

# Basic Statistics

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**PhD in Computer Science**

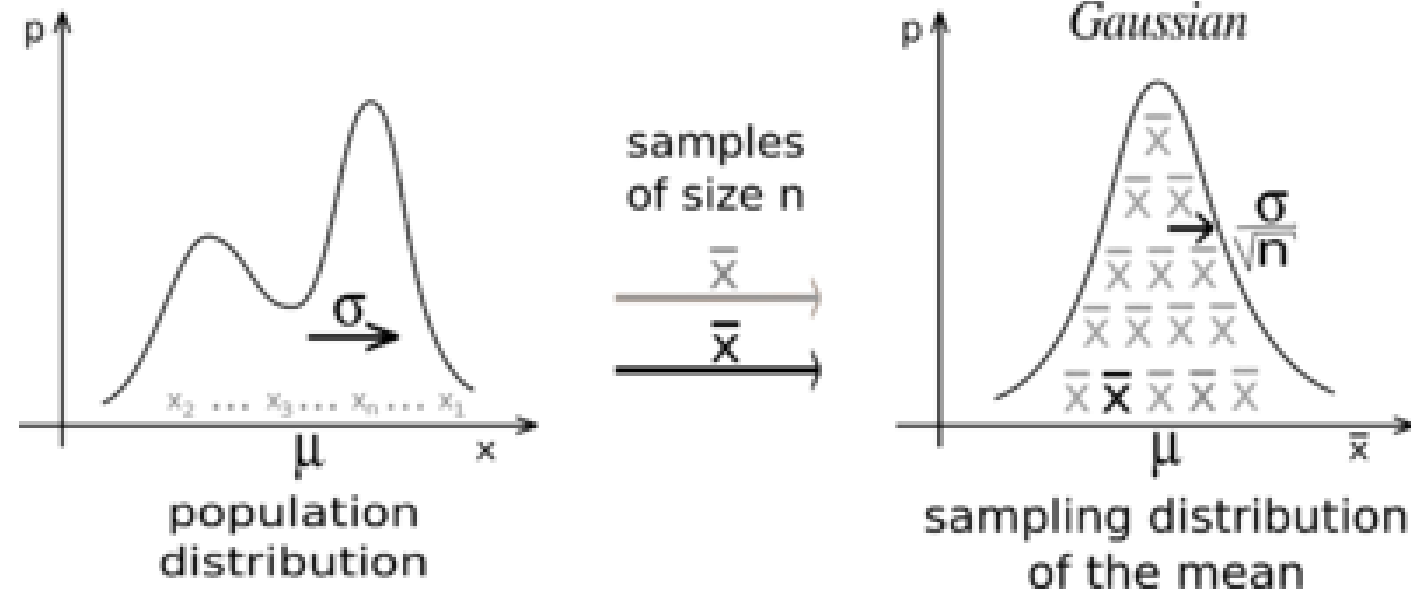
# Outline

- Central limit theorem
- Mean and its Applications
- Median and its Applications
- Variance and its Applications
- Quick Introduction to Correlation

# Central limit theorem

## ❖ Example

When independent random variables are summed up, their properly normalized sum tends toward a normal distribution even if the original variables themselves are not normally distributed.



[https://en.wikipedia.org/wiki/Central\\_limit\\_theorem](https://en.wikipedia.org/wiki/Central_limit_theorem)

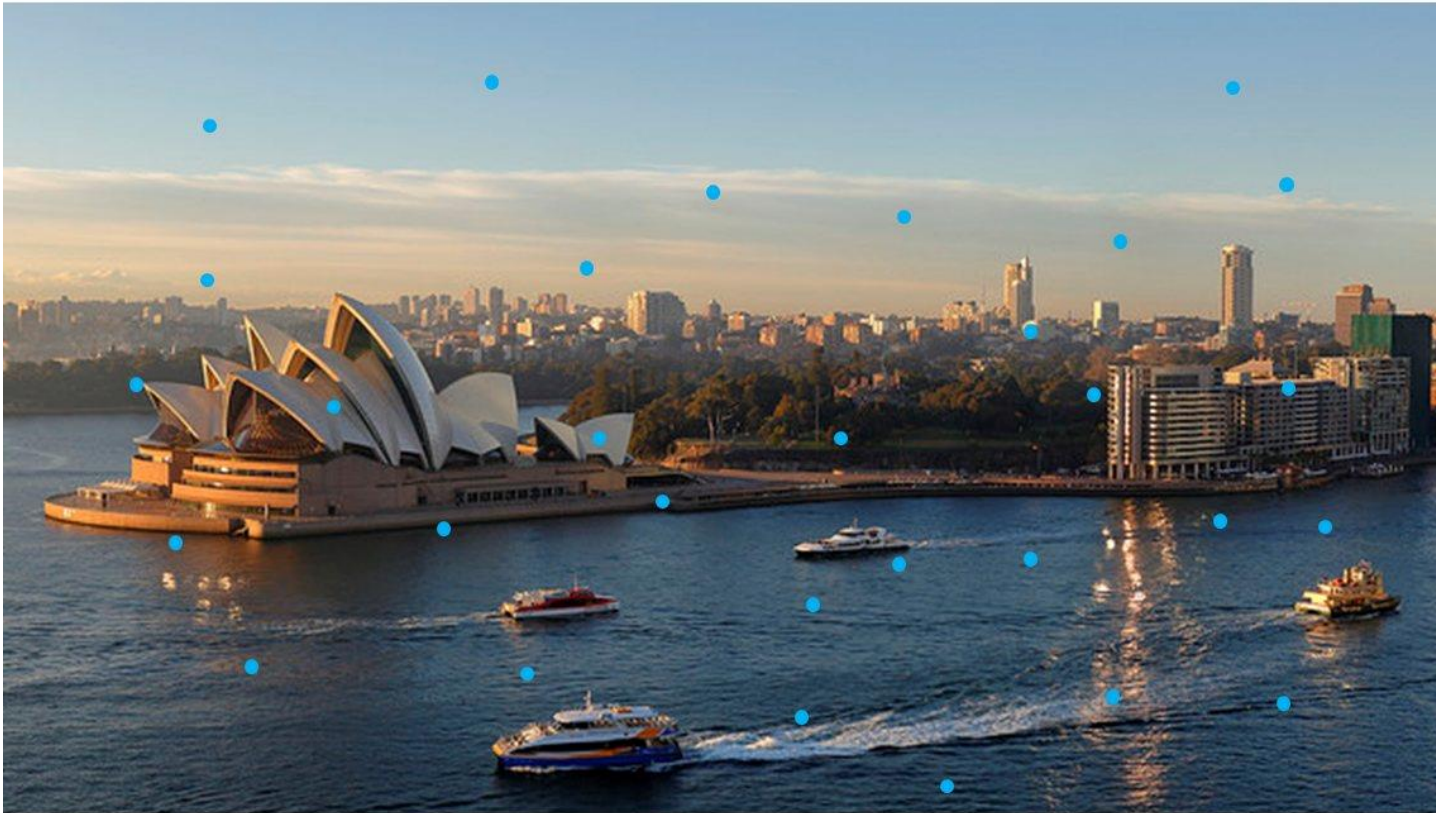
# Central Limit Theorem

Randomly selected 30  
pixels

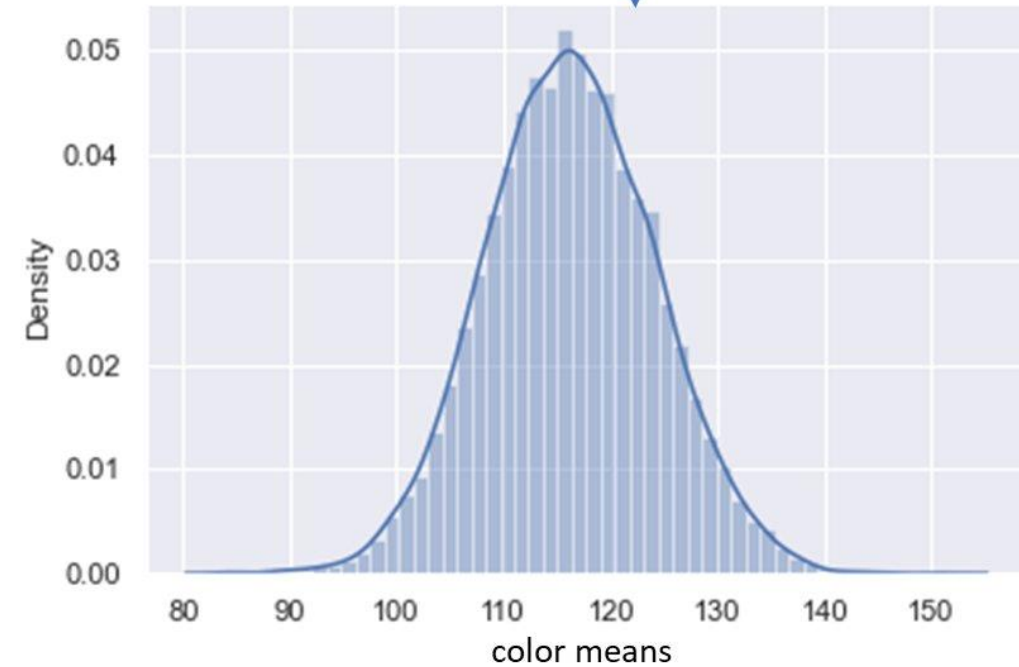
Compute color mean for  
the 30 pixels

Repeat 10000  
times

Histogram of  
the 10000  
color means



Population: pixel colors of the entire image



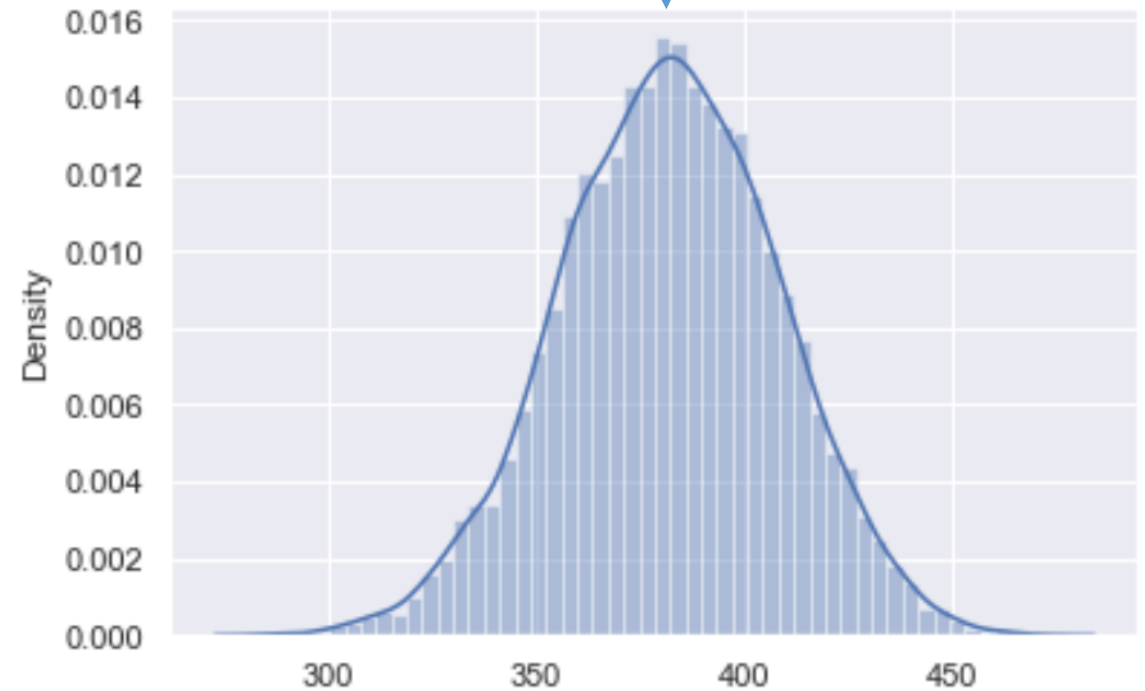
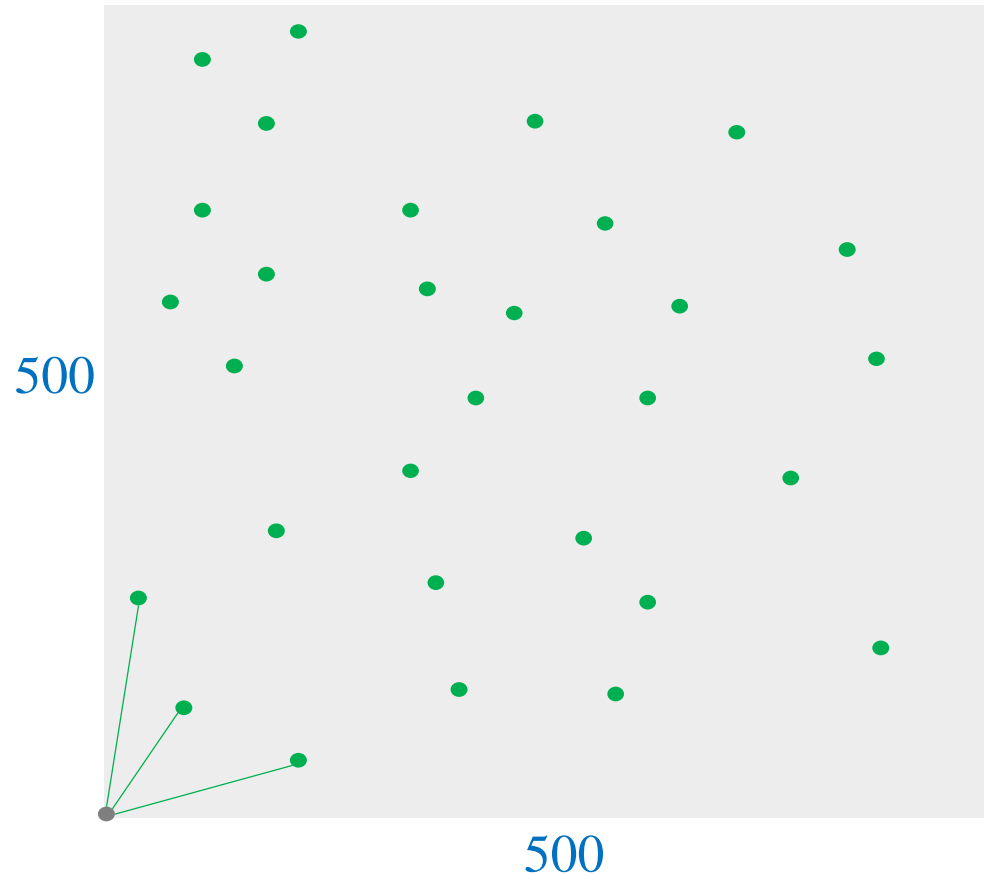
# Example

