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# A data driven approach for detection and isolation of anomalies in a group of UAVs

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**Abstract** The use of groups of unmanned aerial vehicles (UAVs) has greatly expanded UAV's capabilities in a variety of applications, such as surveillance, searching and mapping. As the UAVs are operated as a team, it is important to detect and isolate the occurrence of anomalous aircraft in order to avoid collisions and other risks that would affect the safety of the team. In this paper, we present a data-driven approach to detect and isolate abnormal aircraft within a team of formatted flying aerial vehicles, which removes the requirements for the prior knowledge of the underlying dynamic model in conventional model-based fault detection algorithms. Based on the assumption that normal behaviored UAVs should share similar (dynamic) model parameters, we propose to firstly identify the model parameters for each aircraft of the team based on a sequence of input and output data pairs, and this is achieved by a novel sparse optimization technique. The fault states of the UAVs would be detected and isolated in the second step by identifying the change of model parameters. Simulation results have demonstrated the efficiency and flexibility of the proposed approach.


## 1. Introduction

Unmanned aerial vehicles (UAVs) have received growing popularities in both military and civil applications, since they can offer a great number of advantages over the manned counterparts, such as elimination of the threat to pilot's life and longer endurance time. The capabilities can be further extended when a group of UAVs are deployed, which allows for completing tasks that cannot be achieved by a single vehicle. Hence, UAVs formation and coordination becomes an active area of research in recent years. Despite the high redundancy designs of the aircraft system, both hardware and software faults can also happen due to the complexity of the system and unpredicted application environment. Hence, early detection and localization of the occurrence of the anomalies is important to ensure the safety and reliability of UAVs during formation flying.

Model-driven method, based on the concept of analytical redundancy,[1,2] is one of the most popular approaches for fault detection and isolation (FDI). The application of this type of algorithm was pioneered by Beard and his colleagues.[3] In these

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methods, the system errors were detected by comparing the measured outputs of the system with predicted outputs from a pre-identified model of the system.[4] The analytical redundancy, also called residual, representing the difference between the actual behavior of the system and predicted outputs of a mathematical model, approaches to zero when no anomaly is detected. On the other hand, in case the residuals deviate from zeros (zero means), a subsequent process, based on the statistical test algorithms, would be carried out to determine whether (and where) a fault occurs. In practice, however, the mathematical model, representing the dynamics of the system, is unable to exactly describe the behavior of the system due to unmodeled parameters and unknown disturbances. As a result, the residuals may not be equal to zero under normal conditions, leading to false alarms and deterioration in the performance of the system. To address this issue, the system's observers can be designed based on decoupling principles, where the outputs of the mathematic model are robust to unknown disturbance and system uncertainties while sensitive to faults.[5] Zhu and Feng proposed the use of a full-order observer to detect and locate the occurrence of actuator fault, where the residuals produced by this kind of observer are more robust to model uncertainties.[6] Unknown input observer (UIO) based approaches, firstly developed by Watanabe and Himmelblau,[7] is also a popular choice of observer for model based fault detection. In the proposed method, the unknown inputs of the system can be decoupled when the observer matching condition is met. This constraint, however, is too strict to be satisfied in practical applications. In order to loosen the observer matching condition, algebraically decomposed technique,[8] unscented transformation (UT),[9] population-based approaches,[10] optimization methods[11,12] and neural networks,[13] have been incorporated into the framework of UIO, which greatly improves the performance of the original method in applications. Stochastic filters[11,14,15] such as Kalman filter and particle filter, have been introduced to solve the fault detection and isolation problems in recent years. In these methods, the system dynamics are modeled as a stochastic process, and the residual is generated by comparing the measured output probability density function and the predicted one.

In contrast to model-driven based FDI methods, which rely on the prior knowledge about the dynamics of a system, data-driven based FDI approaches can predict and locate the occurrence of the system errors from a large number of data collected from the dynamical process. Statistical techniques, such as principle component analysis (PCA) and its extensions,[16,17] partial least squares (PLS) and Fisher discriminate analysis (FDA), are one of the most popular approaches for learning the patterns of a dynamic system based on historical data. In these methods, the system dynamics are usually assumed to be linear, which cannot be hold in many real world applications. In order to deal with the nonlinearity of the dynamic system, an iterative procedure is usually applied to solving the associated nonlinear optimization problem in these methods, which may result in undesired local optimal solution. To address this problem, machine learning based algorithms, e.g., support vector machines,[18] artificial neural networks[19] and fuzzy logic,[20,21] are employed to deal with nonlinear system. The unmodeled system parameters and uncertainties, however, are difficult to incorporate into these approaches. Moreover, the machine learning based methods usually require sufficient data to approximate the system's dynamics, which may lead to under-fitting problem if only a small volume of training data is available.

The current study introduces a fast yet robust algorithm for the detection and isolation of the anomaly aircraft within a group of formatted flying UAVs. In contrast to conventional model based approaches rely on prior knowledge about the dynamic/measurement model of the system, the proposed method is able to identify the occurrence of the system abnormalities from inputs and outputs data, which does not require the prior knowledge about the dynamics model of the aircraft. In addition, in this paper the FDI problem is formulated as multiple objectives' convex optimization problem and solved through a fast numerical scheme, which ensures the global optimality of the resulting solution. It should be noted that different from conventional fault detection and isolation algorithms, which try to identify the system failure for each individual aircraft, the proposed work aims to locate the abnormal behavired aircraft within a team of formatted flying aerial vehicles.

The reminder of this paper is organized as follows. In the following section, we firstly present the necessary technical background for data-driven based fault detection method. Then, the proposed method would be described with great detail in the following section. Section 4 presents and analyzes the experimental results, which demonstrate the efficiency and flexibility of the propose approach. Finally, Section 5 is dedicated to conclusions of this research and a discussion about future directions.

