N 7

| | | | | | |
|---|---|---|---|---|---|
| Число смертей | 0 | 1 | 2 | 3 | 4 |
| Частота | 109 | 65 | 22 | 3 | 1 |

$$\lambda = \frac{0+65+44+9+4}{200} = 0{,}61$$

Для Пуассона: $P(X=i) = \dfrac{e^{-\lambda}\lambda^i}{i!}$

$i=0$: $P(0) = \dfrac{e^{-0{,}61}\cdot 0{,}61^0}{1} \simeq 0{,}543$

$E(0) = 200 \times 0{,}543 \simeq 108{,}6$

$i=1$: $P(1) = e^{-0{,}61}\cdot 0{,}61 \simeq 0{,}543\cdot 0{,}61 \simeq 0{,}331$

$E(1) = 200\cdot 0{,}331 \simeq 66{,}2$

$i=2$: $P(2) = \dfrac{e^{-0{,}61}\cdot 0{,}61^2}{2} \simeq 0{,}101$

$E(2) = 200\cdot 0{,}101 \simeq 20{,}2$

$i=3$: $P(3) = \dfrac{e^{-0{,}61}\cdot 0{,}61^3}{6} \simeq 0{,}0205$

$E(2) = 200\cdot 0{,}0205 \simeq 4{,}1$

$i=4+$: $P(4+) = 1 - P(0) - P(1) - P(2) - P(3) \simeq 0{,}0045$

$E(4+) = 200\cdot 0{,}0045 \simeq 0{,}9$

| $i$ | $\vartheta$ | $E_i$ |
|---|---|---|
| 0 | 109 | 108,6 |
| 1 | 65 | 66,2 |
| 2 | 22 | 20,2 |
| 3+ | 4 | 5,0 |

$$\tilde{\Delta} = \sum_{i=0}^{\text{"}3+\text{"}} \frac{(\vartheta_i - E_i)^2}{E_i} = \frac{(109-108{,}6)^2}{108{,}6} + \frac{(65-66{,}2)^2}{66{,}2} + \frac{(22-20{,}2)^2}{20{,}2} + \frac{(4-5)^2}{5} \simeq 0{,}3837$$

$$\text{p-value} = \int_{0{,}3837}^{\infty} p_{\chi^2(2)}(x)\,dx \simeq 0{,}825 > \alpha = 0{,}05 \implies ✓$$

N 8.

| | Малые | Точные | Большие |
|---|---|---|---|
| 1 парт | 25 | 50 | 25 |
| 2 парт | 52 | 41 | 7 |

$$P(B_i) = \frac{1}{200}\sum_{j=1}^{3} n_{ij} \quad \to \quad q_1 = q_2 = \frac{1}{2}$$

$$P(A_j) = \frac{1}{200}\sum_{i=1}^{2} n_{ij} \quad \to \quad p_1 = \frac{77}{200};\; p_2 = \frac{91}{200};\; p_3 = \frac{32}{200}$$

$$\tilde{\Delta} = \sum_{i,j} \frac{(n_{ij} - 200 p_j q_i)^2}{200 p_j q_i}$$

$$\tilde{\Delta} = \frac{\left(25 - \frac{77}{2}\right)^2}{\frac{77}{2}} + \frac{\left(50 - \frac{91}{2}\right)^2}{\frac{91}{2}} + \frac{\left(25 - \frac{32}{2}\right)^2}{\frac{32}{2}} + \frac{\left(52 - \frac{77}{2}\right)^2}{\frac{77}{2}} +$$

$$+ \frac{\left(41 - \frac{91}{2}\right)^2}{\frac{91}{2}} + \frac{\left(7 - \frac{32}{2}\right)^2}{\frac{32}{2}} \simeq 20,479$$

$$! \quad \Delta \sim \chi^2((3-1)(2-1)) = \chi^2(2)$$

$$\text{p-value} = \int_{20,479}^{\infty} p_{\chi^2(2)}(x) \simeq 0 < \alpha = 0,05 \quad \Longrightarrow$$

гипотеза неверна

$$\{\ \text{p-value} \simeq \int_{20,479}^{\infty} \frac{1}{2} e^{-\frac{x}{2}} dx = e^{-\frac{20,479}{2}} \simeq 3,71 \cdot 10^{-5}\ \}$$

№ 9

| | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 поток | 33 | 43 | 80 | 144 |
| 2 поток | 39 | 35 | 72 | 154 |

$n = 300.$

$$p_1 = \frac{72}{600} \qquad p_2 = \frac{78}{600} \qquad p_3 = \frac{152}{600} \qquad p_4 = \frac{298}{600}$$

$$\tilde{\Delta}_1 = \frac{(33-72)^2}{\frac{72}{2}} + \frac{(43-78)^2}{\frac{78}{2}} + \frac{(80-152)^2}{\frac{152}{2}} + \frac{(80-144)^2}{\frac{144}{2}} = 1,04$$

$$\tilde{\Delta}_2 = \ . \ . \ . \ = 1,04$$

$$\tilde{\Delta} = \tilde{\Delta}_1 + \tilde{\Delta}_2 = 2,08$$

$$\Delta \sim \chi^2((4-1)(2-1)) = \chi^2(3)$$

$$p\text{-value} = \int_{2,08}^{\infty} p_{\chi^2(3)}(x)\,dx = 0,556 > \alpha = 0,05$$

✓

№ 10.

