Calculus Baseline Assessment (Version 1.0)

Questions $8, 9, 10, 11$, and 12 are modified versions of questions from the Precalculus Concept Assessment by Carlson et al.

- Carlson, M., Oehrtman, M., & Engelke, N. (2010). The precalculus concept assessment: A tool for assessing students' reasoning abilities and understandings. *Cognition and Instruction*, 28(2), 113-145.

Question 13 is modified from the Calculus Concept Readiness instrument by Carlson et al.

- Carlson, M.P., Madison, B. & West, R.D. A Study of Students' Readiness to Learn Calculus. *Int. J. Res. Undergrad. Math. Ed.* 1, 209–233 (2015).

- Carlson, M. O. Madison B. & West. R. D. The Calculus Concept Readiness (CCR) Instrument: Assessing Student Readiness for Calculus. *arXiv preprint arXiv:1010.2719* (2010).

1. If $\dfrac{1}{a} = \dfrac{1}{b} + \dfrac{1}{c}$, which of the following is equal to a^2?

   (a) $\dfrac{b^2c^2}{b^2 + c^2}$

   (b) $\dfrac{b^2 + c^2}{b^2c^2}$

   (c) $\dfrac{b^2c^2}{(b + c)^2}$

   (d) $b^2 + c^2$

   (e) $(b + c)^2$

2. If $P = Ae^{2t}$, which of the following is equal to t?

   (a) $\dfrac{\ln(P) - \ln(A)}{2}$

   (b) $\dfrac{\ln(P) - \ln(A)}{\ln(2)}$

   (c) $\dfrac{\ln(P)}{2\ln(A)}$

   (d) $\ln\left(\frac{P}{A}\right) - 2$

   (e) $\sqrt{\frac{P}{A}}$

3. Consider the function $f(x) = 5x^2(x^2 - 4)$. How many distinct x-intercepts does the graph of $y = f(x)$ have?

   (a) 0

   (b) 1

   (c) 2

   (d) 3

   (e) 4

4. If $f(x) = \dfrac{x}{x - 1}$, which of the following is equal to $f\left(\frac{1}{x^2}\right)$?

   (a) $\dfrac{1}{1 - x^2}$

   (b) $\dfrac{x^4}{1 - x^2}$

   (c) $\dfrac{(x - 1)^2}{x^2}$

   (d) $\dfrac{1 - x^2}{x^4}$

   (e) $\dfrac{x^2 - 1}{x^2}$

5. Let $f(x) = 8x^2 - 5x + 4$. Find $f(a + h)$.

   (a) $8a^2 + 16ah - 5a + 8h^2 - 5h + 4$

   (b) $8a^2 - 5a + 8h^2 - 5h + 4$

   (c) $8a^2 - a + 8h^2 - h + 4$

   (d) $8a^2 + 16ah - a + 8h^2 - h + 4$

   (e) $8a^2 - 5a + 4$

6. A hot drink is taken outside on a cold winter day when the air temperature is $-10°$C. According to a principle of physics called Newton's Law of Cooling, the temperature T (in degrees Celsius) of the drink t minutes after being taken outside is given by

$$T(t) = -10 + 110e^{-t}.$$

   Which of the following statements is true?

   (a) The temperature of the drink increases with time, but does not exceed 100°C.

   (b) The temperature of the drink increases with time, but does not exceed 110°C.

   (c) The temperature of the drink decreases with time, but does not drop below $-10°$C.

   (d) The temperature of the drink decreases with time, but does drop below 0°C.

   (e) The temperature of the drink fluctuates with time, both increasing and decreasing.

