

# Blockchain System

## 1 Definitions

**Definition 1 (Set).** Let  $\mathbf{Elt}$  be the set of (concrete) elements. Let  $\emptyset$  be an empty set and  $\mathbf{e} \in \mathbf{Elt}$ . A set of elements is expressed as the following syntax:  
 $\mathbf{s} ::= \emptyset \mid \mathbf{e} \mid \mathbf{s} :: \mathbf{s}$

**Definition 2 (Account).** An account is a tuple  $\langle \mathbf{als}, \mathbf{pak}, \mathbf{puk}, \mathbf{pkh} \rangle$ , where  $\mathbf{als}$  is the alias of the account,  $\mathbf{pak}$  is its private key,  $\mathbf{puk}$  is its public key and  $\mathbf{pkh}$  is its public key hash .

**Definition 3 (Contract).** A contract is a tuple  $\langle \mathbf{als}, \mathbf{puh}, \mathbf{code} \rangle$ , where  $\mathbf{als}$  is the alias of the contract,  $\mathbf{puh}$  is its public hash, and  $\mathbf{code}$  is the code of the contract.

**Definition 4 (Manager).** A manager manages a single account. It is represented by a tuple  $\langle \mathbf{puk}, \mathbf{pkh}, \mathbf{bal}, \mathbf{cou} \rangle$ , where  $\mathbf{puk}$  is the public key of an account,  $\mathbf{pkh}$  is its public key hash,  $\mathbf{bal}$  is its balance and  $\mathbf{cou}$  is its counter whose form is a pair  $(n, b)$ , where  $n$  is a natural number and  $b$  is a boolean value.

**Definition 5 (Contractor).** A contractor manages a smart contract. It is represented by a tuple  $\langle \mathbf{puh}, \mathbf{bal}, \mathbf{code}, \mathbf{storage} \rangle$ , where  $\mathbf{puh}$  is the public key hash of the contract,  $\mathbf{bal}$  is its current balance,  $\mathbf{code}$  is its code, and  $\mathbf{storage}$  is its current storage.

**Definition 6 (Operation).** Operations are defined by the following grammar:

$\mathbf{op} ::= \text{transfer } \mathbf{n} \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}$   
|  $\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s} \text{ fee } \mathbf{m}$

**Definition 7 (Query).** Queries are defined by the following grammar:

$\mathbf{qry} ::= \text{get balance for } \mathbf{pkh}/\mathbf{puh}$   
|  $\text{get status for } \mathbf{oph}$   
|  $\text{get contract storage } \mathbf{puh}$   
|  $\text{get code for } \mathbf{puh}$   
|  $\text{get public key for } \mathbf{pkh}$   
|  $\text{get counter for } \mathbf{pkh}$

**Definition 8 (State of a node).** The state of a node is a tuple  $\mathbf{N} = [\mathbf{C}, \mathbf{O}, \mathbf{S}]$  where  $\mathbf{C}$  is a set of accounts,  $\mathbf{O}$  a set of operations, and  $\mathbf{S}$  a set of contracts.

When an operation is injected in a node, it enters a *pending pool* (and is called a *pending operation*).

**Definition 9 (Pending operation).** A pending operation is a tuple  $\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle$ , where  $\mathbf{op}$  is an operation,  $\mathbf{oph}$  is the operation hash, and  $\mathbf{t}$  is the time when the operation was injected.

After some time, a pending operation may be included in the blockchain as an accepted operation.

**Definition 10 (Accepted operation).** An accepted operation is a tuple  $\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle$ , where  $\mathbf{op}$  is an operation,  $\mathbf{oph}$  is the operation hash, and  $\mathbf{t}$  is the time when it was included in the blockchain.

**Definition 11 (Blockchain).** The state of a blockchain is a tuple  $[P, A, K, T, t]$  where  $P$  is a set of pending operations,  $A$  is a set of accepted operations,  $K$  is a set of managers,  $T$  is a set of contractors, and  $t$  is the current time.

