Blockchain System

1 Definitions

Definition 1 (Set). Let **Elt** be the set of (concrete) elements. Let \emptyset be an empty set and $\mathbf{e} \in \mathbf{Elt}$. A set of elements is expressed as the following syntax: $\mathbf{s} ::= \emptyset \mid \mathbf{e} \mid \mathbf{s} :: \mathbf{s}$

Definition 2 (Account). An account is a tuple $\langle als, pak, puk, pkh \rangle$, where als is the alias of the account, pak is its private key, puk is its public key and pkh is its public key hash.

Definition 3 (Contract). An contract is a tuple $\langle als, puh, code \rangle$, where als is the alias of the contract, puh is its public hash, and code is the code of the contract.

Definition 4 (Manager). A manager manages a single account. It is represented by a tuple $\langle puk, pkh, bal, cou \rangle$, where puk is the public key of an account, pkh is its public key hash, bal is its balance and cou is its counter whose form is a pair (n,b), where n is a natural number and b is a boolean value.

Definition 5 (Contractor). A contractor manages a smart contract. It is represented by a tuple $\langle puh, bal, code, storage \rangle$, where puh is the public key hash of the contract, bal is its current balance, code is its code, and storage is its current storage.

Definition 6 (Operation). Operations are defined by the following grammar:

```
op ::= transfer \ n \ from \ pkh \ to \ pkh' \ arg \ s \ fee \ m
\mid originate \ contract \ id \ transferring \ n \ from \ pkh \ running \ code \ init \ s \ fee \ m
```

Definition 7 (Query). Queries are defined by the following grammar:

Definition 8 (State of a node). The state of a node is a tuple N = [C, O, S] where C is a set of accounts, O a set of operations, and S a set of contracts.

When an operation is injected in a node, it enters a *pending pool* (and is called a pending operation).

Definition 9 (Pending operation). A pending operation is a tuple $\langle op, oph, t \rangle$, where op is an operation, oph is the operation hash and t is the time when the operation was injected.

After some time, a pending operation may be included in the blockchain as an accepted operation.

Definition 10 (Accepted operation). An accepted operation is a tuple $\langle op, oph, t \rangle$, where op is an operation, oph is the operation hash and t is the time when it was included in the blockchain.

Definition 11 (Blockchain). The state of a blockchain is a tuple [P, A, K, T, t] where P is a set of pending operations, A is a set of accepted operations, K is a set of managers, T is a set of contractors, and t is the current time of the blockchain.

Definition 12 (Blockchain system). A blockchain system is a pair N || B where

- 1. N = [C, O, S] is the state of a node, and
- 2. B = [P, A, K, T, t] is the state of a blockchain such that $\forall c \in C \implies \exists k \in K, k.pkh = c.pkh$ and $\forall s \in S \implies \exists p \in T, s.puh = p.puh$.

2 Rules

2.1 Transitions on Nodes

Each node has (nondeterministic) rules to propose an operation.

Node-Transfer

$$checkAcc(\mathbf{puk}, \mathbf{C})$$

$$[C, O, S] \longrightarrow_N [C, (transfer n from puk to puk' arg () fee m) :: O, S]$$

Node-Originate

$$\operatorname{checkAcc}(\mathbf{puk}, \mathbf{C})$$
 $\operatorname{checkId}(\mathbf{id}, \mathbf{S})$

$$[\mathbf{C}, \mathbf{O}, \mathbf{S}] \longrightarrow_N$$

[C, (originate contract id transferring n from puk running code init s fee m) :: <math>C, C

$$\frac{\mathbf{N} \longrightarrow_{N} \mathbf{N}'}{\mathbf{N} \| \mathbf{B} \longrightarrow \mathbf{N}' \| \mathbf{B}}$$

2.2 Transfers

Rule 1 [proposal]:

$$\frac{\operatorname{checkAcc}(pkh, \mathbf{C})}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \to} \\ \langle [\mathbf{C}, (\operatorname{transfer} \mathbf{n} \text{ from } \mathbf{pkh} \text{ to } \mathbf{pkh'} \text{ arg } () \text{ fee } \mathbf{m}) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle}{(1)}$$