Randomly selected  
30 pixels

Compute distance  
mean

Repeat 10000  
times

Histogram of 1000  
distance means



# Outline

- Central limit theorem
- Mean and its Applications
- Median and its Applications
- Variance and its Applications
- Quick Introduction to Correlation

# Mean

## ❖ Definition

### Data

$$X = \{X_1, \dots, X_N\}$$

### Given the data

$$X = \{2, 8, 5, 4, 1, 4\}$$

$$N = 6$$

### Formula

$$\mu_X = \frac{1}{N} \sum_{i=1}^N X_i$$

$$\begin{aligned} \mu_X &= \frac{1}{N} \sum_{i=1}^N X_i = \frac{1}{6} (2 + 8 + 5 + 4 + 1 + 4) \\ &= \frac{24}{6} = 4 \end{aligned}$$

# Mean

## Data

$$X = \{X_1, \dots, X_N\}$$

## Formula

$$E(X) = \sum_{i=1}^N X_i P_X(X_i)$$

## Given the data

$$X = \{2, 8, 5, 4, 1, 4\}$$

$$N = 6$$

$$P_X(X = 2) = \frac{1}{6}$$

$$P_X(X = 4) = \frac{2}{6}$$

$$P_X(X = 8) = \frac{1}{6}$$

$$P_X(X = 1) = \frac{1}{6}$$

$$P_X(X = 5) = \frac{1}{6}$$

$$\begin{aligned} E(X) &= 2 \times \frac{1}{6} + 8 \times \frac{1}{6} + 5 \times \frac{1}{6} + 4 \times \frac{2}{6} + 1 \times \frac{1}{6} \\ &= \frac{2}{6} + \frac{8}{6} + \frac{5}{6} + \frac{8}{6} + \frac{1}{6} = 4 \end{aligned}$$



# Mean

## ❖ Application

```
1. def calculate_mean(numbers): #1
2.     s = sum(numbers)         #2
3.     N = len(numbers)         #3
4.     mean = s/N               #4
5.     return mean              #5
6.
7. # Tạo mảng donations đại diện cho số tiền quyên góp trong 12 ngày
8. donations = [100, 60, 70, 900, 100, 200, 500, 500, 503, 600, 1000, 1200]
9.
10. mean_value = calculate_mean(donations)
11. print('Trung bình số tiền quyên góp là: ', mean_value)
```



Làm mờ ảnh  
dựa vào mean



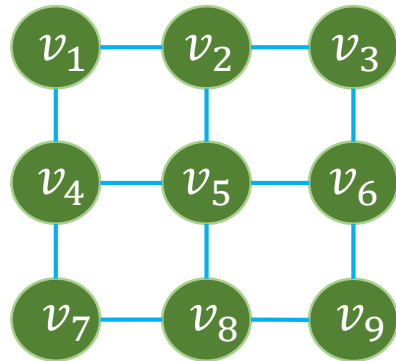
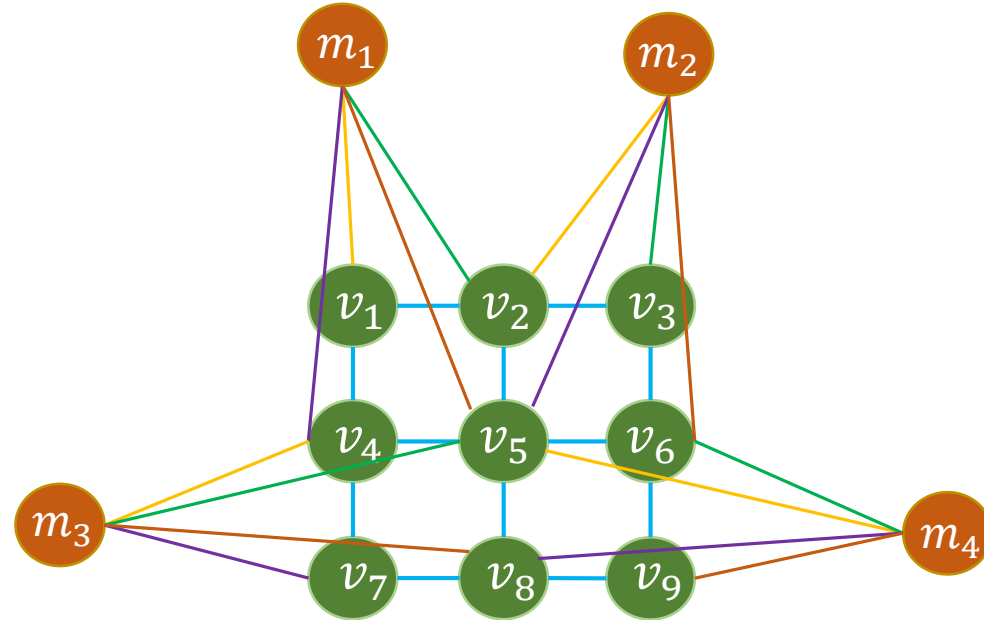
# Mean Applications

## ❖ Correlation (~convolution)



Kernel

$$m_1 = v_1w_1 + v_2w_2 + v_4w_3 + v_5w_4$$

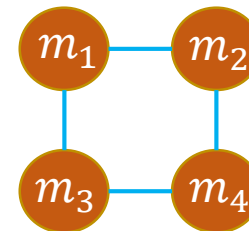


Image

\*



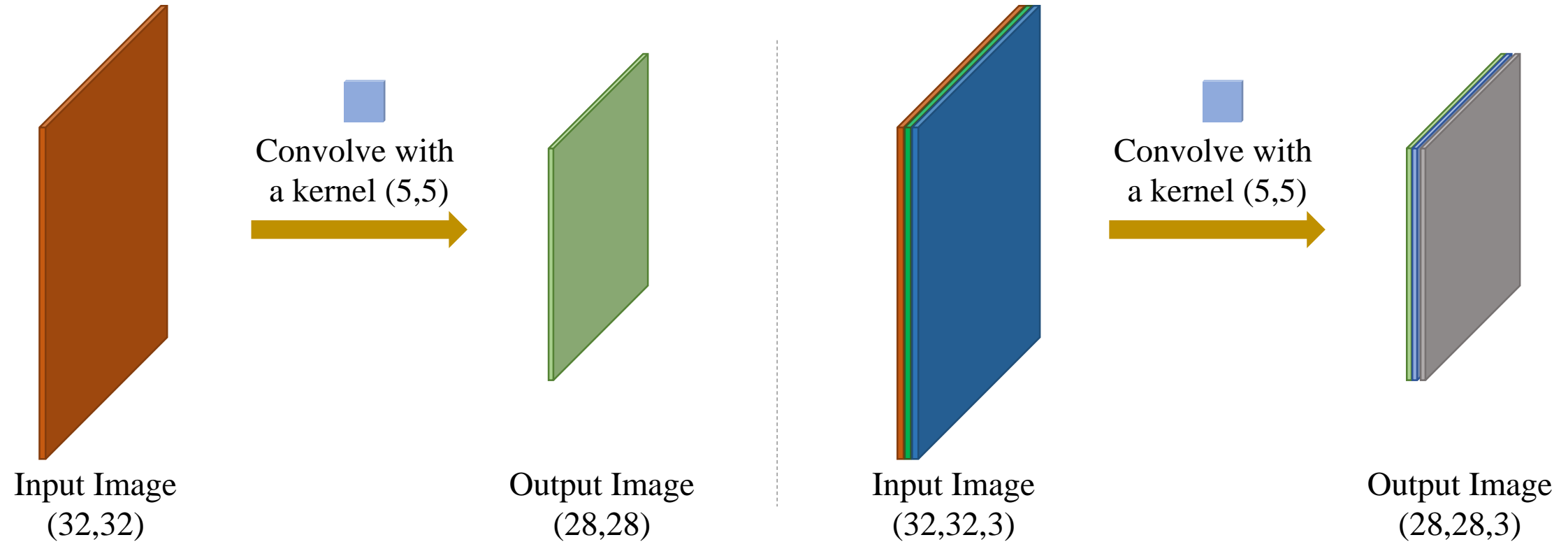
Kernel



Output

# Mean Applications

## ❖ Correlation (~convolution)



# Mean Applications

## ❖ Correlation (~convolution)

Kernels for computing mean

|        |  |
|--------|--|
| Numpy  | <code>np.einsum()</code>               |
| Scipy  | <code>scipy.signal.convolve2d()</code> |
| OpenCV | <code>cv2.filter2D()</code>            |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

(3x3) kernel

|   |   |   |   |   |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$\frac{1}{25}$

(5x5) kernel

```
output_image = cv2.filter2D(input_image, cv2.CV_8U, kernel)
```

# Mean

## ❖ Correlation (~convolution)

```
1  # load image and blurring
2
3  import numpy as np
4  import cv2
5
6  # load image in grayscale mode
7  image = cv2.imread('mrbean.jpg', 0)
8
9  # create kernel
10 kernel = np.ones((5,5), np.float32) / 25.0
11
12 # compute mean for each pixel
13 dst = cv2.filter2D(image, cv2.CV_8U, kernel)
14
15 # show images
16 cv2.imshow('image', image)
17 cv2.imshow('dst', dst)
18
19 # waiting for any keys pressed and close windows
20 cv2.waitKey(0)
21 cv2.destroyAllWindows()
```

Given the face region



# Numpy Review

## ❖ Slicing

`arr[for_axis_0, for_axis_1, ...]`

‘:’: get all the elements

‘a:b’: get the elements from  $a^{th}$  to  $(b^{th}-1)$

data

|   | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

data[0, 1]

|   | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

data[1: 3]

|   | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

data[0: 1, 0]

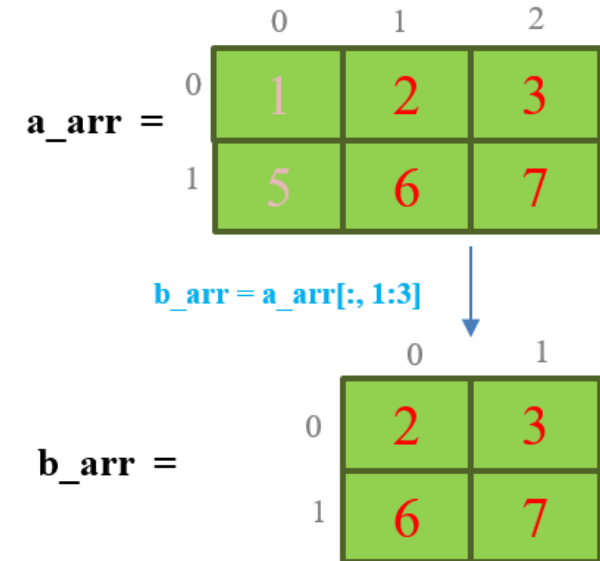
|   | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

data[:, :]

|   | 0 | 1 |
|---|---|---|
| 0 | 1 | 2 |
| 1 | 3 | 4 |
| 2 | 5 | 6 |

```
1 # aivietnam.ai
2 import numpy as np
3
4 # Khởi tạo numpy array a_arr
5 a_arr = np.array([[1, 2, 3],
6                  [5, 6, 7]])
7
8 # Sử dụng slicing để tạo mảng b_arr
9 # bằng cách lấy tất cả các dòng và cột 1, 2
10 b_arr = a_arr[:, 1:3]
11
12 print(a_arr)
13 print(b_arr)
```

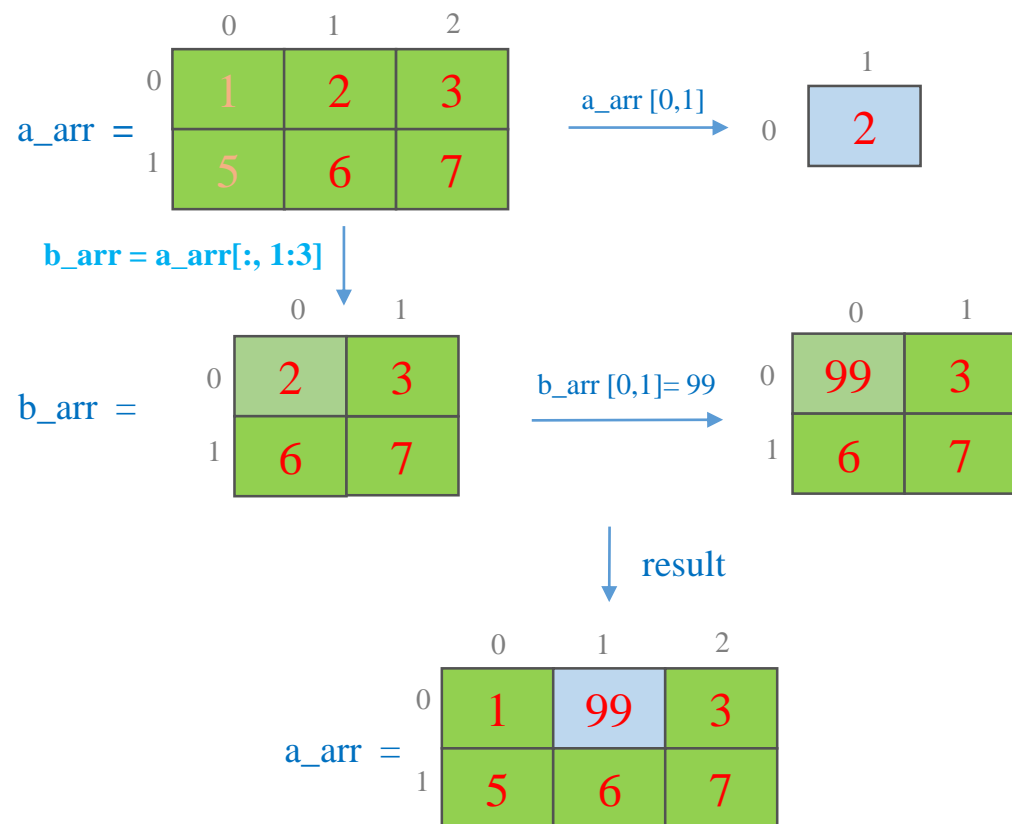
```
[[1 2 3]
 [5 6 7]]
[[2 3]
 [6 7]]
```