**Definition 12 (Blockchain system).** A blockchain system is a pair  $N \parallel B$  where

1.  $N = [C, O, S]$  is the state of a node, and
2.  $B = [P, A, K, T, t]$  is the state of a blockchain such that  $\forall c \in C \implies \exists k \in K, k.pkh = c.pkh$  and  $\forall s \in S \implies \exists p \in T, s.puh = p.puh$ .

## 2 Rules

### 2.1 Transitions on Nodes

Each node has (nondeterministic) rules to propose an operation.

$$\begin{array}{c}
 \text{NODE-TRANSFER} \\
 \frac{\text{checkAcc}(\mathbf{puk}, \mathbf{C})}{[\mathbf{C}, \mathbf{O}, \mathbf{S}] \longrightarrow_N [\mathbf{C}, (\text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } () \text{ fee } \mathbf{m}) :: \mathbf{O}, \mathbf{S}]} \\
 \\
 \text{NODE-ORIGINATE} \\
 \frac{\text{checkAcc}(\mathbf{puk}, \mathbf{C}) \quad \text{checkId}(\mathbf{id}, \mathbf{S})}{[\mathbf{C}, \mathbf{O}, \mathbf{S}] \longrightarrow_N [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{puk} \text{ running } \mathbf{code} \text{ init } \mathbf{s} \text{ fee } \mathbf{m}) :: \mathbf{O}, \mathbf{S}]} \\
 \\
 \text{NODE-SYSTEM} \\
 \frac{\mathbf{N} \longrightarrow_N \mathbf{N}'}{\mathbf{N} \parallel \mathbf{B} \longrightarrow \mathbf{N}' \parallel \mathbf{B}}
 \end{array}$$

## 2.2 Transfers

Rule 2 [injected]:

$$\frac{\text{checkBal}(\mathbf{K}, \mathbf{puk}, \mathbf{n}, \mathbf{m}) \quad \text{checkCou}(\mathbf{K}, \mathbf{puk}) \quad \text{checkPub}(\mathbf{K}, \mathbf{puk}') \quad \begin{array}{l} \mathbf{op} = \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } () \text{ fee } \mathbf{m} \\ \mathbf{oph} = \text{genOpHash}(\mathbf{puk}, \mathbf{puk}', \mathbf{n}, \mathbf{m}) \end{array}}{[\mathbf{C}, \mathbf{op} :: \mathbf{O}, \mathbf{S}] \parallel [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \longrightarrow [\mathbf{C}, \mathbf{O}, \mathbf{S}] \parallel [\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{K}, \mathbf{puk}, \text{True}), \mathbf{T}, \mathbf{t}]}$$

Rule 3 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{K}, \mathbf{pkh})}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}]} \quad (1)$$

Rule 4 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{K}, \mathbf{pkh}, m, n)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}]} \quad (2)$$

Rule 5 [rejected of public key]:

$$\frac{\neg \text{checkPub}(\mathbf{K}, \mathbf{pkh}')}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}]} \quad (3)$$

Rule 6 [included]:

$$\frac{\text{BLOCK-ACCEPT} \quad \mathbf{t}' - \mathbf{t} < 60}{[\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rightarrow [\mathbf{P}, \langle \mathbf{op}, \mathbf{oph}, \mathbf{t}' \rangle :: \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}' + 1]}$$

Rule 7 [timeout]:

$$\frac{\text{BLOCK-TIMEOUT} \quad \mathbf{op} = \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } () \text{ fee } \mathbf{m} \quad \mathbf{t}' - \mathbf{t} \geq 60}{[\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rightarrow [\mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{K}, \mathbf{puk}, \text{False}), \mathbf{T}, \mathbf{t}']}$$