Rule 2 [injected]:

$$\begin{split} \operatorname{checkBal}(\mathbf{K}, \mathbf{puk}, \mathbf{n}, \mathbf{m}) & \operatorname{checkCou}(\mathbf{K}, \mathbf{puk}) & \operatorname{checkPub}(\mathbf{K}, \mathbf{puk}') \\ \mathbf{op} &= \operatorname{transfer} \ \mathbf{n} \ \operatorname{from} \ \mathbf{puk} \ \operatorname{to} \ \mathbf{puk}' \ \operatorname{arg} \ () \ \operatorname{fee} \ \mathbf{m} \\ & \mathbf{oph} &= \operatorname{generateOph}(\mathbf{puk}, \mathbf{puk}', \mathbf{n}, \mathbf{m}) \\ \hline & [\mathbf{C}, \mathbf{op} :: \mathbf{O}, \mathbf{S}] \| [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \longrightarrow \\ [\mathbf{C}, \mathbf{O}, \mathbf{S}] \| [\langle \mathbf{op}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \operatorname{updateCou}(\mathbf{K}, \mathbf{puk}, \mathbf{True}), \mathbf{T}, \mathbf{t}] \end{split}$$

Rule 3 [rejected of counter]:

$$\frac{\neg \operatorname{checkCou}(\mathbf{K}, pkh)}{\langle [\mathbf{C}, (\operatorname{transfer} n \operatorname{from} pkh \operatorname{to} pkh' \operatorname{fee} m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow} \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 4 [rejected of balance]:

$$\frac{\neg \operatorname{checkBal}(\mathbf{K}, pkh, m, n)}{\langle [\mathbf{C}, (\operatorname{transfer} n \text{ from } pkh \text{ to } pkh' \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \to} \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$
(3)

Rule 5 [rejected of public key]:

$$\frac{\neg \operatorname{checkPub}(\mathbf{K}, pkh')}{\langle [\mathbf{C}, (\operatorname{transfer} n \operatorname{from} pkh \operatorname{to} pkh' \operatorname{fee} m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \to} \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 6 [included]:

[< transfer
$$n$$
 from puk to puk' fee m , oph, $\mathbf{t} > :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rightarrow$ [\mathbf{P} , < transfer n from puk to puk' fee m , oph, $\mathbf{t}' > :: \mathbf{A}$, updateSucc(\mathbf{K} , puk, puk', n, m), $\mathbf{T}, \mathbf{t}' + 1$]

Rule 7 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \ge 60}{[< \text{transfer } n \text{ from } puk \text{ to } puk' \text{ fee } m, \text{ oph, } \mathbf{t} > :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \to} [\mathbf{P}, \mathbf{A}, \text{ updateCou}(\mathbf{K}, puk, \text{False}), \mathbf{T}, \mathbf{t}']}$$
(6)

2.3 Smart Contracts

A. Originate

Rule 1 [proposal]:

$$\frac{\operatorname{checkAcc}(pkh, \mathbf{C}) \wedge \operatorname{checkId}(id, \mathbf{S})}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \text{ (originate contract } id \text{ transferring}} \\ n \text{ from } pkh \text{ running } code \text{ init } s) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle}$$

$$(7)$$

Rule 2 [injected]:

checkBal(
$$\mathbf{K}, pkh, n, m$$
) \land checkCou(\mathbf{K}, pkh) \land checkPrg($code, s$)
$$\overline{\langle [\mathbf{C}, (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s)}$$

$$:: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}],$$

$$[(\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s)$$

$$:: \mathbf{P}, \mathbf{A}, \text{ updateCou}(\mathbf{K}, pkh, \text{True}), \mathbf{T}, \mathbf{t}] \rangle$$

Rule 3 [rejected of code]:

$$\frac{\neg \operatorname{checkPrg}(code, s)}{\langle [\mathbf{C}, (\operatorname{originate contract} id \operatorname{transferring} n \operatorname{from} pkh \operatorname{running} code \operatorname{init} s)}$$
 :: $\mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$

