# First Post

## Vorlesung 2

**Definition 0.1.** $F \in L(V, W)$ can be represented by a matrix $A \in C^{n \times m}$ with $n = \dim V$ and $m = \dim W$ when basises are chosen.

**Definition 0.2** (Complex Hilbert space)**.** A complete (every cauchy sequence converges) complex vector space $\mathcal{H}$ with a sequilinear inner product

$$\langle \cdot, \cdot \rangle : \mathcal{H} \times \mathcal{H} \to \mathbb{C}$$

**Definition 0.3.** A Basis $(e_1, \dots, e_n) \subset \mathcal{H}$ with

$$\langle e_i, e_j \rangle = \delta_{ij}$$

**Definition 0.4.** We define the dual space

$$\mathcal{H}^* = \{f : \mathcal{H} \overset{linear}{\to} \mathbb{C}\}$$

**Definition 0.5.**

$$\begin{aligned} \mathcal{H}^* &= \{f : \mathcal{H} \overset{linear}{\to} \mathbb{C}\} \\ &= \{\langle x, \cdot \rangle | x \in \mathcal{H}\} \end{aligned}$$

We will use the bra-ket notation:

**Definition 0.6** (bra-ket Notation)**.** $\varphi, \psi \in \mathcal{H}$:

$$\begin{aligned} \varphi &= |\varphi\rangle \in \mathcal{H} & \text{bra} \\ \langle \varphi, \cdot \rangle &= \langle \varphi | \in L(\mathcal{H}) & \text{ket} \\ \langle \varphi, \psi \rangle &= \langle \varphi | \psi \rangle \end{aligned}$$

**Definition 0.7** (Rank-1 operators)**.** Let $|\psi\rangle, |\varphi\rangle \in \mathcal{H}$ The "*Outer product*" acts on $|\eta\rangle \in \mathcal{H}$ as

$$(|\psi\rangle\langle\varphi|)(|\eta\rangle) = |\psi\rangle\langle\varphi|\eta\rangle$$

**Definition 0.8** (self-adjoint operators, aka. Hermitian operators)**.**

$$\mathrm{Herm}(\mathcal{H}) = \{A \in L(\mathcal{H}) | A^\dagger = A\}$$

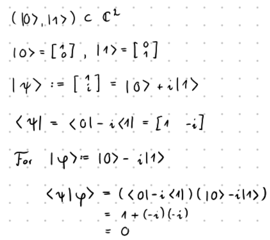


Abbildung 1: Example of BraKet

**Definition 0.9** (unitary operators)**.**

$$U(\mathcal{H}) = \{A \in L(\mathcal{H}) | A^\dagger = A^{-1}\}$$

**Definition 0.10** (normal operators)**.**

$$\{A \in L(\mathcal{H}) | A^\dagger A = AA^\dagger\}$$

**Proposition 0.11.** $A \in L(\mathcal{H})$ *ist diagonalizable if there is a basis of eigenvectors*