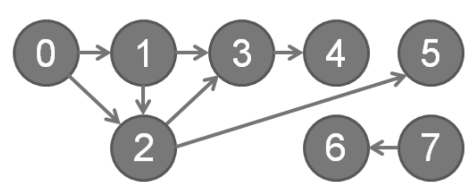
// Breadth First Search
// Complexity: O(|V|+|E|) : for Adjacency Matrix

```
/* Some Important Implements                          up-left, up, u-right, left, right, down-left, down, down-right
/ int path[8][8]  (adjacency matrix path),            int  r[] = {-1, -1, -1, 0, 0, 1, 1, 1}, c[] = {-1, 0, 1, -1, 1, -1, 0, 1}
bool visited[8][8]  (adjacency matrix)
char direction[5] = "NESW";
*/
// To find the second shortest path the edges and nodes which builds the shortest path(s) needs to be cut off
vector<int>parent, G[MAX];                      // This comments are for adjacency matrix
void printPath(int u, int source_node) {        // void pathPrinter(int start_x, int start_y, int end_x, int end_y)
    if(u == source_node) {                      // if(end_y == start_y && end_x == start_x) {
        printf("%d", u);                        // printf("%d", path[end_x][end_y]);
        return;                                 // return;
    }
    printPath(parent[u], source_node);          // if(path[end_x][end_y] == 0)  pathPrinter(start_x, start_y, end_x, end_y+1);
    printf(" %d", u);                           // do this for path_[end_x][end_y] [0, 1, 2, 3] : [up, right, down, left]
}                                               // printf("%c ", direction[path[end_x][end_y]])

int BFS(int source_node, int finish_node, int vertices) {
    vector<int>dist(vertices+5, INF);           // Contains the distance from source to end point
    queue<int>Q;                                // Make pair<int, int> if adjacency matrix is used
    Q.push(source_node);
    parent.resize(vertices+5, -1);              // For path printing

    while(!Q.empty()) {                         // This comments are for adjacency matrix
        int u = Q.front();                      //int _x = Q.front().first,  _y = Q.front().second;
        Q.pop();

        if(u == finish_node)                    // Remove this line if shortest path to all nodes are needed
            return dist[u];                     // if(_x == end_x && _y == end_y)  return;

        for(int i = 0; i < G[u].size(); i++) {  // for(int i = 0; i < total_points; i++) {
            int v = G[u][i];                    // int x = _x + r[i],  y = _y + c[i];
            if(dist[v] == INF) {                // if(x >= 0 && y >= 0 && !visited[x][y]) {
                dist[v] = dist[u] + 1;          // visited[x][y] = 1;
                parent[v] = u;                  // path[x][y] = 1;    --------  [0, 1, 2, 3] : [up, right, down, left]
                Q.push(v);                      // Q.push(make_pair(x, y));
            }
        }
    }
    return -1;    //if not found, return -1
}
```



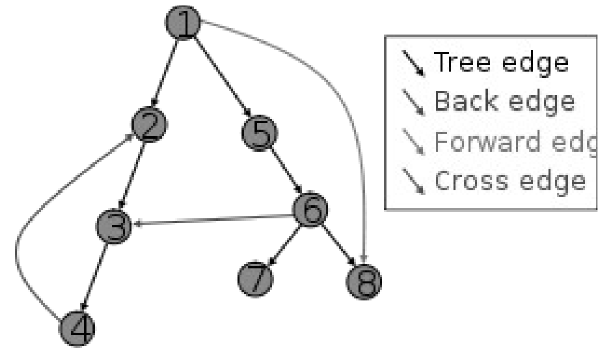
// Topological Sort
// DFS method
// Topological Sorting for a graph is not possible if the graph is not a DAG

```
stack<int>topsort;                              // In this DFS method output is
bool visited[1000];                             // 7 6 0 1 2 5 3 4  (remember that there can be >= 1 valid toposort)
                                                // Khans Algorithm output: 0 7 1 2 6 3 5 4  [In-degree wise sorting]
```

```
void dfs2(int u) {
        visited[u] = 1;                          // Mark the starting node as visited
        for(size_t i = 0; i < G[u].size(); i++) { // For all nodes connected with u (v)
                if(visited[G[u][i]] == 0)        // if not visited v
                        dfs2(G[u][i]);           // dfs on v
        }                                        // Note: paths are saved backwards
        topsort.push(u);                         // push in stack
}

main() { …...
        for(int i = 1; i <= E; i++)
                if(visited[i] == 0)
                        dfs2(i);
        while(!topsort.empty()) {                                          // Top-sort printing
                if(topsort.size() == 1)  printf("%d\n", topsort.top());    // If this is the last topological point
                else printf("%d ", topsort.top());                         // If this is not the last topological point
                topsort.pop();
        }……
}
```

