//1D Max Sum
//Algorithm : Jay Kadane
//Complexity : O(n)

```
main() {
   int n;
   scanf("%d", &n);
   int A[n+1];
   for(int i = 0; i < n; i++)
      scanf("%d", &A[i]);

   //Main part of the code
   int sum = 0, ans = 0;
   for(int i = 0; i < 9; i++) {
      sum += A[i];
      ans = max(sum, ans);              //always take the larger sum
      if(sum < 0)
         sum = 0;                       //if sum is negative, reset it (greedy)
   }
   printf("1D Max Sum : %d\n", ans);
}
```

//2D Max Sum
//DP, Inclusion Exclusion
//Complexity : O(n^4)

```
int main() {
   int row_column, A[100][100];          //A square matrix
   scanf("%d", &row_column);

   for(int i = 0; i < row_column; i++)       //input of the matrix/2D array
      for(int j = 0; j < row_column; j++) {
         scanf("%d", &A[i][j]);
         if(i > 0)
            A[i][j] += A[i-1][j];          //take from right
         if(j > 0)
            A[i][j] += A[i][j-1];          //take from left
         if(i > 0 && j > 0)
            A[i][j] -= A[i-1][j-1];        //inclusion exclusion
      }

   int maxSubRect = -1e7;

   for(int i = 0; i < row_column; i++)            //i & j are the starting coordinate of sub-rectangle
      for(int j = 0; j < row_column; j++)
         for(int k = i; k < row_column; k++)      //k & l are the finishing coordinate of sub-rectangle
            for(int l = j; l < row_column; l++) {
               int subRect = A[k][l];
               if(i > 0)
```

```
                subRect -= A[i-1][l];
            if(j > 0)
                subRect -= A[k][j-1];
            if(i > 0 && j > 0)
                subRect += A[i-1][j-1];        //due to inclusion exclusion
            maxSubRect = max(subRect, maxSubRect);
        }
    printf("2D Max Sum : %d\n", maxSubRect);
    return 0;
}
```

// 0-1 Knapsack
// Dynamic Programming

```
//Note : val array contains element values starting from 1 index, 0 index is empty
int Knapsack(int totalWeight, int val[], int totalElements) {
    int dp[50001][101];                          // DP Table [BagWeight][TotalElements]
    //int track[101] = {0};                      // Use this if you want to print the taken elements
    for(int i = 0; i <= totalWeight; i++)
        dp[i][0] = 0;                            // Base Case

    // Calculating best weight(that will be taken) for every possible element
    for(int i = 1; i <= totalElements; i++) {    // Element starts from 1
        for(int weight = 1; weight <= totalWeight; weight++) {
            if(val[i] > weight)                  // If elements weight is greater than available weight
                dp[weight][i] = dp[weight][i-1]; // Skip this element
            else {                               // If enough space is available for this element
                if(dp[weight][i-1] >= dp[weight-val[i]][i-1] + val[i])
                    dp[weight][i] = dp[weight][i-1];   // If ignoring this element causes good outcome, ignore this
                else {
                    dp[weight][i] = dp[weight - val[i]][i-1] + val[i];   // Otherwise take this element
                    //track[dp[weight][i]] = i;                         // This tracks the taken element
            } } } }

            /* These code outputs the taken values
            int sumWeight = dp[totalWeight][totalElements];
            int lastTakenElement = track[totalWeight];
            while(lastTakenElement != 0) {                  // lastTakenElement is the index of element
                    printf("%d ", val[lastTakenElement]);   // val[lastTakenElement] is taken value
                    sumWeight -= val[lastTakenElement];
                    lastTakenElement = track[sumWeight];
            }
            printf("\n"); */

            return dp[totalWeight][totalElements];
}
```

		capacity j					
	i	0	1	2	3	4	5
	0	0	0	0	0	0	0
$w_1 = 2,\ v_1 = 12$	1	0	0	12	12	12	12
$w_2 = 1,\ v_2 = 10$	2	0	10	12	22	22	22
$w_3 = 3,\ v_3 = 20$	3	0	10	12	22	30	32
$w_4 = 2,\ v_4 = 15$	4	0	10	15	25	30	**37**

FIGURE 8.5 Example of solving an instance of the knapsack problem by the dynamic programming algorithm.

// Coin Change
// All Possible Types

```
int main() {
        int n, coin_amount = 3;                 // n = value to produce
        int coin[] = {1, 2, 3}, test[1000];     // coin[] = coin values

        // Solution for producing amount with coins. Without any co-occurance and
        // coins can be used more than once
        // Bottom Up solution

        memset(test, 0, sizeof(test));
        test[0] = 1;      // Base case

        for(register int i = 0; i < coin_amount; i++)           // This will NOT produce co-occurrence
                for(register int j = 1; j<=n; j++)              // Solution for 4 if there is present 1 & 2 coins would be 3
                        if(j >= coin[i])                        // 1+1+2, 2+2, 1+1+1+1
                                test[j] += test[j - coin[i]];
        printf("Solution without co-occurance : %d\n", test[n]);

        // Solution for producing amount with coins. With co-occurance and
        // Coins can be used more than once
        // Bottom Up solution

        memset(test, 0, sizeof(test));
        test[0] = 1;                                            // Base case

        for(int j = 1; j <= n; j++)                             // This will produce co-occurrence
                for(int i = 0; i < coin_amount; i++)            // Solution for 4 if there is present 1 & 2 coins would be 5
                        if(j >= coin[i])                        // 1+1+2, 2+2, 1+1+1+1
                                test[j] += test[j - coin[i]];   // and also 2+1+1, 1+2+1

        printf("Solution with co-occurrence : %d\n", test[n]);


        //  Solution for producing amount with coins. With co-occurrence and
        //  Coins can be used more than once
        //  Bottom up solution

        for(int i = 0; i <= 1000; i++)
                test[i] = inf;          // Normal case

        test[0] = 0;                    // Base case

        for(int i = 0; i < coin_amount; i++)                    // this will produce co-occurrence
                for(int j = n; j > 0; j--)                      // solution for 4 if there is present 1, 2 & 3 coins would be 2
                        if(j >= coin[i] && (test[j - coin[i]] + 1) < inf)       // 1+3, and 3+1
                                test[j] = test[j-coin[i]] + 1;
        printf("Solution by using coins only once with co-occurrence : %d\n", test[n]);
```