# Numpy Review

## ❖ Slicing

### ❖ Mutable



```

1 # aivietnam.ai
2 import numpy as np
3
4 # Khởi tạo numpy array a_arr
5 a_arr = np.array([[1,2,3],
6                  [5,6,7]])
7 print('a_arr \n', a_arr)
8
9 # Sử dụng slicing để tạo mảng b_arr
10 b_arr = a_arr[:, 1:3]
11 print('b_arr \n', b_arr)
12
13 print('before changing \n', a_arr[0, 1])
14 b_arr[0, 0] = 99
15 print('after changing \n', a_arr[0, 1])

```

```

a_arr
[[1 2 3]
 [5 6 7]]
b_arr
[[2 3]
 [6 7]]
before changing
2
after changing
99

```

# Mean

## ❖ Numpy review

```
1  # numpy review
2  import numpy as np
3
4  arr = np.ones((5,5))
5  print(arr)
6
7  roi = arr[1:4, 1:4]
8  roi = roi + 1
9  print(roi)
10
11 arr[1:4, 1:4] = roi
12 print(arr)
```

```
[[1.  1.  1.  1.  1.]
 [1.  1.  1.  1.  1.]
 [1.  1.  1.  1.  1.]
 [1.  1.  1.  1.  1.]
 [1.  1.  1.  1.  1.]
```

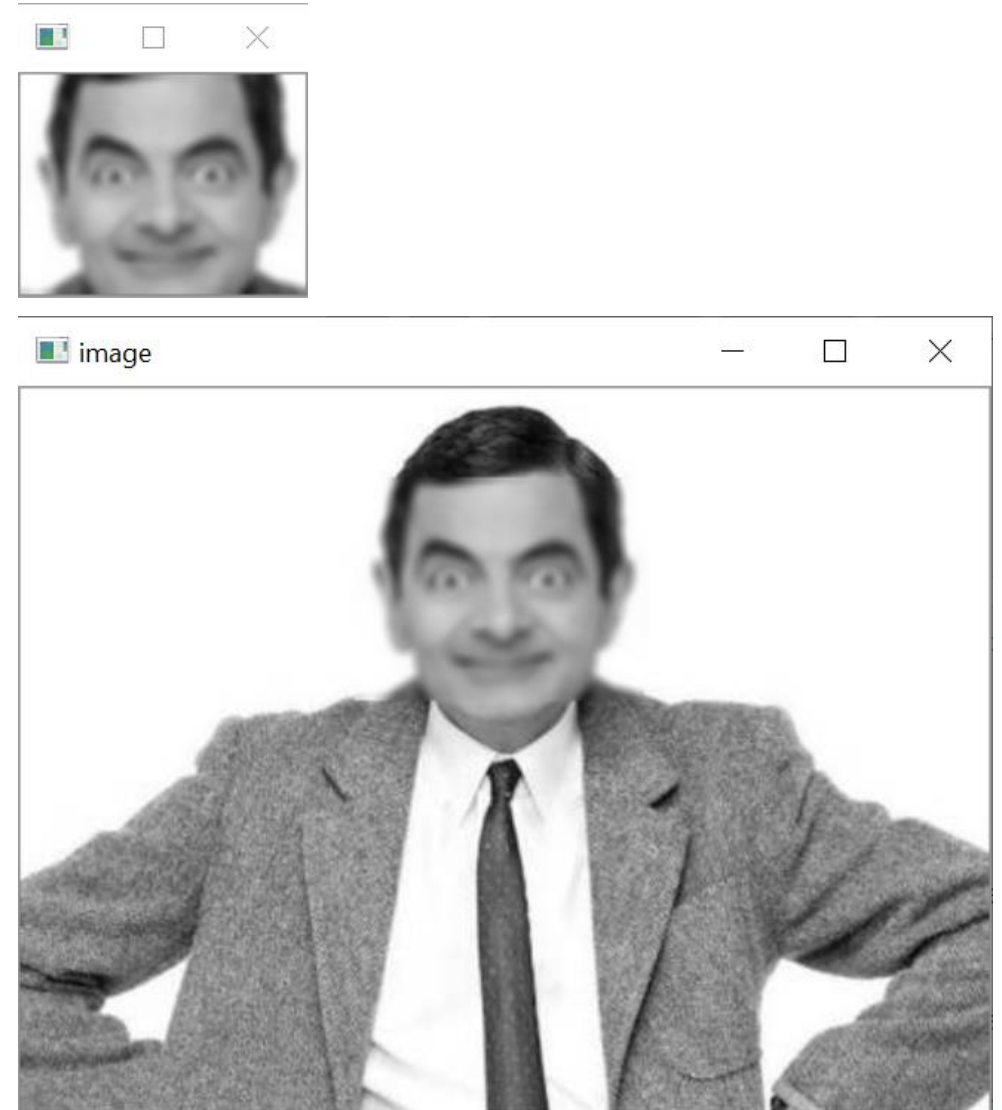
```
[[2.  2.  2.]
 [2.  2.  2.]
 [2.  2.  2.]
```

```
[[1.  1.  1.  1.  1.]
 [1.  2.  2.  2.  1.]
 [1.  2.  2.  2.  1.]
 [1.  2.  2.  2.  1.]
 [1.  1.  1.  1.  1.]
```



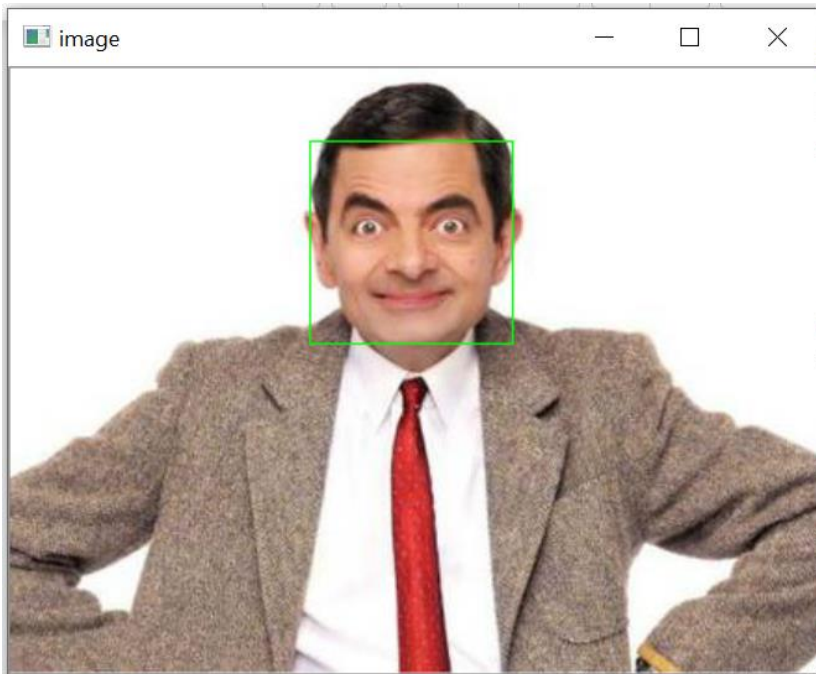
## ❖ Image Blurring

```
1  # load image and blurring using mask-simple
2
3  import numpy as np
4  import cv2
5
6  # load image in grayscale mode
7  image = cv2.imread('mrbean.jpg', 0)
8
9  # create kernel
10 kernel = np.ones((5,5), np.float32) / 25.0
11
12 # Select ROI (top_y,top_x,height, width)
13 roi = image[40:140,150:280]
14
15 # compute mean for each pixel
16 roi = cv2.filter2D(roi, cv2.CV_8U, kernel)
17
18 image[40:140,150:280] = roi
19
20 # show images
21 cv2.imshow('roi', roi)
22 cv2.imshow('image', image)
23
24 # waiting for any keys pressed and close windows
25 cv2.waitKey(0)
26 cv2.destroyAllWindows()
```



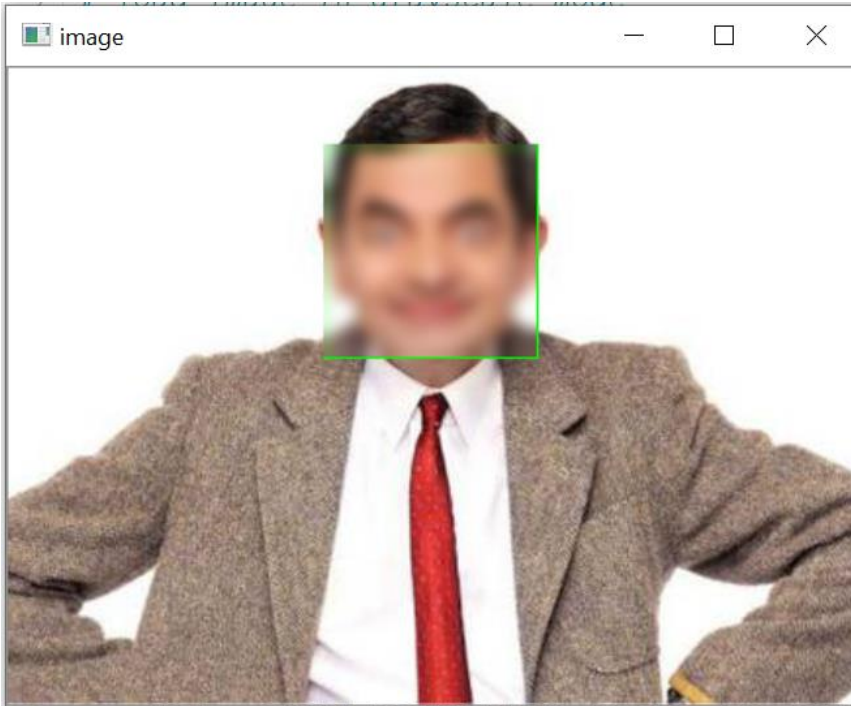
# Mean

## ❖ Image Blurring



```
1  # load image and blurring using face detection
2
3  import numpy as np
4  import cv2
5
6  # face detection setup
7  face_cascade = cv2.CascadeClassifier('haarcascade_frontalface_default.xml')
8
9  # load image in grayscale mode
10 image = cv2.imread('mrbean.jpg', 1)
11
12 # Convert to grayscale
13 gray = cv2.cvtColor(image, cv2.COLOR_BGR2GRAY)
14
15 # face detection
16 faces = face_cascade.detectMultiScale(gray, 1.1, 4)
17
18 # Draw the rectangle around each face
19 for (x, y, w, h) in faces:
20     cv2.rectangle(image, (x, y), (x+w, y+h), (0, 255, 0), 1)
21
22 # show images
23 cv2.imshow('image', image)
24
25 # waiting for any keys pressed and close windows
26 cv2.waitKey(0)
27 cv2.destroyAllWindows()
```

## ❖ Image Blurring



```
1 # load image and blurring using face detection
2
3 import numpy as np
4 import cv2
5
6 # face detection setup
7 face_cascade = cv2.CascadeClassifier('haarcascade_frontalface_default.xml')
8
9 # load image in grayscale mode
10 image = cv2.imread('mrbean.jpg', 1)
11
12 # Convert to grayscale
13 gray = cv2.cvtColor(image, cv2.COLOR_BGR2GRAY)
14
15 # face detection
16 faces = face_cascade.detectMultiScale(gray, 1.1, 4)
17
18 # create kernel
19 kernel = np.ones((7,7), np.float32) / 49.0
20
21 # Draw the rectangle around each face
22 for (x, y, w, h) in faces:
23     cv2.rectangle(image, (x, y), (x+w, y+h), (0, 255, 0), 1)
24     roi = image[y:y+h,x:x+w]
25
26     # compute mean for each pixel
27     roi = cv2.filter2D(roi, cv2.CV_8U, kernel)
28     roi = cv2.filter2D(roi, cv2.CV_8U, kernel)
29     roi = cv2.filter2D(roi, cv2.CV_8U, kernel)
30
31     # update
32     image[y:y+h,x:x+w] = roi
33
34 # show images
35 cv2.imshow('image', image)
36
37 # waiting for any keys pressed and close windows
38 cv2.waitKey(0)
39 cv2.destroyAllWindows()
```



```
35         # update
36         img[y-pad:y+h+pad,x-pad:x+w+pad] = roi # boundary
37
38     # Display
39     cv2.imshow('img', img)
40     cv2.imshow('img2', img2)
41
42     # Stop if escape key is pressed
43     key = cv2.waitKey(30) & 0xff
44     if key==27:
45         break
46
47 # Release the VideoCapture object
48 cap.release()
49 cv2.destroyAllWindows()
```

In [ ]:

1

In [18]:

```
1 import cv2
2 import numpy as np
3
4 # To capture video from webcam.
5 cap = cv2.VideoCapture(0)
6
7 # select region
8 while True:
9     # Read the frame and Convert to grayscale
10    _, img = cap.read()
11    gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
12
13    top_left = (200, 200)
```

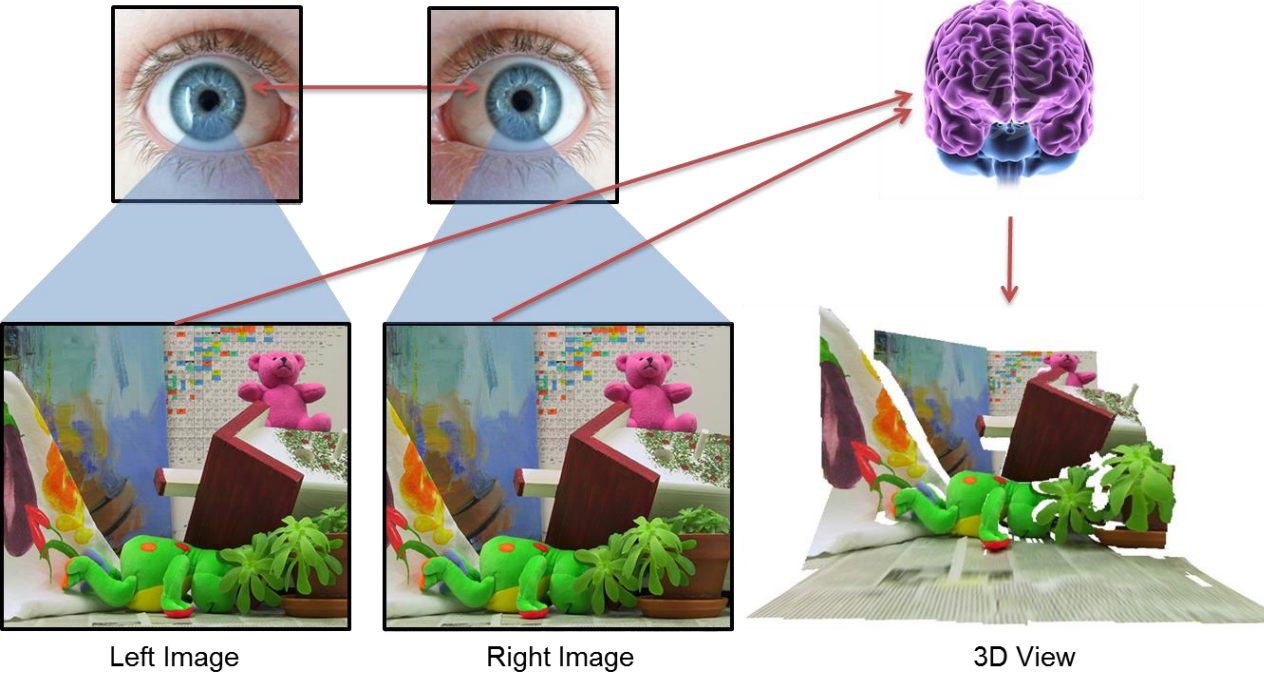


Human-Eye

Brain

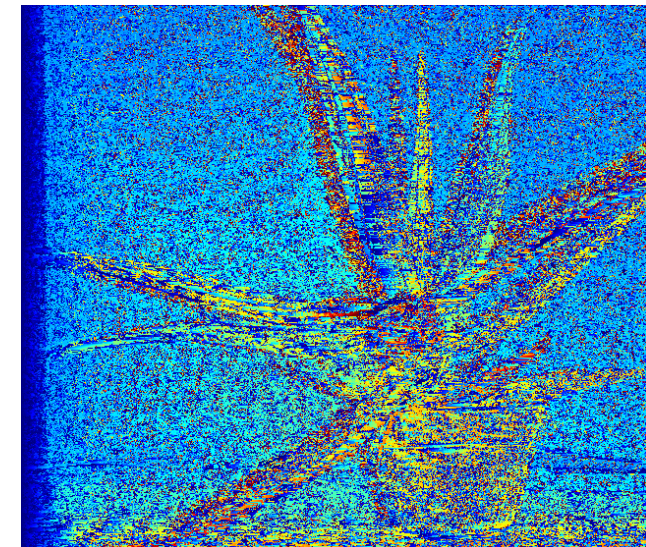
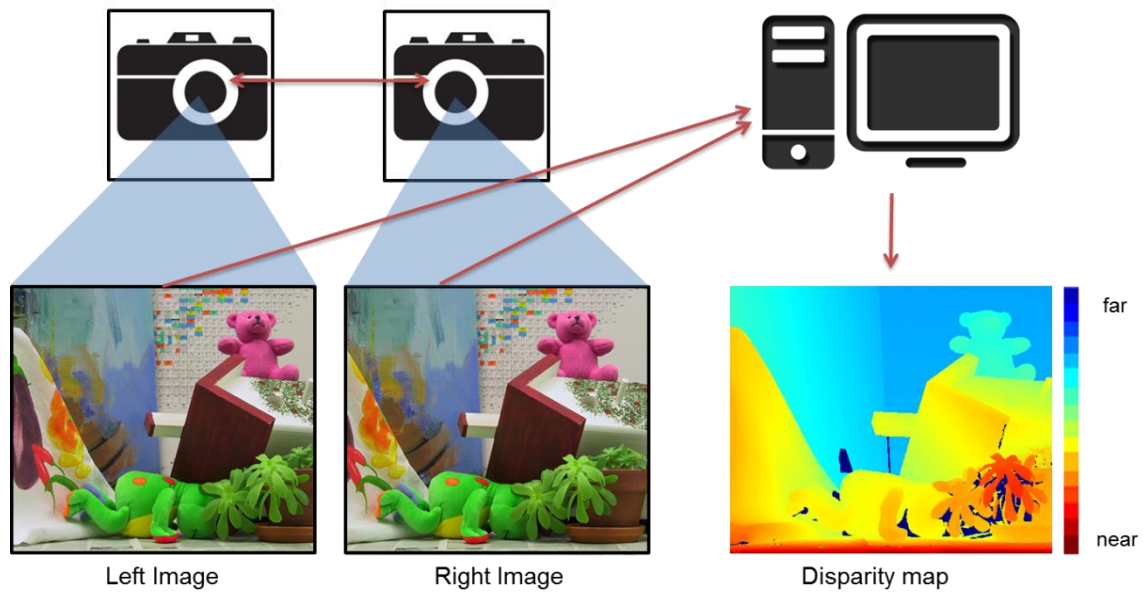
# Mean

❖ Stereo matching



Stereo Camera

Computer





# Stereo Matching

## ❖ Simple Method

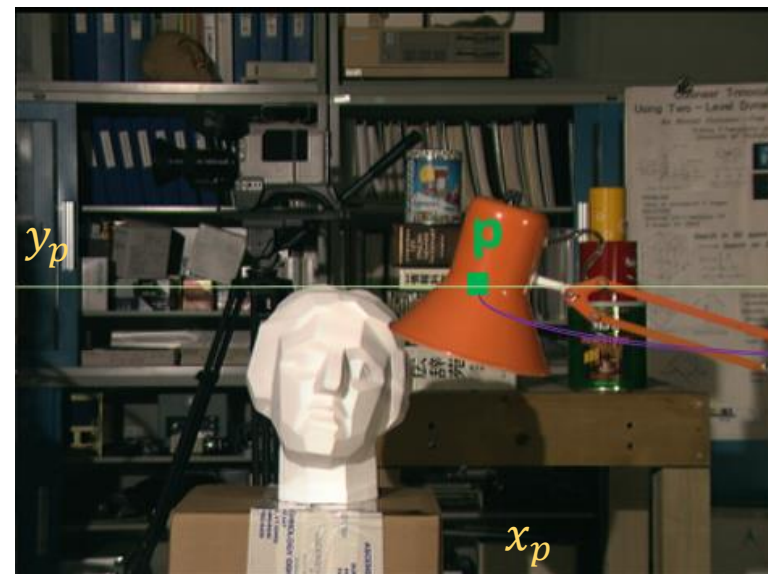
$L$  is the left image

$R$  is the right image

$L(\mathbf{p})$  is the (vector) value of  $\mathbf{p}$

$$\mathbf{p} = \begin{bmatrix} x_p \\ y_p \end{bmatrix} = \begin{bmatrix} 234 \\ 140 \end{bmatrix}$$

$$D = 16$$



Left Image



Right Image

$$\begin{bmatrix} x_p - 0 \\ y_p \end{bmatrix} = \begin{bmatrix} 234 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 1 \\ y_p \end{bmatrix} = \begin{bmatrix} 233 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 2 \\ y_p \end{bmatrix} = \begin{bmatrix} 232 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 3 \\ y_p \end{bmatrix} = \begin{bmatrix} 231 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 4 \\ y_p \end{bmatrix} = \begin{bmatrix} 230 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 5 \\ y_p \end{bmatrix} = \begin{bmatrix} 229 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 6 \\ y_p \end{bmatrix} = \begin{bmatrix} 228 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 7 \\ y_p \end{bmatrix} = \begin{bmatrix} 227 \\ 140 \end{bmatrix}$$

$q$

$$\begin{bmatrix} x_p - 8 \\ y_p \end{bmatrix} = \begin{bmatrix} 226 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 9 \\ y_p \end{bmatrix} = \begin{bmatrix} 225 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 10 \\ y_p \end{bmatrix} = \begin{bmatrix} 224 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 11 \\ y_p \end{bmatrix} = \begin{bmatrix} 223 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 12 \\ y_p \end{bmatrix} = \begin{bmatrix} 222 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 13 \\ y_p \end{bmatrix} = \begin{bmatrix} 221 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 14 \\ y_p \end{bmatrix} = \begin{bmatrix} 220 \\ 140 \end{bmatrix}$$

$$\begin{bmatrix} x_p - 15 \\ y_p \end{bmatrix} = \begin{bmatrix} 219 \\ 140 \end{bmatrix}$$

# Stereo Matching

## ❖ Simple Method

$L$  is the left image

$R$  is the right image

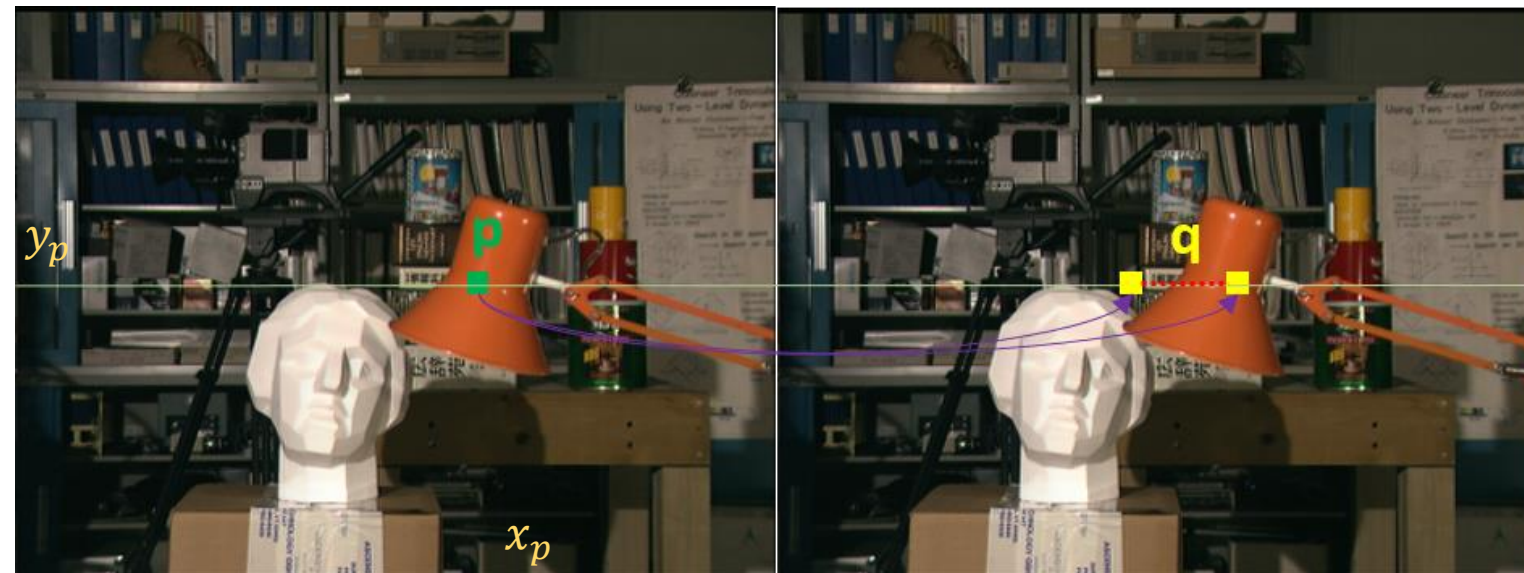
$L(\mathbf{p})$  is the (vector) value of  $\mathbf{p}$

$$\mathbf{p} = \begin{bmatrix} x_p \\ y_p \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

$$d \in D$$

$$C_1(\mathbf{p}, \mathbf{d}) = |L(\mathbf{p}) - R(\mathbf{p} - \mathbf{d})|$$



Left Image

Right Image

Finding  $d$  so that  $C_1(\mathbf{p}, \mathbf{q}, d)$  is minimum.

$$d = \underset{d \in D}{\operatorname{argmin}} (C_1(\mathbf{p}, \mathbf{d}))$$

Then,  $d$  is the value for the pixel  $\mathbf{p}$  in disparity map

# Stereo Matching

## ❖ Simple Method : Result

$$d = \begin{bmatrix} d \\ 0 \end{bmatrix} \quad d \in D$$

$$C_1(p, d) = |L(p) - R(p - d)|$$

Finding  $d$  so that  $C_1(p, d)$  is minimum.