## 2. Preliminaries

### 2.1. Aerodynamics of rigid-body aircraft

Given an input vector $\boldsymbol{x}$ including both control and state values of an aircraft, the dynamics of the aircraft can be written as follows:

$$\boldsymbol{y}_i(t) = \boldsymbol{\theta}_i(t)F(\boldsymbol{x}_i(t)) + \boldsymbol{e}_i(t) \quad i = (1, 2, \ldots, N) \tag{1}$$

where $\boldsymbol{y}_i$ is a column vector representing the measured outputs of the $i$th aircraft and $F(\bullet)$ a regression function describing the dynamics of the aircraft. To simplify the representation, we use $\boldsymbol{x}_i(t)$ to represent the output of the regression function $F(\boldsymbol{x}_i(t))$. $\boldsymbol{\theta}_i$ denotes the unknown (or partially unknown) parameters of the regression function of the $i$th aircraft, which is usually assumed to be full column rank. The recorded data pairs $\{\boldsymbol{y}_i(t), \boldsymbol{x}_i(t)\}_{t=1}^{M}$ denote the measured input–output data at each sampling time $t$. $M$ is the total number of the measurements. $\boldsymbol{e}_i(t)$ represents the total uncertainties and disturbances of the dynamic model, which is assumed to be independent from each vehicle within the group. To simplify the problem, we define $\boldsymbol{e}_i(t)$ as a multivariate zero mean Gaussian random vector with unknown covariance. Since $\boldsymbol{e}_i(t)$ is assumed to be mutually independent, we can expect the inter-data covariance of the model uncertainties $\boldsymbol{e}_i(t)$ to be zero for different aircraft:

$$E(\boldsymbol{e}_i(t)^{\mathrm{T}}\boldsymbol{e}_j(t)) = 0 \quad (i \neq j) \tag{2}$$

### 2.2. Problem description

Assume that a group of formatted flying UAVs consists of $N$ individual aircraft with the same structure and flight control system. The dynamics of each flight, described in terms of the model parameters $\boldsymbol{\theta}_i$, should be similar if all of the vehicles are in normal statues. Ideally, the model parameters $\boldsymbol{\theta}_i$ should be the same for normal aircraft, thus leading to the following condition:

$$\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2 =, \ldots, = \boldsymbol{\theta}_N = \boldsymbol{\theta}_0 \tag{3}$$

where $\boldsymbol{\theta}_0$ denotes the 'mean' model parameters of the aircraft system, which can be calculated by averaging the model parameters $\boldsymbol{\theta}_i$ of each aircraft. The occurrence of the anomaly of the aircraft can be detected by finding the change of the model parameters $\boldsymbol{\theta}_i$. However, due to system uncertainties and unknown disturbances, the model parameters $\boldsymbol{\theta}_i$ for normally operated aircraft would not be exactly the same in practice. Thus, $||\boldsymbol{\theta}_i - \boldsymbol{\theta}_0||$ would not be zero for normal aircraft. The abnormal aircraft can be determined by finding the value of $||\boldsymbol{\theta}_i - \boldsymbol{\theta}_0||$ which exceeds a pre-defined threshold. Since the model parameter $\boldsymbol{\theta}_i$ is unable to be determined based on prior knowledge, we propose to firstly identify the model parameters, based on a sequence of inputs and outputs data pairs, for each aircraft of the team, and the abnormal vehicles are determined in the following stage by finding the changes of parameters in the identified model parameters.

### 2.3. Model parameters' estimation by maximum likelihood (ML)

Since the unknown uncertainties of the aircraft model is assumed as zero mean Gaussian distribution, the probability density function for each $\boldsymbol{e}_i(t)$ can be written as

$$\boldsymbol{p}_{\boldsymbol{e}_i(t)} = \frac{1}{(2\pi)^{0.5q}|\boldsymbol{\Sigma}|^{0.5}} \exp\left[-\frac{\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{e}_i(t)}{2}\right] \tag{4}$$

where $\boldsymbol{\Sigma}$ is the covariance matrix of $\boldsymbol{e}_i$, which is assumed to be invertible and unknown from prior information. $q$ represents the dimension of $\boldsymbol{e}_i$. Given a set of model parameters $\boldsymbol{\theta}_i$, the joint probability function of the measurement error, $\boldsymbol{P}_{\boldsymbol{y}_i(t)}$, at all sampling time can be defined as

$$\begin{aligned}\boldsymbol{P}_{\boldsymbol{y}_i(t)}(\boldsymbol{\theta}_i) &= l(\boldsymbol{\theta}_i, \boldsymbol{\Sigma}) \\ &= \prod_{t=1}^{M} \frac{1}{(2\pi)^{0.5q}|\boldsymbol{\Sigma}|^{0.5}} \cdot \exp\left[-\frac{(\boldsymbol{y}_i(t) - \boldsymbol{\theta}_i(t)\boldsymbol{x}_i(t))\boldsymbol{\Sigma}^{-1}(\boldsymbol{y}_i(t) - \boldsymbol{\theta}_i(t)\boldsymbol{x}_i(t))^{\mathrm{T}}}{2}\right]\end{aligned} \tag{5}$$

where $M$ is the number of sampled data pairs. The joint probability function, $\boldsymbol{P}_{\boldsymbol{y}_i(t)}$, defined in the above equation would reach its maximum when the model parameters $\boldsymbol{\theta}_i$ fully describe the actual behavior of the system. Thus, by maximizing the joint probability defined in Eq. (5), the unknown parameters $\boldsymbol{\theta}_i$ can be determined.

