1. How many ways are there to distribute 40 balls to 4 people Hasan, Mehmet, Ayla, and Büşra if Ayla and Büşra together get no more than 30 balls and Mehmet gets at least 9?

Let H, M, A, B be the number of balls distributed to Hasan, Mehmet, Ayla and Busra respectively such that

$H+M+A+B=40.$

We distribute 9 balls to Mehmet, then $40-9=31$ balls are distributed. Thus only, 31 balls are left.

We know that number of ways of distributing n among balls among r persons are ${}^{n+r-1}C_{r-1}$

Now, $H+M+A+B=40-9$

$\Rightarrow H+M+A+B=31 ;\ 0\le A+B\le 30,\ H\ge 0,\ M\ge 0$

Following cases are

① $H+M=1$, $A+B=30$ $\Rightarrow$ RNOW $=\binom{1+2-1}{2-1}\binom{30+2-1}{2-1}=2\times 31$
$n=1, r=2$; $r=2, n=30$

② $H+M=2$, $A+B=29$ $\Rightarrow$ RNOW $=\binom{2+2-1}{2-1}\binom{29+2-1}{2-1}=3\times 30$
$(r=2, n=2)$; $(r=2, n=29)$

③ $H+M=3$, $A+B=28$ $\Rightarrow$ RNOW $=\binom{3+2-1}{2-1}\binom{28+2-1}{2-1}=4\times 29.$
$(r=2, n=3)$; $(r=2, n=28)$

All Possible cases of distributing the remaining 31 balls
$(H+M, A+B) =$ (1,30), (2,29), (3,28), (4,27), (5,26), (6,25), (7,24), (8,23), (9,22), (10,21), (11,20), (12,19), (13,18), (14,17), (15,16), (16,15), (17,14), (18,13), (19,12), (20,11), (21,10), (22,9), (23,8), (24,7), (25,6), (26,5), (27,4), (28,3), (29,2), (30,1), (31,0)

From (1), (2) & (3), We observed that Required Number Of Ways (RNOW) of distributing 31 balls in above respective cases are
$2\times 31, 3\times 30, 4\times 29, 5\times 28, 6\times 27, \ldots, 30\times 3, 31\times 2, 32\times 1.$

Total number of ways

$= 2\times31 + 3\times30 + 4\times29 + \cdots + 30\times3 + 31\times2 + 32\times1$

$$= \sum_{r=1}^{31} \begin{pmatrix} r\text{'th term of sequence} \\ 2,3,4,\cdots,30,31,32 \end{pmatrix} \begin{pmatrix} r\text{'th term of sequence} \\ 31,30,29,\cdots,3,2,1 \end{pmatrix}$$

$$= \sum_{r=1}^{31} [2+(r-1)(1)]\,[31+(r-1)(-1)]$$

$$= \sum_{r=1}^{31} (r+1)(32-r)$$

$$= \sum(-r^2+31r+32)$$

$$= -\sum r^2 + 31\sum r + \sum 32$$

$\because$ total number of terms $= 31$

$\therefore$ put $r = 31$

$$= -\frac{r(r+1)(2r+1)}{6} + 31\frac{r(r+1)}{2} + 32r$$

$$= -\left(\frac{31\times\cancel{32}^{16}\times\cancel{63}^{21}}{\cancel{6}_3}\right) + 31\left(\frac{31\times\cancel{32}^{16}}{\cancel{2}}\right) + 32\times31$$

$$= 5952.$$

$\boxed{\text{Hence, the required number of ways to distribut 40 balls are 5952.}}$

Answer

2. Given a box containing five balls numbered 1, 2, 3, 4, 5. Let X be the bigger number when two balls are randomly drawn from the box. Determine $E(X)$.

Number of ways to draw 2 balls from the box containing 5 balls = ${}^5C_2$ = 5! / ((5-2)! 2!)

= 5! / (3! 2!)

= (5 * 4) / 2

= 10

When two balls are drawn, the bigger number on the balls can be any number between 2 and 5. The support of X is 2, 3, 4, 5.

