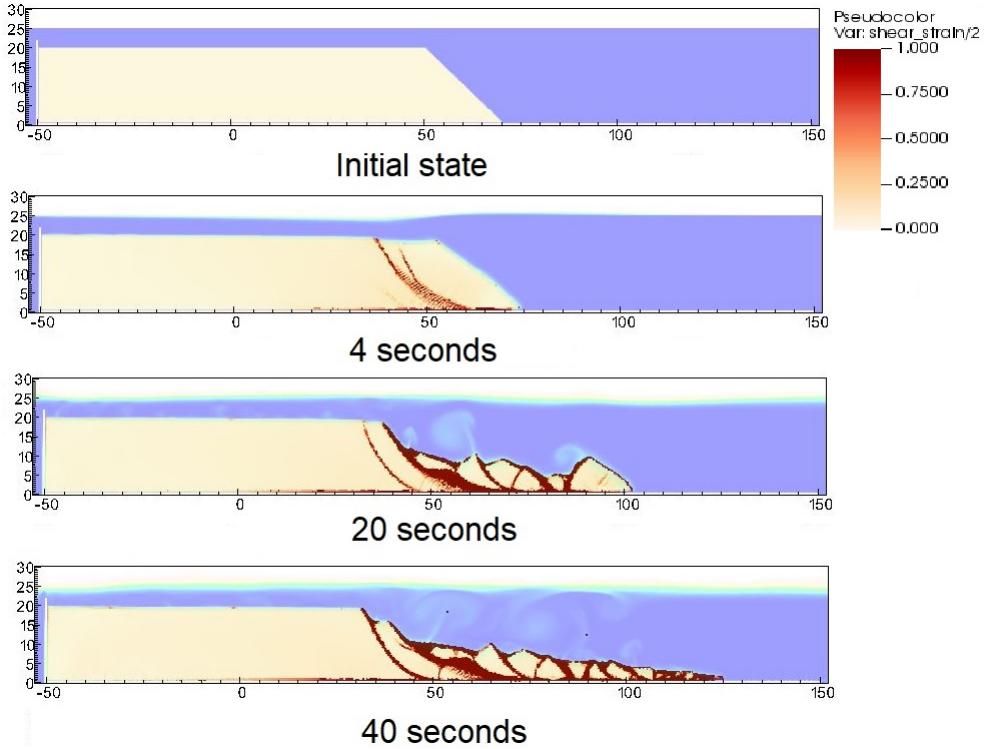
Figure 16: Mobilized friction angle in Mohr Coulomb model

558 A non-associated Mohr-Coulomb model is used for the soil. The soil grain  
 559 has the density of  $2650 \text{ kg/m}^3$ , Young's modulus of 10 kPa and Poisson's  
 560 ratio of 0.3 and zero cohesion. The mobilized friction angle  $\phi'_m$  is governed  
 561 following the softening curve (see Figure 16) with the peak friction angle  $\phi'_p$   
 562 of 45 degrees and the residual friction angle  $\phi'_r$  of 10 degrees. The porosity  
 563 is 0.3 and the average grain size of the soil is around  $0.1 \mu\text{m}$  to mimic the  
 564 undrained behavior. The mobilized dilatancy angle is calculated from the  
 565 Row-stress dilatancy as follow:

$$\sin \psi'_m = \frac{\sin \phi'_m - \sin \phi'_r}{1 - (\sin \phi'_r \sin \phi'_m)} \quad (96)$$