Maximizing the probability function in Eq. (5) is equivalent to minimizing its negative logarithm:

$$\begin{aligned}l(\boldsymbol{\theta}_i, \boldsymbol{\Sigma}) = \underset{(\boldsymbol{\theta}_i, \boldsymbol{\Sigma})}{\arg\min} \Bigg[ &\frac{M}{2} \lg|\boldsymbol{\Sigma}| + \frac{Mq}{2} \lg(2\pi) + \frac{1}{2}\sum_{t=1}^{M}(\boldsymbol{y}_i(t) \\ &- \boldsymbol{\theta}_i(t)\boldsymbol{x}_i(t))^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\boldsymbol{y}_i(t) - \boldsymbol{\theta}_i(t)\boldsymbol{x}_i(t))\Bigg]\end{aligned} \tag{6}$$

Since the covariance of the model uncertainties $\boldsymbol{\Sigma}$ is also unknown, it should be jointly estimated from Eq. (6) and thus the minimum of Eq. (6) can be found if

$$\frac{\partial l(\boldsymbol{\theta}_i, \boldsymbol{\Sigma})}{\partial \boldsymbol{\theta}_i} = \frac{\partial l(\boldsymbol{\theta}_i, \boldsymbol{\Sigma})}{\partial \boldsymbol{\Sigma}} = 0 \tag{7}$$

According to Eqs. (6) and (7), the best estimation of the model parameters and the covariance matrix $\boldsymbol{\Sigma}$ can be determined as

$$\hat{\boldsymbol{\theta}}_i = \sum_{t=1}^{M} \boldsymbol{y}_i(t)\boldsymbol{x}_i^{\mathrm{T}}(t) \left[\sum_{t=1}^{M} \boldsymbol{x}_i(t)\boldsymbol{x}_i^{\mathrm{T}}(t)\right]^{-1} \tag{8}$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{M}\sum_{t=1}^{M}\left[\boldsymbol{y}_i(t) - \hat{\boldsymbol{\theta}}_i\boldsymbol{x}_i(t)\right]\left[\boldsymbol{y}_i(t) - \hat{\boldsymbol{\theta}}_i\boldsymbol{x}_i(t)\right]^{\mathrm{T}} \tag{9}$$

For each aircraft of the team, the model parameters can be estimated based on Eq. (8), and the estimated mean model parameters $\hat{\boldsymbol{\theta}}_0$ can be therefore calculated as

$$\hat{\boldsymbol{\theta}}_0 = \frac{1}{N}\sum_{i=1}^{N} \hat{\boldsymbol{\theta}}_i \tag{10}$$

If the model parameters, i.e., $\{\hat{\boldsymbol{\theta}}_1, \hat{\boldsymbol{\theta}}_2, \ldots, \hat{\boldsymbol{\theta}}_N\}$ , for each constitute aircraft of the team can be estimated, then, the aircraft with abnormal behavior can be determined by finding the model parameters that deviate from the standard reference $(\hat{\boldsymbol{\theta}}_0)$ in terms of some distance metrics, such as Euclidian and Mahalanobis distance.[12] The proposed procedure for detection of the anomaly of the flight is shown in Fig. 1, where the abnormal aircraft are shown within the circles. The proposed approach begins with identifying the system dynamics from a sequence of input and output data pairs, using a sparse optimization algorithm, described in Section 3. Then, an unsupervised classifier would be used to detect the abnormal aircraft, based on the model parameters estimated from the first stage of the proposed method.

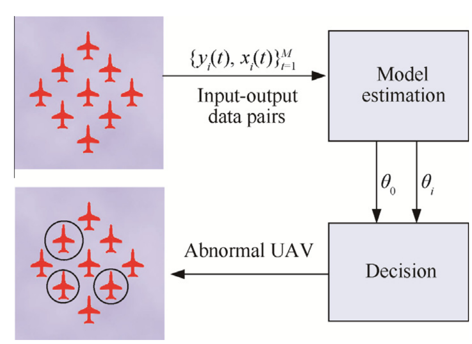


**Fig. 1** Proposed procedure for detecting and isolating the anomaly of UAVs.

## 3. Proposed method

### 3.1. Anomaly detection and isolation through sparse optimization

As discussed previously in Section 2.2, the model parameters estimated by using maximum likelihood (ML) based algorithms assume the model uncertainties subject to Gaussian distribution with zero means, which cannot be held in many real world applications. In this section, we present a sparse optimization based technique to estimate the model parameters based on input and output data pairs alone. Assuming that there are $r$ out of $N$ aircraft encountering some system fault during formatted flying, the model parameters can be identified by solving the following constrained optimization problem:

$$\min_{\theta_1,\theta_2,\ldots,\theta_N,\theta_0} \sum_{i=1}^{N}\sum_{t=1}^{M}(\|\boldsymbol{y}_i(t)-\boldsymbol{\theta}_i\boldsymbol{x}(t)\|_2)^2$$
$$\text{subject to } \left\|\left(\|\boldsymbol{\theta}_1-\boldsymbol{\theta}_0\|_p,\ \|\boldsymbol{\theta}_2-\boldsymbol{\theta}_0\|_p,\ \cdots,\ \|\boldsymbol{\theta}_N-\boldsymbol{\theta}_0\|_p\right)\right\|_0 = r \tag{11}$$

where the notion $\|\bullet\|_p$ denotes $L_p$ Euclidian norm, and $L_0$ norm is defined as the number of non-zero elements, indicating the number of abnormal aircraft.