$$d = \underset{d \in D}{\operatorname{argmin}}(C_1(p, d))$$

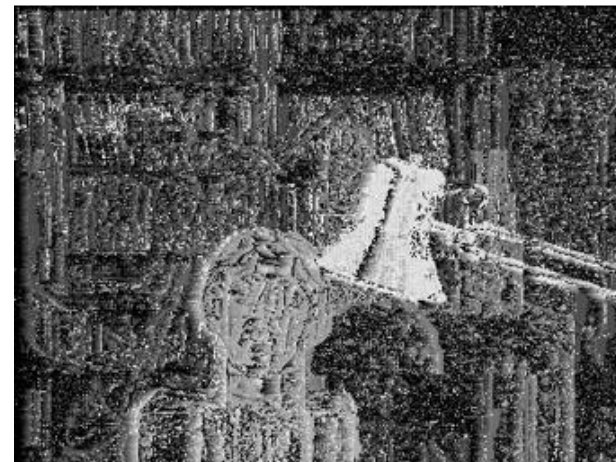
Then,  $d$  is the value for the pixel  $p$  in disparity map



Left Image



Right Image



Disparity Map



Ground Truth



# Stereo Matching

## ❖ Method 1: Implementation

$$\mathbf{d} = \begin{bmatrix} d \\ 0 \end{bmatrix} \quad d \in D$$

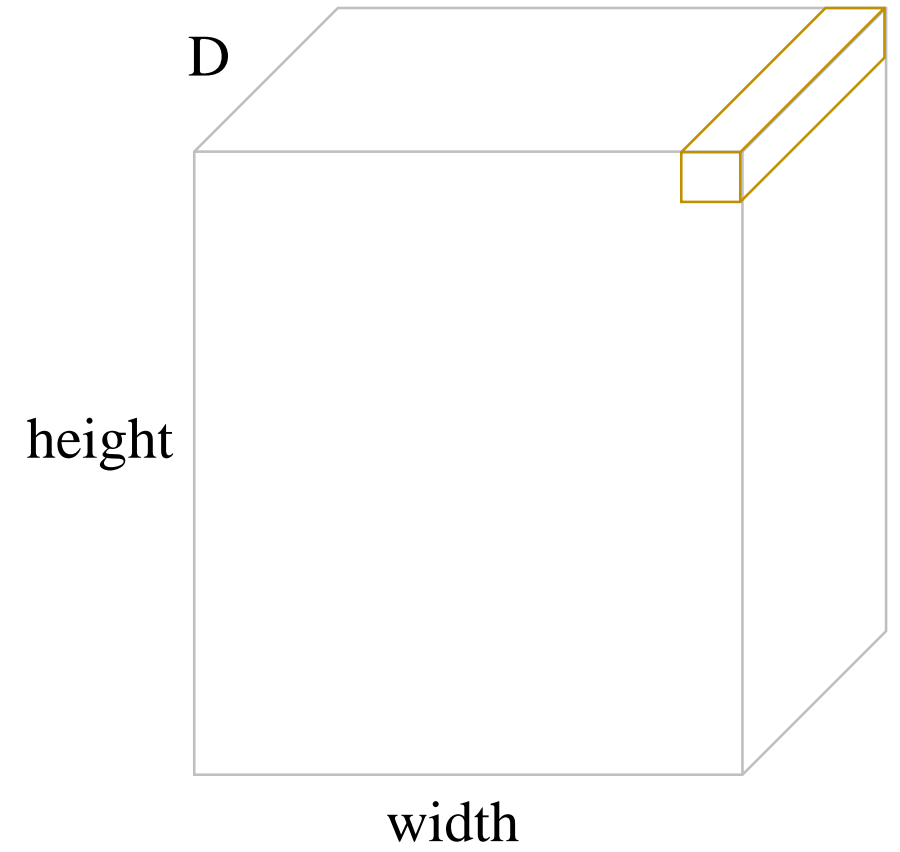
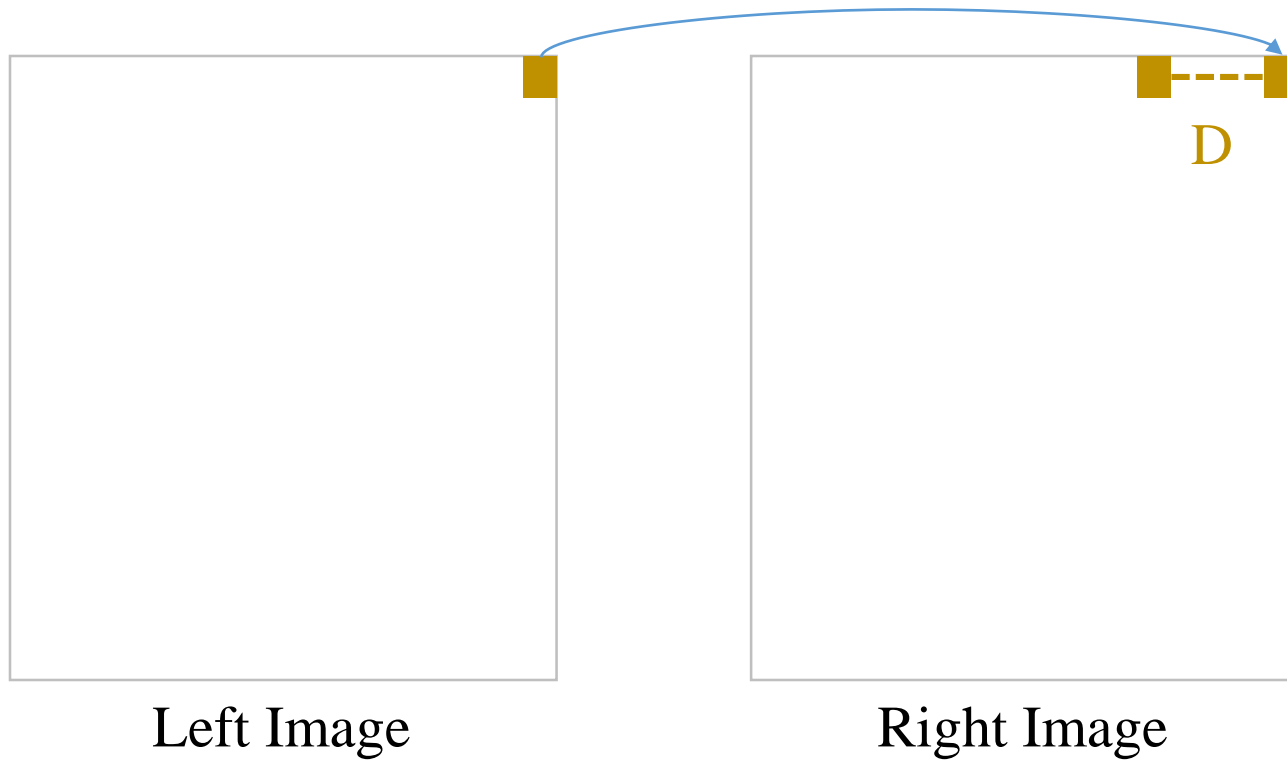
$$C_1(\mathbf{p}, \mathbf{d}) = |L(\mathbf{p}) - R(\mathbf{p} - \mathbf{d})|$$

Finding  $d$  so that  $C_1(\mathbf{p}, \mathbf{d})$  is minimum.

$$d = \underset{d \in D}{\operatorname{argmin}}(C_1(\mathbf{p}, \mathbf{d}))$$

Then,  $d$  is the value for the pixel  $\mathbf{p}$  in disparity map

```
16 # disparity map
17 depth = np.zeros((height, width), np.uint8)
18 scale = 255 / disparity_range
19
20 for y in range(height):
21     for x in range(width):
22
23         # find argmin_d
24         disparity = 0
25         cost_min = abs(left[y, x] - right[y, x])
26         for d in range(disparity_range):
27             if (x - d) < 0:
28                 cost = 255
29             else:
30                 cost = abs(left[y, x] - right[y, x - d])
31
32             # update cost_min
33             if cost < cost_min:
34                 cost_min = cost
35                 disparity = d
36
37         # set to the disparity map
38         depth[y, x] = disparity*scale
39
40 # save
41 cv2.imwrite('images/disparity_ad.png', depth)
```

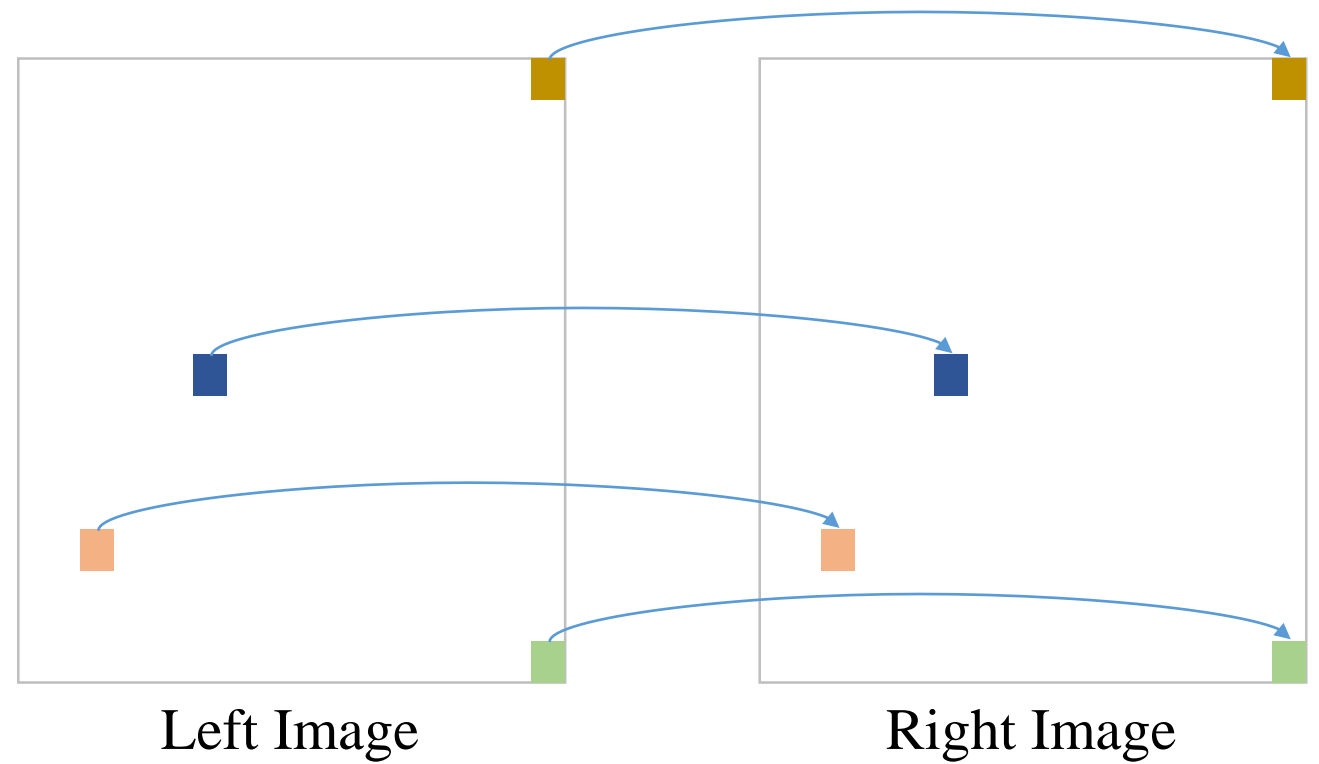
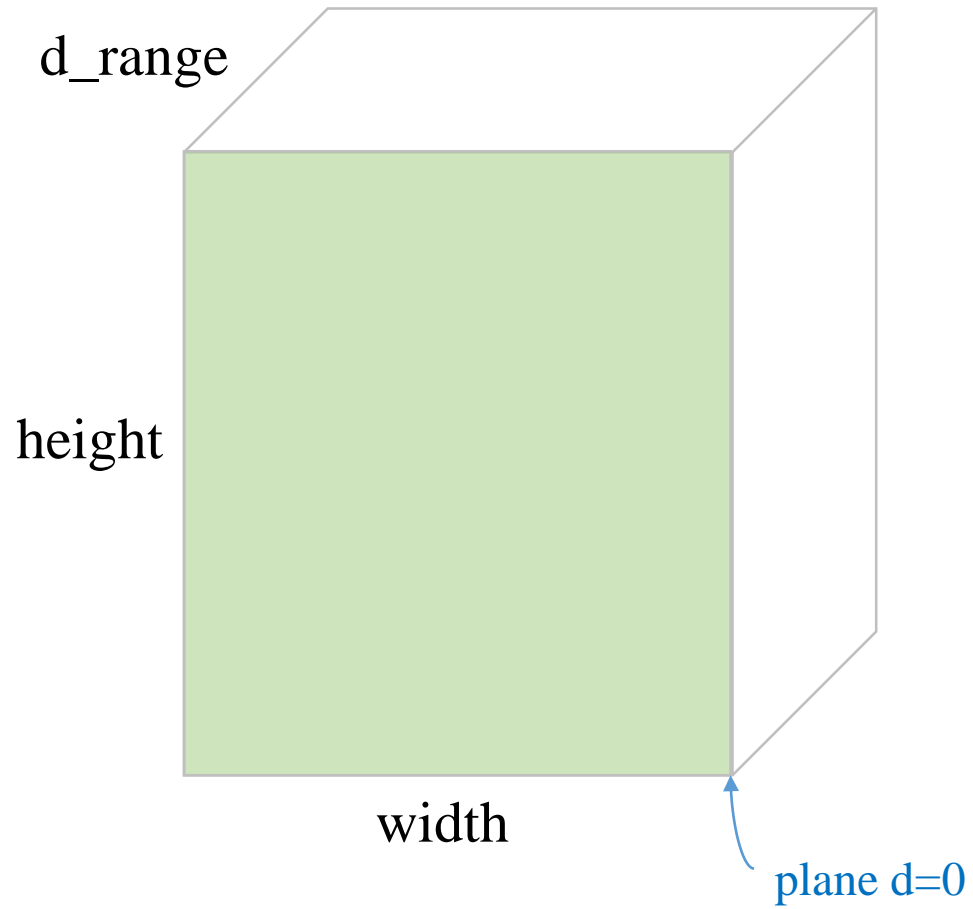


Normal approach

Each pixel  $p$  in the left image has  $D$  candidate pixel  $q$  in the right image.