566 The solid plane is modeled as a rigid body acted as a shaking table. The  
 567 contact between horizontal plane and the sand is the frictional contact with  
 568 the friction coefficient of 0.1. No artificial damping is applied in the simu-  
 569 lation. The contact between vertical plane and the sand is considered to be  
 570 smooth with zero friction coefficient. Under gravity, the density of the water  
 571 at the surface is  $999.8 \text{ kg/m}^3$  at the pressure of 1 atm. At the top boundary,  
 572 the air has a density of  $1.17 \text{ kg/m}^3$  at the atmospheric pressure of 1 atm.  
 573 At 5 Celcius degrees, air and water have viscosity of  $18.45e^{-3} \text{ mPa s}$  and  
 574  $1 \text{ mPa s}$  respectively. On all boundary faces, the symmetric boundary con-  
 575 dition is imposed, while the Neuman boundary condition is imposed at the  
 576 top boundary for pressure ( $d\text{p}/dx = 0 \text{ kPa}$ ) and density ( $d\rho/dx = 0 \text{ kg/m}^3$ ).  
 577 The mesh size is  $0.25 \times 0.25 \text{ m}$  with 300852 element cells and 142316 material  
 578 points. The simulation takes a couple of hours to perform 60 seconds of the  
 579 simulation using 4096 CPUs.

580

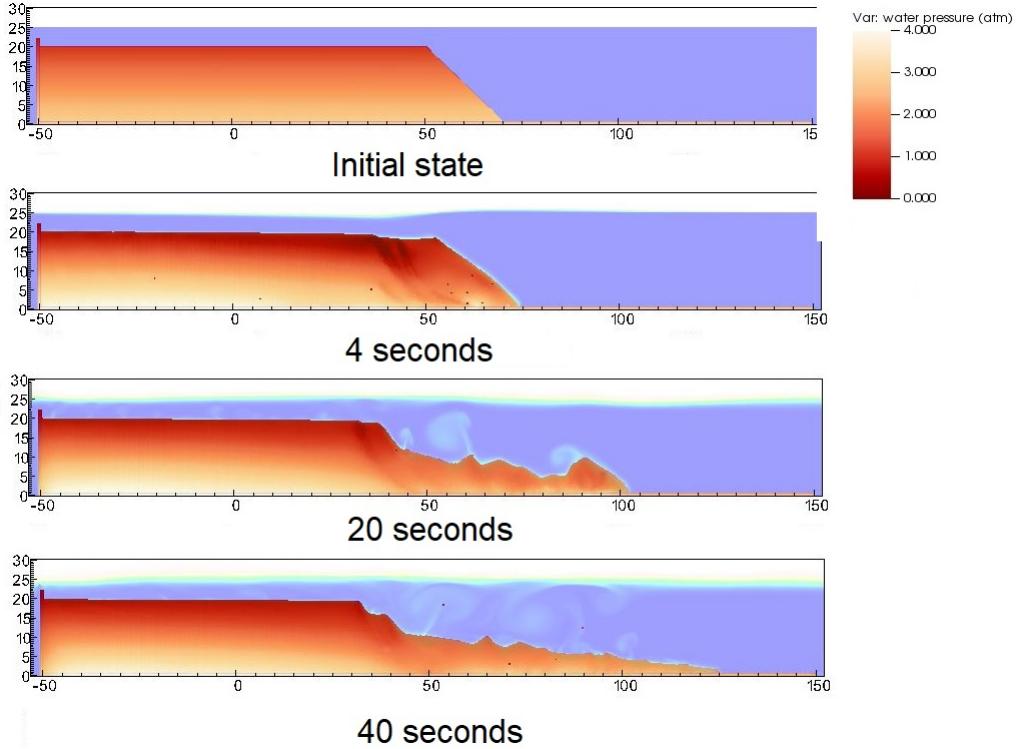


*Figure 17: Shear strain during the earthquake-induced submarine landslides*

We demonstrate the entire process and the mechanism of the earthquake-induced submarine landslides by showing the shear strain (Figure 17), the pore water pressure in atm (Figure 18) and the velocity (Figure 19). The failure mechanism can be characterized as the progressive failure mechanism. Here are some numerical observations:

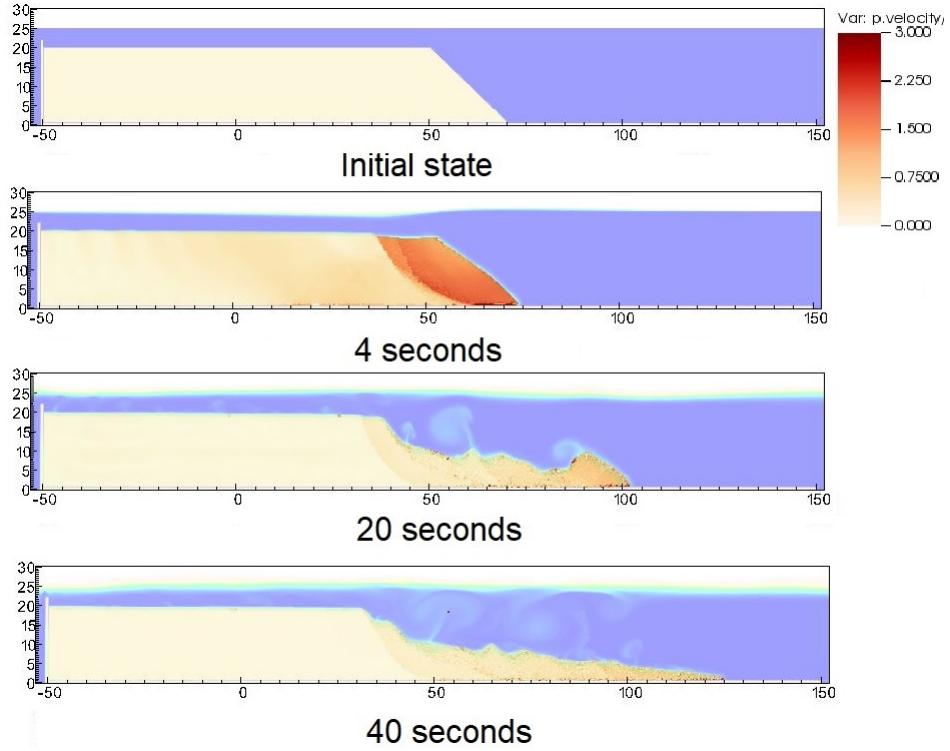
586

- 587 1. At the initial of the seismic event, the seismic loading triggers the  
 588 first slide at 3 seconds. At 4 seconds, the debris start to move with the  
 589 maximum speed of around 2-3 m/s with multiple shear band developed  
 590 in the slope. The wave generated from the submarine slide is around  
 591 2-3m towards the slide direction.



*Figure 18: pore water pressure during the earthquake-induced submarine landslides*

- 592     2. When the onset of the shear band occurs in the slope (for example  
 593       at 4 seconds and 20 seconds), the negative excess pore water pressure  
 594       is developed along this shear band with pore water pressure is under  
 595       1atm. This is a typical dilatancy behavior when the soil is sheared  
 596       rapidly in the undrained behavior.
- 597     3. When the seismic loading ends at 23 seconds, the last shear band is  
 598       mobilized and the slope soon reaches to the final deposition. No more  
 599       progressive failure developed in the slope. The turbulent flow developed  
 600       as the interaction between debris flow and seawater.
- 601     Overall, we show the completed process of the earthquake-induced submarine  
 602       landslides involving (1) earthquake triggering mechanism, (2) the onset of the  
 603       shear band with the delvelopment of negative excess pore water pressure, (3)  
 604       progressive failure mechanism, (4) submarine landslide induced wave to final  
 605       deposition.



*Figure 19: Velocity during the earthquake-induced submarine landslides*

## 606 Conclusions

607 We have presented a numerical approach MPMICE for the simulation  
 608 of large deformation soil-fluid-structure interaction, emphasizing the simu-  
 609 lation of the earthquake-induced submarine landslides. The model uses (1)  
 610 the Material Point Method for capturing the large deformation of iso-thermal  
 611 porous media and solid structures and (2) Implicit Continuous Eulerian (com-  
 612 pressible, conservative multi-material CFD formulation) for modeling the  
 613 complex fluid flow including turbulence. This model is implemented in the  
 614 high-performance Uintah computational framework and validated against an-  
 615 alytical solution and experiment. We then demonstrate the capability of the  
 616 model to simulate the entire process of the earthquake induced submarine  
 617 landslides.