According to optimal control theory, the constrained optimization problem defined in Eq. (11) can be translated to an unconstrained problem as

$$\min_{\theta_1,\theta_2,\cdots,\theta_N,\theta_0}\sum_{i=1}^{N}\sum_{t=1}^{M}(\|\boldsymbol{y}_i(t)-\boldsymbol{\theta}_i\boldsymbol{x}_i(t)\|_2)^2+\lambda\|\left[\|\boldsymbol{\theta}_1-\boldsymbol{\theta}_0\|_p,\ \|\boldsymbol{\theta}_2-\boldsymbol{\theta}_0\|_p,\ \cdots,\ \|\boldsymbol{\theta}_N-\boldsymbol{\theta}_0\|_p\right]\|_0 \tag{12}$$

where $\lambda$ is a constant and can be determined based on convex optimization technique.[22] By solving the optimization problem in Eq. (12), the unknown model parameters $\{\hat{\boldsymbol{\theta}}_1,\hat{\boldsymbol{\theta}}_2,\ldots,\hat{\boldsymbol{\theta}}_N,\hat{\boldsymbol{\theta}}_0\}$ can be determined. The optimization problem defined in Eq. (12), however, is impractical to solve. The reasons are two-folds. Firstly, the cost function defined in Eq. (12) is not convex due to the existence of the $L_0$ norm, which may cause difficulties in determining the optimal solution. Secondly, as discussed previously it is impractical for different aircraft to have the same model parameters, and thus $\|\boldsymbol{\theta}_i-\boldsymbol{\theta}_0\|$ would not be zero for normal aircraft. Yet, since we have assumed that normal behaviored aircraft share similar dynamical model parameters, the magnitude of the $\|\boldsymbol{\theta}_i-\boldsymbol{\theta}_0\|$ should be small for normal aircraft, while the value of $\|\boldsymbol{\theta}_i-\boldsymbol{\theta}_0\|$ would be large for abnormal aircraft. Based on this assumption, we use $L_1$ norm to replace the $L_0$ norm operator in Eq. (12), leading to the following convex cost function:

$$\min_{\theta_1,\theta_2,\cdots,\theta_N,\theta_0}\sum_{i=1}^{N}\sum_{t=1}^{M}(\|\boldsymbol{y}_i(t)-\boldsymbol{\theta}_i\boldsymbol{x}_i(t)\|_2)^2+\lambda\sum_{i=1}^{N}\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p \tag{13}$$

Let $\boldsymbol{\varphi}$ denote all of the model parameters, including the mean model parameters $\boldsymbol{\theta}_0$, need to be estimated:

$$\boldsymbol{\varphi}=[\boldsymbol{\theta}_1,\quad \boldsymbol{\theta}_2,\quad \ldots,\quad \boldsymbol{\theta}_N,\quad \boldsymbol{\theta}_0] \tag{14}$$

and rewrite the system's inputs and measured outputs data as the following form:

$$\begin{cases} \boldsymbol{Y}_i=[\boldsymbol{y}_i(1),\boldsymbol{y}_i(2),\ldots,\boldsymbol{y}_i(M)] \\ \boldsymbol{X}_i=[\boldsymbol{x}_i(1),\boldsymbol{x}_i(2),\ldots,\boldsymbol{x}_i(M)] \end{cases} \tag{15}$$

where $\boldsymbol{X}_i$ and $\boldsymbol{Y}_i$ are matrix, and each column implies input vector and output vector recorded at each sampling time, respectively. Hence, Eq. (13) can be reformed, by substituting the according elements using Eqs. (14) and (15), as follows:

$$\begin{cases} \min\limits_{\boldsymbol{\varphi}} G(\boldsymbol{\varphi})=\sum\limits_{i=1}^{N}G_i(\boldsymbol{\varphi}) \\ G_i(\boldsymbol{\varphi})=(\|\boldsymbol{Y}_i-\boldsymbol{\theta}_i\boldsymbol{X}_i\|_2)^2+\lambda\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p \end{cases} \tag{16}$$

According to the necessary optimality condition of convex optimization problem, we have

$$\begin{cases} \dfrac{\partial G_i(\boldsymbol{\varphi})}{\partial \boldsymbol{\theta}_j}=0 \quad (i\neq j) \\ \dfrac{\partial G_i(\boldsymbol{\varphi})}{\partial \boldsymbol{\theta}_i}=-2\boldsymbol{X}_i(t)(\boldsymbol{Y}_i-\boldsymbol{\theta}_i\boldsymbol{X}_i(t))^{\mathrm{T}}+\dfrac{\lambda}{p}\left[\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p\right]^{1-p} \\ \dfrac{\partial G(\boldsymbol{\varphi})}{\partial \boldsymbol{\theta}_0}=\sum\limits_{i=1}^{N}\dfrac{\lambda}{p}\left[\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p\right]^{1-p} \end{cases} \tag{17}$$

The best estimation of the unknown model parameters can be determined as

$$\begin{cases} 2\boldsymbol{X}_i(t)(\boldsymbol{Y}_i-\boldsymbol{\theta}_i\boldsymbol{X}_i(t))^{\mathrm{T}}+\frac{\lambda}{p}\left[\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p\right]^{1-p}=0 \\ \sum\limits_{i=1}^{N}\frac{\lambda}{p}[\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_p]^{1-p}=0 \end{cases} \tag{18}$$

Let $p = 2$ and Eq. (18) can be written as

$$\begin{cases} \dfrac{\partial G(\boldsymbol{\varphi})}{\partial \boldsymbol{\theta}_i}=2\boldsymbol{X}_i(t)(\boldsymbol{Y}_i-\boldsymbol{\theta}_i\boldsymbol{X}_i(t))^{\mathrm{T}}+2\lambda\|\boldsymbol{\theta}_0-\boldsymbol{\theta}_i\|_2=0 \\ \dfrac{\partial G(\boldsymbol{\varphi})}{\partial \boldsymbol{\theta}_0}=\sum\limits_{i=1}^{N}2\lambda(\boldsymbol{\theta}_0-\boldsymbol{\theta}_i)=0 \end{cases} \tag{19}$$

