Decimation

$$x(n) \longrightarrow \boxed{\downarrow M} \longrightarrow x_d(n) = x(nM)$$

In CT-sampling:

$$X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

similarly, $x_d(n) = x(nM) = x_c(nT')$ with $T' = MT$

$$X_d(e^{j\omega}) = \frac{1}{T'}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T'} - \frac{2\pi r}{T'}\right)\right)$$

$$= \frac{1}{MT}\sum_{r=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{MT} - \frac{2\pi r}{MT}\right)\right)$$

$$\Rightarrow \boxed{X_d(e^{j\omega}) = \frac{1}{M}\sum_{i=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)}$$

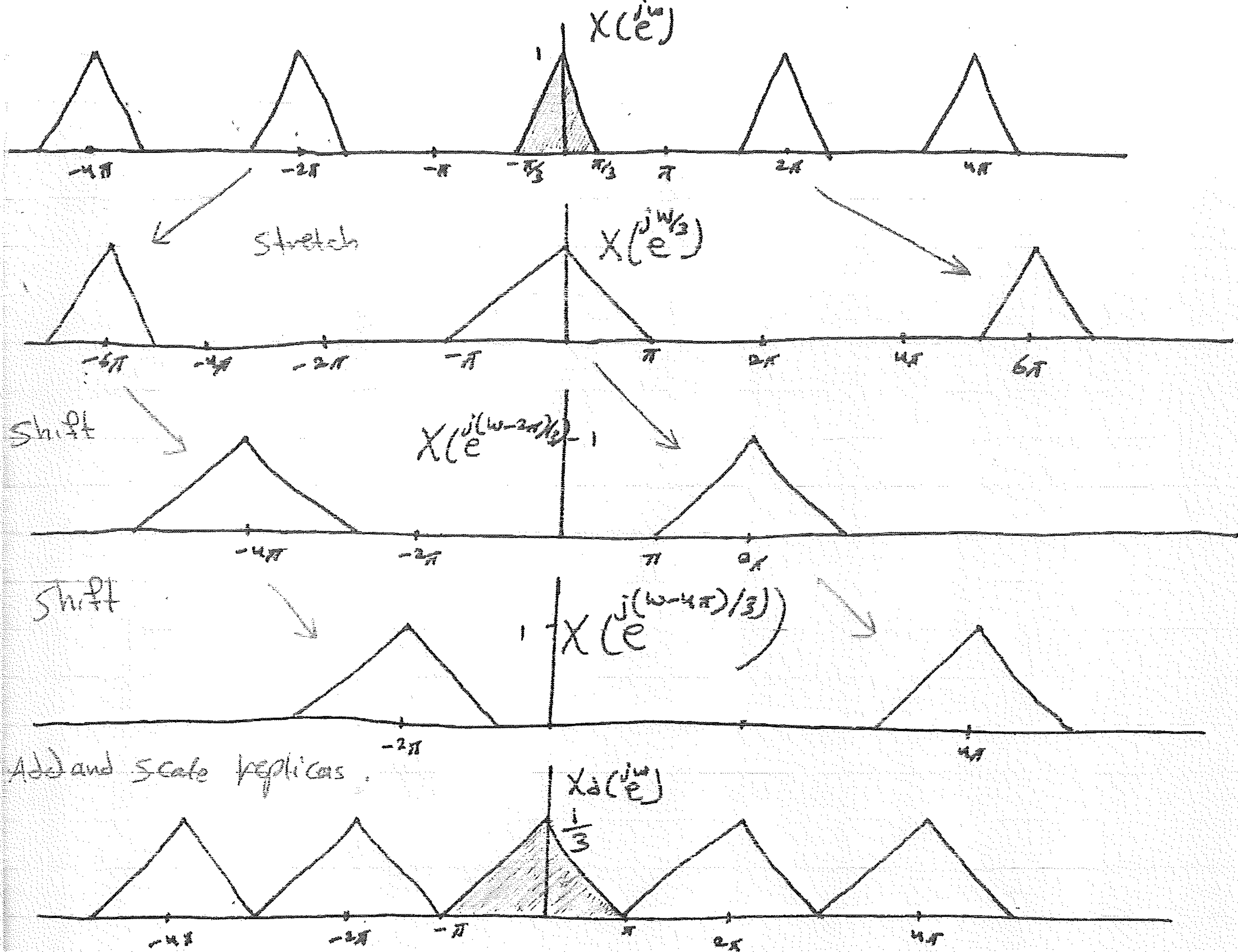
So, To sketch $X_d(e^{j\omega})$ directly from $X(e^{j\omega})$ follow 3 steps:

1. Stretch $X(e^{j\omega})$ by a factor M to obtain $X(e^{j\frac{\omega}{M}})$. note that the highest freq. of $X(e^{j\omega})$, ω_H, is "repositioned" to freq. $\omega = \omega_H \cdot D$.