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 630 for High Performance Computing and Data Storage in Norway.

631 **Appendix: Equation derivation**

632 Before deriving the governing equation, we define the Lagrangian deriva-  
 633 tive for a state variable  $f$  as:

$$\frac{D_f f}{Dt} = \frac{\partial f}{\partial t} + \mathbf{U}_f \cdot \nabla f \quad \frac{D_s f}{Dt} = \frac{\partial f}{\partial t} + \mathbf{U}_s \cdot \nabla f \quad (97)$$

we use some definition following [16] as below:

$$-\frac{1}{V} \left[ \frac{\partial V_f}{\partial p} \right] \equiv \kappa_f \quad \text{isothermal compressibility of fluid} \quad (98)$$

$$\frac{1}{V} \left[ \frac{\partial V_f}{\partial T} \right] \equiv \alpha_f \quad \text{constant pressure thermal expansivity of fluid} \quad (99)$$

634 Then, the rate of volume with incompressible solid grains are calculated as  
 635 below:

$$\frac{1}{V} \frac{D_f V_f}{Dt} = \frac{1}{V} \left( \left[ \frac{\partial V_f}{\partial p} \right] \frac{D_f P_{eq}}{Dt} + \left[ \frac{\partial V_f}{\partial T_f} \right] \frac{D_f T_f}{Dt} \right) = \frac{1}{V} \left( -\kappa_f \frac{D_f P_{eq}}{Dt} + \alpha_f \frac{D_f T_f}{Dt} \right) \quad (100)$$

636 *Evolution of porosity*

637 Solving the solid mass balance equation (4) with the definition of solid  
 638 mass in equation (2), it leads to the rate of porosity as below:

$$\frac{D_s m_s}{Dt} = \frac{D_s (\phi_s \rho_s V)}{Dt} = \rho_s V \frac{D_s \phi_s}{Dt} + \phi_s V \frac{D_s \rho_s}{Dt} + \phi_s \rho_s \frac{D_s V}{Dt} = 0 \quad (101)$$

639 The soil grains are assumed to be incompressible, therefore, term 2 in the  
 640 right hand side is zero.

$$V \frac{D_s \phi_s}{Dt} + \phi_s \frac{D_s V}{Dt} = 0 \quad (102)$$

641 Dividing all terms with  $V$  with the equation  $\frac{1}{V} \frac{D_s V}{Dt} = \nabla \cdot \mathbf{U}_s$ , it leads to:

$$\frac{D_s n}{Dt} = \frac{\partial n}{\partial t} + \mathbf{U}_s \cdot \nabla n = \phi_s \nabla \cdot \mathbf{U}_s \quad (103)$$

642 *Momentum conservation*

643 The linear momentum balance equation for the fluid phases based on  
 644 mixture theory is:

645

$$\frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} = \nabla \cdot (-\phi_f p_f \mathbf{I}) + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d + \mathbf{f}_b \quad (104)$$

646 On the right hand sand, the first term is the divergence of partial fluid phase  
 647 stress, the third term is the body force, the fourth term is the drag force  
 648 (momentum exchange) and the fifth term is the buoyant force described in  
 649 [25] for the immiscible mixtures. The buoyant force is in the form:

650

$$\mathbf{f}_b = \boldsymbol{\sigma}_f \nabla(n) \quad (105)$$

651 As a result, the linear momentum balance equation for the fluid phases be-  
 652 comes:

$$\frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} = \frac{1}{V} \left[ \frac{\partial(m_f \mathbf{U}_f)}{\partial t} + \nabla \cdot (m_f \mathbf{U}_f \mathbf{U}_f) \right] = -\phi_f \nabla p_f + \nabla \cdot \boldsymbol{\tau}_f + \bar{\rho}_f \mathbf{b} + \sum \mathbf{f}_d \quad (106)$$