## 2.3 Smart Contracts

A. Originate

Rule 2 [injected]:

$$\frac{\text{BLOCK-ORIGINATE} \quad \text{checkBal}(\mathbf{K}, \mathbf{puk}, \mathbf{n}, \mathbf{m}) \quad \text{checkCou}(\mathbf{K}, \mathbf{puk}) \quad \text{checkPrg}(\mathbf{code}, \mathbf{s}) \quad \mathbf{orig} = \text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{puk} \text{ running } \mathbf{code} \text{ init } \mathbf{s} \text{ fee } \mathbf{m}}{\langle [\mathbf{C}, \mathbf{orig} :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{orig} :: \mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{K}, \mathbf{puk}, \mathbf{True}), \mathbf{T}, \mathbf{t}] \rangle}$$

Rule 3 [rejected of code]:

$$\frac{\neg \text{checkPrg}(\mathbf{code}, \mathbf{s})}{\langle [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s})] :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (4)$$

Rule 4 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{K}, \mathbf{pkh})}{\langle [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s})] :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (5)$$

Rule 5 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{K}, \mathbf{pkh}, \mathbf{n}, \mathbf{m})}{\langle [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s})] :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (6)$$

Rule 6 [included]:

$$\frac{}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\langle (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s}), \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, (\langle \mathbf{id}, \text{generateHash}(\mathbf{id}, \mathbf{code}, \mathbf{s}, \mathbf{t}'), \mathbf{code} \rangle :: \mathbf{S})], [\mathbf{P}, \langle (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s}), \mathbf{t}' \rangle :: \mathbf{A}, \text{updateSucc}(\mathbf{K}, \mathbf{puk}, \mathbf{n}, \mathbf{m}), (\langle \text{generateHash}(\mathbf{id}, \mathbf{code}, \mathbf{s}, \mathbf{t}'), \mathbf{n}, \mathbf{code}, \text{getStorage}(\mathbf{code}, \mathbf{s}) \rangle :: \mathbf{T}), \mathbf{t}' + 1] \rangle} \quad (7)$$

Rule 7 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \geq 60}{\begin{array}{l} \langle \langle \text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running code init } s \rangle \rangle \\ :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}' \rangle \rightarrow [\mathbf{P}, \mathbf{A}, \text{updateCou}(\mathbf{K}, puk, \text{False}), \mathbf{T}, \mathbf{t}'] \end{array}} \quad (8)$$

## B. Transfer

Rule 1 [proposal]:

$$\frac{\text{checkAcc}(pkh, \mathbf{C})}{\begin{array}{l} \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \\ \text{fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \end{array}} \quad (9)$$

Rule 2 [injected]:

$$\frac{\text{checkBal}(\mathbf{K}, pkh, n, m) \wedge \text{checkCou}(\mathbf{K}, pkh) \wedge \text{checkPuh}(\mathbf{T}, puh) \wedge \text{checkArg}(\mathbf{T}, puh, s)}{\begin{array}{l} \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \\ \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [(\langle \text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m \rangle, \\ \text{generateOph}(pkh, puh, s, n, m, \mathbf{t}), \mathbf{t} \rangle) :: \mathbf{P}, \mathbf{A}, \text{updateCou}(\mathbf{K}, pkh, \text{True}), \\ \mathbf{T}, \mathbf{t}] \rangle \end{array}} \quad (10)$$

Rule 3 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{K}, pkh)}{\begin{array}{l} \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \\ \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \end{array}} \quad (11)$$

Rule 4 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{K}, pkh, n, m)}{\begin{array}{l} \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \\ \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \end{array}} \quad (12)$$

Rule 5 [rejected of public hash]:

$$\frac{\neg \text{checkPuh}(\mathbf{T}, \text{puh})}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \text{pkh} \text{ to } \text{puh} \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (13)$$

Rule 6 [rejected of argument]:

$$\frac{\neg \text{checkArg}(\mathbf{T}, \text{puh}, s)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \text{pkh} \text{ to } \text{puh} \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (14)$$

