## MATH 6320, Theory of Functions of a Real Variable Assignment 1

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- 1. Solution: pass
- 2. Solution: pass
- 3. **Solution:** Let  $\mathcal{M}$  be a  $\sigma$ -algebra of subsets of X which is at most countable. Let  $E_i$  be a well ordering of elements of  $\mathcal{M}$ . Now let

$$B_x = \bigcap_{x \in E_i} E_i$$

Then  $B_x \in \mathcal{M}$  since each  $E_i \in \mathcal{M}$ . Now let y be any element X distinct from x with  $y \in B_x$ . Since every element in  $\mathcal{M}$  is indexed by  $\mathbb{N}$ , and  $B_y \in \mathcal{M}$ ,  $B_y = E_j$  for some  $j \in \mathbb{N}$ .