The PMF of X is,

P(X = 2) = Number of ways to choose ball numbered 1 and ball numbered 2 / Total number of ways to draw 2 balls from the box containing 5 balls

= 1 / 10 = 0.1

P(X = 3) = Number of ways to choose ball numbered 2 and any ball numbered less than 3 / Total number of ways to draw 2 balls from the box containing 5 balls

= 2 / 10 = 0.2 (There are two numbers less than 3)

P(X = 4) = Number of ways to choose ball numbered 4 and any ball numbered less than 4 / Total number of ways to draw 2 balls from the box containing 5 balls

= 3 / 10 = 0.3 (There are three numbers less than 4)

P(X = 5) = Number of ways to choose ball numbered 5 and any ball numbered less than 5 / Total number of ways to draw 2 balls from the box containing 5 balls

= 4 / 10 = 0.4 (There are four numbers less than 5)

The PMF of X is,

X	P(X)
2	0.1
3	0.2
4	0.3
5	0.4

E(X) = Σ x * P(X = x)

= 2 * 0.1 + 3 * 0.2 + 4 * 0.3 + 5 * 0.4

= **4**

3. Use mathematical induction to prove that 43 divides $6^{n+1} + 7^{2n-1}$ for every positive integer n.

The objective is to use the mathematical induction to prove that 43 divides $6^{n+1}+7^{2n-1}$ for every positive integer n.

Comment

Step 2 of 3

Let $P(n)$ be the proposition that 43 divides $6^{n+1}+7^{2n-1}$ for every positive integer.

Basis step:

For $n=1$, the expression $6^{n+1}+7^{2n-1}$ becomes,

$6^2+7=43$.

So, it is clear that $P(1)$ is divisible by 43.

Therefore, it is true for $n=1$.

Step 3 of 3

Inductive step:

Let us assume that $P(k)$ is true for all positive integers $n=k$, that is, $6^{k+1}+7^{2k-1}$ is divisible by 43.

It is to be shown that $P(k+1)$ is true under this assumption, that is, $6^{k+2}+7^{2k+1}$ is divisible by 43.

As,

$$\begin{aligned}
6^{(k+1)+1}+7^{2(k+1)-1} &= 6^{k+2}+7^{2k+1} \\
&= 6\cdot 6^{k+1}+49\cdot 7^{2k-1} \\
&= 6\cdot 6^{k+1}+6\cdot 7^{2k-1}+43\cdot 7^{2k-1} \\
&= 6\left(6^{k+1}+7^{2k-1}\right)+43\cdot 7^{2k-1}
\end{aligned}$$

By the inductive hypothesis, the first part of the above equation is divisible by 43.

The second part is clearly divisible by 43. Hence it is also true for $n=k+1$.

Thus, it will true for all values of n. Therefore, the sum is divisible by 43.

Hence proved.

4. Use mathematical induction to prove $\sum_{i=1}^{n} i^2 . 2^i = n^2 . 2^{n+1} - n . 2^{n+2} + 3 . 2^{n+1} - 6$ for every positive integer n.

$$\sum_{i=1}^{n} i^2 \cdot 2^i = n^2 \cdot 2^{n+1} - n \cdot 2^{n+2} + 3 \cdot 2^{n+1} - 6 = F(n)$$

For $n=1$

$LHS = 1^2 \cdot 2^1 = 2$

$RHS = 1 \cdot 2^2 - 1 \cdot 2^3 + 3 \cdot 2^2 - 6$

$= 4 - 8 + 12 - 6 = 2$

$\therefore$ True for $n=1$

Let $F(n)$ be true for some natural number k

$$F(k) = 1^2 \cdot 2^1 + 2^2 \cdot 2^2 + 3^2 \cdot 2^3 + \dots + k^2 2^k = k^2 \cdot 2^{k+1} - k \cdot 2^{k+2} + 3 \cdot 2^{k+1} - 6 \quad \text{①}$$

Now,

$$F(k+1) = 1^2 \cdot 2^1 + 2^2 \cdot 2^2 + 3^2 \cdot 2^3 + \dots + k^2 2^k + (k+1)^2 \cdot 2^{k+1}$$

$$= k^2 \cdot 2^{k+1} - k \cdot 2^{k+2} + 3 \cdot 2^{k+1} - 6 + (k+1)^2 \cdot 2^{k+1} \quad (\text{from ①})$$

$$= k^2 \cdot 2^{k+1} - k \cdot 2^{k+2} + 3 \cdot 2^{k+1} - 6 + k^2 \cdot 2^{k+1} + 2k \cdot 2^{k+1} + 2^{k+1}$$