Thus, the optimal estimation of $\hat{\boldsymbol{\theta}}_0$ and $\hat{\boldsymbol{\theta}}_i$ can be calculated as

$$\hat{\boldsymbol{\theta}}_0=\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\theta}_i \tag{20}$$

$$\hat{\boldsymbol{\theta}}_i=\left[\boldsymbol{X}_i\boldsymbol{X}_i^{\mathrm{T}}+\lambda\boldsymbol{I}\right]^{-1}\left[\boldsymbol{X}_i\boldsymbol{Y}_i^{\mathrm{T}}+\lambda\boldsymbol{\theta}_0\right] \tag{21}$$

The unknown parameter $\boldsymbol{\theta}_0$ can be replaced by its estimation defined in Eq. (20), leading to the following solution:

$$\hat{\boldsymbol{\theta}}_i=\left[\boldsymbol{X}_i\boldsymbol{X}_i^{\mathrm{T}}+\lambda\boldsymbol{I}\right]^{-1}\left[\boldsymbol{X}_i\boldsymbol{Y}_i^{\mathrm{T}}+\lambda\hat{\boldsymbol{\theta}}_0\right] \tag{22}$$

By iteratively calculating $\hat{\boldsymbol{\theta}}_0$ and $\hat{\boldsymbol{\theta}}_i$, in terms of Eqs. (20) and (22), the optimal estimation of model parameters can be reached. Once the model parameters of all the aircraft have been identified, we can detect and isolate the abnormal flight by comparing their model parameters. For each flight within the group, if its model parameters, $\boldsymbol{\theta}_i$, are close to the team's mean parameters $\boldsymbol{\theta}_0$, i.e., $|\boldsymbol{\theta}_i-\boldsymbol{\theta}_0|<\varepsilon$($\varepsilon$ is a threshold value), then it would be considered as a normal flight. In contrast, if the model parameters of the these aircraft are not consistent with the reference model $\boldsymbol{\theta}_0$, i.e., $|\boldsymbol{\theta}_i-\boldsymbol{\theta}_0|\geqslant\varepsilon$ , they would be recognized as anomaly flight.

**Remark 1.** It should be noted that $\lambda$ in Eq. (13) is also an unknown tuning parameter that can be optimized. However, since Eq. (13) is a convex function, it guarantees the global optimality of the resulting solution, despite the choice of $\lambda$. Yet, the parameter $\lambda$ should be chosen to be relatively small in magnitude in order to prevent over-fitting of the model.

### *3.2. Numerical scheme*

The model parameters for each aircraft cannot always be determined through Eqs. (20) and (22), since the first element in Eq. (22) is not always invertible. To address this problem, we present an improved fast gradient numerical scheme to solve the optimization problem defined in Eq. (16).

In Eq. (16), the first order derivative of $G(\boldsymbol{\varphi})$ with respect to unknown parameters $\boldsymbol{\theta}_i$ can be calculated as

$$\frac{\partial G(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}} = \begin{bmatrix} -2\boldsymbol{X}_1(\boldsymbol{Y}_1 - \boldsymbol{\theta}_1\boldsymbol{X}_1)^{\mathrm{T}} - \frac{\lambda}{p}\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p\right]^{1-p} \\ -2\boldsymbol{X}_2(\boldsymbol{Y}_2 - \boldsymbol{\theta}_2\boldsymbol{X}_2)^{\mathrm{T}} - \frac{\lambda}{p}\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_2\|_p\right]^{1-p} \\ \vdots \\ -2\boldsymbol{X}_N(\boldsymbol{Y}_N - \boldsymbol{\theta}_N\boldsymbol{X}_N)^{\mathrm{T}} - \frac{\lambda}{p}\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_N\|_p\right]^{1-p} \\ \sum_{i=1}^{N}\frac{\lambda}{p}\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_i\|_p\right]^{1-p} \end{bmatrix} \tag{23}$$

The associated Hessian matrix can be then computed as

The optimization problem defined in Eq. (30) can be solved by using the improved fast gradient method and the numerical procedure is given in Algorithm 1 as follows:

**Algorithm 1. Numerical scheme of the proposed method**

Numerical scheme for solving sparse optimization problem

1. Calculate the Hessian matrix of $G(\boldsymbol{\varphi})$ based on Eq. (24), and determine the eigenvalues $\mu_1$ and $\mu_2$ using Eq. (25)
2. Estimate the model parameters $\boldsymbol{\varphi}$:
   $\boldsymbol{\varphi}_{k+1} = \arg\min_{\boldsymbol{\varphi}} \|\boldsymbol{\varphi}_k - \boldsymbol{h}_k + \frac{1}{\mu_1}\nabla G(\boldsymbol{h}_k)\|^2$
3. Calculate $c_{k+1} \in (0,1)$through:
   $c_{k+1}^2 = (1 - c_{k+1})c_k^2 + \dfrac{\mu_2 c_{k+1}}{\mu_1}$
4. Compute $\boldsymbol{h}_{k+1} = \boldsymbol{\varphi}_{k+1} + b_k(\boldsymbol{\varphi}_{k+1} - \boldsymbol{\varphi}_k)$, where
   $b_k = \frac{c_k(1-c_{k+1})}{c_k^2 + c_{k+1}}$
5. If not converge, go to 1

$$\frac{\partial G^2(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}^2} = \begin{bmatrix} 2\boldsymbol{X}_i\boldsymbol{X}_i^{\mathrm{T}} - \frac{\lambda}{p}(1-p)\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p\right]^{-p}\dfrac{\partial\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p}{\partial \boldsymbol{\theta}_1} & \cdots & \frac{\lambda}{p}\frac{1}{1-p}\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p\right]^{-p}\frac{\partial\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p}{\partial \boldsymbol{\theta}_0} \\ \vdots & \vdots & \vdots \\ \frac{\lambda}{p}(1-p)\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p\right]^{-p}\dfrac{\partial\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p}{\partial \boldsymbol{\theta}_1} & \cdots & \sum_{i=1}^{N}\frac{\lambda}{p}(1-p)\left[\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_i\|_p\right]^{-p}\frac{\partial\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_1\|_p}{\partial \boldsymbol{\theta}_0} \end{bmatrix} \tag{24}$$

The eigenvalues of the Hessian matrix defined in Eq. (24), $\mu_1$ and $\mu_2$ ,can be determined as follows:

$$\begin{cases} \mu_1 = \mathrm{eig}_{\max}\left[\dfrac{\partial G^2(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}^2}\right] \\ \mu_2 = \mathrm{eig}_{\min}\left[\dfrac{\partial G^2(\boldsymbol{\varphi})}{\partial \boldsymbol{\varphi}^2}\right] \end{cases} \tag{25}$$

where $\mathrm{eig}_{\max}$ and $\mathrm{eig}_{\min}$ denote the maximum and minimum eigenvalues of the Hessian matrix, respectively.

