Blockchain System

1 Definitions

Definition 1 (Set). *Let* $\mathbf{Elt}$ *be the set of (concrete) elements. Let* $\emptyset$ *be an empty set and* $\mathbf{e} \in \mathbf{Elt}$*. A set of elements is expressed as the following syntax:*
$\mathbf{s} ::= \emptyset \mid \mathbf{e} \mid \mathbf{s} :: \mathbf{s}$

Definition 2 (Account). *An account is a tuple* $\langle \boldsymbol{als}, \boldsymbol{pak}, \boldsymbol{puk}, \boldsymbol{pkh} \rangle$*, where* $\boldsymbol{als}$ *is the alias of the account,* $\boldsymbol{pak}$ *is its private key,* $\boldsymbol{puk}$ *is its public key and* $\boldsymbol{pkh}$ *is its public key hash .*

Definition 3 (Contract). *An contract is a tuple* $\langle \boldsymbol{als}, \boldsymbol{puh}, \boldsymbol{code} \rangle$*, where* $\boldsymbol{als}$ *is the alias of the contract,* $\boldsymbol{puh}$ *is its public hash, and* $\boldsymbol{code}$ *is the code of the contract.*

Definition 4 (Manager). *A manager manages a single account. It is represented by a tuple* $\langle \boldsymbol{puk}, \boldsymbol{pkh}, \boldsymbol{bal}, \boldsymbol{cou} \rangle$*, where* $\boldsymbol{puk}$ *is the public key of an account,* $\boldsymbol{pkh}$ *is its public key hash,* $\boldsymbol{bal}$ *is its balance and* $\boldsymbol{cou}$ *is its counter whose form is a pair* (n, b)*, where* n *is a natural number and* b *is a boolean value.*

Definition 5 (Contractor). *A contractor manages a smart contract. It is represented by a tuple* $\langle \boldsymbol{puh}, \boldsymbol{bal}, \boldsymbol{code}, \boldsymbol{storage} \rangle$*, where* $\boldsymbol{puh}$ *is the public key hash of the contract,* $\boldsymbol{bal}$ *is its current balance,* $\boldsymbol{code}$ *is its code, and* $\boldsymbol{storage}$ *is its current storage.*

Definition 6 (Operation). *Operations are defined by the following grammar:*

$$
\begin{aligned}
\boldsymbol{op} ::= & \textit{ transfer } \boldsymbol{n} \textit{ from } \boldsymbol{pkh} \textit{ to } \boldsymbol{pkh}' \textit{ arg } \boldsymbol{s} \textit{ fee } \boldsymbol{m} \\
& \mid \textit{originate contract } \boldsymbol{id} \textit{ transferring } \boldsymbol{n} \textit{ from } \boldsymbol{pkh} \textit{ running } \boldsymbol{code} \textit{ init } \boldsymbol{s} \textit{ fee } \boldsymbol{m}
\end{aligned}
$$

Definition 7 (Query). *Queries are defined by the following grammar:*

$$
\begin{aligned}
\boldsymbol{qry} ::= & \textit{ get balance for } \boldsymbol{pkh}/\boldsymbol{puh} \\
& \mid \textit{get status for } \boldsymbol{oph} \\
& \mid \textit{get contract storage } \boldsymbol{puh} \\
& \mid \textit{get code for } \boldsymbol{puh} \\
& \mid \textit{get public key for } \boldsymbol{pkh} \\
& \mid \textit{get counter for } \boldsymbol{pkh}
\end{aligned}
$$

Definition 8 (State of a node). *The state of a node is a tuple* $[\boldsymbol{C}, \boldsymbol{O}, \boldsymbol{S}]$ *where* $\boldsymbol{C}$ *is a set of accounts,* $\boldsymbol{O}$ *a set of operations, and* $\boldsymbol{S}$ *a set of contracts.*

When an operation is injected in a node, it enters a *pending pool* (and is called a pending operation).

Definition 9 (Pending operation). *A pending operation is a tuple* $\langle \boldsymbol{op}, \boldsymbol{oph}, \boldsymbol{t} \rangle$*, where* $\boldsymbol{op}$ *is an operation,* $\boldsymbol{oph}$ *is the operation hash and* $\boldsymbol{t}$ *is the time when the operation was injected.*

After some time, a pending operation may be included in the blockchain as an accepted operation.