// Topological Sort
//Khans Algorithm [Sorts in-degree wise]
// Complexity : O(V+E)

```
void khansTopsort() {
   int indegree[110];                        // This algorithm uses in-degree for every node
   memset(indegree, 0, sizeof(indegree));
   for(int i = 0; i < Edges; i++) {          // Calculating in-degree for every node
      for(int j = 0; j < G[u].size(); j++)
         v = G[u][j], indegree[v]++;
   }

   priority_queue<int, vector<int>, greater<int> >pq;        //Normally queue can be used
   for(int i = Edges-1; i >= 0; i--) {
      if(indegree[i] == 0)
         pq.push(node);                                      // All zero in-degree nodes are inserted in a queue
   }

   //int cnt = 0;          // To detect if the graph is Acyclic, check below
   vector<int>ans;      // This contains the topological answer
   while(!pq.empty()) {
      int u = pq.top();
      pq.pop();
      ans.push_back(u);

      for(int i = 0; i < G[u].size(); i++)
         if(--indegree[G[u][i]] == 0)
            pq.push(G[u][i]);
      //cnt++;
   }
```




```
//if(cnt != to_int.size())      //it has no topological order (Acyclic)
    for(int i = 0; i < ans.size(); i++)
            if(i == 0)  printf("%d", ans[i]);
            else printf(" %d", ans[i]);
}
```

//Dijkstra Algorithm (Greedy)
// Complexity : O(E logV)

```
vector<int>dist, G[MAX], W[MAX];
void printPath(int u) {
    if (u == s) {                          // Extract information from 'vi p'
        printf("%d", s);                   // Base case, at the source s
        return;
    }
    printPath(p[u]);                       // Recursive: to make the output format: s -> ... -> t
    printf(" %d", u);
}

void dikjstra(int source, int destination, int nodes) {
    dist.resize(nodes+1, INF);                        // dist[v] contains the distance from u to v
    dist[source] = 0;
    priority_queue<pair<int, int> > pq;               // pq is sorted in ascending(low→hi) order according to weight and edge
    pq.push({0, -source});

    while(!pq.empty()) {
        int u = -pq.top().second;
        int wu = -pq.top().first;
        pq.pop();

        if(u == destination) return;                  // If we only need distance of destination, then we may return here
        if(wu > dist[u]) continue;                    // Skipping the longer edges, if we have found shorter edge earlier

        for(int i = 0; i < G[u].size(); i++) {
            int v = G[u][i];
            int wv = W[u][i];
            // Path relaxation
            if(wu + wv < dist[v])  {
                dist[v] = wu + wv;
                pq.push({-dist[v], -v});
}}}}
```