In convex programming problems, $\mu_1$ is a constant that satisfies Lipschitz continuity condition and $\mu_2$ are convex parameters of the convex function $G(\boldsymbol{\varphi})$. We define the estimation sequence of $G(\boldsymbol{\varphi})$ as

$$\begin{cases} \{\psi_k\}_{k=0}^{\infty} \\ \{\gamma_k\}_{k=0}^{\infty} \\ \gamma_k \geqslant 0 \end{cases} \tag{26}$$

According to the properties of convex function, when $\gamma_k \to 0$,

$$\psi_k(\boldsymbol{\varphi}) \leqslant (1-\gamma_k)G(\boldsymbol{\varphi}) + \gamma_k\psi_0(\boldsymbol{\varphi}) \quad \forall k \geqslant 0 \tag{27}$$

Based on the concept of estimation sequence, it should meet the following requirements:

$$G(\boldsymbol{\varphi}_k) \leqslant \psi_k^* = \min_{\boldsymbol{\varphi}}\psi_k(\boldsymbol{\varphi}) \tag{28}$$

Thus, Eq. (27) can be rewritten as

$$G(\boldsymbol{\varphi}_k) - G^* \geqslant \gamma_k[\psi_0(\boldsymbol{\varphi}^*) - G^*] \to 0 \tag{29}$$

where

$$\begin{cases} G^* = \min_{\boldsymbol{\varphi}} G(\boldsymbol{\varphi}) \\ \boldsymbol{\varphi}^* = \arg\min_{\boldsymbol{\varphi}}\psi_k(\boldsymbol{\varphi}) \end{cases} \tag{30}$$

where $\boldsymbol{h}_k$ denotes the estimation sequence of unknown parameter matrix $\boldsymbol{\varphi}$, and we assign $\boldsymbol{h}_0$ to the initial estimation of the model parameters, i.e., $\boldsymbol{h}_0 = \boldsymbol{\varphi}_0$. $c_i$ represents the optimization coefficient, and we assume $c_0$ satisfies the following condition:

$$1 > c_0 \geqslant \sqrt{\frac{\mu_2}{\mu_1}} > 0$$

### 3.3. Convergence analysis of the proposed method

In this section, the global convergence of the proposed numerical scheme for solving the convex optimization problem is analyzed. Assuming the sequence of the estimated unknown parameters is given as $\{\boldsymbol{\varphi}_k\}_{k=0}^{\infty}$, it should subject to the following condition:

$$G(\boldsymbol{\varphi}_k) - G^* \leqslant \lambda_k\left[G(\boldsymbol{\varphi}_0) - G^* + \frac{\gamma_0}{2}\|\boldsymbol{\varphi}_0 - \boldsymbol{\varphi}^*\|^2\right] \tag{31}$$

where $\lambda_0 = 1$ and $\lambda_k = \Pi_{n=0}^{k-1}(1 - c_n)$, $n$ represents the number of iteration up to $k-1$, when we chose $\gamma_0 \geqslant \mu_2$, leading to the following condition.

$$\lambda_k \leqslant \min\left(\left(1 - \sqrt{\frac{\mu_2}{\mu_1}}\right)^k, \frac{4\mu_1}{(2\sqrt{\mu_1} + k\sqrt{\gamma_0})^2}\right) \tag{32}$$

If we let $\gamma_0 = \mu_1$ and combine Eq. (31) with Eq. (32), then

$$G(\boldsymbol{\varphi}_k) - G^* \leqslant \min\left(\left(1 - \sqrt{\frac{\mu_2}{\mu_1}}\right)^k, \frac{4}{(2+k)^2}\right)\|\boldsymbol{\varphi}_0 - \boldsymbol{\varphi}^*\|^2 \tag{33}$$

when the coefficient $c_0$ is selected to be $c_0 \geqslant \sqrt{\frac{\mu_2}{\mu_1}}$, the estimated sequence obtained through fast gradient algorithm should satisfy the following condition:

$$G(\boldsymbol{\varphi}_k) - G^* \leqslant \min\left\{\left(1-\sqrt{\frac{\mu_2}{\mu_1}}\right)^k, \frac{4\mu_1}{(2\sqrt{\mu_1}+k\sqrt{\gamma_0})^2}\right\} \times \left[G(\boldsymbol{\varphi}_0) - G^* + \frac{\gamma_0}{2}\|\boldsymbol{\varphi}_0 - \boldsymbol{\varphi}^*\|^2\right] \tag{34}$$

where

$$\gamma_0 = \frac{c_0(c_0\mu_1 - \mu_2)}{1-c_0}$$

Since we have chosen that $1 > c_0 \geqslant \sqrt{\frac{\mu_2}{\mu_1}} > 0$, the first term of the right hand side of Eq. (34) would approach to zero when the number of iteration $k$ increases. Hence, the model parameters estimated from Algorithm 1 would converge to the global optimality of the convex function defined in Eq. (16).