7. Which of the following best describes the behaviour of the function f defined by,

$$f(x) = \frac{8x^2}{\sqrt{4x^4 + 7}}?$$

   (a) The value of f increases without bound

   (b) The value of f approaches 8

   (c) The value of f approaches 4

   (d) The value of f approaches 2

   (e) The value of f approaches 0

8. Which of the following formulas defines the area, A, of a square in terms of its perimeter, p?

   a) $A = \frac{p^2}{16}$

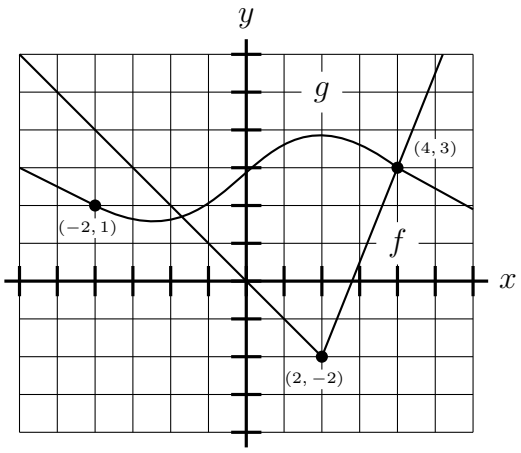
   b) $A = s^2$

   c) $A = \frac{p^2}{4}$

   d) $A = 16x^2$

   e) $p = 4\sqrt{A}$

9. Use the graphs of f and g to evaluate $g(f(2))$.



a) 2

b) 1

c) 0

d) -2

e) none of the above

10. The distance s (in feet) of a car moving in a straight line is given by $s = t^2 + t$, where t is measured in seconds. Find the average rate of change (velocity) for the time period from $t = 1$ to $t = 4$.

a) 5 ft/sec

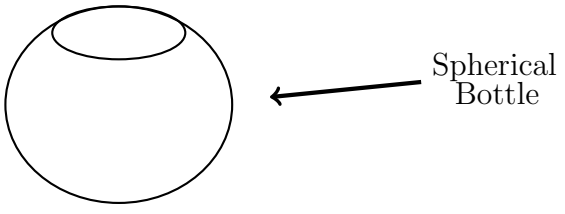
b) 6 ft/sec

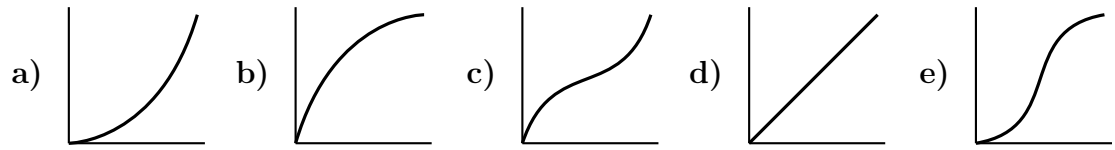
c) 9 ft/sec

d) 10 ft/sec

e) 11 ft/sec

11. Assume that water is poured into a spherical bottle at a constant rate. Which of the following graphs best represents the height of water in the bottle as a function of the amount of water in the bottle?

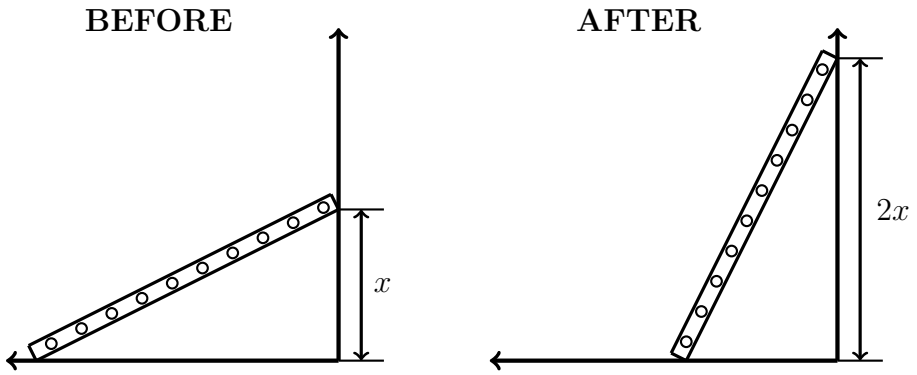




12. A ball is thrown into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area, A, of the circle in terms of the time, t, (in seconds) that have passed since the ball hits the lake.

    a) $A(t) = 25\pi t$
    b) $A(t) = \pi t^2$
    c) $A(t) = 25\pi t^2$
    d) $A(t) = 2\pi t^2$
    e) $A(t) = 10\pi t$

13. A ladder that is leaning against a wall is adjusted so that the distance of the top of the ladder from the floor is twice as high as it was before it was adjusted.



    The slope of the ladder is:

    a) Less than twice what it was
    b) Exactly twice what it was
    c) More than twice what it was
    d) The same as what it was before
    e) There is not enough information to determine if any of a through d is correct.

14. If θ is a solution to the equation $\cos(\theta) = \frac{1}{2}\sin(\theta)$, then what is $\tan(\theta)$?

    (a) 2
    (b) $\dfrac{1}{2}$
    (c) $\sqrt{3}$
    (d) $\dfrac{\sqrt{3}}{2}$
    (e) Not enough information to determine

15. How does the graph of $g(x) = \cos(x)$ compare to the graph of $h(x) = \cos(2x)$?

    (a) The period of $h(x)$ is double the period of $g(x)$.