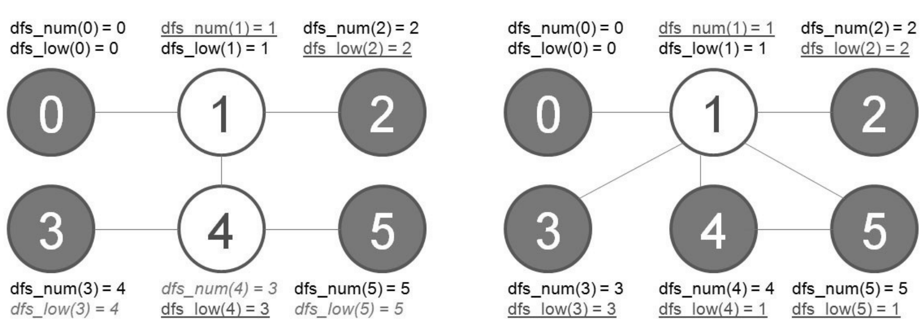


Fig: Finding Articulation Point

// Articulation Point
// Complexity O(V+E)
// Tarjan, DFS

```
vector<int>G[101];
int dfs_num[101], dfs_low[101], parent[101], isAtriculationPoint[101], dfsCounter, rootChildren, dfsRoot;
```

```
// dfs_num[] : how many times dfs found this node;  dfs_low[] : dfs_low = the shortest counter on which the node was found
void articulationPoint(int u) {
    dfs_low[u] = dfs_num[u] = ++dfsCounter;
    for(int i = 0; i < G[u].size(); i++) {
        int v = G[u][i];
        if(dfs_num[v] == 0) {
            parent[v] = u;
            if(u == dfsRoot)                // Special case for root node
                rootChildren++;             // If root node has child, increment counter
            articulationPoint(v);

          //1 : if dfs_num[u] == dfs_low[v], then it is a back edge
          //2 : if dfs_num[u] < dfs_low[v], then u is ancestor of v and there is no back edge
          //so, if u is not root node, then we can chose u for Articulation Point

            if(dfs_num[u] <= dfs_low[v] && u != dfsRoot)    //Avoiding root node
                isArticulationPoint[u]++;

          //if there is any child node of u that is a back edge of a previous node
          //then the value of dfs_low[v] might be less than the present dfs_low[u]
          //we try to save the lowest value possible

            dfs_low[u] = min(dfs_low[v], dfs_low[u]);
        }
      //As nodes are bi-directional, avoiding direct child node
      //if it is not direct child node, and visited, then there is a back edge
      //so we try to decrease the value of dfs_low[u] with the dfs_num[v]
      //the dfs_num[v] is less than dfs_num[u] (as it it a back edge)

        else if(parent[u] != v)     dfs_low[u] = min(dfs_low[u], dfs_num[v]);
    }
}

int main() { …...
    dfsCounter = 0;
    memset(dfs_num, 0, sizeof(dfs_num));
    isArticulationPoint.reset();
    for(int i = 1; i <= n; i++) {
        if(dfs_num[i] == 0) {
            dfsCounter = rootChildren = 0;
            dfsRoot = i;
            articulationPoint(i);
            isArticulationPoint[i] = (rootChildren > 1);
        }
        isAtriculationPoint + 1 = number of nodes that is disconnected                //Important
    }

    /*for(int i = 0; i < 101; i++)                                                    //Printing Articulation Points
        if(isArticulationPoint[i])
            printf("%d ", i);
    printf("\n");*/
```

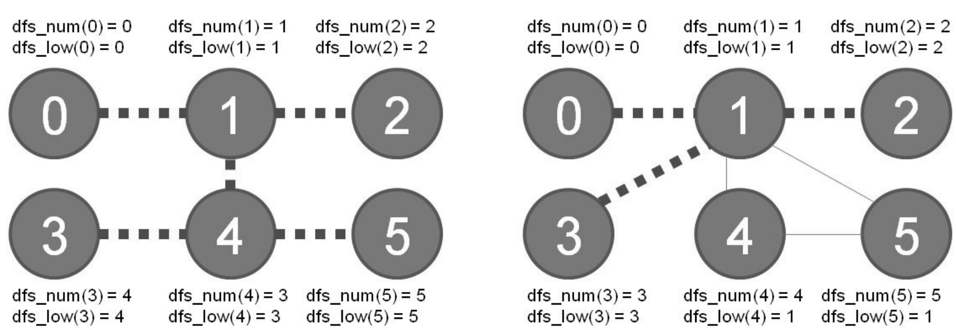


Fig: Finding Bridges

```
    printf("%d\n", (int)isArticulationPoint.count());
    return 0;
}
```

