

Blockchain System

1 Definitions

Definition 1. Let \mathbf{Elt} be the set of (concrete) elements. Let \emptyset be an empty set and $\mathbf{e} \in \mathbf{Elt}$. A set of elements is expressed as the following syntax:

$\mathbf{s} ::= \emptyset \mid \mathbf{e} \mid \mathbf{s} :: \mathbf{s}$

Definition 2. An account is a tuple $\langle \mathbf{als}, \mathbf{pak}, \mathbf{puk}, \mathbf{pkh} \rangle$, where \mathbf{als} is the alias of the account, \mathbf{pak} is its private key, \mathbf{puk} is its public key and \mathbf{pkh} is its public key hash .

Definition 3. An contract is a tuple $\langle \mathbf{als}, \mathbf{puh}, \mathbf{code} \rangle$, where \mathbf{als} is the alias of the contract, \mathbf{puh} is its public hash, and \mathbf{code} is the code of the contract.

Definition 4. A manager manages a single account. It is represented by a tuple $\langle \mathbf{puk}, \mathbf{pkh}, \mathbf{bal}, \mathbf{cou} \rangle$, where \mathbf{puk} is the public key of an account, \mathbf{pkh} is its public key hash, \mathbf{bal} is its balance and \mathbf{cou} is its counter whose form is a pair (n, b) , where n is a natural number and b is a boolean value.

Definition 5. A contractor manages a smart contract. It is represented by a tuple $\langle \mathbf{puh}, \mathbf{bal}, \mathbf{code}, \mathbf{storage} \rangle$, where \mathbf{puh} is the public key hash of the contract, \mathbf{bal} is its current balance, \mathbf{code} is its code, and $\mathbf{storage}$ is its current storage.

Definition 6. Operations are defined by the following grammar:

$\mathbf{op} ::= \text{transfer } \mathbf{n} \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}$
| $\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s} \text{ fee } \mathbf{m}$

Definition 7. A query is defined by the following grammar:

$\mathbf{qry} ::= \text{get balance for } \mathbf{pkh}/\mathbf{puh}$
| $\text{get status for } \mathbf{oph}$
| $\text{getStorage } \mathbf{puh}$
| $\text{get code for } \mathbf{puh}$
| $\text{get public key for } \mathbf{pkh}$
| $\text{get counter for } \mathbf{pkh}$

Definition 8. The state of a node is a tuple $\mathbf{N} = [\mathbf{C}, \mathbf{O}, \mathbf{S}]$ where \mathbf{C} is a set of accounts, \mathbf{O} a set of operations, and \mathbf{S} a set of contracts.

When an operation is injected in a node, it enters a *pending pool* (and is called a *pending operation*).

Definition 9. A pending operation is a tuple $\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle$, where \mathbf{op} is an operation, \mathbf{oph} is the operation hash, and \mathbf{t} is the time when the operation was injected.

After some time, a pending operation may be included in the blockchain as an accepted operation.

Definition 10. An accepted operation is a tuple $\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle$, where \mathbf{op} is an operation, \mathbf{oph} is the operation hash, and \mathbf{t} is the time when it was included in the blockchain.

Definition 11. The state of a blockchain is a tuple $[P, A, M, T, t]$ where P is a set of pending operations, A is a set of accepted operations, M is a set of managers, T is a set of contractors, and t is the current time.

Definition 12. A blockchain configuration is a pair $N \parallel B$ where

1. $N = [C, O, S]$ is the state of a node, and
2. $B = [P, A, M, T, t]$ is the state of a blockchain such that $\forall c \in C \implies \exists k \in M, k.pkh = c.pkh$ and $\forall s \in S \implies \exists p \in T, s.puh = p.puh$.