2. Create and put M copies of $X(e^{j\frac{\omega}{M}})$ at freq. $\omega = 2\pi i$ (integer multiple of 2π) for $i = 0, 1, 2, \ldots, M-1$.

3. Add the M streched and shifted replicas and then divide by M to obtain the spectrum $X_d(e^{j\omega})$ of the downsampled sequence $x_d(n) = x(nM)$.

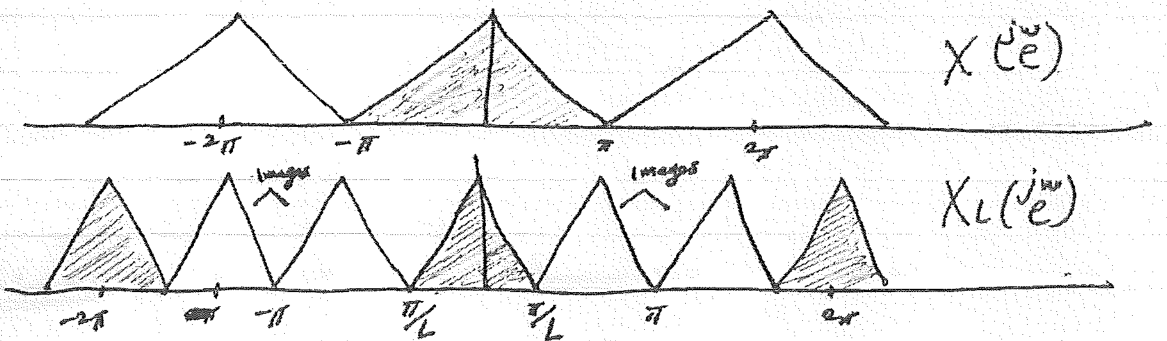
<u>Example</u>. Let $M=3$ and Bandwidth of signal $x(n)$ is $\pi/3$.