//Finding Bridges (Graph)
//Complexity : O(V+E)

```
vector<int> G[MAX];
vector<pair<int, int> >ans;
int dfs_num[MAX], dfs_low[MAX], parent[MAX], dfsCounter;

void bridge(int u) {
   //dfs_num[u] is the dfs counter of u node
   //dfs_low[u] is the minimum dfs counter of u node (it is minimum if a back-edge exists)
    dfs_num[u] = dfs_low[u] = ++dfsCounter;
    for(int i = 0; i < G[u].size(); i++) {
        int v = G[u][i];
        if(dfs_num[v] == 0) {
            parent[v] = u;

            bridge(v);
            //if dfs_num[u] is lower than dfs_low[v], then there is no back edge on u node
            //so u - v can be a bridge
            if(dfs_num[u] < dfs_low[v])
                ans.push_back(make_pair(min(u, v), max(u, v)));

            //obtainig lower dfs counter (if found) from child nodes
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
        }
       //if v is not parent of u then it is a back edge
       //also dfs_num[v] must be less than dfs_low[u]
       //so we update it
        else if(parent[u] != v)
            dfs_low[u] = min(dfs_low[u], dfs_num[v]);
    }
}

int main() { ……..
        memset(dfs_num, 0, sizeof(dfs_num));
        dfsCounter = 0;
        for(int i = 0; i < n; i++)
            if(dfs_num[i] == 0)
                bridge(i);

        sort(ans.begin(), ans.end());      //Output
        for(int i = 0; i < ans.size(); i++)
            printf("%d - %d\n", ans[i].first, ans[i].second);
        printf("\n");
    return 0;
}
```

// Floyd Warshal (All Pair Shortest Path)
// Complexity : O(V^3) (Use if V <= 400)

```
int G[MAX][MAX], parent[MAX][MAX];
void graphINIT() {
   for(int i = 0; i < MAX; i++)
      for(int j = 0; j < MAX; j++)
         G[i][j] = INF;
   for(int i = 0; i < MAX; i++)
      G[i][i] = 0;
}

void floydWarshall(int V){
         for(int i = 0; i < V; i++)                 //path printing matrix initialization
                  for(int j = 0; j < V; j++)
                           parent[i][j] = i;        //we can go to j from i by only obtaining i (by default)

   for(int k = 0; k < V; k++)                       //Selecting a middle point as k
      for(int i = 0; i < V; i++)                    //Selecting all combination of source (i) and destination (j)
         for(int j = 0; j < V; j++)
            if(G[i][k] != INF && G[k][j] != INF) {
                  //if the graph contains negative edges, then min(INF, INF+ negative edge) = +-INF!
               G[i][j] = min(G[i][j], G[i][k]+G[k][j]);          //if G[i][i] = negative, then node i is in negative circle
               parent[i][j] = parent[k][j];                      //if path printing needed
}}

void printPath(int i, int j) {
         if(i != j)  printPath(i, parent[i][j]);
         printf(" %d", j);
}

void minMax(int V) {                                             // Maximum edge weight in minimum distance path
         for(int k = 0; k < V; k++)
                  for(int i = 0; i < V; i++)
                           for(int j = 0; j < V; j++)
                                    G[i][j] = min(G[i][j], max(G[i][k], G[k][j]));
}

void transitiveClosure(int V) {                                  // Determine if u is connected to v directly or indirectly
         for(int k = 0; k < V; k++)
                  for(int i = 0; i < V; i++)
                           for(int j = 0; j < V; j++)
                                    G[i][j] |= (G[i][k] & G[k][j]);
}
```

