Alg. Homework.2. Qutaiba ALASHQAR, 2020d0036.

Soru1

(a) $2n^3 + 5n + 10 = \Theta(n^3)$

$g(n) = n^3$, $n \geq 1$ için

$c1.(n^3) \leq 2n^3 + 5n + 10 \leq c_2.(n^3)$

$c_1 \leq \frac{2n^3+5n+10}{n^3}$, $c1 \leq 2 + \frac{5}{n^2} + \frac{10}{n^3}$ artan fonksiyon

$c_1 = 2$ ise doğru

$\frac{2n^3+5n+10}{n^3} \leq c_2$, $2 + \frac{5}{n^2} + \frac{10}{n^3} \leq c_2$

$n = 1$ için $(2)(1)^3 \leq 2(1)^3 + 5(1) + 10$

$2 \leq 17$

o zaman $c_2 = 17$

Doğru

(b) $n^2 + 3n + 4 = O(n^3)$

$n^2 + 3n + 4 \leq cn^3$

eğer $n \geq 1$ and $c = 8$ ise

$n=1$ $1 + 3 + 4 \leq 8$ doğru → if we have 8 will be a good guessing

$n=2$ $4 + 6 + 4 \leq 64$ doğru ✓

$n=3$ $9 + 9 + 4 \leq 216$ doğru ✓

o zaman doğru

(c)

(c) $n^2 + 4n + 3 = \Omega(n)$

$n^2 + 4n + 3 \geq c \cdot n$

eğer $n \geq 1$ ve $C = ?$ için ise doğrumu

$n=1$, $1 + 4 + 3 \geq ??$ → so here we can guess any number we want but less than 8, so would I like to guess "1"

As $n \geq 1$ and $c = 1$

$n=1$, $1+4+3 \geq 1$ ✓

$n=2$, $4 + 8 + 3 \geq 2$ ✓

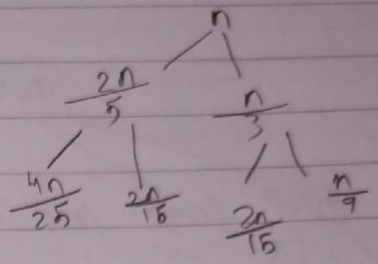
$n=3$ $9 + 12 + 3 \geq 3$ ✓

DOĞRU.

Soru 2// $T(n) = T(2n/5) + T(n/3) + n$, $n > 1$ iken

$T(1) = 1$

$T(n) = O(n)$ için , tahmnin doğru olduğunu tümevarım yöntem ile ispatlayın



$T(1) = O(n)$

$T(1) = O(1)$

$1 \leq c \cdot 1$, $c > 1$ ise

$1 \leq 1$ $c = 1$ için doğrun

$1 \leq k \leq n$, $\forall k$ için eşitsizlik doğru olsun.

$T(k) \leq c \cdot k$

$T(n/3) \leq c(n/3)$

$T(2n/5) \leq c(2n/5)$

$T(n) \leq c(n)$

$T(n) = T(2n/5) + T(n/3) + n$

$T(n) \leq c(2n/5) + c(n/3) + n$

$T(n) \leq 2c\left(\frac{6n+5n}{15}\right) + n$

$T(n) \leq 2c\left(\frac{11n}{15}\right) + n$

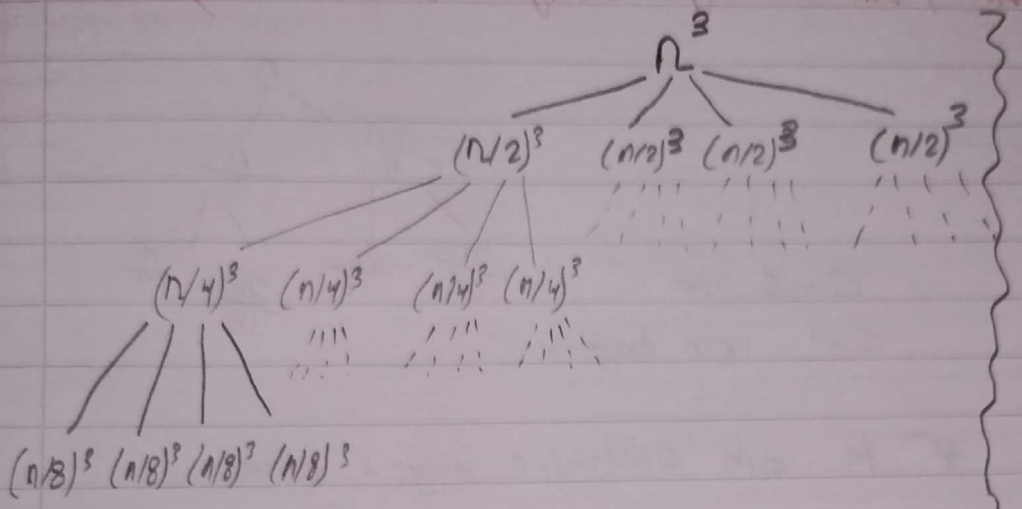
$T(n) \leq 2c\left(\frac{11n}{15}\right) + n$

$2c \cdot \frac{11n}{15} + n \leq c \cdot n \Rightarrow c \cdot 11n + \frac{15}{2}n \leq \frac{15}{2} c \cdot n$

$c11n + 7.5n \leq 7.5\,cn \Rightarrow 7.5n \leq -3.5\,cn$???

$7.5 \leq -3.5c$ $c = -2.15$

Soru 3. $T(n) = 4T(n/2) + n^3$, $n \geq 1$ iken
$T(1) = 1$, ağaç ile.



n^3

$n^3/2$

$n^3/4$

$n^3/8$

$$\sum_{k=0}^{k} \left(\frac{n^3}{2^k}\right)$$

~~$T(1) = 4T(1) + \sum_{k=0} \left(\frac{n^3}{2^k}\right)$~~

$T(n/2) = 4T(n/4) + \left(\frac{n}{2}\right)^3$

$T(n) = 4\left(4T(n/4) + (n/2)^3\right) + n^3$

$T(n/4) = 4T(n/8) + (n/4)^3$

$T(n) = 4\left(4\left(4T(n/2) + (n/2)^3\right) + (n/3)^3\right)$

$T(n) = n^3 + 4\left(\frac{n}{2}\right)^3 + 4^2\left(\frac{n}{2^2}\right)^3 + \dots + 4^n\left(\frac{n}{2^n}\right)^3$

$$\sum_{k=0}^{k=0} \left(\frac{n}{2^k}\right)^3$$

Soru 4. $x(n) = x(n/2) + 1 \quad , n > 1$

$x(1) = 1$ rekürensini $n = 2^k$ değerleri için kesin çözüm bulun.

$n = 2^k$

$x(2^k) = x(2^{k-1}) + 1$

$x(2^{k-1}) = x(2^{k-2}) + 1$

$\vdots$

$x(2^1) = x(2^0) + 1 \rightarrow x(2) = x(1) + 1$

$x(2^k) = x(1) + 1 + \cdots + 1$

$x(2^k) = 1 + 1 \cdot (n-1) = n$