Equations Vacuum Break Model

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1 2D NS Cylindrical Coordinates

General equations:

Divergence (cylindrical)

$$\begin{bmatrix} \nabla_r & \nabla_\theta & \nabla_x \\ \frac{1}{r}\frac{\partial}{\partial r}(r) & \frac{1}{r}\frac{\partial}{\partial\theta} & \frac{\partial}{\partial x} \end{bmatrix}$$

Laplacian (cylindrical)

$$\begin{bmatrix} \nabla^{2}(\mathbf{v}) \\ (\frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^{2}}\frac{\partial^{2}}{\partial\theta^{2}} + \frac{\partial d}{\partial dx^{2}})(\mathbf{v}) \end{bmatrix}$$

Continuity:

$$\frac{\partial\rho}{\partial t} + \nabla\,.\,(\rho\,\mathbf{v}) = S \tag{1}$$

Momentum:

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t} + \mathbf{v}\,.\,\nabla\,\mathbf{v}\right) = -\nabla\,p + \mu\,\nabla^{2}\,\mathbf{v} + \frac{\mu}{3}\,\nabla\,(\nabla\cdot\,\mathbf{v}) \tag{2}$$

Energy:

$$\frac{\partial\,E}{\partial t} + \nabla\,\cdot\,(E\,\mathbf{v}) = S \tag{3}$$

Vacuum Break

Continuity:

within flow field

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho\,u_x)+\frac{1}{r}\frac{\partial}{\partial r}(\rho\,r\,u_r)=\,0 \tag{4}$$

$$\rho^{t+1}_{m,n}=\rho^{t}_{m,n}-\frac{\partial t}{n\,\Delta\,r\,\partial r}([\rho\,(n{+}1)\,\Delta\,r\,u_r]^{t}_{m,n+1}-[\rho\,n\,\Delta\,r\,u^{t}_{r}]_{m,n})-\frac{\partial t}{\partial x}([\rho\,u^{t}_{x}]_{m+1,n}-[\rho\,u^{t}_{x}]_{m,n})$$

(Boundary- cylinder wall)

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho\,r\,u_r)=-\frac{4}{D_1}\,\dot{m}_c \tag{5}$$

$$\rho^{t+1}_{n,m}=\rho^{t}_{m,n}-\frac{\partial t}{n\,\Delta\,r\,\partial r}([\rho\,(n+1)\,\Delta\,r\,u^{t}_{r}]_{m,n+1}-[\rho\,n\,\Delta\,r\,u^{t}_{r}]_{m,n})-\frac{4}{D_1}\,\dot{m}_c\,\Delta\,t$$

Is the m dot at same grid point? m or m+1?

Momentum X:

within flow field

$$\begin{aligned}\frac{\partial}{\partial t}(u_x)+u_x\frac{\partial}{\partial x}(u_x)+u_r\frac{\partial}{\partial r}(u_x)=&-\frac{\partial p}{\rho\,\partial x}\\&+\frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial u_x}{\partial r})+\frac{\partial u_x^2}{\partial\,x^2}\right)\\&+\frac{\mu}{3\,\rho}\,\frac{\partial}{\partial\,x}\left(\frac{1}{r}\,\frac{\partial}{\partial\,r}\,(r\,u_r)+\frac{\partial\,u_x}{\partial x}\right)\end{aligned} \tag{6}$$

$$\begin{aligned}u_x^{t+1}{}_{m,n}=u^{t}_{x\,m,n}-\Delta\,t\frac{(p^{t}_{m+1,n}-p^{t}_{m,n})}{\rho^{t}_{m,n}\,\Delta\,x}+\mu\,\frac{\Delta\,t}{\rho^{t}_{m,n}}&\left[(\frac{u^{t}_{x\,m,n+1}+u^{t}_{x\,m,n-1}-2\,u^{t}_{x\,m,n}}{\Delta\,r^2})\right.\\&\left.+\frac{1}{r}\,(\frac{u_{x\,m,n+1}\,-\,u_{x\,m,n}}{\Delta\,r})+\frac{u^{t}_{x\,m+1,n}+u^{t}_{x\,m-1,n}-2\,u^{t}_{x\,m,n}}{\Delta\,x^2}\right]\\&-\Delta\,t\,u^{t}_{x\,m,n}\frac{u^{t}_{x\,m+1,n}-u^{t}_{x\,m,n}}{\Delta\,x}-\Delta\,t\,u^{t}_{r\,m,n}\frac{u^{t}_{x\,m,n+1}-u^{t}_{x\,m,n}}{\Delta\,r}\Bigg]\end{aligned} \tag{7}$$

Note the mu/3 term is not considered.
No momentum x equation on the boundaries.