Rearranging the terms, we get

$$F(k+1) = 2^{k+1}(k^2 + 3 + k^2 + 2k + 1) + 2^{k+2}(-k) - 6$$

So $LHS = 2^{k+1}(2k^2 + 2k + 4) + 2^{k+2}(-k) - 6$

$= 2^{k+2}(k^2 + k + 2) - k\, 2^{k+2} - 6$

$= (k^2 + 2)\, 2^{k+2} - 6$

$RHS = (k+1)^2 \cdot 2^{k+2} - (k+1) \cdot 2^{k+3} + 3 \cdot 2^{k+2} - 6$

$= 2^{k+2}(k^2 + 2k + 1 - 2k - 2 + 3) - 6$

$= 2^{k+2}(k^2 + 2) - 6$

$\therefore$ LHS = RHS, $F(k+1)$ is valid

So, the above equation is valid for all positive integer

5. Suppose that we roll a fair die until a 6 comes up.

a) What is the probability that we roll the die n times?

b) What is the expected number of times we roll the die?

(a)

If X is the number of times we roll the dice, then X has a geometric distribution with,

$$p = \frac{1}{6}$$

Therefore, the probability, that we roll the die n times is,

$$p(X = n) = (1 - p)^{n-1} p$$

$$p(X = n) = \left(\frac{5}{6}\right)^{n-1} \left(\frac{1}{6}\right)$$

$$\boxed{p(X = n) = \frac{5^{n-1}}{6^n}}$$

(b)

From the definition of the expectations we know that,

$$E(X) = \frac{1}{p}$$

$$\boxed{E(X) = 6}$$

6. Let R be the relation defined on $A = Z \times Z$ in the following way :

$$((x_1, y_1), (x_2, y_2)) \in R \Leftrightarrow x_1 . y_2 = x_2 . y_1$$

Determine whether the relation R is an equivalence relation on A or not.

$A = Z \times Z$ by $\{(x_1, y_1)(x_2, y_2)\} \in R \Leftrightarrow x_1 y_2 = x_2 y_1$

Reflexive :-

For any ordered pair (a, b) then $ab = ba$ and $(a, b) R (a, b)$. It is reflexive.

Symmetry :-

Let Given (x_1, y_1) and (x_2, y_2) with $(x_1, y_1) \sim (x_2, y_2)$ then an sd

$x_1 y_2 = x_2 y_1$

$x_2 y_1 = x_1 y_2$

which shows that $(x_2, y_2) \sim (x_1, y_1)$. It is symmetric

Transitive :-

Given (x_1, y_1) (x_2, y_2) and (x_3, y_3) with $(x_1, y_1) \sim (x_2, y_2)$ and $(x_2, y_2) \sim (x_3, y_3)$ then the equations, $x_1 y_2 = x_2 y_1$ and $x_2 y_3 = x_3 y_2$.

Multiply first equation by y_3 and second equation by y_1 we get

$x_1 y_2 y_3 = x_2 y_1 y_3 = x_3 y_1 y_2$

Since $y_2 \neq 0$, cancelling gives

$x_1 y_3 = x_3 y_1$ gives

$(x_1, y_1) \sim (x_3, y_3)$

$\therefore$ It is an equivalence Relation on A.

7. Let A be a set and let R and S be symmetric relations defined on A. Determine whether $R \circ S$ (the composition of R and S) is symmetric or not.

Let $x, y, z \in A$

If $R \circ S = S \circ R$ and $(x, z) \in S \circ R$

then $\exists\ y : (x, y) \in R$ and $(y, z) \in S$ but

then $(z, y) \in S$

and $(y, x) \in R$ by symmetry

so $(z, x) \in R \circ S = S \circ R$

Thus $S \circ R$ is symmetric.

On the other hand, suppose $S \circ R$ is symmetric.

$(z, x) \in R \circ S \Leftrightarrow (x, z) \in S \circ R$

$\Leftrightarrow (z, x) \in S \circ R$

Thus $S \circ R = R \circ S$.

8. What is the maximum possible number of vertices for a connected undirected graph with 19 edges such that each vertex has degree at least 4 ? Draw a graph to demonstrate one possible case.