//Strongly Connected Component (Tarjan)
//Complexity : O(V+E)

```
vector<int>G[30], SCC;
int dfs_num[30], dfs_low[30], dfsCounter, SCC_no = 1;
```

```
bitset<30>visited;
void tarjanSSC(int u) {
   SCC.push_back(u);                    // Generally it is stack data structure, here, it is implemented as vector instead

   //visited[u] marks if the node u is usable in a SCC and not used on other SCC
   //if visited[u] is false, then it is used in other SCC

   visited[u] = 1;
   dfs_num[u] = dfs_low[u] = ++dfsCounter;

   for(int i = 0; i < G[u].size(); i++) {          //for all Strongly Connected Component (directed graph), dfs_low[u] is same
      int v = G[u][i];
      if(dfs_num[v] == 0) {                        //if it is not visited yet, backtrack it
         tarjanSSC(v);
      }
      if(visited[v])      //if node v (visited[v]) is not visited, we can use it to minimize the dfs_low[u] value from dfs_low[v]
         dfs_low[u] = min(dfs_low[u], dfs_low[v]);
   }

   if(dfs_low[u] == dfs_num[u]) {
      bool first = 1;              //in a SCC the first node of the SCC, node u is the first node in a SCC if dfs_low[u] == dfs_low[v]
      printf("SCC %d\n", SCC_no++);
      while(1) {                   //as we implementing stack like data structure, the nodes from top to u are on the same SCC
         int v = SCC.back();
         SCC.pop_back();
         visited[v] = 0;                  //node v is used, so marking it as false, so that the ancestor nodes
         printf("%d\n", v);               //doesn't use this node to update it's value
         if(u == v)
            break;
      }
      printf("\n");
}}

int main() { ………...
   memset(dfs_num, 0, sizeof(dfs_num));
   dfsCounter = 0;
   visited.reset();
   for(int i = 1; i < indx; i++) {
      if(dfs_num[i] == 0)
         tarjanSSC(i);
}
```

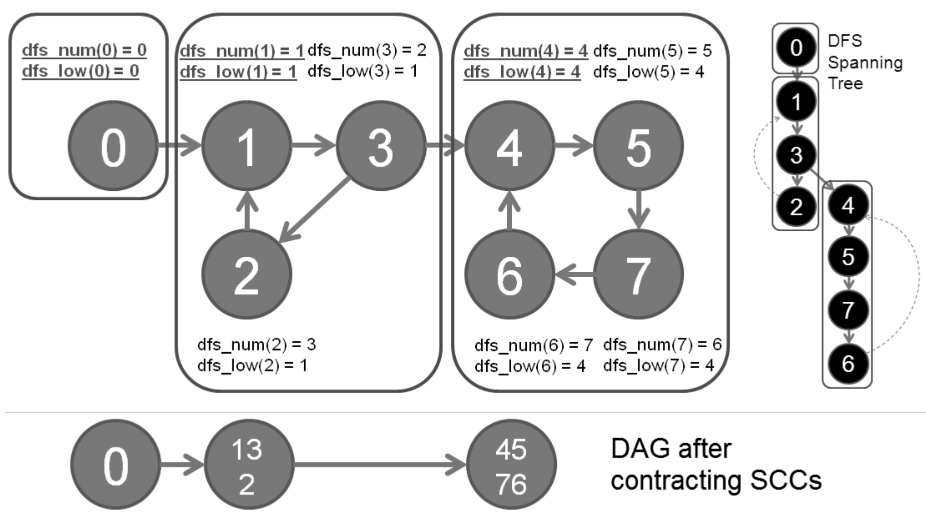


Fig: Strongly Connected Component

//Bipartite Graph

```
bool bipatite(int n) {
        queue<int>q;
        q.push(n), visited[n] = 1;
        while(!q.empty()) {
                int u = q.front();
                for(unsigned int i = 0; i < mat[u].size(); i++) {
                        if(visited[mat[u][i]] == -1) {
                                if(visited[u] == 1)          visited[mat[u][i]] = 2;
```

```
                    else        visited[mat[u][i]] = 1;
                    q.push(mat[u][i]);
                }
                if(visited[u] == visited[mat[u][i]])
                    return false;
            }
            q.pop();
        }
        return true;
}
```