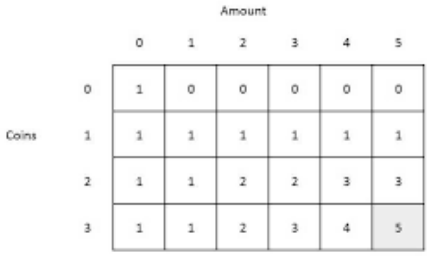
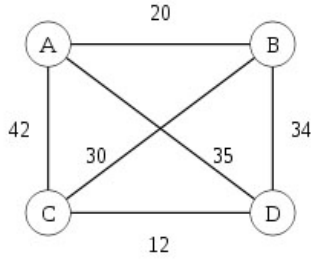


Fig: Coin Change Table

```
	//  Solution for producing amount with coins. With co-occurrence and
	//  coins can be used more than once
	//  Bottom up solution

	for(int i = 0; i <= 1000; i++)
		test[i] = inf;			// Normal case

	test[0] = 0;				// Base case

	for(register int i = n; i > 0; i--)					// this will NOT produce co-occurrence
		for(register int j = 0; j < coin_amount; j++)	// solution for 4 if there is present 1, 2 & 3 coins would be 1
			if(i >= coin[j] && (test[i - coin[j]] + 1) < inf)	// 1+3 only
				test[i] = test[i - coin[j]] + 1;

	printf("Solution by using coins only once without co-occurrence : %d\n", test[n]);
	return 0;
}
```

// Traveling Salesman
// Time Complexity :

```
//dist[u][v] = distance from u to v
//dp[u][bitmask] = dp[node][set_of_taken_nodes]  (saves the best(min/max) path)
//call with tsp(starting node, 1)
```



	A	B	C	D
A	0	20	42	35
B	20	0	30	34
C	42	30	0	12
D	35	34	12	0

Fig: Traveling Salesman Problem State (A-B-C-D-A)

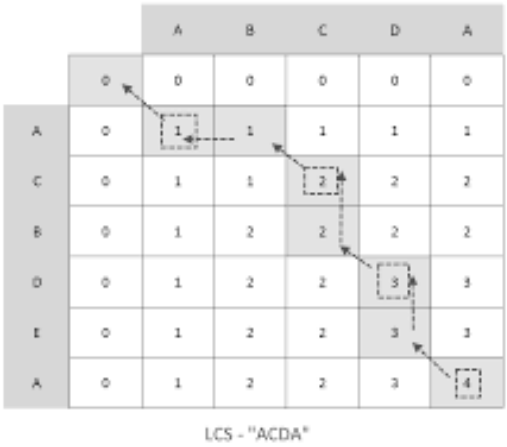
```
int n, x[11], y[11], dist[11][11], memo[11][1 << 11], dp[11][1 << 11];

int tsp(int u, int bitmask) {			// Starting node and bitmask of taken nodes
	if(bitmask == ((1 << (1+n)) - 1))	// When it steps in this node, if all nodes are visited
		return dist[u][0];		// Then return to 0'th (starting) node [as the path is Hamiltonian]

	//or use return dist[u][start]  if starting node is not 0
	if(dp[u][bitmask] != -1)			// If we have previous value set up
		return dp[u][bitmask];			// Use that previous value

	int ans = 1e9;					// Set an infinite value
	for(int v = 0; v <= n; v++)			// For all the nodes
		if(u != v && !(bitmask & (1 << v)))	// if this node is not the same node, and if this node is not used
							// yet(in bitmask)

			ans = min(ans, dist[u][v] + tsp(v, bitmask | (1 << v)));
							//min(past_val, dist u->v + dist(v->all other untaken nodes))
	return dp[u][bitmask] = ans;			//save in dp and return
}
```

//Longest Common SubSequence
//Dynamic Programming

```
char a[210], b[210];
int dp[210][210], len_a, len_b;
```

```
//LCS is the same sequence in two strings:  a s x z  and  s x z a. Here LCS is 3 {a, sx, z}
int LCS(char a[], char b[], int len_a, int len_b) {
   dp[210][210] = 0;
   for(register int i = 1; i <= len_a; i++)
      for(register int j = 1; j <= len_b; j++) {
         if(i == 0 || j == 0)                    //base case
            dp[i][j] = 0;
         else if(a[i-1] == b[j-1])               //if a match found
            dp[i][j] = dp[i-1][j-1] + 1;
         else
            dp[i][j] = max(dp[i-1][j], dp[i][j-1]);       // dp[i][j] = max(ignoring
b[j-1] (taking b[j]), ignoring a[i-1] (taking a[i]))
      }
   return dp[len_a][len_b];
}
```

Fig: Longest Common Subsequence



LCS - "ACDA"