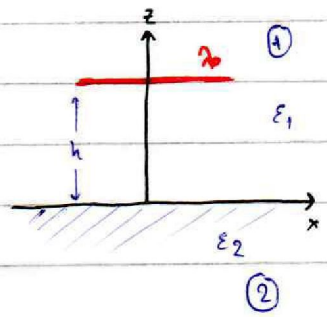
ΗΜΜΥ ΕΜΠ
5ο ΕΞΑΜΗΝΟ

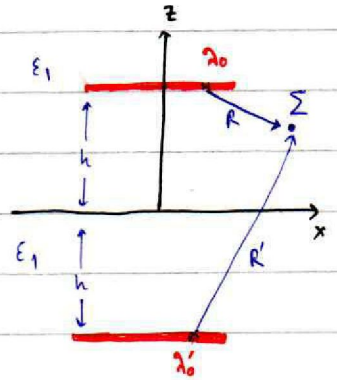
ΗΜ ΠΕΔΙΑ Β'
2η Σειρά Ασκήσεων

[redacted]

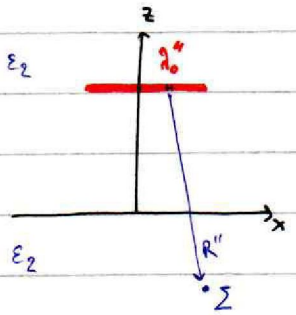
Άσκηση 7



Υποπρόβλημα 1



Υποπρόβλημα 2



(α) Χρησιμοποιούμε τη μέθοδο των ειδώλων

Για να βρούμε το δυναμικό στην περιοχή ① παίρνουμε είδωλο γυμνόσημο λ_0' συμμετρικά από τον x άξονα. Θεωρώντας παντού $\varepsilon = \varepsilon_1$ (υποπρόβλημα 1) με $\lambda_0' = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}\lambda_0$

$$\vec{R} = (x-x')\hat{i}_x + y\hat{i}_y + (z-h)\hat{i}_z \Rightarrow R = \left[(x-x')^2 + y^2 + (z-h)^2\right]^{1/2}$$
$$\vec{R}' = (x-x')\hat{i}_x + y\hat{i}_y + (z+h)\hat{i}_z \Rightarrow R' = \left[(x-x')^2 + y^2 + (z+h)^2\right]^{1/2}$$

Από αρχή επαλληλίας είναι $\Phi_1 = \frac{\lambda_0}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'}{R} + \frac{\lambda_0'}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'}{R'} \Rightarrow$

$$\Rightarrow \Phi_1(x,y,z) = \frac{\lambda_0}{4\pi\varepsilon_1}\ln\frac{L-x+\left[(x-L)^2+y^2+(z-h)^2\right]^{1/2}}{-L-x+\left[(x+L)^2+y^2+(z-h)^2\right]^{1/2}} + \frac{\lambda_0'}{4\pi\varepsilon_1}\ln\frac{L-x+\left[(x-L)^2+y^2+(z+h)^2\right]^{1/2}}{-L-x+\left[(x+L)^2+y^2+(z+h)^2\right]^{1/2}}, \quad z \geq 0$$

Για να βρούμε το δυναμικό στην περιοχή ② θεωρούμε παντού $\varepsilon = \varepsilon_2$ και γυμνόσημα $\lambda_0'' = \frac{2\varepsilon_2}{\varepsilon_1+\varepsilon_2}\lambda_0$ (υποπρόβλημα 2)

$$\vec{R}'' = (x-x'')\hat{i}_x + y\hat{i}_y + (z-h)\hat{i}_z \Rightarrow R'' = \left[(x-x'')^2 + y^2 + (z-h)^2\right]^{1/2}$$

Από αρχή επαλληλίας είναι $\Phi_2 = \frac{\lambda_0''}{4\pi\varepsilon_2}\int_{-L}^{L}\frac{dx''}{R''} \Rightarrow \Phi_2(x,y,z) = \frac{\lambda_0''}{4\pi\varepsilon_2}\ln\frac{L-x+\left[(x-L)^2+y^2+(z-h)^2\right]^{1/2}}{-L-x+\left[(x+L)^2+y^2+(z-h)^2\right]^{1/2}}, \quad z \leq 0$