653 The Reynolds stress component can be included in the term  $\boldsymbol{\tau}_f$  to consider the  
 654 turbulent effects if needed. To derive the linear momentum balance equation  
 655 for the solid phase, we begin with the linear momentum balance equation for  
 656 the mixture as:

$$\frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} + \frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}) + \bar{\rho}_f \mathbf{b} + \bar{\rho}_s \mathbf{b} \quad (107)$$

657 Combining Terzaghi's equation (3) and subtracting both sides with equation  
 658 (106), we obtain the linear momentum balance equation for the solid phase  
 659 as:

$$\frac{1}{V} \frac{D_s(m_s \mathbf{U}_s)}{Dt} = \nabla \cdot (\boldsymbol{\sigma}') - \phi_s \nabla p_f + \bar{\rho}_s \mathbf{b} - \sum \mathbf{f}_d + \sum \mathbf{f}_{fric} \quad (108)$$

660 Here the  $\mathbf{f}_{fric}$  stems from the soil-structure interaction following the contact  
 661 law between the soil/structure interfaces.

662 *Energy conservation*

663 We adopt the general form of the total energy balance equation for the  
 664 porous media from [26], the total energy balance equations for the fluid phases  
 665 are:

$$\frac{1}{V} \frac{D_f(m_f(e_f + 0.5\mathbf{U}_f^2))}{Dt} = \nabla \cdot (-np_f \mathbf{I}) \cdot \mathbf{U}_f + \nabla \cdot \mathbf{q}_f + (\bar{\rho}_f \mathbf{b}) \cdot \mathbf{U}_f + \sum \mathbf{f}_d \cdot \mathbf{U}_f + \sum q_{sf} \quad (109)$$

666 Applying the product rule  $D(m\mathbf{U}^2) = D(m\mathbf{U} \cdot \mathbf{U}) = 2\mathbf{U} \cdot D(m\mathbf{U})$ , the left  
 667 hand side of equation (109) becomes:

$$\frac{1}{V} \frac{D_f(m_f(e_f + 0.5\mathbf{U}_f^2))}{Dt} = \frac{1}{V} \frac{D_f(m_f e_f)}{Dt} + \frac{1}{V} \frac{D_f(m_f \mathbf{U}_f)}{Dt} \cdot \mathbf{U}_f \quad (110)$$

669 Combining equations (106), (109), (110), we obtain the final form of the  
 670 internal energy balance equation for the fluid phases as:

$$\frac{1}{V} \frac{D_f(m_f e_f)}{Dt} = \frac{1}{V} \left[ \frac{\partial(m_f e_f)}{\partial t} + \nabla \cdot (m_f e_f \mathbf{U}_f) \right] = \bar{\rho}_f p_f \frac{D_f v_f}{Dt} + \nabla \cdot \mathbf{q}_f + \sum q_{sf} \quad (111)$$

671 On the right hand side, the terms include the average pressure-volume work,  
 672 the average viscous dissipation, the thermal transport and the energy ex-  
 673 change between solid and fluid respectively. The heat flux is  $\mathbf{q}_f = \bar{\rho}_f \beta_f \nabla T_f$   
 674 with  $\beta_f$  being the thermal conductivity coefficient. To derive the internal  
 675 energy balance equation for the solid phase, we introduce the rate of the  
 676 internal energy for the thermoelastic materials as a function of elastic strain  
 677 tensor  $\boldsymbol{\epsilon}_s^e$  and temperature  $T_s$ :

$$\frac{m_s}{V} \frac{D_s(e_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + \frac{D_s(e_s)}{D_s(T_s)} \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\boldsymbol{\epsilon}_s^e)}{Dt} + c_v \frac{D_s(T_s)}{Dt} \quad (112)$$