Then,  $D$  cost values are computed from  $D$  pairs  $(p, q)$

# Stereo Matching

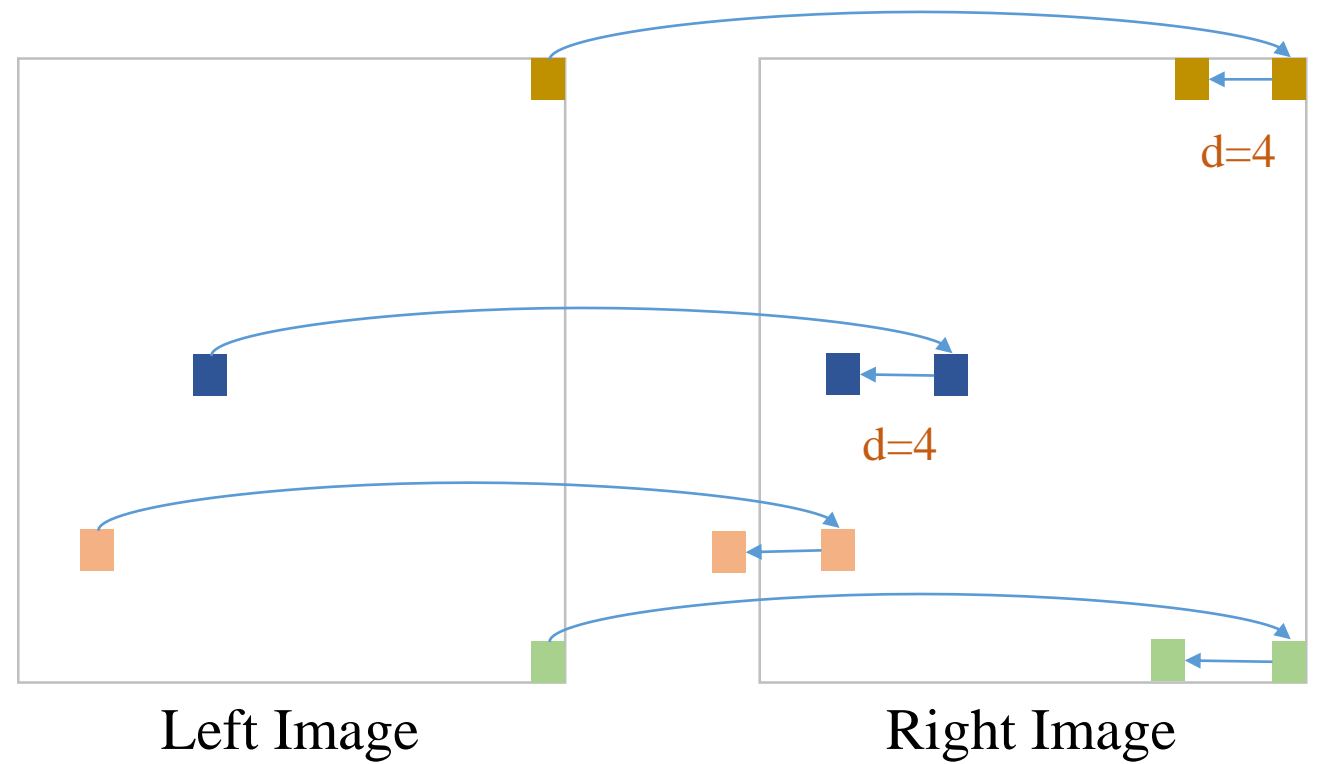
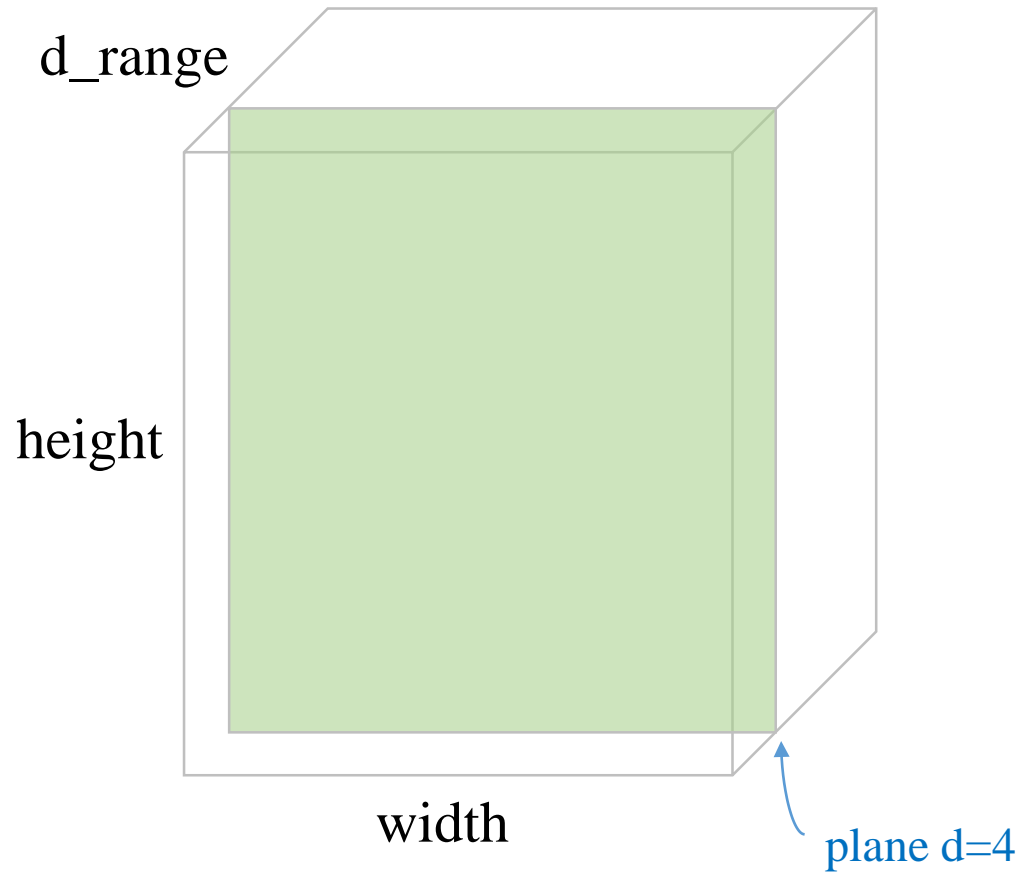


Cost values for a disparity  $d=0$

Given  $C_d$  is a cost plane for a disparity  $d$

$$C_0 = |L - R|$$

# Stereo Matching



Cost values for a disparity  $d=4$

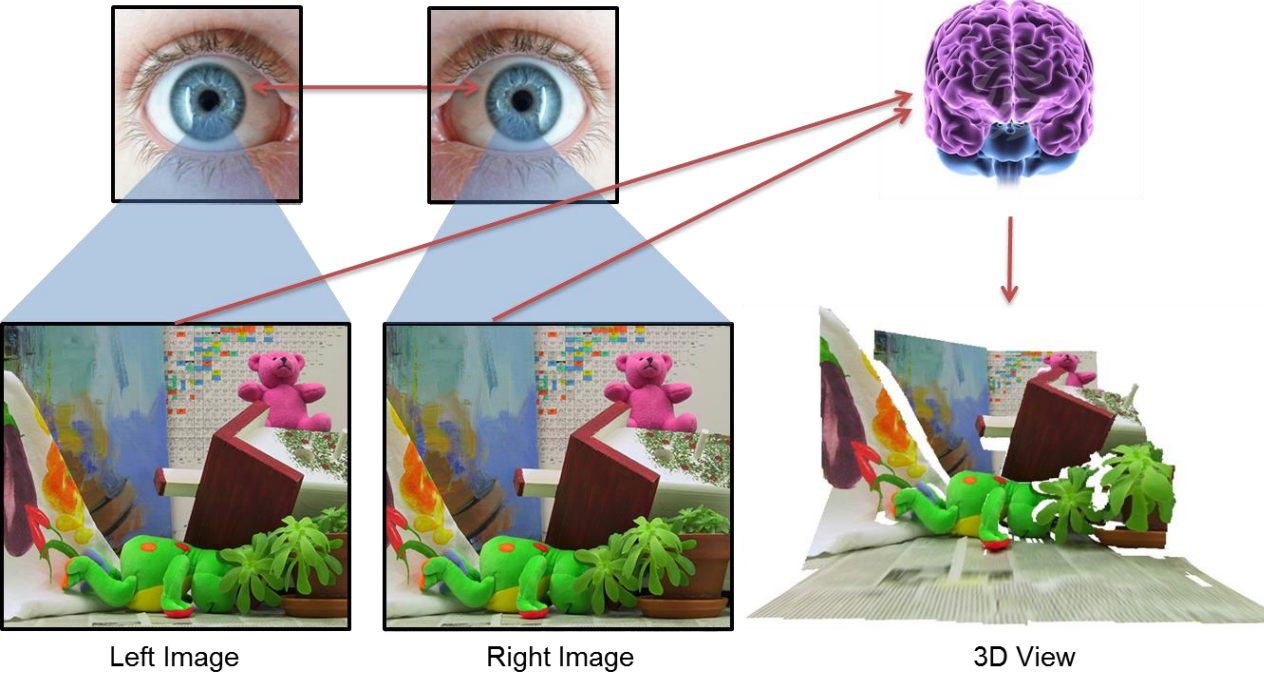


Human-Eye

Brain

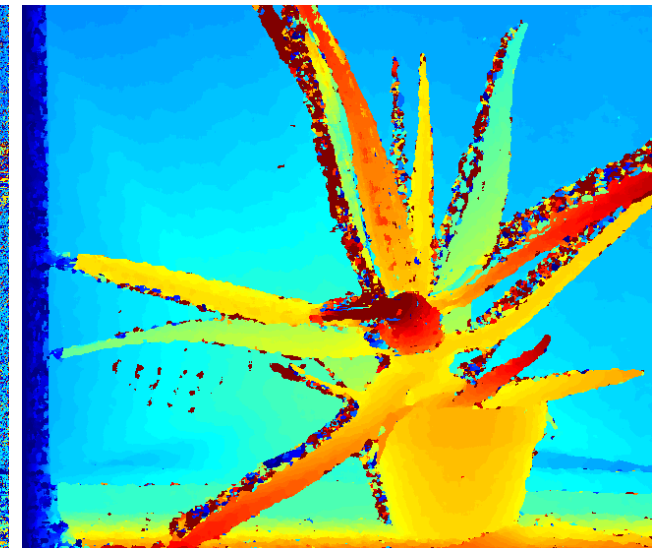
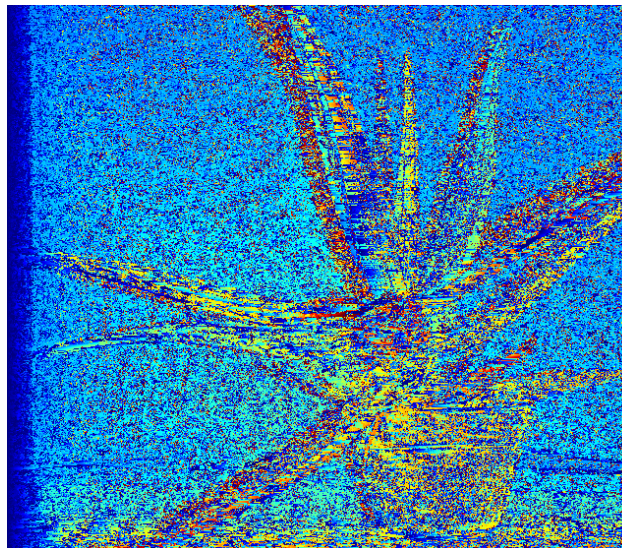
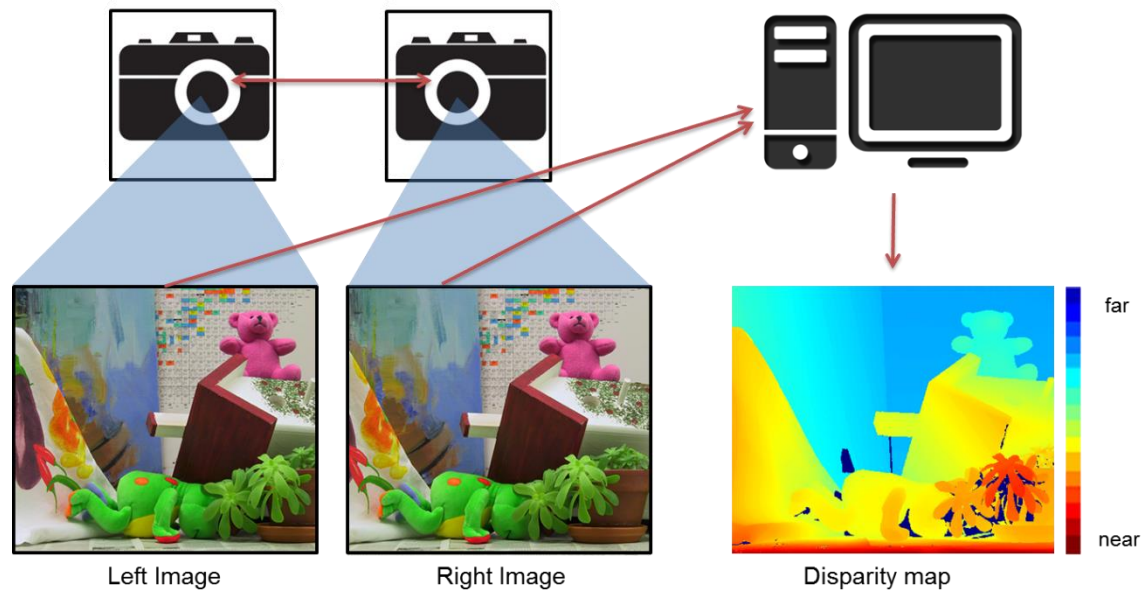
Mean