// Bellman Ford's Algorithm for Single Source Shortest Path (Negative Cycle)
//Complexity : O(VE)

```
vector<int>G[MAX], W[MAX];
int V, E, dist[MAX];

void bellmanFord() {
   for(int i = 0; i <= V; i++)              //set to ( -INF ) if max distance is needed
      dist[i] = INF;

   for(int i = 0; i < V-1; i++)             //relax all edges V-1 times
      for(int u = 0; u < V; u++)      //all the nodes
         for(int j = 0; j < (int)G[u].size(); j++) {
            int v = G[u][i];
            int w = W[u][i];

            // Relax edges
            if(dist[u] != INF)              //if there is a negative weight, then INF + negative weight < INF and INF becomes +-INF
               dist[v] = min(dist[v], dist[u]+w);           //set to max if max distance needed
}}

bool hasNegativeCycle() {
   for(int u = 0; u < V; u++)
      for(int i = 0; i < G[u].size(); i++) {
         int v = G[u][i];
         int w = W[u][i];

         //if bellmanFord is run for max distance, then this code will return true for positive cycle by adding this line
         //if(dist[v] < dist[u] + w)
         if(dist[v] > dist[u] + w)
            return 1;
}
return 0; }
```

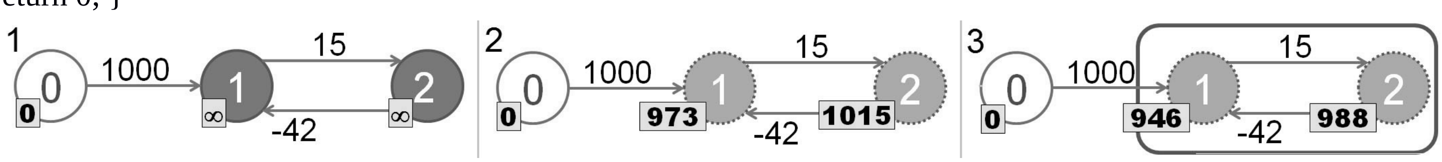


Fig : Single Source Shortest Path (Negative Cycle)

//Minimum Spanning Tree (Kruskal)
//NOTE: IMPLEMENT UNION DISJOINT FUNCTIONS FIRST

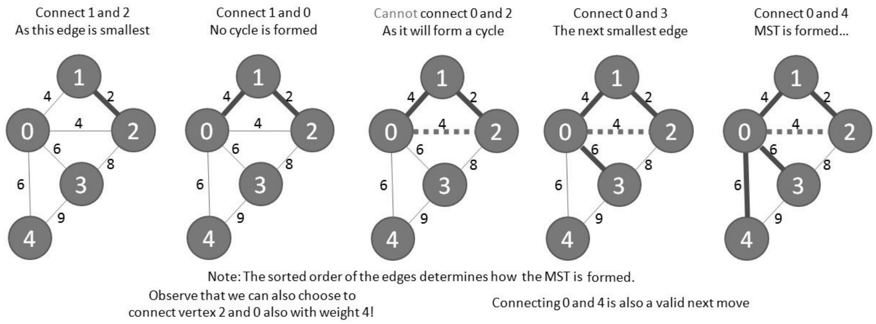
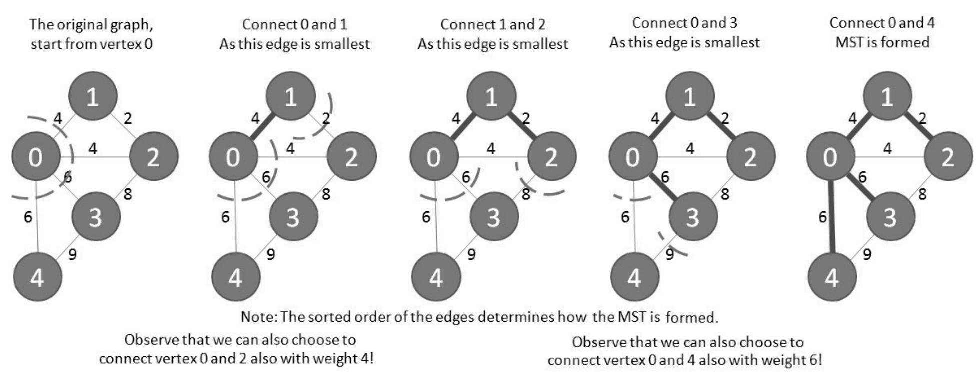