Definition 10 (Accepted operation). *An accepted operation is a tuple* $\langle \boldsymbol{op}, \boldsymbol{oph}, \boldsymbol{t} \rangle$*, where* $\boldsymbol{op}$ *is an operation,* $\boldsymbol{oph}$ *is the operation hash and* $\boldsymbol{t}$ *is the time when it was included in the blockchain.*

Definition 11 (Blockchain). *The state of a blockchain is a tuple* $[\boldsymbol{P}, \boldsymbol{A}, \boldsymbol{K}, \boldsymbol{T}, \boldsymbol{t}]$ *where* $\boldsymbol{P}$ *is a set of pending operations,* $\boldsymbol{A}$ *is a set of accepted operations,* $\boldsymbol{K}$ *is a set of managers,* $\boldsymbol{T}$ *is a set of contractors, and* $\boldsymbol{t}$ *is the current time of the blockchain.*

Definition 12 (Blockchain system). *A blockchain system is a pair* $\langle \mathbf{M}, \mathbf{B} \rangle$ *where*

1. $\mathbf{M} = [\boldsymbol{C}, \boldsymbol{O}, \boldsymbol{S}]$ *is the state of a node, and*
2. $\mathbf{B} = [\boldsymbol{P}, \boldsymbol{A}, \boldsymbol{K}, \boldsymbol{T}, \boldsymbol{t}]$ *is the state of a blockchain such that* $\forall c \in \boldsymbol{C} \implies \exists k \in \boldsymbol{K}, k.pkh = c.pkh$ *and* $\forall s \in \boldsymbol{S} \implies \exists p \in \boldsymbol{T}, s.puh = p.puh$.

2 Rules

2.1 Transfers

Rule 1 [proposal]:

$$\frac{\text{checkAcc}(pkh, \mathbf{C})}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } pkh' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \quad (1)$$

Rule 2 [injected]:

$$\frac{\text{checkBan}(\mathbf{K}, pkh, n, m) \wedge \text{checkCou}(\mathbf{K}, pkh) \wedge \text{checkPub}(\mathbf{K}, pkh')}{\begin{array}{l}\langle [\mathbf{C}, (\text{transfer } n \text{ from } pkh \text{ to } pkh' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \\ \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [(< \text{transfer } n \text{ from } puk \text{ to } puk' \text{ fee } m, \text{generateOph}(pkh, \\ pkh', n, m, \mathbf{t}), \mathbf{t}>) :: \mathbf{P}, \mathbf{A}, \text{updateCou}(\mathbf{K}, pkh, \text{True}), \mathbf{T}, \mathbf{t}] \rangle\end{array}} \quad (2)$$

Rule 3 [rejected of counter]:

$$\frac{\neg\ \text{checkCou}(\mathbf{K},\ pkh)}{\langle[\mathbf{C},\ (\text{transfer}\ n\ \text{from}\ pkh\ \text{to}\ pkh'\ \text{fee}\ m)\ ::\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \quad (3)$$

Rule 4 [rejected of balance]:

$$\frac{\neg\ \text{checkBan}(\mathbf{K},\ pkh, m, n)}{\langle[\mathbf{C},\ (\text{transfer}\ n\ \text{from}\ pkh\ \text{to}\ pkh'\ \text{fee}\ m)\ ::\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \quad (4)$$

Rule 5 [rejected of public key]:

$$\frac{\neg\ \text{checkPub}(\mathbf{K},\ pkh')}{\langle[\mathbf{C},\ (\text{transfer}\ n\ \text{from}\ pkh\ \text{to}\ pkh'\ \text{fee}\ m)\ ::\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \quad (5)$$

Rule 6 [included]:

$$\frac{}{[<\text{transfer}\ n\ \text{from}\ puk\ \text{to}\ puk'\ \text{fee}\ m,\ \text{oph},\ \mathbf{t}>\ ::\ \mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}'\ ] \rightarrow [\mathbf{P},\ <\text{transfer}\ n\ \text{from}\ puk\ \text{to}\ puk'\ \text{fee}\ m,\ \text{oph},\ \mathbf{t}'\ >\ ::\ \mathbf{A},\ \text{updateSucc}(\mathbf{K}, puk, puk', n, m),\ \mathbf{T},\ \mathbf{t}'\ +\ 1]} \quad (6)$$

Rule 7 [timeout]:

$$\frac{\mathbf{t}'\ -\ \mathbf{t} \geq 60}{[<\text{transfer}\ n\ \text{from}\ puk\ \text{to}\ puk'\ \text{fee}\ m,\ \text{oph},\ \mathbf{t}>\ ::\ \mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}'\ ] \rightarrow [\mathbf{P},\ \mathbf{A},\ \text{updateCou}(\mathbf{K},\ puk,\ \text{False}),\ \mathbf{T},\ \mathbf{t}']} \quad (7)$$