678  $c_v$  is the specific heat at the constant volume of the solid materials. The total  
 679 energy balance equation for the mixture based on [26] can be written as:

$$\begin{aligned} & \frac{1}{V} \frac{D_f(m_f(e_f + 0.5\mathbf{U}_f^2))}{Dt} + \frac{1}{V} \frac{D_s(m_s(e_s + 0.5\mathbf{U}_s^2))}{Dt} = \nabla \cdot (-\phi_f p_f \mathbf{I}) \cdot \mathbf{U}_f \\ & + \nabla \cdot (\boldsymbol{\sigma}' - \phi_s p_f \mathbf{I}) \cdot \mathbf{U}_s + (-\phi_f p_f \mathbf{I}) : \nabla \mathbf{U}_f + \boldsymbol{\sigma}' : \nabla \mathbf{U}_s \\ & + (\bar{\rho}_f \mathbf{b}) \cdot \mathbf{U}_f + (\bar{\rho}_s \mathbf{b}) \cdot \mathbf{U}_s + \nabla \cdot \mathbf{q}_f + \nabla \cdot \mathbf{q}_s + \sum \mathbf{f}_d \cdot (\mathbf{U}_f - \mathbf{U}_s) \end{aligned} \quad (113)$$

681 Subtracting equation (113), (112) to equations (109) and (108), we obtained  
682 the internal energy balance equation for solid phase as:

$$\boldsymbol{\sigma}' : \frac{D_s(\epsilon_s^e)}{Dt} + \frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \Delta W_s + \Delta W_{friction} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (114)$$

683 On the right hand side, the terms include the work rate from frictional sliding  
684 between solid materials  $\Delta W_{friction}$ , thermal transport and energy exchange  
685 between solid and fluid respectively. The heat flux is  $\mathbf{q}_s = \bar{\rho}_s \beta_s \nabla T_s$  with  $\beta_s$   
686 being the thermal conductivity of the solid materials, the mechanical work  
687 rate  $\Delta W_s = \boldsymbol{\sigma}' : \frac{D_s(\epsilon_s)}{Dt} = \boldsymbol{\sigma}' : (\frac{D_s(\epsilon_s^e)}{Dt} + \frac{D_s(\epsilon_s^p)}{Dt})$  computed from the constitutive  
688 model with  $\epsilon_s^p$  is the plastic strain tensor, . By subtracting the term  $\boldsymbol{\sigma}' : \frac{D_s(\epsilon_s^e)}{Dt}$ ,  
689 we get the final form of the energy balance equation as:

$$\frac{m_s}{V} c_v \frac{D_s(T_s)}{Dt} = \boldsymbol{\sigma}' : \frac{D_s(\epsilon_s^p)}{Dt} + \Delta W_{friction} + \nabla \cdot \mathbf{q}_s - \sum q_{sf} \quad (115)$$

### 690 Advanced Fluid Pressure

691 The discretization of the pressure equation begins with the Lagrangian  
692 face-centered velocity and the equation for the pressure

$$\bar{\rho}_{f,FC} \frac{\mathbf{U}_{f,FC}^{n+1} - \mathbf{U}_{f,FC}^n}{dt} = n \nabla^{FC} P_{fc}^{n+1} + \bar{\rho}_{f,FC} \mathbf{b} \quad (116)$$

693

$$\kappa \frac{dP}{dt} = \nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} \quad (117)$$

694 The divergence of the equation (116) with  $\nabla \cdot \mathbf{b} = 0$  is

$$\nabla^c \cdot \mathbf{U}_{f,FC}^{n+1} - \nabla^c \cdot \mathbf{U}_{f,FC}^n = \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} (P_{fc}^n + \Delta P_{fc}^n) \quad (118)$$