❖ Stereo matching



Stereo Camera

Computer



# Outline

- Central limit theorem
- Mean and its Applications
- Median and its Applications
- Variance and its Applications
- Quick Introduction to Correlation

# Median

## ❖ Definition

### Data

$$X = \{X_1, \dots, X_N\}$$

### Formula

Step 1: Sort  $X \rightarrow S$

Step 2

If  $N$  is odd, then  $m = S_{\left(\frac{N+1}{2}\right)}$

If  $N$  is even, then  $m = \left(S_{\left(\frac{N}{2}\right)} + S_{\left(\frac{N}{2}+1\right)}\right) / 2$

### Given the data

$$X = \{2, 8, 5, 4, 1\}$$

$$N = 5$$

Step 1

$$S = \{1, 2, 4, 5, 8\}$$

1   2   3   4   5

Step 2;  $N = 5$

$$k = \frac{N + 1}{2} = 3$$

$$m = S_k = 4$$



# Median

## ❖ Definition

### Data

$$X = \{X_1, \dots, X_N\}$$

### Formula

Step 1: Sort  $X \rightarrow S$

Step 2

If  $N$  is odd, then  $m = S_{\left(\frac{N+1}{2}\right)}$

If  $N$  is even, then  $m = \left(S_{\left(\frac{N}{2}\right)} + S_{\left(\frac{N}{2}+1\right)}\right) / 2$

### Given the data

$$X = \{2, 8, 5, 4, 1, 8\}$$

$$N = 6$$

Step 1

$$S = \{1, 2, 4, 5, 8, 8\}$$

1   2   3   4   5   6

Step 2;  $N = 6$

$$\begin{aligned} m &= \frac{S_3 + S_4}{2} \\ &= \frac{4 + 5}{2} = 4.5 \end{aligned}$$



# Median

## ❖ Code

Data  $X = \{X_1, \dots, X_N\}$

### Formula

Step 1: Sort  $X \rightarrow S$

Step 2

If  $N$  is odd, then  $m = S_{\left(\frac{N+1}{2}\right)}$

If  $N$  is even, then  $m = \left(S_{\left(\frac{N}{2}\right)} + S_{\left(\frac{N}{2}+1\right)}\right) / 2$

```
1. def calculate_median(numbers):
2.     N = len(numbers)
3.     numbers.sort()
4.     if N%2 == 0:
5.         m1 = N/2
6.         m2 = (N/2) + 1
7.         m1 = int(m1)-1
8.         m2 = int(m2)-1
9.         median = (numbers[m1] + numbers[m2])/2
10.    else:
11.        m = (N+1)/2
12.        m = int(m)-1
13.        median = numbers[m]
14.    return median
```

# Median

## ❖ Image Denoising



Input Image



(3x3) kernel



(5x5) kernel

# Median

## ❖ Image Denoising

```
1 import numpy as np
2 import cv2
3
4 img1 = cv2.imread('mrbean_noise.jpg')
5 img2 = cv2.medianBlur(img1, 3)
6
7 # show images
8 cv2.imshow('img1', img1)
9 cv2.imshow('img2', img2)
10
11 # waiting for any keys pressed and clo
12 cv2.waitKey(0)
13 cv2.destroyAllWindows()
```

Input Image



(3x3) kernel



# Mean and Median

## ❖ Comparison

### Data

$$X = \{X_1, \dots, X_N\}$$

### Formula

$$\mu = \frac{1}{N} \sum_{i=1}^N X_i$$

### Formula

Step 1: Sort  $X \rightarrow S$

Step 2

If  $N$  is odd, then  $m = S_{\left(\frac{N+1}{2}\right)}$

If  $N$  is even, then  $m = \left(S_{\left(\frac{N}{2}\right)} + S_{\left(\frac{N}{2}+1\right)}\right) / 2$

# Outline

- **Central limit theorem**
- **Mean and its Applications**
- **Median and its Applications**
- **Variance and its Applications**
- **Quick Introduction to Correlation**

# Variance

## ❖ Definition

**Formula:**

$$\text{mean } \mu = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\text{variance } \text{var}(X) = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2$$

$$\text{Standard deviation } \sigma = \sqrt{\text{var}(X)}$$

**Example:**  $X = \{5, 3, 6, 7, 4\}$

$$\mu = \frac{1}{5} \sum_{i=1}^n (5 + 3 + 6 + 7 + 4) = \frac{25}{5} = 5$$

$$\begin{aligned} \text{var}(X) &= \frac{1}{5} [(5 - 5)^2 + (3 - 5)^2 + (6 - 5)^2 + \\ &\quad (7 - 5)^2 + (4 - 5)^2] \\ &= \frac{1}{5} (0 + 4 + 1 + 4 + 1) = 2 \end{aligned}$$

$$\sigma = \sqrt{\text{var}(X)} = 1.41$$

Ignore the differences between population and sample!!!



# Variance

## Formula

mean

$$E(X) = \sum_{i=1}^N X_i P_X(X_i)$$

variance

$$\begin{aligned} \text{var}(X) &= E\left((X - E(X))^2\right) \\ &= \sum_{i=1}^N (X_i - E(X))^2 P_X(X_i) \end{aligned}$$

Standard  
deviation

$$\sigma = \sqrt{\text{var}(X)}$$

**Example:**  $X = \{5, 3, 6, 7, 4\}$

$$\begin{aligned} E(X) &= 5 \times \frac{1}{5} + 3 \times \frac{1}{5} + 6 \times \frac{1}{5} + 7 \times \frac{1}{5} + 4 \times \frac{1}{5} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{var}(X) &= \frac{1}{5} [(5 - 5)^2 + (3 - 5)^2 + (6 - 5)^2 + \\ &\quad (7 - 5)^2 + (4 - 5)^2] \\ &= \frac{1}{5} (0 + 4 + 1 + 4 + 1) = 2 \end{aligned}$$

$$\sigma = \sqrt{\text{var}(X)} = 1.41$$

# Variance

## Formula

### mean

$$E(X) = \sum_{i=1}^N X_i P_X(X_i)$$

### variance

$$\begin{aligned} \text{var}(X) &= E\left((X - E(X))^2\right) \\ &= \sum_{i=1}^N (X_i - E(X))^2 P_X(X_i) \end{aligned}$$

### Standard deviation

$$\sigma = \sqrt{\text{var}(X)}$$

$$\begin{aligned} \text{var}(X) &= \sum_{i=1}^N (X_i - E(X))^2 P_X(X_i) \\ &= \sum_{i=1}^N (X_i^2 - 2X_i E(X) + E(X)^2) P_X(X_i) \\ &= \sum_{i=1}^N X_i^2 P_X(X_i) - \sum_{i=1}^N 2X_i E(X) P_X(X_i) \\ &\quad + \sum_{i=1}^N E(X)^2 P_X(X_i) \\ &= E(X^2) - 2E(X) \left[ \sum_{i=1}^N X_i P_X(X_i) \right] + E(X)^2 \\ &= E(X^2) - (E(X))^2 \end{aligned}$$



# Variance

```
1 import numpy as np
2
3 x = np.array([5, 3, 6, 7, 4])
4
5 var = (x - x.mean())**2
6 var = np.mean(var)
7 print(var)
8
9 sdv = np.sqrt(var)
10 print(sdv)
```

2.0

1.4142135623730951

$$\text{mean } \mu = \frac{1}{n} \sum_{k=1}^n x_i$$

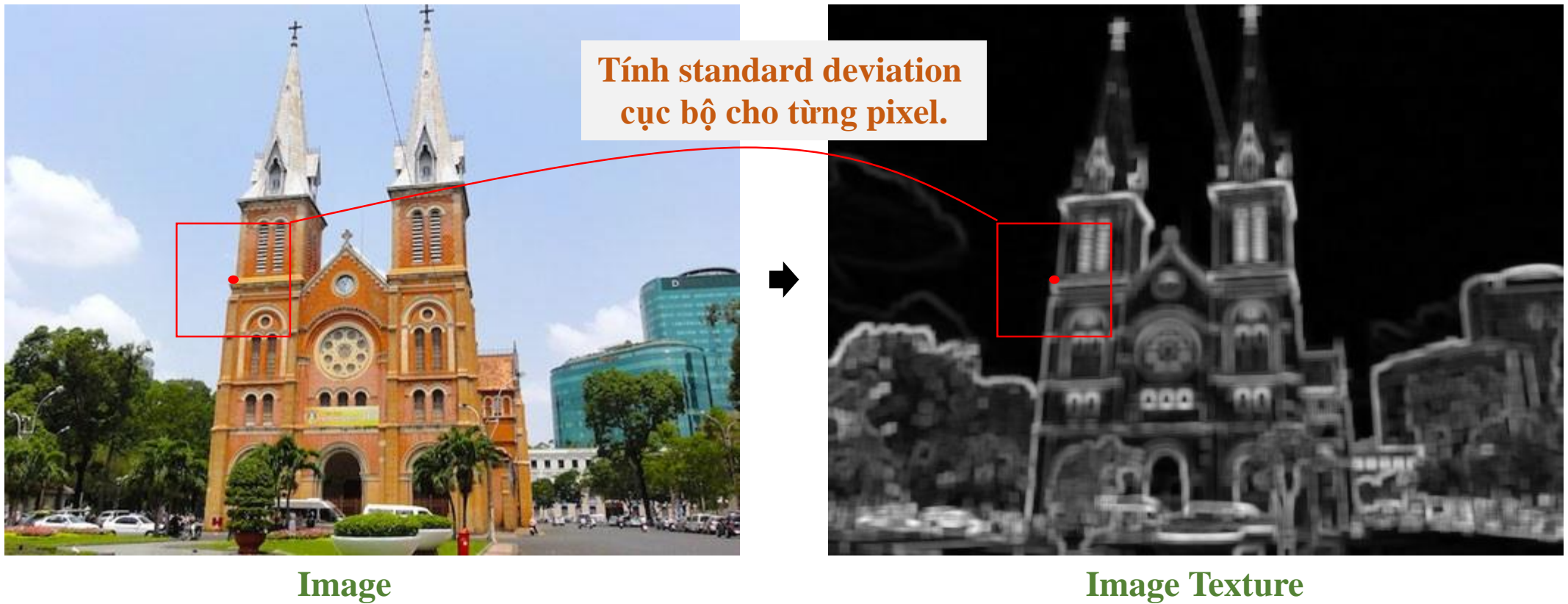
$$\text{variance } \text{var}(X) = \frac{1}{n} \sum_{k=1}^n (x_i - \mu)^2$$

$$\text{Standard deviation } \sigma = \sqrt{\text{var}(X)}$$

```
1 # variance
2 def calculate_mean(numbers):
3     s = sum(numbers)
4     N = len(numbers)
5     mean = s/N
6     return mean
7
8 def caculate_variance(numbers):
9     mean = calculate_mean(numbers)
10
11     diff = []
12     for num in numbers:
13         diff.append(num-mean)
14
15     squared_diff = []
16     for d in diff:
17         squared_diff.append(d**2)
18
19     sum_squared_diff = sum(squared_diff)
20     variance = sum_squared_diff/len(numbers)
21
22     return variance
```

# Variance

Ứng dụng tính chất của variance (~standard deviation) để tìm texture cho một hình