2.2 Smart Contracts

A. Originate

Rule 1 [proposal]:

$$\frac{\text{checkAcc}(pkh,\ \mathbf{C}) \wedge \text{checkId}(id,\ \mathbf{S})}{\langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ (\text{originate contract}\ id\ \text{transferring}\ n\ \text{from}\ pkh\ \text{running}\ code\ \text{init}\ s)\ ::\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \quad (8)$$

Rule 2 [injected]:

$$\frac{\text{checkBan}(\mathbf{K},\, pkh, n, m) \wedge \text{checkCou}(\mathbf{K},\, pkh) \wedge \text{checkPrg}(code, s)}{\langle[\mathbf{C},\, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\ \rangle \rightarrow \langle\ [\mathbf{C},\, \mathbf{O},\, \mathbf{S}],\, [(\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{P},\, \mathbf{A},\, \text{updateCou}(\mathbf{K},\, pkh,\, \text{True}),\, \mathbf{T},\, \mathbf{t}]\ \rangle} \quad (9)$$

Rule 3 [rejected of code]:

$$\frac{\neg\ \text{checkPrg}(code, s)}{\langle[\mathbf{C},\, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\, \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle} \quad (10)$$

Rule 4 [rejected of counter]:

$$\frac{\neg\ \text{checkCou}(\mathbf{K},\, pkh)}{\langle[\mathbf{C},\, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s)) :: \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle \rightarrow \langle\ [\mathbf{C},\, \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle} \quad (11)$$

Rule 5 [rejected of balance]:

$$\frac{\neg\ \text{checkBan}(\mathbf{K},\, pkh, n, m)}{\langle[\mathbf{C},\, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\, \mathbf{O},\, \mathbf{S}],\, [\mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t}]\rangle} \quad (12)$$

Rule 6 [included]:

$$\frac{}{\langle[\mathbf{C},\, \mathbf{O},\, \mathbf{S}],\, [< (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s),\, \mathbf{t}> :: \mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t'}\ ]\rangle \rightarrow \langle[\mathbf{C},\, \mathbf{O}, (< id,\, \text{generateHash}(id, code, s,\, \mathbf{t'}),\, code > :: \mathbf{S})],\, [\mathbf{P},\, <\ (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s),\, \mathbf{t'} > :: \mathbf{A},\, \text{updateSucc}(\mathbf{K},\, puk, n, m),\, (< \text{generateHash}(id, code, s,\, \mathbf{t'}),\, n, code,\, \text{getStorage}(code, s)\ >) :: \mathbf{T}),\, \mathbf{t'} + 1]\rangle} \quad (13)$$

Rule 7 [timeout]:

$$\frac{\mathbf{t'} - \mathbf{t} \geq 60}{[< (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s)\ > :: \mathbf{P},\, \mathbf{A},\, \mathbf{K},\, \mathbf{T},\, \mathbf{t'}\ ] \rightarrow [\mathbf{P},\, \mathbf{A},\, \text{updateCou}(\mathbf{K},\, puk,\, \text{False}),\, \mathbf{T},\, \mathbf{t'}]} \quad (14)$$

B. **Transfer**

Rule 1 [proposal]:

$$\frac{\text{checkAcc}(pkh,\ \mathbf{C})}{\langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \tag{15}$$

Rule 2 [injected]:

$$\frac{\text{checkBan}(\mathbf{K},\ pkh, n, m) \wedge \text{checkCou}(\mathbf{K},\ pkh) \wedge \text{checkPuh}(\mathbf{T},\ puh) \wedge \text{checkArg}(\mathbf{T},\ puh, s)}{\begin{array}{l}\langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \\ \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [(< (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m), \\ \text{generateOph}(pkh, puh, s, n, m,\ \mathbf{t}),\ \mathbf{t}>) :: \mathbf{P},\ \mathbf{A},\ \text{updateCou}(\mathbf{K},\ pkh,\ \text{True}), \\ \mathbf{T},\ \mathbf{t}]\ \rangle\end{array}} \tag{16}$$

Rule 3 [rejected of counter]:

$$\frac{\neg\ \text{checkCou}(\mathbf{K},\ pkh)}{\langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \tag{17}$$