695 To solve this equation, we define the face-centered intermediate velocity  
696  $\mathbf{U}_{f,FC}^*$  as:

$$\bar{\rho}_{f,FC} \frac{\mathbf{U}_{f,FC}^* - \mathbf{U}_{f,FC}^n}{\Delta t} = n \nabla^{FC} P_{fc}^n + \bar{\rho}_{f,FC} \mathbf{b} \quad (119)$$

697 The divergence of the equation (119) is

$$\nabla^c \cdot \mathbf{U}_{f,FC}^* - \nabla^c \cdot \mathbf{U}_{f,FC}^n = \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} P_{fc}^n \quad (120)$$

698 Combining equations (117, 118, 120), it leads to

$$\left( \kappa - \nabla^c \frac{\Delta t}{\rho_{f,FC}^n} \cdot \nabla^{FC} \right) \Delta P_{fc}^n = -\nabla^c \cdot \mathbf{U}_{f,FC}^* \quad (121)$$

699 When the fluid is incompressible,  $\kappa$  approaches to zero and the equation  
700 (121) becomes the Poisson's equation for the incompressible fluid flow.

701 *Momentum and Energy exchange with an implicit solver*

702 Considering the fluid momentum balance equation as

$$(m\mathbf{U})_{f,FC}^{n+1} = (m\mathbf{U})_{f,FC}^n - \Delta t(Vn\nabla^{FC}P_{fc}^n + m_f\mathbf{b}) + VK\Delta t(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (122)$$

703 Assuming  $m_{f,FC}^{n+1} = m_{f,FC}^n$  we get

$$\mathbf{U}_{f,FC}^{n+1} = \mathbf{U}_{f,FC}^n - \Delta t\left(\frac{\nabla^{FC}P_{fc}^n}{\rho_{f,FC}^n} + \mathbf{b}\right) + \frac{\Delta tK}{\bar{\rho}_{f,FC}^n}(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (123)$$

704 As defined in the section 'Advanced Fluid Pressure', the face-centered inter-  
705 mediate fluid velocity  $\mathbf{U}_{f,FC}^* = \Delta t(\nabla^{FC}P_{fc}^n/\rho_{f,FC}^n + \mathbf{b})$  leading to

$$\mathbf{U}_{f,FC}^{n+1} = \mathbf{U}_{f,FC}^* + \frac{\Delta tK}{\bar{\rho}_{f,FC}^n}(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (124)$$

706 Considering the solid momentum balance equation as

$$(m\mathbf{U})_{s,FC}^{n+1} = (m\mathbf{U})_{s,FC}^n - \Delta t(V\nabla^{FC} \cdot \boldsymbol{\sigma}'^n - V(1-n)\nabla^{FC}P_{fc}^n + m_s\mathbf{b}) - VK\Delta t(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (125)$$

707 We define the face-centered intermediate solid velocity as  $\mathbf{U}_{s,FC}^* = \Delta t(\nabla^{FC} \cdot \boldsymbol{\sigma}_c'^n / \bar{\rho}_{s,FC}^n - \nabla^{FC}P_{fc}^n / \rho_s + \mathbf{b})$  leading to

$$\mathbf{U}_{s,FC}^{n+1} = \mathbf{U}_{s,FC}^* - \frac{\Delta tK}{\bar{\rho}_{s,FC}^n}(\mathbf{U}_{s,FC}^{n+1} - \mathbf{U}_{f,FC}^{n+1}) \quad (126)$$

709 Combining equation (124) and (126) we get

$$\begin{aligned} \mathbf{U}_{f,FC}^* + \Delta\mathbf{U}_{f,FC} &= \mathbf{U}_{f,FC}^* + \frac{\Delta tK}{\bar{\rho}_{f,FC}^n}(\mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} - \mathbf{U}_{f,FC}^* - \Delta\mathbf{U}_{f,FC}) \\ \mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} &= \mathbf{U}_{s,FC}^* - \frac{\Delta tK}{\bar{\rho}_{s,FC}^n}(\mathbf{U}_{s,FC}^* + \Delta\mathbf{U}_{s,FC} - \mathbf{U}_{f,FC}^* - \Delta\mathbf{U}_{f,FC}) \end{aligned} \quad (127)$$