A) As per hand shaking lemma,

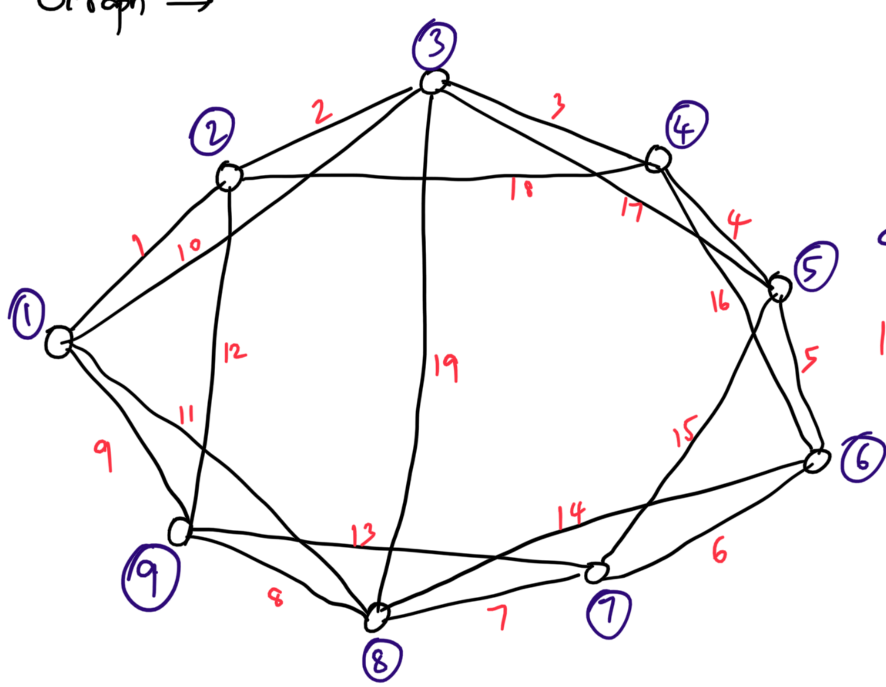
$$\sum_{u \in V} \text{degree}(v) = 2|E|$$

Given, degree ≥ 4 , $|E| = 19$

$$\therefore \sum_{u \in V} 4 \leq 2 \times 19 , \quad 4 \times n \leq 38$$

$$n \leq 9.5 \qquad \therefore \boxed{n_{max} = 9}$$

Graph →



9 → Vertices

19 → Edges

$\therefore$ {Maximum possible number of vertices = 9}

9. Consider the poset $(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\},\subseteq)$.

a) Find the maximal elements of the poset.
b) Find the minimal elements of the poset.
c) Find the all upper bounds of $\{\{2\},\{4\}\}$.
d) Find the all lower bounds of $\{\{1,3,4\},\{2,3,4\}\}$.

consider the poset

$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\}$
$\{1,3,4\},\{2,3,4\},\subseteq\}$

(a.) Maximal elements of poset are $\{1,3,4\}$ and $\{2,3,4\}$

(b.) Minimal element of poset are $\{1\},\{4\},\{2\}$

(c.) all upper bounds are $\{\{2,4\},\{2,3,4\}\}$

(d.) all lower bound are $\{\{3,4\},\{1\},\{4\}\}$

10. Let $n \in \mathbb{Z}^+$ and $n \leq 500$. How many such n are there which are not divisible by 3, 5, or 8?

let events

A= divisible by 3

B= divisible by 5

C= divisible by 8

now

n(A)= 166 (divide 500 by 3)

n(B)=100

n(C)=62

now

n(A∩B)=33 (divide 500 by LCM)

3	3	5
5	1	5
	1	1

LCM of $3, 5 = 3 \times 5 = 15$

n(A∩C)=20

2	3	8
2	3	4
2	3	2
3	3	1
	1	1

LCM of $3, 8 = 2 \times 2 \times 2 \times 3 = 24$

n(B∩C)=12

2	5	8
2	5	4
2	5	2
5	5	1
	1	1

LCM of $5, 8 = 2 \times 2 \times 2 \times 5 = 40$

n(A∩B∩C)=4

2	3	5	8
2	3	5	4
2	3	5	2
3	3	5	1
5	1	5	1
	1	1	1

LCM of $3, 5, 8 = 2 \times 2 \times 2 \times 3 \times 5 = 120$

then number which are divisible by 3,5 or 8 is given as

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$n(A \cup B \cup C) = 166 + 100 + 62 - 33 - 20 - 12 + 4$$