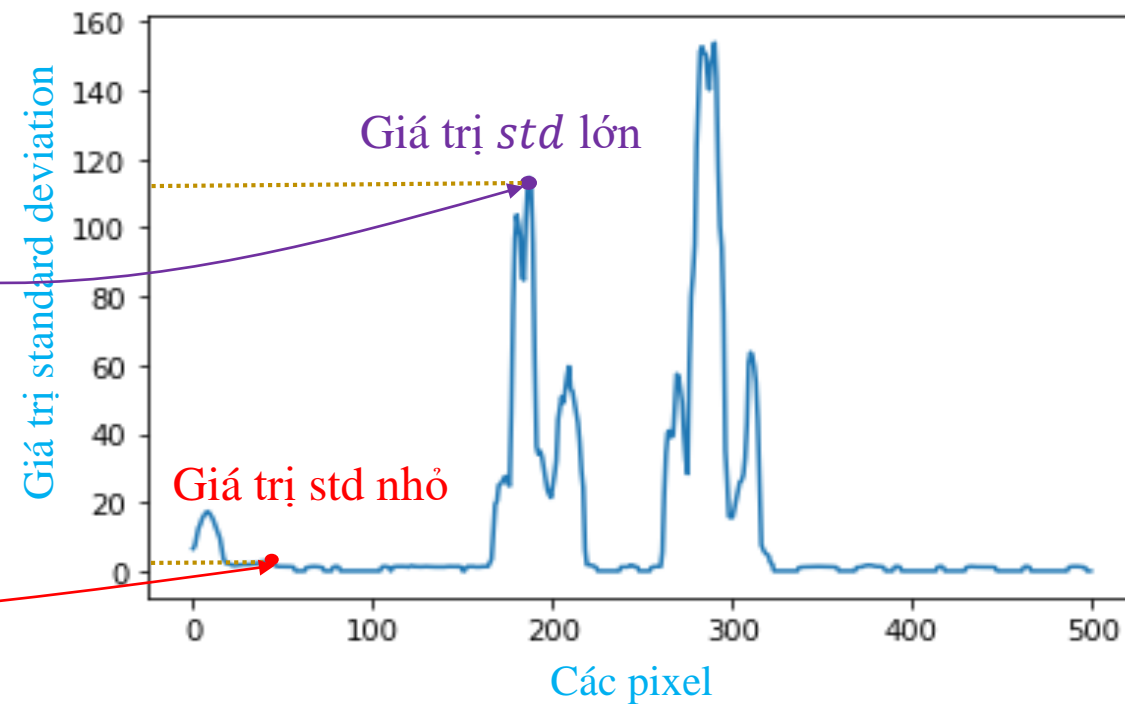
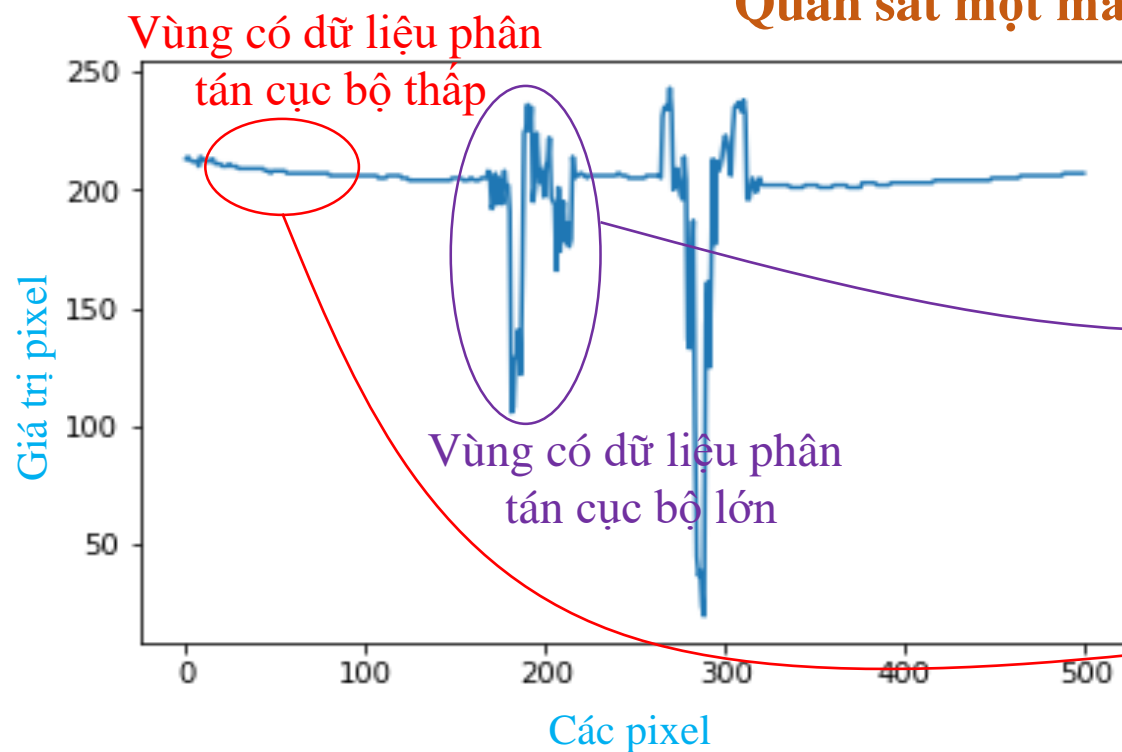
Ảnh gốc

Tính standard deviation  
cục bộ cho từng pixel.



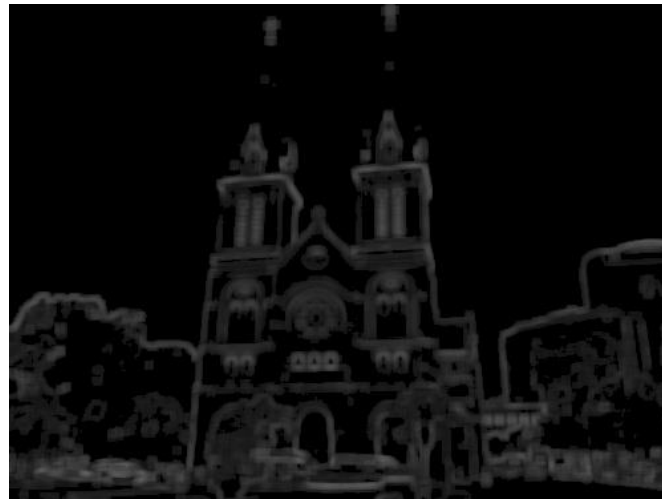
Ảnh thông tin texture

## Quan sát một mảng các giá trị pixel.



# Variance

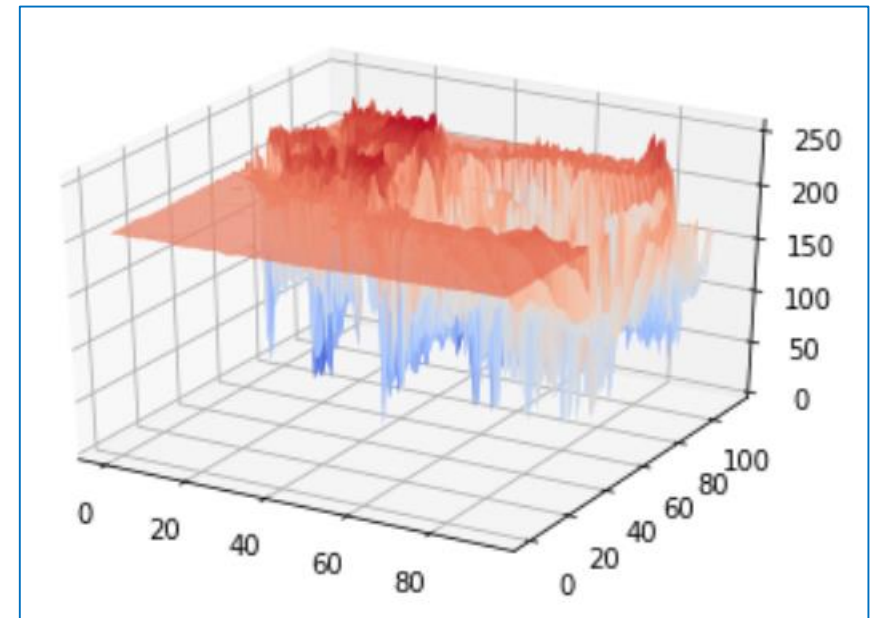
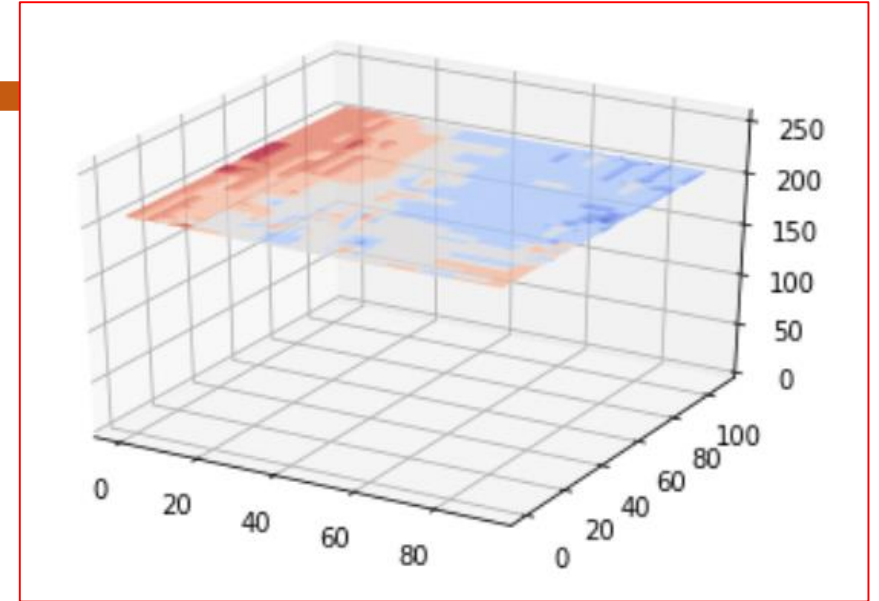
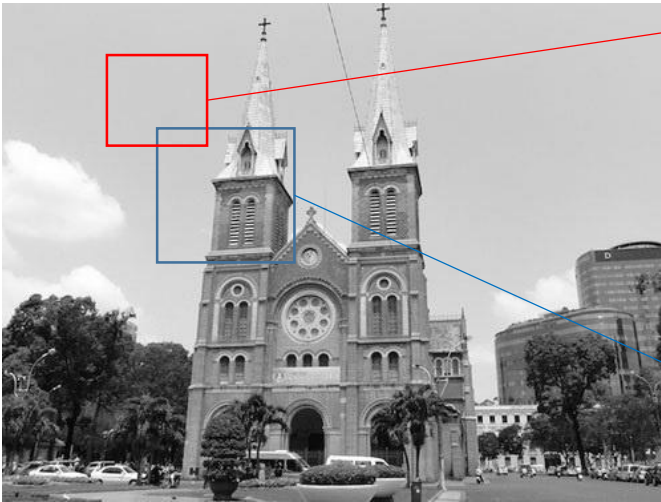
## ❖ Implementation





# Variance

## ❖ Implementation



# Variance

## ❖ Implementation

```
1 import numpy as np
2 import cv2
3 import math
4 from scipy.ndimage.filters import generic_filter
5
6 img = cv2.imread('img.jpg')
7 gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
8 cv2.imwrite('edge_s1.jpg', gray)
9
10 x = gray.astype('float')
11 x_filt = generic_filter(x, np.std, size=7)
12 cv2.imwrite('edge_s2.jpg', x_filt)
13
14 x_filt[x_filt < 20] = 0
15 cv2.imwrite('edge_s3.jpg', x_filt)
16
17 maxv = np.max(x_filt)
18 print(maxv)
19
20 x_filt = x_filt*2.5
21 cv2.imwrite('edge_s4.jpg', x_filt)
```



# Outline

- **Central limit theorem**
- **Mean and its Applications**
- **Median and its Applications**
- **Variance and its Applications**
- **Quick Introduction to Correlation**

# Correlation Coefficient

## ❖ Definition

ignore the differences between  
population and sample

**Công thức:** Gọi  $x, y$  là hai biến ngẫu nhiên

$$\begin{aligned}\rho_{xy} &= \frac{E[(x - \mu_x)(y - \mu_y)]}{\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}} \\ &= \frac{n(\sum_i x_i y_i) - (\sum_i x_i)(\sum_i y_i)}{\sqrt{n \sum_i x_i^2 - (\sum_i x_i)^2} \sqrt{n \sum_i y_i^2 - (\sum_i y_i)^2}}\end{aligned}$$

### Tính chất 1

$$\begin{array}{ccc} -1 & \leq & \rho_{xy} \leq 1 \\ \leftarrow & & \rightarrow \\ \text{Tương quan} & & \text{Tương quan} \\ \text{nghịch} & & \text{thuận} \end{array}$$

### Tính chất 2

$$\rho_{xy} = \rho_{uv}$$

trong đó

$$u = ax + b$$

$$v = cy + d$$

### Ví dụ 1

$$x = [7, 18, 29, 2, 10, 9, 9]$$

$$y = [1, 6, 12, 8, 6, 21, 10]$$

$$\begin{aligned}\rho_{xy} &= \frac{E[(x - \mu_x)(y - \mu_y)]}{\sqrt{\text{var}(x)}\sqrt{\text{var}(y)}} \\ &= \frac{n * 818 - 84 * 64}{\sqrt{n * 1480 - 7056} \sqrt{n * 822 - 4096}} = 0.149\end{aligned}$$

### Ví dụ 2

$$u = 2 * x - 14 = [0, 22, 44, -10, 6, 4, 4]$$

$$v = y + 2 = [3, 8, 14, 10, 8, 23, 12]$$

$$\begin{aligned}\rho_{uv} &= \frac{E[(u - \mu_u)(v - \mu_v)]}{\sqrt{\text{var}(u)}\sqrt{\text{var}(v)}} \\ &= \frac{n * 880 - 70 * 78}{\sqrt{n * 2588 - 4900} \sqrt{n * 1106 - 6084}} = 0.149\end{aligned}$$



mean-median/ Mean-Demo - Jupyter Notebook NCC - Jupyter Notebook Thông tin cuộc họp - Zoom

localhost:8888/notebooks/mean-median/Mean-Demo.ipynb 110%

Bắt đầu Google IELTS Writing Task 1 S... Intensive IELTS Vietnamese-German ... Video Conferencing, ...

jupyter Mean-Demo Last Checkpoint: 9 giờ trước (unsaved changes) Logout

File Edit View Insert Cell Kernel Widgets Help Trusted Python 3

```
17
18     # Display
19     cv2.imshow('img', img)
20
21     # Stop if escape key is pressed
22     if cv2.waitKey(33) == ord('a'):
23         break
24
25     template = gray[300:400, 200:400]
26     w, h = template.shape[::-1]
27     cv2.imshow('template', template)
28     cv2.waitKey(0)
29
30     while True:
31         # Read the frame and Convert to grayscale
32         _, img = cap.read()
33         gray = cv2.cvtColor(img, cv2.COLOR_BGR2GRAY)
34
35         corr_map = cv2.matchTemplate(gray, template, cv2.TM_CCOEFF_NORMED)
36         min_val, max_val, min_loc, max_loc = cv2.minMaxLoc(corr_map)
37
38         # take minimum
39         top_left = max_loc
40         bottom_right = (top_left[0] + w, top_left[1] + h)
41
42         # draw
43         cv2.rectangle(img, top_left, bottom_right, (0, 255, 0), 2)
44
45
46         # Display
47         cv2.imshow('img', img)
48
49         # Stop if escape key is pressed
50         if cv2.waitKey(33) == ord('a'):
51             break
52
53     # Release the VideoCapture object
54     cap.release()
55     cv2.destroyAllWindows()
```

In [ ]: 1