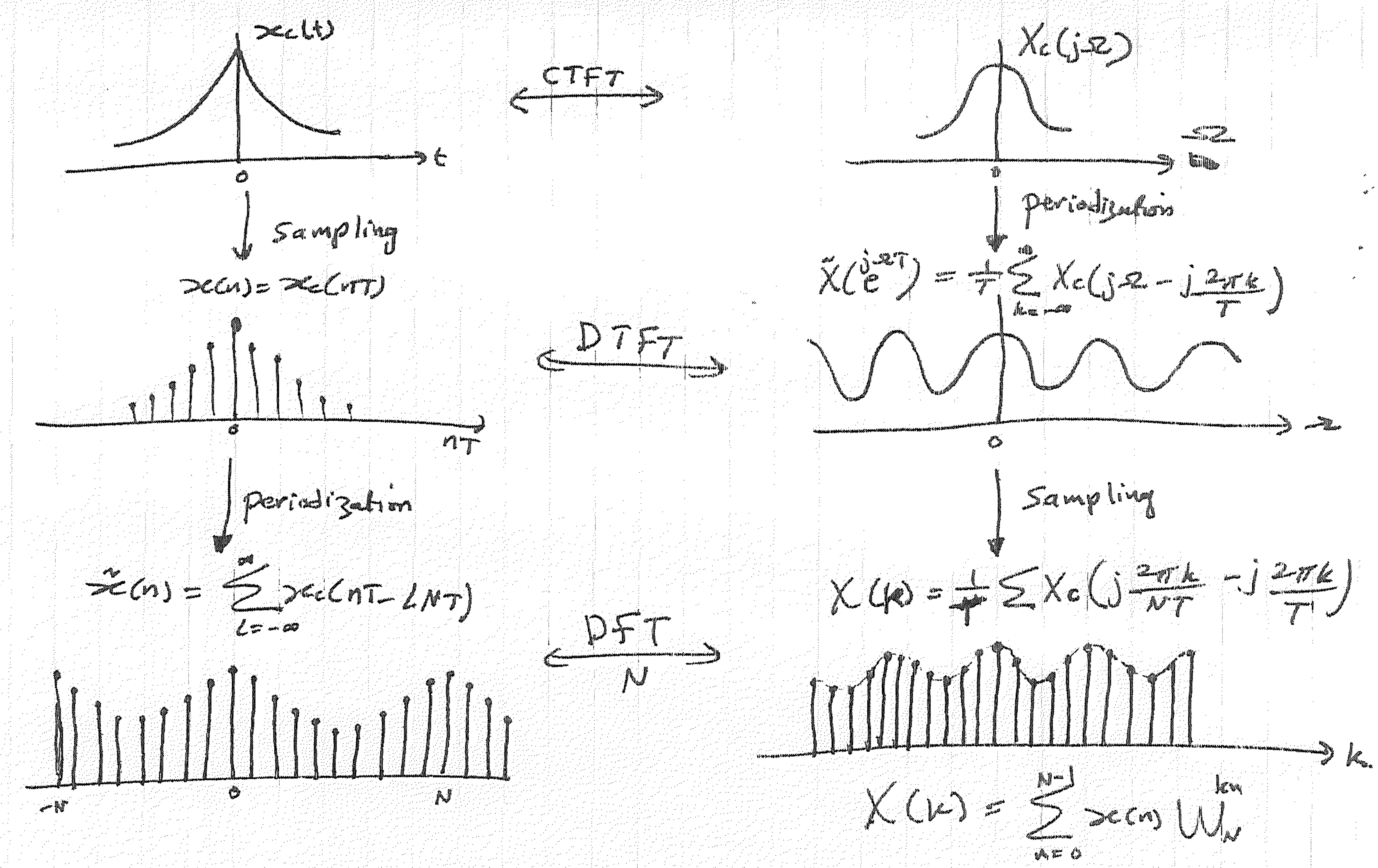


<u>Interpolation:</u>

$$x(n) \rightarrow \boxed{\uparrow L} \rightarrow x_L(n) = x(n/L)$$

$$X_L(e^{jw}) = X(e^{jwL})$$

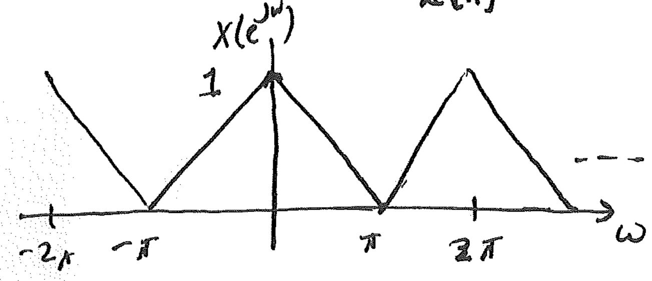