Rule 4 [rejected of counter]:

$$\frac{\neg \operatorname{checkCou}(\mathbf{K}, pkh)}{\langle [\mathbf{C}, (\operatorname{originate contract} id \operatorname{transferring} n \operatorname{from} pkh \operatorname{running} code \operatorname{init} s))}^{(10)} :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 5 [rejected of balance]:

$$\frac{\neg \operatorname{checkBal}(\mathbf{K}, pkh, n, m)}{\langle [\mathbf{C}, (\operatorname{originate contract} id \operatorname{transferring} n \operatorname{from} pkh \operatorname{running} code \operatorname{init} s)} (11) \\ :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 6 [included]:

$$\overline{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [< (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running}})$$
 (12) $code \text{ init } s), \mathbf{t} > :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rangle \rightarrow \langle [\mathbf{C}, \mathbf{O}, (< id, \mathbf{generateHash}(id, code, s, \mathbf{t}'), code > :: \mathbf{S})], [\mathbf{P}, < (\text{originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s), \mathbf{t}' > :: \mathbf{A}, \text{ updateSucc}(\mathbf{K}, puk, n, m), (< \text{generateHash}(id, code, s, \mathbf{t}'), n, code, \text{getStorage}(code, s) >) :: \mathbf{T}), \mathbf{t}' + 1] \rangle}$

Rule 7 [timeout]:

$$\frac{\mathbf{t'} - \mathbf{t} \ge 60}{[< \text{(originate contract } id \text{ transferring } n \text{ from } pkh \text{ running } code \text{ init } s)} > (13)$$
:: $\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t'}] \rightarrow [\mathbf{P}, \mathbf{A}, \text{ updateCou}(\mathbf{K}, puk, \text{False}), \mathbf{T}, \mathbf{t'}]$

B. Transfer

Rule 1 [proposal]:

$$\frac{\operatorname{checkAcc}(pkh, \mathbf{C})}{\langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle \to \langle [\mathbf{C}, (\operatorname{transfer} n \text{ from } pkh \text{ to } puh \text{ arg } s) \\ \text{fee } m) :: \mathbf{O}, \mathbf{S}, [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle}$$
(14)

Rule 2 [injected]:

$$\frac{\operatorname{checkBal}(\mathbf{K}, pkh, n, m) \wedge \operatorname{checkCou}(\mathbf{K}, pkh) \wedge \operatorname{checkPuh}(\mathbf{T}, puh) \wedge \operatorname{checkArg}(\mathbf{T}, puh, s)}{\langle [\mathbf{C}, (\operatorname{transfer} n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} \rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [(< (\operatorname{transfer} n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m), \\ \operatorname{generateOph}(pkh, puh, s, n, m, \mathbf{t}), \mathbf{t} >) :: \mathbf{P}, \mathbf{A}, \operatorname{updateCou}(\mathbf{K}, pkh, \operatorname{True}), \\ \mathbf{T}, \mathbf{t}] \rangle$$

$$(15)$$

Rule 3 [rejected of counter]:

$$\frac{\neg \operatorname{checkCou}(\mathbf{K}, pkh)}{\langle [\mathbf{C}, (\operatorname{transfer} n \operatorname{from} pkh \operatorname{to} puh \operatorname{arg} s \operatorname{fee} m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} (16)$$

$$\rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 4 [rejected of balance]:

$$\frac{\neg \operatorname{checkBal}(\mathbf{K}, pkh, n, m)}{\langle [\mathbf{C}, (\operatorname{transfer} n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} (17)$$

$$\rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 5 [rejected of public hash]:

$$\frac{\neg \operatorname{checkPuh}(\mathbf{T}, \operatorname{puh})}{\langle [\mathbf{C}, (\operatorname{transfer} n \operatorname{from} \operatorname{pkh} \operatorname{to} \operatorname{puh} \operatorname{arg} s \operatorname{fee} m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} (18)$$

$$\rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 6 [rejected of argument]:

$$\frac{\neg \text{ checkArg}(\mathbf{T}, puh, s)}{\langle [\mathbf{C}, \text{ (transfer } n \text{ from } pkh \text{ to } puh \text{ arg } s \text{ fee } m) :: \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle} (19)$$

$$\rightarrow \langle [\mathbf{C}, \mathbf{O}, \mathbf{S}], [\mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}] \rangle$$