## 4. Simulation results and analysis

In this section, we demonstrate the efficiency and flexibility of the proposed anomaly detection and isolation method in terms of a series of simulation. The dynamics of the unmanned aircraft is modeled based on Beaver airframe in MATLAB, which allows for generation of inputs–outputs data pairs. The input and output vectors are defined as

$$\begin{aligned} \boldsymbol{x}_i &= [v_x, v_y, v_z, \alpha, \beta, p, q, r, \delta_e, \delta_a, \delta_r, \delta_{th}]^T \\ \boldsymbol{y}_i &= [\omega_x, \omega_y, \omega_z, a_x, a_y, a_z]^T \end{aligned} \tag{35}$$

where $\boldsymbol{x}$ consists of three measured linear velocities of the aircraft along its body axes ($v_x$, $v_y$, $v_z$), angle of attack ($\alpha$) and slide slip ($\beta$), three angular speeds ($\omega_x$, $\omega_y$, $\omega_z$) as well as the four actuator positions, i.e., the elevator ($\delta_e$), ailerons ($\delta_a$), rudders ($\delta_r$) and the throttle ($\delta_{th}$), a total number of twelve elements. The output vector $\boldsymbol{y}$ is defined by the rotational

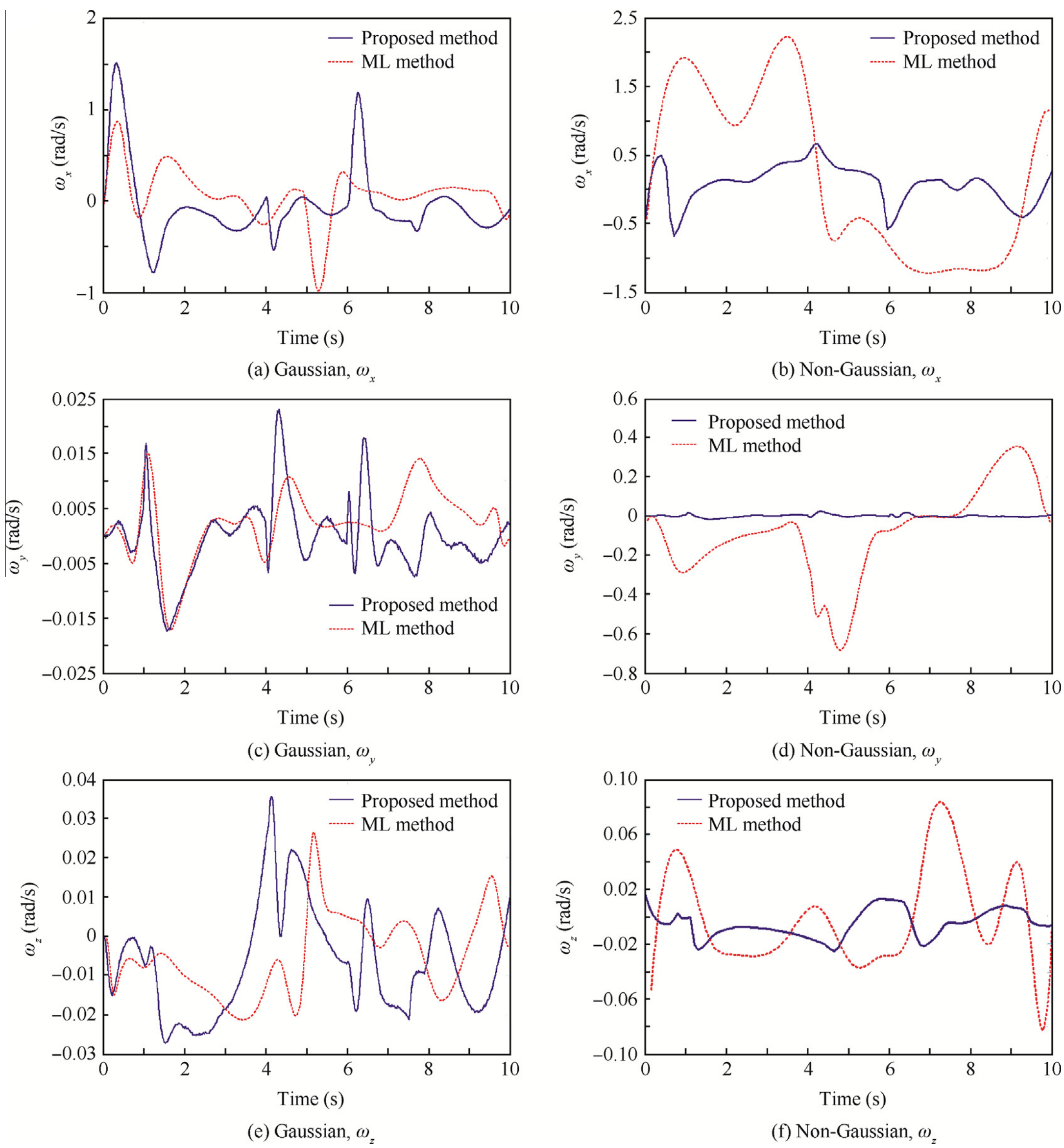


**Fig. 2** Diagram of model prediction error for output data with Gaussian and non-Gaussian disturbance.

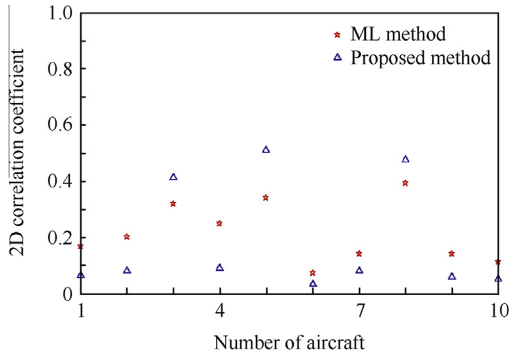


**Fig. 3** Inversed 2D correlation coefficient metric between the estimated model of each aircraft and reference model.

velocities and the three linear accelerations ($a_x$, $a_y$, $a_z$) along its body axes. The regression function is chosen as a linear affine function for simplify. A team of UAVs comprising of 10 identical UAVs are used throughout the experiments.