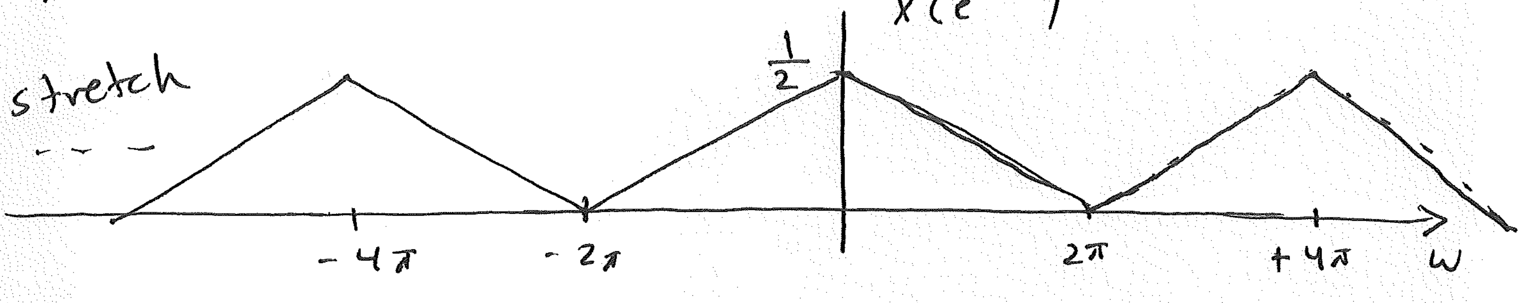
relationship between CTFT, DTFT and DFT.

Example : Decimation by 2

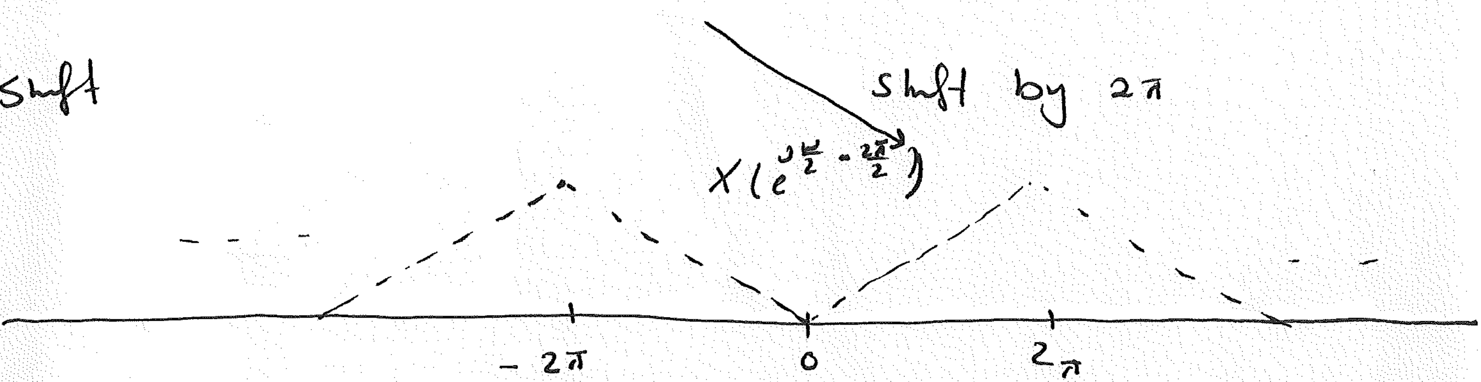
$x[n] \longrightarrow \boxed{2\downarrow} \longrightarrow y[n] = x[2n]$