2 Rules

2.1 Transitions on Nodes

Each node has (nondeterministic) rules to propose an operation.

$$\frac{\text{NODE-TRANSFER} \quad \text{checkAcc}(\mathbf{puk}, \mathbf{C}) \quad \mathbf{op} = \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}}{[\mathbf{C}, \mathbf{O}, \mathbf{S}] \longrightarrow_N [\mathbf{C}, \mathbf{op} :: \mathbf{O}, \mathbf{S}]}$$

$$\frac{\text{NODE-ORIGINATE} \quad \text{checkAcc}(\mathbf{puk}, \mathbf{C}) \quad \text{checkId}(\mathbf{id}, \mathbf{S}) \quad \mathbf{op} = \text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{puk} \text{ running code init } \mathbf{s} \text{ fee } \mathbf{m}}{[\mathbf{C}, \mathbf{O}, \mathbf{S}] \longrightarrow_N [\mathbf{C}, \mathbf{op} :: \mathbf{O}, \mathbf{S}]}$$

$$\frac{\text{NODE-SYSTEM} \quad \mathbf{N} \longrightarrow_N \mathbf{N}'}{\mathbf{N} \parallel \mathbf{B} \longrightarrow \mathbf{N}' \parallel \mathbf{B}} \quad \frac{\text{BLOCK-SYSTEM} \quad \mathbf{B} \longrightarrow_B \mathbf{B}'}{\mathbf{N} \parallel \mathbf{B} \longrightarrow \mathbf{N} \parallel \mathbf{B}'}$$

2.2 Transfers

Rule 2 [injected]:

$$\frac{\text{checkBal}(\mathbf{M}, \mathbf{puk}, \mathbf{n}, \mathbf{m}) \quad \text{checkCou}(\mathbf{M}, \mathbf{puk}) \quad \text{checkPub}(\mathbf{M}, \mathbf{puk}') \quad \mathbf{op} = \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } () \text{ fee } \mathbf{m} \quad \mathbf{oph} = \text{genOpHash}(\mathbf{puk}, \mathbf{puk}', \mathbf{n}, \mathbf{m})}{[\mathbf{C}, \mathbf{op} :: \mathbf{O}, \mathbf{S}] \parallel [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \longrightarrow [\mathbf{C}, \mathbf{O}, \mathbf{S}] \parallel [\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{M}, \mathbf{puk}, \mathbf{True}), \mathbf{T}, \mathbf{t}]}$$

Rule 3 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{M}, \mathbf{pkh})}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (1)$$

Rule 4 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{M}, \mathbf{pkh}, m, n)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh}' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (2)$$

Rule 5 [rejected of public key]:

$$\frac{\neg \text{checkPub}(\mathbf{M}, pkh')}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } pkh' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (3)$$

Rule 6 [included]:

$$\frac{\text{BLOCK-ACCEPT} \quad \mathbf{t}' - \mathbf{t} < 60}{\langle [\mathbf{op}, \mathbf{oph}, \mathbf{t}] :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}' \rangle \longrightarrow_B [\mathbf{P}, \langle \mathbf{op}, \mathbf{oph}, \mathbf{t}' \rangle :: \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}' + 1]} \quad (4)$$

Rule 7 [timeout]:

$$\frac{\text{BLOCK-TIMEOUT} \quad \mathbf{op} = \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puk}' \text{ arg } () \text{ fee } \mathbf{m} \quad \mathbf{t}' - \mathbf{t} \geq 60}{\langle [\mathbf{op}, \mathbf{oph}, \mathbf{t}] :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}' \rangle \longrightarrow_B [\mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{M}, \mathbf{puk}, \mathbf{False}), \mathbf{T}, \mathbf{t}']}$$