    (b) The period of $h(x)$ is half the period of $g(x)$.

    (c) The amplitude of $h(x)$ is double the amplitude of $g(x)$.

    (d) The amplitude of $h(x)$ is half the amplitude of $g(x)$.

    (e) There is no difference between the graphs.

16. Two quilters, a master and their apprentice, are completing a project together. For every three quilt squares made by the apprentice, the master makes five squares in the same amount of time. If the two work together to complete a 64-square, what is the difference between the number of squares made by the master and the apprentice?

    (a) 2 squares

    (b) 8 squares

    (c) 16 squares

    (d) 40 squares

    (e) None of the above

17. What is the domain of the function $f(x) = \sqrt{x^2 - 36}$?

    (a) All real numbers

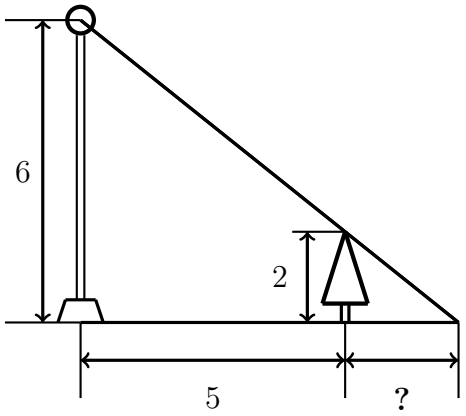
    (b) All real numbers such that $x \neq 0$

    (c) All real numbers such that $0 \leq x \leq 6$

    (d) All real numbers such that $-6 \leq x \leq 6$

    (e) All real numbers such that $x \leq -6$ or $x \geq 6$

18. A 2-metre tall Douglas fir tree is planted 5 metres from a lighted streetlight whose lamp is 6 metres above the ground. How many metres long is the shadow of that tree?



(a) 1 metre

(b) $\frac{5}{4}$ metres

(c) $\frac{5}{3}$ metres

(d) $\frac{12}{5}$ metres

(e) $\frac{5}{2}$ metres

19. Solve the equation $2x - 3y = 4$.

(a) $(x, y) = (0, -\frac{4}{3})$ only

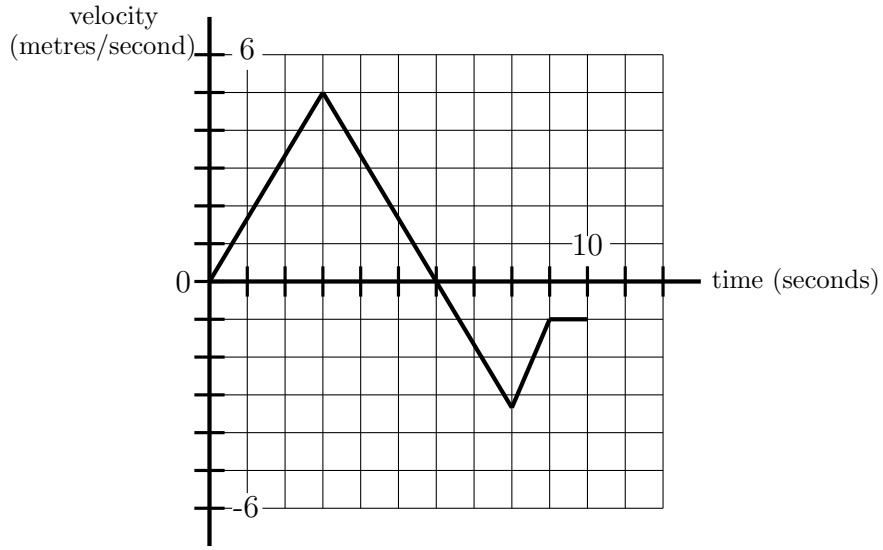
(b) $(x, y) = (5, 2)$ only

(c) $(x, y) = (2, 0)$ only

(d) no solutions

(e) infinitely many solutions

20. A vehicle is being test driven along a straight road. Its velocity over time is modeled using the function provided.



Based on this information, at what time is the vehicle farthest away from its starting point at time $t = 0$?

(a) $t = 3$ seconds

(b) $t = 5$ seconds

(c) $t = 6$ seconds

(d) $t = 8$ seconds

(e) $t = 10$ seconds