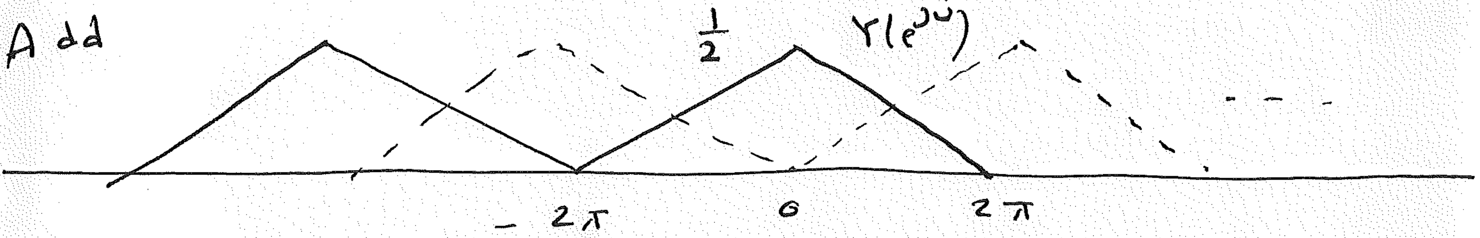
stretch



shift

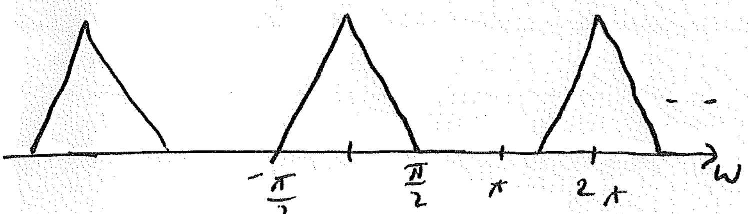


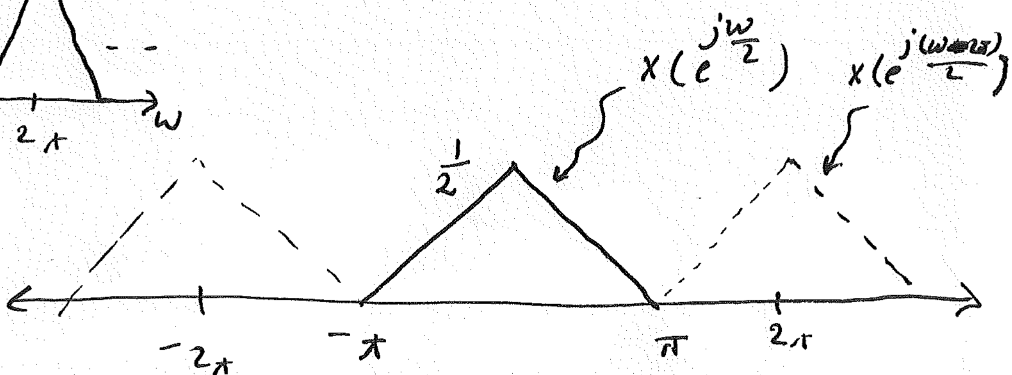
Add



Extreme Aliasing

let $x[n]$ have $X(e^{j\omega})$ as follows :-



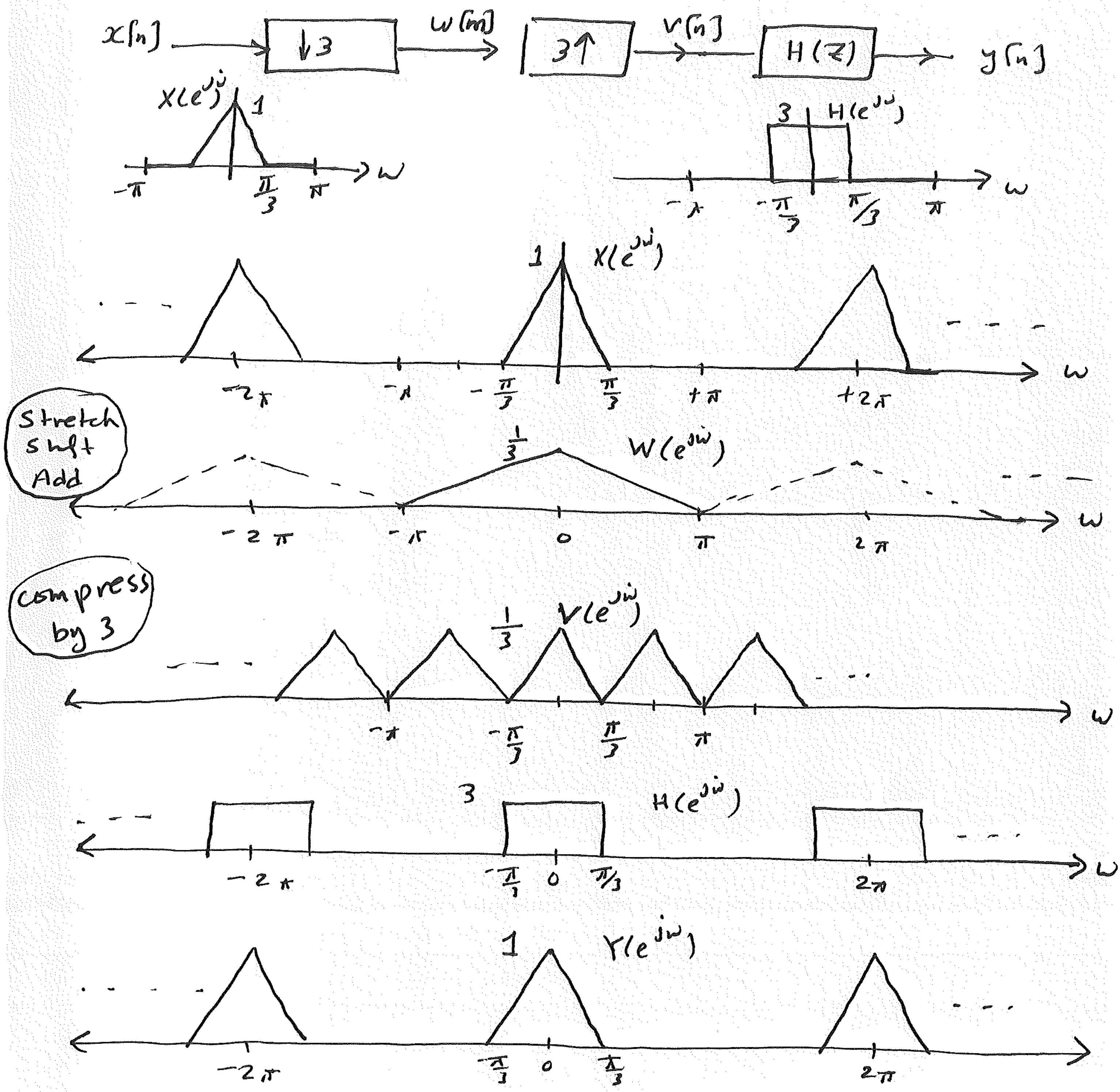


No Aliasing

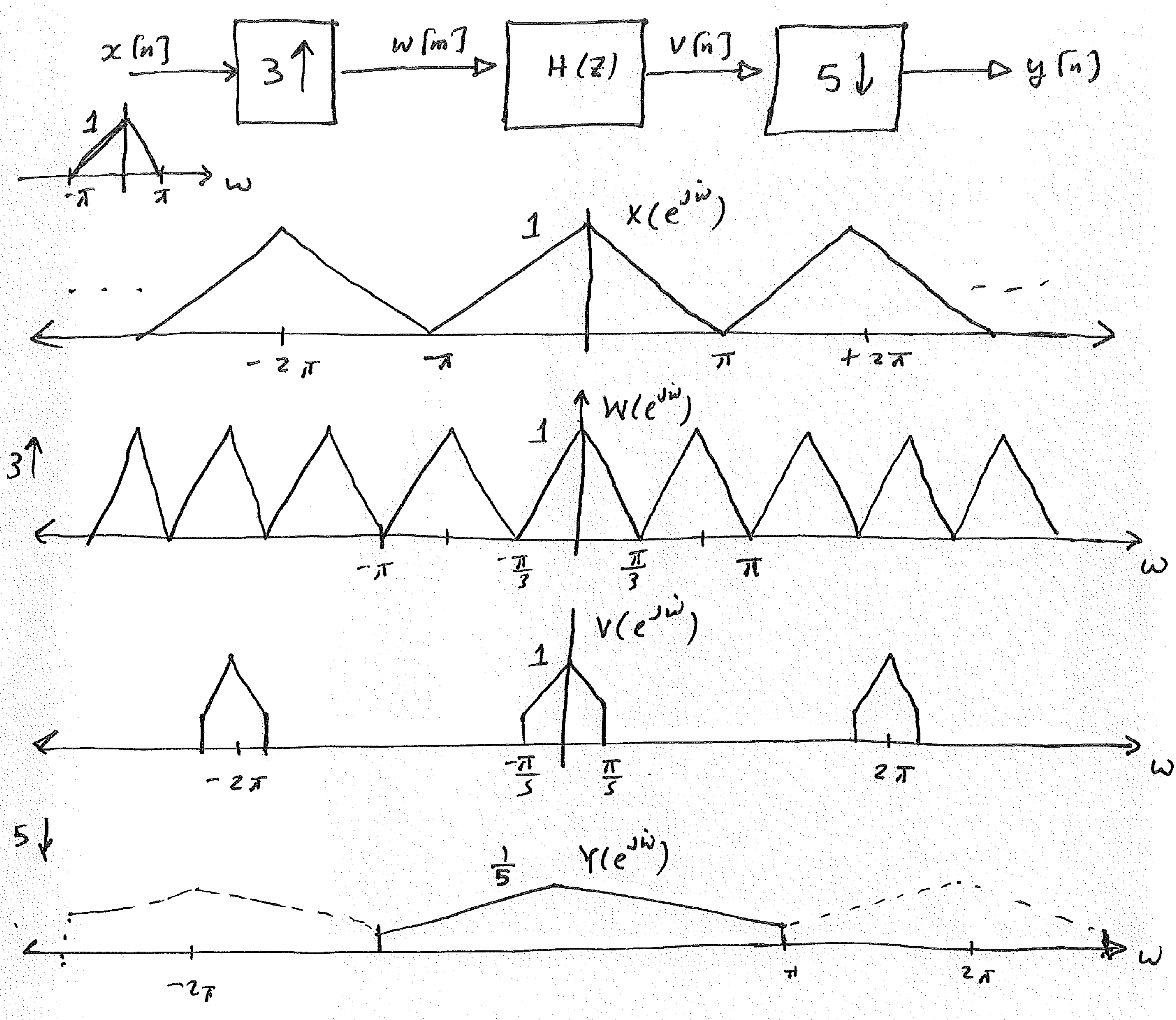
Example

An input signal $x[n]$ with spectrum $X(e^{j\omega})$ is shown below. The input is applied to the system shown below.

Sketch $X(e^{j\omega})$, $W(e^{j\omega})$, $V(e^{j\omega})$, $Y(e^{j\omega})$.



Example The ideal LPF $H(z)$ has a gain factor of 1 in the pass band & a cut-off freq. $w_c = \frac{\pi}{5}$. Consider the following system plot $X(e^{jw})$, $W(e^{jw})$, $V(e^{jw})$, $Y(e^{jw})$



a) Consider the system shown in figure 2-1 below. The input to this system is a bandlimited signal whose Fourier transform is shown in figure 2-2 below with $\Omega_0 = \frac{\pi}{T}$. The discrete-time LTI system in figure 2-1 has the frequency response shown in figure 2-3 below.