Rule 4 [rejected of balance]:

$$\frac{\neg\ \text{checkBan}(\mathbf{K},\ pkh, n, m)}{\langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \tag{18}$$

Rule 5 [rejected of public hash]:

$$\frac{\neg\ \text{checkPuh}(\mathbf{T},\ puh)}{\langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \tag{19}$$

Rule 6 [rejected of argument]:

$$\frac{\neg\ \text{checkArg}(\mathbf{T},\ puh, s)}{\langle[\mathbf{C},\ (\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle \rightarrow \langle[\mathbf{C},\ \mathbf{O},\ \mathbf{S}],\ [\mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}]\rangle} \quad (20)$$

Rule 7 [included]:

$$\frac{}{[<\text{transfer } n \text{ from } puk \text{ to } puh \text{ fee } m,\ \text{oph},\ \mathbf{t}> :: \mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}'\ ] \rightarrow [\mathbf{P},\ <(\text{transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m),\ \text{oph},\ \mathbf{t}'> :: \mathbf{A},\ \text{updateSucc}(\mathbf{K},\ \text{puk},\ ",\ n, m),\ \text{updateConstr}(\mathbf{T},\ puh, n,\ s),\ \mathbf{t}' + 1]} \quad (21)$$

Rule 8 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \geq 60}{[\text{transfer } n \text{ from } puk \text{ to } puh \text{ fee } m,\ \mathbf{t}> :: \mathbf{P},\ \mathbf{A},\ \mathbf{K},\ \mathbf{T},\ \mathbf{t}'\ ] \rightarrow [\mathbf{P},\ \mathbf{A},\ \text{updatecou}(\mathbf{K},\ puk,\ \text{False}),\ \mathbf{T},\ \mathbf{t}']} \quad (22)$$

3 Functions

1. Function `checkAcc(pkh, C)` checks whether an account *pkh* exists in **C**
2. Function `checkPub(K, pkh)` checks whether the public key of the public key hash *pkh* is reveled to the blockchain.
3. Function `checkBan(K, pkh, n, m)` checks whether the balance of the account *pkh* is greater or equal to $m + n$
4. Function `checkCou(K, pkh)` checks whether the current counter of an account *phk* is used
5. Function `updateSuc(K, pkh, pkh', n, m)` updates the balance and the counter of the account *phk* and the balance of the account phk', where
   - `< puk, pkh, bal, (n, True) > =>`
     `< puk, pkh, bal - n - m, (n + 1, False) >`
   - `< puk', pkh', bal', cou' > => < puk', pkh', bal' + n, cou' >`
6. Function `updateCou(K, puk, b')` updates the counter of the account *phk*, where
   - `< puk, pkh, bal, (n, b) > => < puk, pkh, bal, (n, b') >`
7. Function `checkId(id, S)` checks whether a contract *id* does not already exist in **S**
8. Function `checkPrg(code, s)` checks whether the code *code* are well type and *s* is well type input
9. Function `generateOph(pkh,pkh', n, m, t)` generates a operation hash
10. Function `generateHash(S, id, puh, code, t)` generates the public hash of a contract
11. Function `getStorage(code, s)` gets the storage for the code *code* and the input *s*

4 Some implementations

Function `checkAcc(puh, C)` checks whether an account exists and and `checkPuk(puh, K)` checks the revelation of its public key to the blockchain.

```
let rec checkAcc puh C =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: C' ->
    if (puh = puh')  then true
    else checkAcc (puh, C')

let rec checkPuk puh K =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: K' ->
    if (puh = puh') and (puk =/= nil) then true
    else 5checkPuk (puh, K')
```

The following functions interact with **K**.

```
let rec checkBal K puk n m =
  match K with
  | 0 -> true
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (n + m) <= bal then true
    else checkBal (K', puk, n, m)

let rec checkPub K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then true
    else checkExi (K', puk)

let rec checkCou K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (cou = T) then true
    else checkCou (K', puk)

let rec updateCou K puk =
  match K with
  | 0 -> 0
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then < puk', bal, F > :: K'
    else < puk', bal, cou > :: updateCou (K', puk)
```

```
let rec updateSuc K puk puk' m n =
  match K with
  | 0 -> 0
  | < puk'', bal, cou > :: K' ->
    if (puk = puk'') then < puk'', bal - (n + m), T >
       :: updateSuc (K', puk, puk', n, m)
    else if (puk' = puk'') then < puk'', bal + n, cou > :: K'
         else  < puk'', bal, cou >
               ::updateSuc (K', puk, puk', n, m)
```