Rule 7 [included]:

[
$$\langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \rightarrow$$
[$\mathbf{P}, \langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{oph}, \mathbf{t}' \rangle :: \mathbf{A},$
updateSucc($\mathbf{K}, \mathbf{puk}, ", \mathbf{n}, \mathbf{m}$), updateConstr($\mathbf{T}, \mathbf{puh}, \mathbf{n}, \mathbf{s}$), $\mathbf{t}' + 1$]

Rule 8 [timeout]:

$$\frac{\mathbf{t}' - \mathbf{t} \ge 60}{[\langle \text{transfer } \mathbf{n} \text{ from } \mathbf{puk} \text{ to } \mathbf{puh} \text{ arg } \mathbf{s} \text{ fee } \mathbf{m}, \mathbf{t} \rangle :: \mathbf{P}, \mathbf{A}, \mathbf{K}, \mathbf{T}, \mathbf{t}'] \to}$$

$$[\mathbf{P}, \mathbf{A}, \text{updateCou}(\mathbf{K}, \mathbf{puk}, \mathbf{False}), \mathbf{T}, \mathbf{t}']$$
(21)

3 Functions

- 1. Function checkAcc(pkh, C) checks whether an account pkh exists in C
- 2. Function checkPub(K, pkh) checks whether the public key of the public key hash pkh is reveled to the blockchain.
- 3. Function checkBal(K, pkh, n, m) checks whether the balance of the account pkh is greater or equal to m+n

- 4. Function checkCou(K, pkh) checks whether the current counter of an account phk is unlocked (i.e., its flag is False)
- 5. Function updateSucc(K, pkh, pkh', n, m) updates the balance and the counter of the account phk and the balance of the account phk', where

```
- < puk, pkh, bal, (n, True) > =>
< puk, pkh, bal - n - m, (n + 1, False) >
```

- < puk', pkh', bal', cou' > => < puk', pkh', bal' + n, cou' >
- 6. Function updateCou(K, puk, b') updates the counter lock of the account phk (True = lockeed, False = unlocked), where
 - < puk, pkh, bal, (n, b) > => < puk, pkh, bal, (n, b') >
- 7. Function checkId(id, S) checks whether a contract id does not already exist in S
- 8. Function checkPrg(code, s) checks whether the code code are well type and s is well type input
- 9. Function generateOph(pkh,pkh', n, m, t) generates a operation hash
- 10. Function generateHash(S, id, puh, code, t) generates the public hash of a contract
- 11. Function getStorage(code, s) gets the storage for the code code and the input s

4 Some implementations

Function checkAcc(puh, C) checks whether an account exists and and checkPuk(puh, K) checks the revelation of its public key to the blockchain.

```
let rec checkAcc puh C =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: C' ->
    if (puh = puh') then true
    else checkAcc (puh, C')
let rec checkPuk puh K =
  match C with
  | 0 -> false
  | < als, pak, puk, pkh' > :: K' ->
    if (puh = puh') and (puk =/= nil) then true
    else 5checkPuk (puh, K')
  The following functions interact with K.
let rec checkBal K puk n m =
  match K with
  | 0 -> true
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (n + m) <= bal then true
    else checkBal (K', puk, n, m)
```

```
let rec checkPub K puk =
 match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then true
    else checkExi (K', puk)
let rec checkCou K puk =
  match K with
  | 0 -> false
  | < puk', bal, cou > :: K' ->
    if (puk = puk') and (cou = T) then true
    else checkCou (K', puk)
let rec updateCou K puk =
  match K with
  | 0 -> 0
  | < puk', bal, cou > :: K' ->
    if (puk = puk') then < puk', bal, F > :: K'
    else < puk', bal, cou > :: updateCou (K', puk)
let rec updateSucc K puk puk' m n =
 match K with
  | 0 -> 0
  | < puk'', bal, cou > :: K' ->
    if (puk = puk'') then < puk'', bal - (n + m), T >
       :: updateSucc (K', puk, puk', n, m)
    else if (puk' = puk'') then < puk'', bal + n, cou > :: K'
         else < puk'', bal, cou >
               ::updateSucc (K', puk, puk', n, m)
```