The simulated flight path consists of three segments. The group of aircraft firstly climbs to 2000 m from their initial cursing altitude of 1500 m. Then they maintain their flight level at 2000 m for 20 s in the second segment of the planned route. In the last flight segment, these flights descend to 1500 m. The input–outputs data pairs are recorded by 10 s, with a sampling rate at 50 Hz, for each segment. Hence, for each flight segment the data contains a 500 samples in variables $\boldsymbol{x}$ and $\boldsymbol{y}$, including 12 inputs and 6 outputs. The model parameters $\boldsymbol{\theta}_i$ are determined based on Eq. (13) for each flight of the team, where $\lambda$ is set and fixed to 0.1 throughout the simulations. The ML based model estimation approach, described in Section 2.1, is also implemented. Both Gaussian and non-Gaussian noise (a mixed Gaussian model in this paper) are added to the measured data.

Fig. 2 present the comparisons of the measured outputs and simulated outputs (based on the identified model) errors with Gaussian and mixture Gaussian model for three output channels, i.e., three angular velocities, respectively. It can be seen that both of the recognized models can accurately represent the dynamics of the flight given the input data with Gaussian disturbances (Fig. 2(a), (c) and (e)). The proposed approach outperforms ML based method, in terms of model prediction error, in the case where unknown non-Gaussian disturbances are presented, as shown in Fig. 2(b), (d) and (f).

To demonstrate the ability of the proposed scheme to detect and isolate the occurrence of abnormity in the aircraft, we randomly choose three aircraft (#3, #5 and #8) and set different types of system failures, i.e., the non functional elevators, low engine power and angular sensory errors. To measure the similarity between identified models, 2D correlation coefficient metric is used. It approaches to unity when two matrices are identical. Zero value of this metric, on the other hand, indicates that two matrices are entirely different.

Fig. 3 depicts the inversed correlation coefficient between the estimated model of each aircraft, the mean model $\boldsymbol{\theta}_i$ and the reference model $\boldsymbol{\theta}_0$. K-means algorithm[23] is then applied to determining the abnormal aircraft based on the correlation coefficient metric. The classification results obtained through K-means algorithm is shown in Fig. 4.

Fig. 4(a) depicts the resulting classification (a team of aircraft is divided into two groups, i.e., normal flights and abnormal flights) based on the dynamic model estimated using the proposed method, and Fig. 4(b) shows the classification results obtained by using the dynamic model predicted by ML algorithm. It can be seen that K-means algorithm can successfully identify the abnormal behaviored aircraft using the model parameters estimated from the proposed method. On the other hand, erroneous classification is found when using the estimated model obtained from ML based approach (as shown in Fig. 4(b)). This is due to the fact that ML based model identification methods can only deal with Gaussian noise (disturbance). In the case where the unknown noise/disturbance does not subject to Gaussian assumption, the optimality of the estimated model parameters cannot be guaranteed. Furthermore, the within-cluster sums of point to centroid distances calculated based on ML algorithm estimated model are found to be (0.0095, 0.0105), which are significantly larger than the proposed method (0.0023, 0.0049), indicating that the model parameters estimated by the proposed approach is insensitive to the unknown noise in the flight dynamics, and therefore improving the classification accuracy in the presence of dynamic disturbances and model uncertainties.

## 5. Conclusions

In this paper, a novel two-stage algorithm is proposed to efficiently detect and isolate the anomaly UAV in a formation fleet. The dynamical model of each UAV is firstly identified

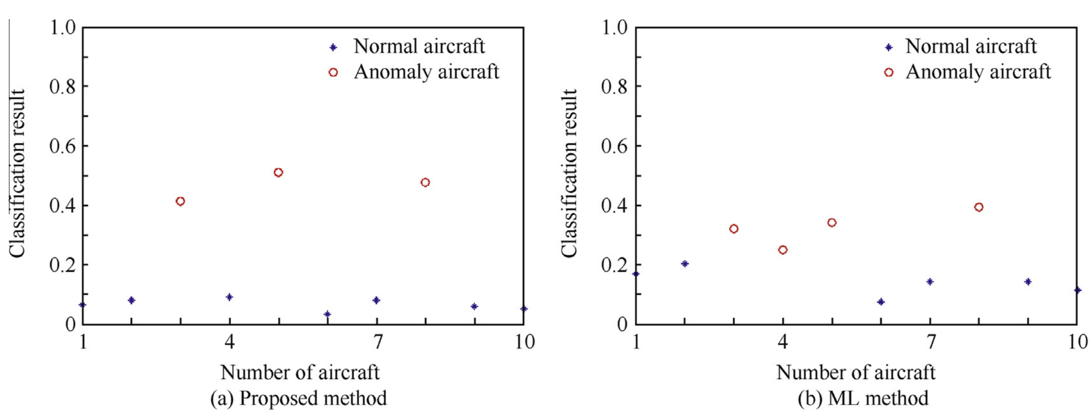


**Fig. 4** K-means classification results obtained based on correlation coefficient calculated using different method.

from a sequence of input–output data pairs using a sparse optimization based technique. An improved fast gradient based numerical scheme is developed to solve the associated optimization problem, and its convergence is analyzed. The anomalies of UAVs are determined at the second stage by finding the changes of the model parameters, using K-means algorithm. Comparative studies have demonstrated the efficiency and flexibility of the proposed approach. In terms of future research, we intend to develop an on-line scheme to estimate the dynamics of the aircraft, which would further improve the applicability of the proposed algorithm in real-world applications.

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