[redacted]

Midterm

The exam should be done individually. You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten. **Solutions must be submitted electronically before 4 pm on August 11.** No credit will be given to solutions obtained verbatim from the Internet or other sources.

1. (20p) Prove that if $n^2 + 2n$ is odd integer, then $n + 1$ is even integer.

2. (20p) Let R be the relation on the set of integers defined as $\forall a, b \in \mathbb{Z}, (a, b) \in R$ if $a.b < 0$. Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation satisfies. Justify your answer.

Employ your id to calculate a specific number that will be used in the further questions as follows *('18YZ0345' will be used here as an example to show you how the number is calculated):*

- remove the letters from your number (if it does not contain any letter, just keep it as it is)

  $18YZ0345 \longrightarrow 180345$

- multiply the result with '12345'

  $180345 * 12345 \longrightarrow 2226359025$

- remove all the zeros from the resulting number

  $2226359025 \longrightarrow 222635925$

- cut out the last 4 digits and assign them to the letters A, B, C, D, respectively.

  $5 \longrightarrow A, \quad 9 \longrightarrow B, \quad 2 \longrightarrow C, \quad 5 \longrightarrow D$

- put the numbers in place of the corresponding letters to solve the following questions.

3. (15p) Solve the recurrence relation $a_n = Aa_{n-1} + Ba_{n-2}$ where $a_0 = C$ and $a_1 = D$.

4. (15p) How many integer solutions are there for the equation $x_1 + x_2 + x_3 + x_4 = 30$ if $x_1 \geq A$, $x_2 \geq B$, and $x_2, x_3 \geq 0$?

5. (15p) How many bit strings (that consist of the symbols '0' and '1') of length $(A + B)$ have more zeroes than ones? *(-the bit string '0101100' of length 7 has more zeros than ones-)*

6. (15p) Suppose A% of the people in a community has a particular disease and there is a fairly accurate diagnostic test for it. B% of the time this test gives a positive result for the people having this disease, and C% of the time this test gives a negative result for the people not having this disease. What is the probability that a person, who had a positive result from the test, has the disease?

1. $n^2+2n=2k+1$

$n(n+2)=2k+1$

n should be odd because of multiplication

↳ proof:

$n=2a+1$

$$(2c+1)(2d+1)=4cd+2d+2c+1$$

$$=\underbrace{\underbrace{2(2cd+d+c)}_{\text{even}}+1}_{\text{odd}}$$

$n+1=2a+2$

$\boxed{n+1=2m}$

2. Reflexivity:

$a.b<0$

$a.a \nless 0$

$a^2 \nless 0$

$\boxed{\text{non-reflexive}}$

Symmetry:

$(a,b)\in R$ $\quad a.b<0$

$(b,a)\in R$ $\quad b.a<0$

$\boxed{\text{symmetric}}$

Transitivity:

$\overset{+}{a}.\overset{-}{b}<0$

$\overset{-}{b}.\overset{+}{c}<0$

$\overset{+}{a}.\overset{+}{c}<0$

if we assume $a\in \mathbb{Z}^+$ and $b\in \mathbb{Z}^-$ then c should be $\in \mathbb{Z}^+$

so $a.c>0$

$\boxed{\text{non-transitive}}$

School number: [redacted]

$238138\,3\underline{3377}\cancel{0}$

$A=3 \quad C=7$

$B=3 \quad D=7$

3.

$a_n = 3a_{n-1} + 3a_{n-2} \qquad a_0 = 7 \qquad a_1 = 7$

$a_n - 3a_{n-1} - 3a_{n-2} = 0$

$r^2 - 3r - 3 = 0$

$\Delta = b^2 \pm 4ac \qquad r = \dfrac{-b \mp \sqrt{\Delta}}{2a}$

$$r = \frac{3 \mp \sqrt{21}}{2}$$

$$a_n = C_1\left(\frac{3+\sqrt{21}}{2}\right)^n + C_2\left(\frac{3-\sqrt{21}}{2}\right)^n$$

$a_0 = 7$

$a_1 = 7$ > We can find C_1, C_2 Thus,

:

:

$$\boxed{a_n = \frac{1}{6}(21+\sqrt{21})\left(\frac{1}{2}(3-\sqrt{21})\right)^n - (\sqrt{21}-21)\left(\frac{1}{2}(3+\sqrt{21})\right)^n}$$

4. $x_1+x_2+x_3+x_4=30$ $\quad x_1, x_2, x_3, x_4 \in \mathbb{Z}^+ \cup \{0\}$

$x_1 \geq 3,\ x_2 \geq 3$

S_1 = Assume $x_1 < 3$

$x=2,\quad x_2+x_3+x_4=28 \quad \binom{30}{28} \Rightarrow 870$

$x=1,\quad x_2+x_3+x_4=29 \quad \binom{31}{29} \Rightarrow 930$

$x=0,\quad x_2+x_3+x_4=30 \quad \binom{32}{30} \Rightarrow 992$

$+$ 2792

S_2 = Assume $x_2 < 3$

same as $S_1 \Rightarrow 2792$

S_3 = We should substract where $x_1<3$ & $x_2<3$

$x_1=2 \quad x_2=2,\quad x_3+x_4=26 \quad \binom{27}{26} \Rightarrow 27$

$x_1=2 \quad x_2=1,\quad x_3+x_4=27 \quad \binom{28}{27} \Rightarrow 28$

$x_1=2 \quad x_2=0,\quad x_3+x_4=28 \quad \binom{29}{28} \Rightarrow 29$

$x_1=1 \quad x_2=2,\quad x_3+x_4=27 \quad \binom{28}{27} \Rightarrow 28$

$x_1=1 \quad x_2=1,\quad x_3+x_4=28 \quad \binom{29}{28} \Rightarrow 29$

$x_1=1 \quad x_2=0,\quad x_3+x_4=29 \quad \binom{29}{28} \Rightarrow 28$

$x_1=0 \quad x_2=2,\quad x_3+x_4=28 \quad \binom{29}{28} \Rightarrow 29$

$x_1=0 \quad x_2=1,\quad x_3+x_4=29 \quad \binom{30}{29} \Rightarrow 30$

$x_1=0 \quad x_2=0,\quad x_3+x_4=30 \quad \binom{31}{30} \Rightarrow 31$

$\Rightarrow 259$

$$U = \binom{33}{30} \to U - (S_1 + S_2 - S_3)$$

$$= 32736 - (2792 \cdot 2 - 259)$$

$$= \boxed{27411}$$

5.

$A+B=6$

more zeroes than ones

= atleast 4 zero

= 4 zero + 5 zero + 6 zero

$$\frac{6!}{4!2!} + \frac{6!}{5!} + 1 = \boxed{22}$$

6.

positive and has disease $\frac{3}{100} \cdot \frac{3}{100} = \frac{9}{10^4}$

positive and not has disease $\frac{97}{100} \cdot \frac{93}{100} = \frac{9021}{10^4}$

sum $\frac{9030}{10^4}$ positive

$$\frac{\frac{3}{10^4}}{\frac{9030}{10^4}} = \boxed{\frac{3}{9030}}$$