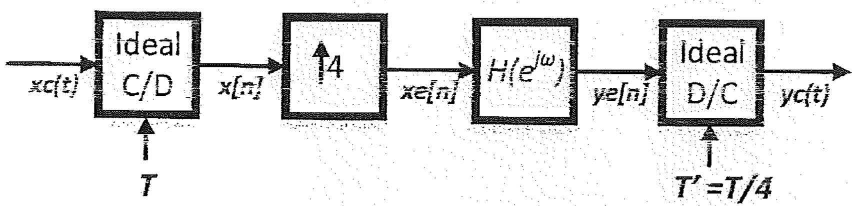


Figure 2-1

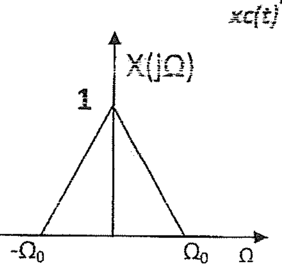


Figure 2-2

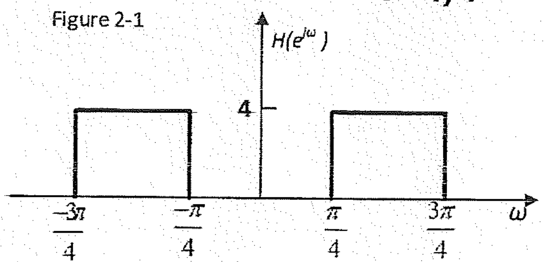
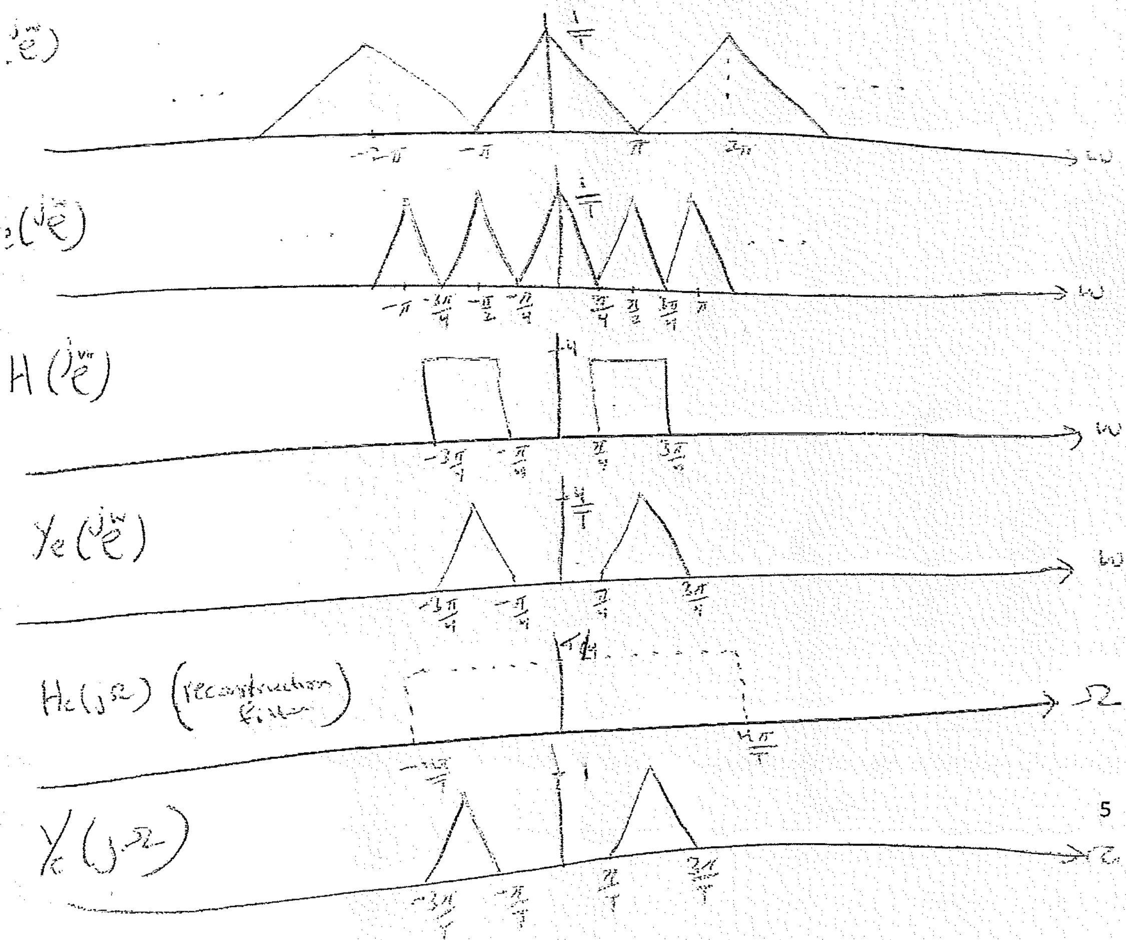


Figure 2-3

i) Sketch the Fourier transforms of $X(e^{jw}),\ X_e(e^{jw}),\ Y_e(e^{jw})\ and\ Y_c(e^{jw})$. [8pts]



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ii) For the general case when $X_c(j\Omega) = 0 \;\; for \; |\Omega| \geq \frac{\pi}{T}$, express $Y_c(j\Omega)$ in terms of $X_c(j\Omega)$. Also give a general expression for $y_c(t)$ in terms of $x_c(t)$, when $x_c(t)$ is bandlimited in this manner. [4pts]

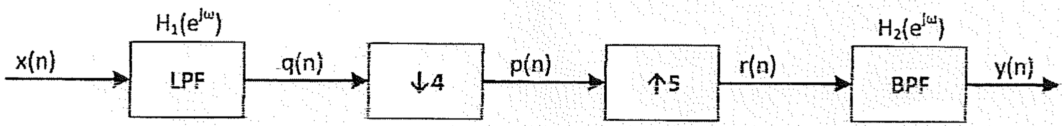
From Figure in Part (i) [Spectrums of $X_c(j\Omega)$ and $Y_c(j\Omega)$]

$$\Rightarrow \quad Y_c(j\Omega) = X_c\left(j\left(\Omega - \frac{2\pi}{T}\right)\right) + X_c\left(j\left(\Omega + \frac{2\pi}{T}\right)\right)$$

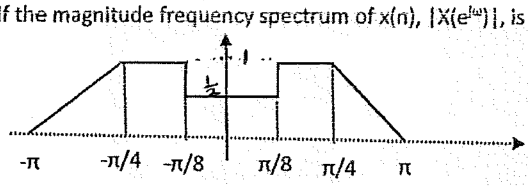
Therefore

$$y_c(t) = 2\, x_c(t) \cos\left(\frac{2\pi}{T} t\right)$$

a) Consider the multi-rate system shown in the Figure below: [12pts]