$$n(A \cup B \cup C) = 267$$

then n which are not divisible by 3,5 or 8 is given as

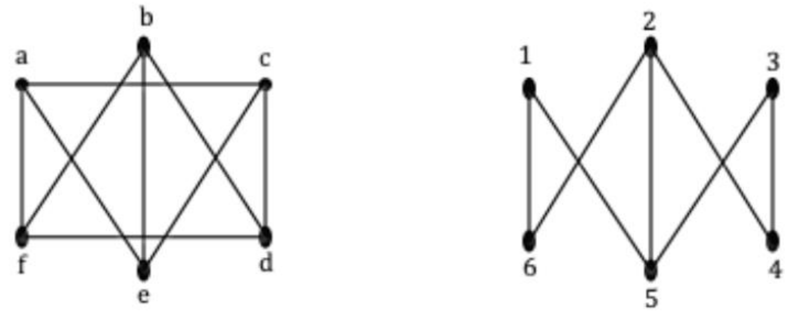
=total numbers - numbers which are divisible by 3,5 or 8

=500-267

=233

hence , 233 number of not divisible by 3,5 or 8.

11.



Determine whether two given graphs are isomorphic or not.

To satisfy the isomorphism of any 2 graphs, those both should have same number of vertices, edges, and same degree of sequence.

According to given problem given both graphs are having same vertices, degree of sequence. But these are having different edges. So these 2 graphs are not isomorphic.

12. Let S be a subset of Z^+ and $|S| \geq 3$. Show that there exist distinct $x, y \in S$ such that $x + y$ is even.

Sol.

Let S be a subset of $Z^+ = \{1, 2, 3, ---\}$

and $|S| \geq 3 \Rightarrow \exists$ atleast three elements $a, b, c \in Z^+$

Case I

If all elements of S are even

So, $\exists$ $x, y \in S$ so that $x+y$ is even

Case II

If ~~[illegible]~~ only one element is even

then it has two odd (atleast) say a, b

$\Rightarrow$ $a+b$ is even

Case III

If S has no even number

then for $a, b \in S$, $a+b$ is even

Thus in each case $\exists$ $x, y \in S$ so that $x+y$ is even.

hence $\exists$ $x, y \in S$ such that $x+y$ is even

13. Let A be a nonempty set and B be a fixed subset of A. Define a relation R on $P(A)$ such that for any $X, Y \in P(A)$, $(X, Y) \in R$ if $B \cap X = B \cap Y$. Show that R is an equivalence relation.

5:32

$A \neq \emptyset$

$B \subset A$

For $x, y \in P(A)$, $(x, y) \in R$ if $B \cap x = B \cap y$

- R is reflexive

  because $B \cap x = B \cap x$.

  Thus $(x, x) \in R \ \forall x \in P(A)$

- R is symmetric

  If $(x, y) \in R$ then $B \cap x = B \cap y$

  $\Rightarrow B \cap y = B \cap x$

  $\Rightarrow (y, x) \in R$

- R is transitive

  If $(x, y), (y, z) \in R$ then

  $B \cap x = B \cap y$ and $B \cap y = B \cap z$

  $\Rightarrow B \cap x = B \cap z$

  $\Rightarrow (x, z) \in R$

$\therefore$ R is equivalence

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14. Find the number of permutations of the letters a b c d e … x y z (26 letters) in which none of the patterns ‘spin’ or ‘net’ occurs.

Total number of permutations:
= 26!

Total permutations with spin:
= 23! given that we consider spin as one group.

Total permutations with game:
= 23!

Total permutations with same:
= 23!

Total permutations with end:
= 24!

Total permutations with spin and game:
= (26 - 8 + 2) !
= 20!

Total permutations with spin and same: 0 since we only have 1 s.

Total permutations with spin and end is 0 since we cannot have both i and e behind n.

Total permutations with game and same is 0 since we cannot have both g and s before a.

Total permutations with game and end:
= (26 - 7 + 2)!
= 21!

Total permutations with same and end:
= 21!

Total permutations with spin, game and same: 0
Total permutations with spin, game and end: 0
Total permutations with spin, same and end: 0
Total permutations with game, same and end: 0

Therefore, total required permutations here is computed as:
= 26! - (23! - 20!) - (23! - 20! - 21!) - (23! - 21!) - (24! - 21! - 21!) - 20! - 21! - 21!