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| Цит | 5 | 8 | 6 | 12 | 14 | 18 | 11 | 6 | 13 | 7 |

a) $\chi^2$ и Колмогорова

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p: | $\frac{1}{9}$ | $\frac{1}{9}$ | . . . . . . | | | | . . . | | $\frac{1}{9}$ | $\frac{1}{9}$ |
| $\nu$ | 0,05 | 0,08 | 0,06 | 0,12 | 0,14 | 0,18 | 0,11 | 0,06 | 0,13 | 0,07 |

$$\tilde{\Delta} = \sum_{i=1}^{10} \frac{100}{\frac{1}{9}} \left(\nu_i - \frac{1}{9}\right)^2 = 900\left(\frac{5}{100} - \frac{1}{9}\right)^2 + \ldots + 900\left(\frac{7}{100} - \frac{1}{9}\right)^2 = 15{,}871$$

$$\Delta \rightsquigarrow \chi^2(9)$$

$$\text{p-value} = \int_{15{,}871}^{\infty} p_{\chi^2(9)}(x)\,dx = 0{,}07 > \alpha = 0{,}05$$

$$\tilde{\Delta} = \max_i \left(\max\left(|F(x_i) - \tilde{F}(x_i - 0)|, |F(x_i) - \tilde{F}(x_i + 0)|\right)\sqrt{n}\right.$$

$$\Delta \rightsquigarrow K(x) = 1 + 2\sum_{k=1}^{\infty} (-1)^k e^{-2k^2x^2}$$

$$\Rightarrow \tilde{\Delta} = 1{,}42 \longrightarrow \text{p-value} = \int_{1{,}42}^{\infty} K(x)\,dx = 0{,}033 < \alpha = 0{,}05$$

По критерию Колмагорова гипотеза неверна

ОМПГ

b) $$p_{N(a;\sigma^2)}(x) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(a-x)^2}{\sigma^2}\right)$$

$$L(a;\sigma^2) = \prod_{i=1}^{n} P(x_i; (a;\sigma^2))$$

$$P(x_i; (a;\sigma^2)) = \int_a^b \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\frac{(a-x_i)^2}{\sigma^2}\right)dx \qquad x_i \in [a;b]$$

$$\text{p-value} = P(\Delta > 9{,}802 \mid H_0) = 0{,}2 > \alpha = 0{,}05$$

т.к. Max. ф-ии правдоподобия $L$ $(a;\sigma) = (5{,}29; 2{,}68)$

Вер-сти событий $\{0{,}055; 0{,}055; 0{,}087; 0{,}119; 0{,}142; 0{,}148; 0{,}134; 0{,}106; 0{,}073; 0{,}083\}$.

$$\text{p-value} = P(\Delta > 9{,}802 \mid H_0) = 0{,}2 > \alpha = 0{,}05$$

ОММ

$$\alpha_k = M[x^k] = \int_{-\infty}^{+\infty} x^k p(x;\vec{\theta})\,dx$$

$$\tilde{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} x_i^k$$

$$\{\tilde{\alpha}_k = \alpha_k(\hat{\theta})\} \longrightarrow \vec{\theta} \qquad k = \dim(\vec{\theta})$$

$$\alpha_1 = M_x = \int_{-\infty}^{+\infty} x\, P_{N(a;\sigma^2)}(x)\,dx = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{+\infty} x\, e^{-\frac{(x-a)^2}{2\sigma^2}}dx =$$

$$= \int_{-\infty}^{+\infty} \frac{\sqrt{2\sigma^2}\,t + a}{\sqrt{\pi}} e^{-t^2}dt = \frac{a}{\sqrt{\pi}}\int_{-\infty}^{+\infty} e^{-t^2}dt + \frac{\sqrt{2\sigma^2}}{\sqrt{\pi}}\int_{-\infty}^{+\infty} t e^{-t^2}dt = a$$

$$\overline{x} = \tilde{\alpha}_1 \to a = \overline{x}$$

$$\alpha_2 = Mx^2 = \int_{-\infty}^{+\infty} x^2 P_{N(a;\sigma^2)}(x)\,dx = \sigma^2 + a^2 = \overline{x^2} \implies \sigma = \sqrt{\overline{x^2} - \overline{x}^2}$$

! $\qquad \xi \sim N(a;\sigma^2)$

ОМПР (кр. Колмогорова)

p-value $= P(\Delta > 0{,}258 \mid H_0) = 0{,}175$

ОММ (Кр. Колмогорова)

p-value $= P(\Delta > 0{,}258 \mid H_0) = 0{,}109$

Оба критерия не опровергают гипотезу