Rule 7 [included]:

$$\frac{}{\langle \langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } s \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}' \rangle \rightarrow [\mathbf{P}, \langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } s \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t}' \rangle :: \mathbf{A}, \text{updateSucc}(\mathbf{K}, \mathbf{puk}, ", \mathbf{n}, \mathbf{m}), \text{updateConstr}(\mathbf{T}, \mathbf{puh}, \mathbf{n}, s), \mathbf{t}' + 1] \rangle} \quad (15)$$

Rule 8 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \geq 60}{\langle \langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } s \text{ fee } \mathbf{m}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}' \rangle \rightarrow [\mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{K}, \mathbf{puk}, \text{False}), \mathbf{T}, \mathbf{t}'] \rangle} \quad (16)$$

### 3 Functions

1. Function `checkAcc(pkh, C)` checks whether an account *pkh* exists in **C**
2. Function `checkPub(K, pkh)` checks whether the public key of the public key hash *pkh* is reveled to the blockchain.
3. Function `checkBal(K, pkh, n, m)` checks whether the balance of the account *pkh* is greater or equal to  $m + n$
4. Function `checkCou(K, pkh)` checks whether the current counter of an account *pkh* is unlocked (i.e., its flag is False)
5. Function `updateSucc(K, pkh, pkh', n, m)` updates the balance and the counter of the account *pkh* and the balance of the account *pkh'*, where
  - $\langle \text{puk}, \text{pkh}, \text{bal}, (\mathbf{n}, \text{True}) \rangle \Rightarrow \langle \text{puk}, \text{pkh}, \text{bal} - \mathbf{n} - \mathbf{m}, (\mathbf{n} + 1, \text{False}) \rangle$
  - $\langle \text{puk}', \text{pkh}', \text{bal}', \text{cou}' \rangle \Rightarrow \langle \text{puk}', \text{pkh}', \text{bal}' + \mathbf{n}, \text{cou}' \rangle$
6. Function `updateCou(K, puk, b')` updates the counter lock of the account *pkh* (True = locked, False = unlocked), where
  - $\langle \text{puk}, \text{pkh}, \text{bal}, (\mathbf{n}, \mathbf{b}) \rangle \Rightarrow \langle \text{puk}, \text{pkh}, \text{bal}, (\mathbf{n}, \mathbf{b}') \rangle$

7. Function `checkId(id, S)` checks whether a contract *id* does not already exist in **S**
8. Function `checkPrg(code, s)` checks whether the code *code* are well type and *s* is well type input
9. Function `generateOph(pkh, pkh', n, m, t)` generates a operation hash
10. Function `generateHash(S, id, puh, code, t)` generates the public hash of a contract
11. Function `getStorage(code, s)` gets the storage for the code *code* and the input *s*

## 4 Some implementations

Function `checkAcc(puh, C)` checks whether an account exists and `checkPuk(puh, K)` checks the revelation of its public key to the blockchain.

```
let rec checkAcc puh C =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: C' ->
    if (puh = puh') then true
    else checkAcc (puh, C')
```

```
let rec checkPuk puh K =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: K' ->
    if (puh = puh') and (puk != nil) then true
    else 5checkPuk (puh, K')
```

The following functions interact with **K**.

```
let rec checkBal K puk n m =
  match K with
  | 0 -> true
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (n + m) <= bal then true
    else checkBal (K', puk, n, m)
```

```
let rec checkPub K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then true
    else checkExi (K', puk)
```

```

let rec checkCou K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (cou = T) then true
    else checkCou (K', puk)

let rec updateCou K puk =
  match K with
  | 0 -> 0
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then < puk', bal, F > :: K'
    else < puk', bal, cou > :: updateCou (K', puk)

let rec updateSucc K puk puk' m n =
  match K with
  | 0 -> 0
  | < puk'', bal, cou > :: K' ->
    if (puk = puk'') then < puk'', bal - (n + m), T >
      :: updateSucc (K', puk, puk', n, m)
    else if (puk' = puk'') then < puk'', bal + n, cou > :: K'
      else < puk'', bal, cou >
        :: updateSucc (K', puk, puk', n, m)

```