710 Rearranging the equation (127), it leads to the linear system of equations

$$\begin{vmatrix} (1 + \beta_{12,FC}) & -\beta_{12,FC} \\ -\beta_{21,FC} & (1 + \beta_{21,FC}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_{f,FC} \\ \Delta \mathbf{U}_{s,FC} \end{vmatrix} = \begin{vmatrix} \beta_{12,FC}(\mathbf{U}_{s,FC}^* - \mathbf{U}_{f,FC}^*) \\ \beta_{21,FC}(\mathbf{U}_{f,FC}^* - \mathbf{U}_{s,FC}^*) \end{vmatrix}$$

711 Solving this linear equations with  $\beta_{12,FC} = (\Delta t K) / \bar{\rho}_{f,FC}^n$  and  $\beta_{21,FC} =$   
 712  $(\Delta t K) / \bar{\rho}_{s,FC}^n$  with K is the momentum exchange coefficient. Similar derivation  
 713 can be performed to computed the cell-center velocity increment leading  
 714 to

$$\begin{vmatrix} (1 + \beta_{12c}) & -\beta_{12c} \\ -\beta_{21c} & (1 + \beta_{21c}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_{fc} \\ \Delta \mathbf{U}_{sc} \end{vmatrix} = \begin{vmatrix} \beta_{12c}(\mathbf{U}_{sc}^* - \mathbf{U}_{fc}^*) \\ \beta_{21c}(\mathbf{U}_{fc}^* - \mathbf{U}_{sc}^*) \end{vmatrix}$$

715 with  $\beta_{12c} = (\Delta t K) / \bar{\rho}_{fc}^n$  and  $\beta_{21c} = (\Delta t K) / \bar{\rho}_{sc}^n$  and the cell-centered interme-  
 716 diate velocity can be calculated by

$$\begin{aligned} \mathbf{U}_{fc}^* &= \mathbf{U}_{fc}^n + \Delta t \left( -\frac{\nabla P_{fc}^{n+1}}{\rho_{fc}^n} + \frac{\nabla \cdot \boldsymbol{\tau}_{fc}^n}{\bar{\rho}_{fc}^n} + \mathbf{b} \right) \\ \mathbf{U}_{sc}^* &= \mathbf{U}_{sc}^n + \Delta t \left( \frac{\nabla \cdot \boldsymbol{\sigma}_c'^n}{\bar{\rho}_{sc}^n} - \frac{\nabla P_{sc}^{n+1}}{\rho_s} + \mathbf{b} \right) \end{aligned} \quad (128)$$

717 For generalize multi materials i,j = 1:N, the linear equations is in the form:

$$\begin{vmatrix} (1 + \beta_{ij}) & -\beta_{ij} \\ -\beta_{ji} & (1 + \beta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta \mathbf{U}_i \\ \Delta \mathbf{U}_j \end{vmatrix} = \begin{vmatrix} \beta_{ij}(\mathbf{U}_i^* - \mathbf{U}_j^*) \\ \beta_{ji}(\mathbf{U}_j^* - \mathbf{U}_i^*) \end{vmatrix}$$

718 Similar approach applied for the energy exchange term leading to:

$$\begin{vmatrix} (1 + \eta_{ij}) & -\eta_{ij} \\ -\eta_{ji} & (1 + \eta_{ji}) \end{vmatrix} \begin{vmatrix} \Delta T_i \\ \Delta T_j \end{vmatrix} = \begin{vmatrix} \eta_{ij}(T_i^n - T_j^n) \\ \eta_{ji}(T_j^n - T_i^n) \end{vmatrix}$$

719 with  $\eta$  is the energy exchange coefficient.

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