2.3 Smart Contracts

A. Originate

Rule 2 [injected]:

$$\frac{\text{BLOCK-ORIGINATE} \quad \text{checkBal}(\mathbf{M}, \mathbf{puk}, \mathbf{n}, \mathbf{m}) \quad \text{checkCou}(\mathbf{M}, \mathbf{puk}) \quad \text{checkPrg}(\mathbf{code}, \mathbf{s}) \quad \mathbf{orig} = \text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{puk} \text{ running } \mathbf{code} \text{ init } \mathbf{s} \text{ fee } \mathbf{m}}{\langle [\mathbf{C}, \mathbf{orig} :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\langle \mathbf{orig}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{M}, \mathbf{puk}, \mathbf{True}), \mathbf{T}, \mathbf{t}] \rangle}$$

Rule 3 [rejected of code]:

$$\frac{\neg \text{checkPrg}(\mathbf{code}, \mathbf{s})}{\langle [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s}) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (4)$$

Rule 4 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{M}, \mathbf{pkh})}{\langle [\mathbf{C}, (\text{originate contract } \mathbf{id} \text{ transferring } \mathbf{n} \text{ from } \mathbf{pkh} \text{ running } \mathbf{code} \text{ init } \mathbf{s}) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (5)$$

Rule 5 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{M}, pkh, n, m)}{\langle [\mathbf{C}, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (6)$$

Rule 6 [included]:

BLOCK-ACCEPT

$$\frac{\begin{array}{l} \mathbf{op} = \text{originate contract } id \text{ transferring } n \text{ from } puk \text{ running } code \text{ init } s \text{ fee } m \\ \mathbf{oph} = \text{genHash}(id, code, s, t') \quad t' - t < 60 \end{array}}{\begin{array}{l} \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\langle \mathbf{op}, t \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, t'] \rangle \longrightarrow \langle [\mathbf{C}, \mathbf{O}, \langle id, \mathbf{oph}, code \rangle :: \mathbf{S}], \\ [\mathbf{P}, \langle \mathbf{op}, t' \rangle :: \mathbf{A}, \text{updSucc}(\mathbf{M}, puk, n, m), \langle \mathbf{oph}, \text{getStorage}(code, s) \rangle :: \mathbf{T}, t' + 1] \rangle \end{array}}$$

Rule 7 [timeout]:

BLOCK-TIMEOUT

$$\frac{\begin{array}{l} \mathbf{op} = \text{originate contract } id \text{ transferring } n \text{ from } puk \text{ running } code \text{ init } s \text{ fee } m \\ t' - t \geq 60 \end{array}}{\langle \langle \mathbf{op}, t \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, t' \rangle \longrightarrow_B [\mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{M}, puk, \mathbf{False}), \mathbf{T}, t']}$$

B. Transfer

Rule 1 [proposal]:

$$\frac{\text{checkAcc}(pkh, \mathbf{C})}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (7)$$

Rule 2 [injected]:

$$\frac{\text{checkBal}(\mathbf{M}, pkh, n, m) \wedge \text{checkCou}(\mathbf{M}, pkh) \wedge \text{checkPuh}(\mathbf{T}, puh) \wedge \text{checkArg}(\mathbf{T}, puh, s)}{\begin{array}{l} \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \\ \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [(< (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m), \\ \text{generateOph}(pkh, puh, s, n, m, t), t >) :: \mathbf{P}, \mathbf{A}, \text{updateCou}(\mathbf{M}, pkh, \mathbf{True}), \\ \mathbf{T}, t] \rangle \end{array}} \quad (8)$$

Rule 3 [rejected of counter]:

$$\frac{\neg \text{checkCou}(\mathbf{M}, pkh)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (9)$$

Rule 4 [rejected of balance]:

$$\frac{\neg \text{checkBal}(\mathbf{M}, pkh, n, m)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (10)$$

Rule 5 [rejected of public hash]:

$$\frac{\neg \text{checkPuh}(\mathbf{T}, puh)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (11)$$

Rule 6 [rejected of argument]:

$$\frac{\neg \text{checkArg}(\mathbf{T}, puh, s)}{\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}] \rangle} \quad (12)$$