Momentum R:

No momentum r equation on the boundaries.

$$\rho\, u_r \;= \dot{m}_c \tag{8}$$

within flow field

$$\begin{aligned}\frac{\partial}{\partial t}(u_r) + u_x \frac{\partial}{\partial x}(u_r) + u_r \frac{\partial}{\partial r}(u_r) = &-\frac{\partial\, p}{\rho\, \partial\, r} \\ &+ \frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial\, r}(r \frac{\partial}{\partial\, r}\, u_r) + \frac{\partial\, u_r^2}{\partial\, x^2} - \frac{u_r}{r^2}\right) \\ &+ \frac{\mu}{3\,\rho}\frac{\partial}{\partial\, r}\left(\frac{1}{r}\frac{\partial}{\partial\, r}(r\, u_r) + \frac{\partial\, u_x}{\partial\, x}\right)\end{aligned} \tag{9}$$

$$\begin{aligned}{u_r^{t+1}}_{m,n} = u^t_{r\,m,n} - \Delta\, t \frac{({p_{m,n+1}}^t - {p_{m,n}}^t)}{{\rho_{m,n}}^t\,\Delta\, r} + \mu\, \frac{\Delta\, t}{\rho^t_{m,n}} \Bigg[&- \frac{u_{r\,m,n}}{dr^2 * n^2} + (\frac{u^t_{r\,m,n+1} + u^t_{r\,m,n-1} - 2\, u^t_{r\,m,n}}{\Delta\, r^2}) \\ &+ \frac{1}{r}(\frac{u_{r\,m,n+1}\; - \; u_{r\,m,n}}{\Delta\, r}) + \frac{u^t_{r\,m+1,n} + u^t_{r\,m-1,n} - 2\, u^t_{r\,m,n}}{\Delta\, x^2}\Bigg] \\ &- \Delta\, t\, u^t_{x\,m,n} \frac{u^t_{r\,m+1,n} - u^t_{r\,m,n}}{\Delta\, x} - \Delta\, t\, u^t_{r\,m,n} \frac{u^t_{r\,m,n+1} - u^t_{r\,m,n}}{\Delta\, r}\end{aligned} \tag{10}$$

Note the mu/3 term is not considered.

Energy

**Note: No viscous dissipation taken into account.

within flow field

$$\frac{\partial}{\partial t}\left[\rho(\epsilon + \frac{v^2}{2})\right] + \frac{\partial}{\partial x}\left[\rho\, u_x(\epsilon + \frac{u^2}{2} + \frac{P}{\rho})\right] + \frac{\partial}{r\,\partial r}\left[\rho\, r\, u_r(\epsilon + \frac{u^2}{2} + \frac{P}{\rho})\right] = 0 \tag{11}$$

$$\rho\,(\epsilon+\frac{u^2}{2})^{t+1}_{m,n}=\rho\,(\epsilon+\frac{u^2}{2})^{t}_{m,n}+\frac{\Delta\,t}{\Delta\,x}\left[\rho\,u_x\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m+1,n}-\rho\,u_r\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n}\right]-\frac{\Delta\,t}{r\,\Delta\,r}\left[\rho\,r\,u_r\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n+1}-\rho\,r\,u_r\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n}\right] \tag{12}$$

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at boundaries (radial wall)

$$\frac{\partial}{\partial t}\left[\rho(\epsilon+\frac{u^2}{2})\right]+\frac{\partial}{r\,\partial r}\left[\rho\,u_r\,r(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})\right]=-\frac{4}{D_1}\,\dot{m}_c\left(\epsilon+\frac{u^2}{2}+\frac{P}{\rho}\right) \tag{13}$$

$$\rho\,(\epsilon+\frac{u^2}{2})^{t+1}_{m,n}=+\rho\,(\epsilon+\frac{u^2}{2})^{t}_{m,n}-\frac{\Delta\,t}{r\,\Delta\,r}\left[\rho\,r\,u_r\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n+1}-\rho\,r\,u_r\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n}\right]-\frac{4\,\Delta\,t}{D_1}\,\dot{m}_c\,(\epsilon+\frac{u^2}{2}+\frac{P}{\rho})^{t}_{m,n} \tag{14}$$

where **v** is the total velocity of the fluid.
also,

$$p=\rho\,R\,T \tag{15}$$

Radial heat transfer

$$q_{dep}=\dot{m}_c\,[\frac{v^2}{2}+h_g-h_s] \tag{16}$$

There is no convection heat transfer into the wall.
Frost layer temperature change

$$\rho_{SN}\,C_{SN}\,\delta\,\frac{\partial\,T_c}{\partial\,t}=q_{dep}-q_i \tag{17}$$

But q_i is only up a preset minimum value.

q_i

$$q_i \;=\; k_{SN}(T_S - T_w)/\delta \tag{18}$$

Copper tube wall temperature variation

$$\rho_{SN}\, C_w\, \frac{D_2^2 - D_1^2}{4\, D_1}\, \frac{\partial\, T_w}{\partial\, t} \;=\; q_i \;-\; q_H e\, \frac{D_2}{D_1} \;+\; \frac{D_2^2 - D_1^2}{4\, D_1}\, k_w\, \frac{\partial^2\, T_w}{\partial\, x^2} \tag{19}$$

But q_i is not always present.

Boundary conditions (radial wall)

Here is a list of boundary conditions used for our equations: **(to be continued)**