15. Let R be the relation defined on Z in the following way :

$$(x, y) \in R \Leftrightarrow x - y \text{ is a multiple of } 3$$

Determine which properties (reflexive, symmetric, antisymmetric, transitive) the relation satisfies. Justify your answer.

Given

$(x,y) \in R \Leftrightarrow (x-y)$ is a multiple of 3 $\forall\, x, y \in \mathbb{Z}$

R is reflexive

let $x \in \mathbb{Z}$

Now $x - x = 0 = 3 \cdot 0$

So $(x-x)$ is a multiple of 3.

So $(x, x) \in R$

So R is reflexive

R is Symmetric

let $x, y \in \mathbb{Z}$

and $(x,y) \in R$ which implies $(x-y) = 3K$; $K \in \mathbb{Z}$

$\Rightarrow -(y-x) = 3K$

or, $(y-x) = -3K$

or, $(y-x) = 3 \cdot (-K)$; $-K \in \mathbb{Z}$

So $(y-x)$ is also multiple of 3.

So $(y,x) \in R$

So R is Symmetric

R is Transitive

let $x, y, z \in \mathbb{Z}$

and $(x,y), (y,z) \in R$ which implies

$(x-y) = 3k_1$ and $y - z = 3k_2$ $\forall\, k_1, k_2 \in \mathbb{Z}$

Now
$$\begin{aligned} x - z &= x - y + y - z \\ &= (x-y) + (y-z) \\ &= 3k_1 + 3k_2 \\ &= 3(k_1 + k_2) \quad \forall\, k_1 + k_2 \in \mathbb{Z} \end{aligned}$$

So $(x-z)$ is also multiple of 3

So $(x,z) \in R$. Hence R is transitive.

Antisymmetricity

let us consider $3, 9 \in \mathbb{Z}$

Now $(3,9) \in R$ because $3-9 = -6$

$= 3 \cdot (-2)$; $-2 \in \mathbb{Z}$

$\therefore$ $(3-9)$ is multiple of 3.

and $(9,3) \in R$ because $9-3 = 6$

$= 3 \cdot 2$; $2 \in \mathbb{Z}$

$\therefore$ $(9-3)$ is also multiple of 3.

But $9 \neq 3$.

So R is not antisymmetric.

16. Consider an ordinary deck of 52 playing cards such that there are 4 suits: diamond, heart, spade, and club, and there are 13 kinds for each suit: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. The cards are to be drawn successively at random and without replacement. What is the probability that the second diamond (not second of diamonds) appears on the sixth draw?

Ans :-

Total cards = 52
Number of Suits = 4
Cards in each suit = 13
Number of cards of Diamond Suit = 13

Six cards are drawn successively at random and without replacement from the deck of 52 playing cards.

we have to find the probability that the second diamond appears on the sixth draw.

Let A be the event of drawing exactly one (1) diamond in the first 5 cards drawn, and let B be the event of ~~se~~ drawing a diamond on the sixth draw.

the probability that we wish to draw is $P(A \cap B)$. Out of $^{52}C_5$ total number of ways of choosing 5 cards out of 52 cards, $^{13}C_1 \times {^{39}C_4}$ of them consist of ~~two spade~~ 1 diamond

So,

$$P(A) = \frac{^{13}C_1 \times {^{39}C_4}}{^{52}C_5} = 0.4114$$

and $P(B/A) = \frac{12}{47}$ (Note: $P(B/A)$ is the probability of event B after event A has occurred i.e. after event A has occurred only 12 diamonds remain out of 47 cards)

So,

$$P(A \cap B) = P(A) \times P(B/A) = 0.4114 \times \frac{12}{47} = 0.1050$$

therefore, the probability of getting second diamond on 6th draw is 0.1050.

17. Suppose that a Bayesian spam filter is trained on a set of 10,000 spam messages and 5000 messages that are not spam. The word “enhancement” appears in 1500 spam messages and 20 messages that are not spam, while the word “herbal” appears in 800 spam messages and 200 messages that are not spam. Estimate the probability that a received message containing both the words “enhancement” and “herbal” is spam.Will the message be rejected as spam if the threshold for rejecting spam is 0.9?

Step 1 of 3

Consider there are 10000 spam messages and 5000 messages that are not spam.

The word “enhancement” appears in 1500 spam messages and 20 messages that are not spam.

The word “herbal” appears in 800 spam messages and 200 messages that are not spam.