Fig: Animation of Kruskal's Algorithm

```
vector<pair<int, pair<int, int> > > Edge;
int main() {
        int V, E, u, v, w;
        scanf("%d %d", &V, &E);
        for(int i = 0; i < E; i++) {
        scanf("%d %d %d", &u, &v, &w);
        Edge.push_back(make_pair(w, make_pair(u, v)));
        }
        sort(Edge.begin(), Edge.end());        // Sort according to weight min to max

        int mst_cost = 0, selected_edge = 0;
        unionInit(V);                          // My union disjoint set initialization

        for(int i = 0; i < E && selected_edge < V; i++) {
                u = Edge[i].second.first;
                v = Edge[i].second.second;
                w = Edge[i].first;
                if(!isSameSet(u, v)) {
                        selected_edge++;
                        mst_cost += w;
                        makeUnion(u, v);
                }}
        printf("MST in Kruskal : %d\n", mst_cost);
        Edge.clear();
        return 0; }
```

//Minimum Spanning Tree (Prim's)
//Complexity : O(E logV)

```
vector<int> G[MAX], W[MAX];
priority_queue<pair<int, int> >pq;
bitset<MAX>taken;                 //priority queue returns the minimum node first, if tie, then the first node

void process(int u) {
   taken[u] = 1;                              //mark this node as taken
   for(int i = 0; i < G[u].size(); i++) {     //get all the edges of this node on a priority queue
      int v = G[u][i];
      int w = W[u][i];
      if(!taken[v])
         pq.push(make_pair(-w, -v));
   } }
```




```
int main() {
   int V, E, u, v, w;
   scanf("%d %d", &V, &E);
   while(E--) {
```

```
        scanf("%d %d %d", &u, &v, &w);
        G[u].push_back(v);
        W[u].push_back(w);
        G[v].push_back(u);
        W[v].push_back(w);
    }
    taken.reset();                  //Main Prim's MST code
    process(0);                     //taking 0 node as default
    int mst_cost = 0;
    while(!pq.empty()) {
        w = -pq.top().first;
        v = -pq.top().second;
        pq.pop();

        if(!taken[v]) {             //if the node is not taken, then use this node
            mst_cost += w;          //as it contains the minimum edge
            process(v);
        } }
    printf("Prim's MST cost : %d\n", mst_cost);
    return 0; }
```

//Dikjstra State-Space Graph

```
int dikjstra(int start_node, int end_node, int gas_capacity) {
    for(int i = 0; i <= V; i++)
        for(int j = 0; j <= 100; j++)
            cost[i][j] = INF;

    priority_queue<pair<int, pair<int, int> > >pq;      //total_cost, gas, node
    cost[start_node][0] = 0;            //node, gas
    pq.push(make_pair(0, make_pair(0, -start_node)));       //at starting city, cost and gas is zero
    while(!pq.empty()) {
        int cost_u = -pq.top().first;
        int gas = -pq.top().second.first;
        int u = -pq.top().second.second;
        pq.pop();

        if(u == end_node)    return cost_u;

        if(cost[u][gas] < cost_u)       continue;

        if(gas < gas_capacity) {        //taking 1 gallon of gas if possible
            int new_cost = cost_u + price[u];
            if(new_cost < cost[u][gas+1]) {
                cost[u][gas+1] = new_cost;
                pq.push(make_pair(-new_cost, make_pair(-(gas+1), -u)));
            }}

        for(int i = 0; i < G[u].size(); i++) {
            int v = G[u][i], w = W[u][i];
            if(w <= gas) {
                int gas_left = gas - w;
                if(cost_u < cost[v][gas_left]) {
                    cost[v][gas_left] = cost_u;
                    pq.push(make_pair(-cost_u, make_pair(-gas_left, -v)));
                } } } } return -1; }
```