Rule 7 [included]:

$$\frac{}{[\langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}'] \rightarrow [\mathbf{P}, \langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t}' \rangle :: \mathbf{A}, \text{updateSucc}(\mathbf{M}, \mathbf{puk}, ", \mathbf{n}, \mathbf{m}), \text{updateConstr}(\mathbf{T}, \mathbf{puh}, \mathbf{n}, \mathbf{s}), \mathbf{t}' + 1]} \quad (13)$$

Rule 8 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \geq 60}{[\langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{M}, \mathbf{T}, \mathbf{t}'] \rightarrow [\mathbf{P}, \mathbf{A}, \text{updCount}(\mathbf{M}, \mathbf{puk}, \mathbf{False}), \mathbf{T}, \mathbf{t}']} \quad (14)$$

3 Functions

1. Function `checkAcc(pkh, C)` checks whether an account `pkh` exists in `C`

2. Function `checkPub(K, pkh)` checks whether the public key of the public key hash *pkh* is revealed to the blockchain.
3. Function `checkBal(K, pkh, n, m)` checks whether the balance of the account *pkh* is greater or equal to $m + n$
4. Function `checkCou(K, pkh)` checks whether the current counter of an account *pkh* is unlocked (i.e., its flag is False)
5. Function `updateSucc(K, pkh, pkh', n, m)` updates the balance and the counter of the account *pkh* and the balance of the account *pkh'*, where
 - $\langle \text{puk}, \text{pkh}, \text{bal}, (n, \text{True}) \rangle \Rightarrow \langle \text{puk}, \text{pkh}, \text{bal} - n - m, (n + 1, \text{False}) \rangle$
 - $\langle \text{puk}', \text{pkh}', \text{bal}', \text{cou}' \rangle \Rightarrow \langle \text{puk}', \text{pkh}', \text{bal}' + n, \text{cou}' \rangle$
6. Function `updateCou(K, puk, b')` updates the counter lock of the account *pkh* (True = locked, False = unlocked), where
 - $\langle \text{puk}, \text{pkh}, \text{bal}, (n, b) \rangle \Rightarrow \langle \text{puk}, \text{pkh}, \text{bal}, (n, b') \rangle$
7. Function `checkId(id, S)` checks whether a contract *id* does not already exist in **S**
8. Function `checkPrg(code, s)` checks whether the code *code* are well type and *s* is well type input
9. Function `generateOph(pkh, pkh', n, m, t)` generates a operation hash
10. Function `generateHash(S, id, puh, code, t)` generates the public hash of a contract
11. Function `getStorage(code, s)` gets the storage for the code *code* and the input *s*

4 Some implementations

Function `checkAcc(puh, C)` checks whether an account exists and `checkPuk(puh, K)` checks the revelation of its public key to the blockchain.

```
let rec checkAcc puh C =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: C' ->
    if (puh = puh') then true
    else checkAcc (puh, C')
```

```
let rec checkPuk puh K =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: K' ->
    if (puh = puh') and (puk != nil) then true
    else 5checkPuk (puh, K')
```

The following functions interact with **M**.

```

let rec checkBal K puk n m =
  match K with
  | 0 -> true
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (n + m) <= bal then true
    else checkBal (K', puk, n, m)

let rec checkPub K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then true
    else checkExi (K', puk)

let rec checkCou K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (cou = T) then true
    else checkCou (K', puk)

let rec updateCou K puk =
  match K with
  | 0 -> 0
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then < puk', bal, F > :: K'
    else < puk', bal, cou > :: updateCou (K', puk)

let rec updateSucc K puk puk' m n =
  match K with
  | 0 -> 0
  | < puk'', bal, cou > :: K' ->
    if (puk = puk'') then < puk'', bal - (n + m), T >
      :: updateSucc (K', puk, puk', n, m)
    else if (puk' = puk'') then < puk'', bal + n, cou > :: K'
      else < puk'', bal, cou >
        :: updateSucc (K', puk, puk', n, m)

```