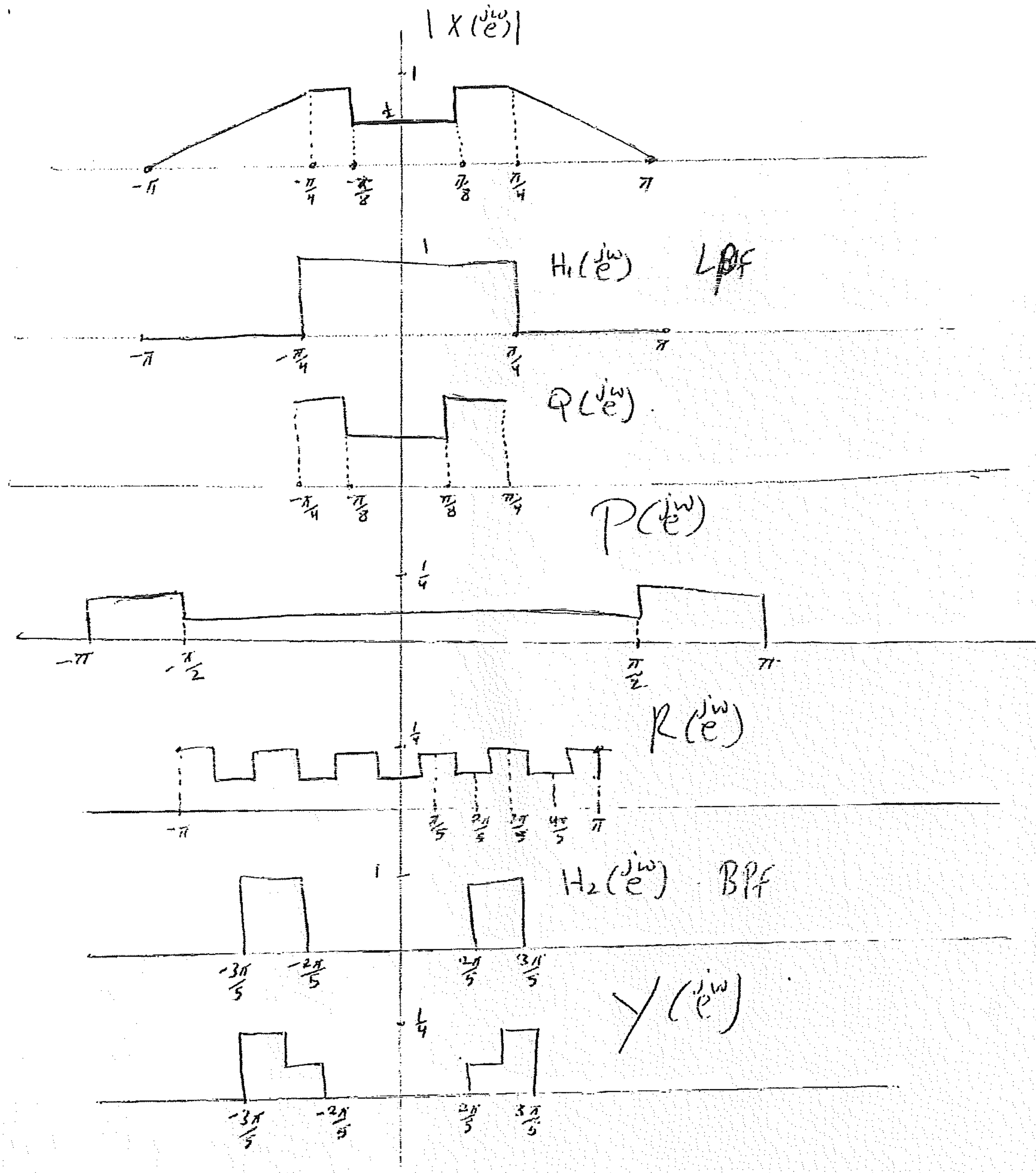
If the magnitude frequency spectrum of x(n), $|X(e^{j\omega})|$, is given as follows:



And frequency response of LowPass filter, $H_1(e^{j\omega})$, and BandPass filter $H_2(e^{j\omega})$ are as follow:

$$H_1(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq \frac{\pi}{4} \\ 0, & otherwise \end{cases} \qquad H_2(e^{j\omega}) = \begin{cases} 1, & \frac{\pi}{3} \leq |\omega| \leq \frac{3\pi}{5} \\ 0, & otherwise \end{cases}$$

Sketch $|Q(e^{j\omega})|$, $|P(e^{j\omega})|$, $|R(e^{j\omega})|$, and $|Y(e^{j\omega})|$?



Consider a first-order digital filter, which is described by the following difference equation:

$y(n) = x(n) - ax(n-1),\ a \neq 0$

1- Is this filter IIR or FIR filter? Justify?

2- Find the coefficient, a, such that this filter attenuates signal -6 db at frequency $\omega = \frac{\pi}{4}$ **?**