Use Bayes’ Theorem to find the probability.

Suppose S be the sample space.

There are two events E and F and the probabilities of them are non-zero.

That is $P(E) \neq 0$ and $P(F) \neq 0$

Then, the conditional probability;

$$P(F \mid E) = \frac{P(E \mid F)P(F)}{P(E \mid F)P(F) + P(E \mid \overline{F})P(\overline{F})}.$$

Step 2 of 3

Use the counts that the word “enhancement” appears in spam messages and messages that are not spam to find the probabilities.

Here the word “enhancement” occurs in 1500 of 10000 messages known to be spam and 20 of 5000 messages know not to be spam.

Here the word “herbal” occurs in 800 of 10000 messages known to be spam and 200 of 5000 messages know not to be spam.

$$p(\text{enhancement}) = \frac{1500}{10000} = 0.15$$

And $q(\text{enhancement}) = \frac{20}{5000} = 0.004$

$$p(\text{herbal}) = \frac{800}{10000} = 0.08$$

$$q(\text{herbal}) = \frac{200}{5000} = 0.04$$

Assume that it is equally likely that an incoming message is or is not spam.

Step **3** of 3

Estimate the probability that the message is spam by;

$$r(\text{enhancement, herbal}) = \frac{p(\text{enhancement})\,p(\text{herbal})}{p(\text{enhancement})\,p(\text{herbal}) + q(\text{enhancement})\,q(\text{herbal})}$$

$$= \frac{(0.15)(0.08)}{(0.15)(0.08)+(0.004)(0.04)}$$

$$= \frac{0.012}{0.012+0.00016}$$

$$= \frac{0.012}{0.01216}$$

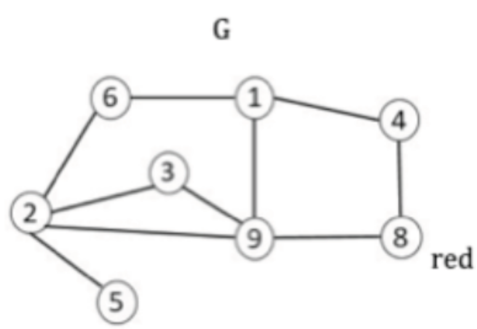
$$\approx 0.9868$$

Because $r(\text{enhancement, herbal})$ is greater than the threshold 0.9.

$\boxed{\text{Yes}}$, such messages will be rejected by the filter.

Comment

18.



a) Give the adjacency matrix of the graph G

b) Does the graph have an Euler path? If so, give such a path. If not, determine the minimum number edges that must be deleted to form a graph which has an Euler path.

c) What is the Chromatic number $\chi(G)$ of G? To get full credit, you need to write the color you assign to each node (as shown in the graph for the node 8).

d) What is $\kappa(G)$, the minimum number of vertices in a vertex cut of G? To get full credit, you need to write the corresponding vertex cut set.

e) What is $\lambda(G)$, the minimum number of edges in an edge cut of G? To get full credit, you need to write the corresponding edge cut set.

1. Adjacency matrix for the given graph is as below. Numbers in green are vertices. As from the image it seems 7 is repeated thrice, so it is represented as 7.1,7.2,7.3. 7.1 is connected to both 6 and 3. 7.2 is the one connected to 1. 7.3 is connected to 7.1 and 7.2 and is diagonally present to 1.

	1	3	5	6	7.1	7.2	7.3	9
1	0	0	0	0	1	1	0	1
3	0	0	0	1	1	0	0	0
5	0	0	0	1	0	0	0	0
6	0	1	1	0	1	0	0	1
7.1	1	1	0	1	0	0	1	0
7.2	1	0	0	0	0	0	1	0
7.3	0	0	0	0	1	1	0	0
9	1	0	0	1	0	0	0	0

2. Euler Path - A Path that travels using evry edge of a graph eaxtly one time.

The Euler path is 5-6-7.1-3-6-9-1-7.2-7.3-7.1-1

3.Chromatic number is 3

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5,9,3,7.3	red
6,1	blue
7.1,7.2	green

4. the vertex cut - Removing minimum number of vertices to make the graph diconnected.

K(G)=1	Removing vertex 6 will make the given graph disconnected.

5. the edge cut - Removing minimum number of edges to make the graph diconnected.

lambda(G)=1, Removing the edge between the verices 5 and 6 will make the graph diconnected.