(6) Στην περιοχή (1) είναι $\vec{E}_1 = \frac{\lambda_0}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,\hat{i}_R}{R^2} + \frac{\lambda_0'}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,\hat{i}_{R'}}{R'^2} = \frac{\lambda_0}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,\vec{R}}{R^3} + \frac{\lambda_0'}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,\vec{R}'}{R'^3} =$

$$= \frac{\lambda_0}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,[(x-x')\hat{i}_x + y\hat{i}_y + (z-h)\hat{i}_z]}{[(x-x')^2+y^2+(z-h)^2]^{3/2}} + \frac{\lambda_0'}{4\pi\varepsilon_1}\int_{-L}^{L}\frac{dx'\,[(x-x')\hat{i}_x + y\hat{i}_y + (z+h)\hat{i}_z]}{[(x-x')^2+y^2+(z+h)^2]^{3/2}} \Rightarrow$$

$$\vec{E}_1(x,y,z) = (x\hat{i}_x + y\hat{i}_y + (z-h)\hat{i}_z)\cdot\left(\frac{\lambda_0}{4\pi\varepsilon_1}\frac{L-x}{(y^2+(z-h)^2)[(x-L)^2+y^2+(z-h)^2]^{1/2}} - \frac{\lambda_0}{4\pi\varepsilon_1}\frac{-L-x}{(y^2+(z-h)^2)[(x+L)^2+y^2+(z-h)^2]^{1/2}}\right) +$$

$$+ (x\hat{i}_x + y\hat{i}_y + (z+h)\hat{i}_z)\cdot\left(\frac{\lambda_0'}{4\pi\varepsilon_1}\frac{L-x}{(y^2+(z+h)^2)[(x-L)^2+y^2+(z+h)^2]^{1/2}} - \frac{\lambda_0'}{4\pi\varepsilon_1}\frac{-L-x}{(y^2+(z+h)^2)[(x+L)^2+y^2+(z+h)^2]^{1/2}}\right)$$

για $z>0$

Στην περιοχή (2) είναι $\vec{E}_2 = \frac{\lambda_0''}{4\pi\varepsilon_2}\int_{-L}^{L}\frac{dx''\,\hat{i}_{R''}}{R''^2} = \frac{\lambda_0''}{4\pi\varepsilon_2}\int_{-L}^{L}\frac{dx''\,\vec{R}''}{R''^3} = \frac{\lambda_0''}{4\pi\varepsilon_2}\int_{-L}^{L}\frac{dx''\,[(x-x'')\hat{i}_x + y\hat{i}_y + (z-h)\,i_z]}{[(x-x'')^2+y^2+(z-h)^2]^{3/2}} \Rightarrow$

$$\vec{E}_2(x,y,z) = (x\hat{i}_x + y\hat{i}_y + (z-h)\hat{i}_z)\cdot\left(\frac{\lambda_0''}{4\pi\varepsilon_2}\frac{L-x}{(y^2+(z-h)^2)[(x-L)^2+y^2+(z-h)^2]^{1/2}} - \frac{\lambda_0''}{4\pi\varepsilon_2}\frac{-L-x}{(y^2+(z-h)^2)[(x+L)^2+y^2+(z-h)^2]^{1/2}}\right)$$

για $z<0$

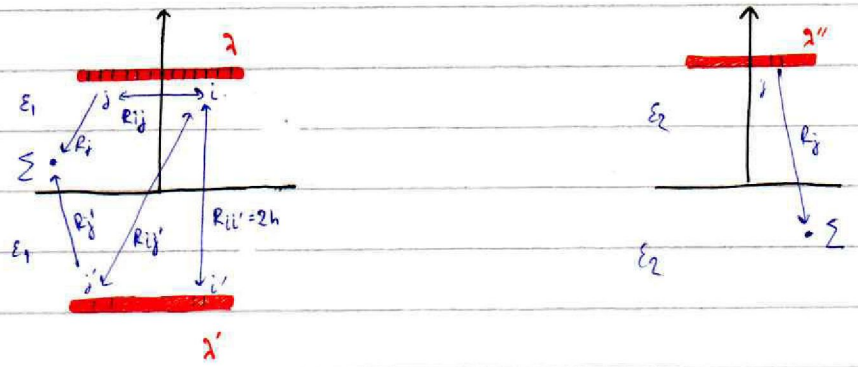
Ειδικη περιπτωση πανω στον αξονα z:

$$\vec{E}_1(x=0,y=0,z) = \hat{i}_z\left(\frac{\lambda_0}{4\pi\varepsilon_1}\frac{2L}{\sqrt{L^2+(z-h)^2}} + \frac{\lambda_0'}{4\pi\varepsilon_1}\frac{2L}{\sqrt{L^2+(z+h)^2}}\right), \quad z>0 \neq h$$

και $\vec{E}_1(x=0,y=0,z=h) = \infty\,\hat{i}_z$

$$\vec{E}_2(x=0,y=0,z) = \hat{i}_z\frac{\lambda_0''}{4\pi\varepsilon_2}\frac{2L}{\sqrt{L^2+(z-h)^2}}, \quad z<0$$

(γ)



Χωρίζουμε τον αγωγό σε N κομμάτια μήκους $\Delta x = \frac{2L}{N}$ όπου το i-οστό έχει γραμμικό λ_i

Τον θεωρούμε ισοδυναμικό αφού $a \ll L$

Συνολικό φορτίο $Q = \sum_{j=1}^{N} \lambda_j \Delta x$

Το δυναμικό στο κέντρο ενός οριζοντίου κομματιού x_i είναι

$$\Phi_i = \sum_{j=1}^{N} \frac{\lambda_j \Delta x}{4\pi\varepsilon_1 R_{ij}} + \sum_{j=1}^{N} \frac{\lambda_j \Delta x}{4\pi\varepsilon_1 R_{ij'}} = \sum_{j=1}^{N} \frac{\lambda_j \Delta x}{4\pi\varepsilon_1} \left(\frac{1}{R_{ij}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{1}{R_{ij'}} \right)$$

Σε μορφή πίνακα: $[\Phi] = [A][\lambda] \xrightarrow{\text{ισοδυναμική}} [\lambda] = [A]^{-1}[\mathbb{1}]\Phi$ $\Big\} \Rightarrow [\lambda] = \frac{\Phi}{\Phi_0}[\lambda_0]$

Θεωρούμε γνωστό δυναμικό Φ_0 και βρίσκουμε $[\lambda_0] = [A]^{-1}[\mathbb{1}]\Phi_0$

Άρα $Q = \frac{\Phi}{\Phi_0} \sum_{j=1}^{N} \lambda_{0j} \Delta x \Rightarrow \Phi = \Phi_0 \frac{Q}{\sum_{j=1}^{N} \lambda_{0j} \Delta x}$

και $[\lambda] = \frac{Q}{\sum_{j=1}^{N} \lambda_{0j} \Delta x} \Phi_0 [A]^{-1}[\mathbb{1}]$ Θέτουμε $\Phi_0 = 1$ για ευκολία

όπου για $i \neq j$ είναι $A_{ij} = \frac{\Delta x}{4\pi\varepsilon_1} \left(\frac{1}{R_{ij}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{1}{R_{ij'}} \right)$ με $R_{ij} = |x_j - x_i|$ και $R_{ij'} = \left[(x_j - x_i)^2 + (2h)^2 \right]^{1/2}$

ενώ για $i = j$ είναι $A_{ii} = \frac{1}{4\pi\varepsilon_1} \left(\ln \frac{\Delta x/2 + \left[a^2 + (\Delta x/2) \right]^{1/2}}{-\Delta x/2 + \left[a^2 + (\Delta x/2) \right]^{1/2}} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{\Delta x}{2h} \right) \overset{\Delta x \ll a}{\approx} \frac{1}{4\pi\varepsilon_1} \left(\ln \left(\frac{\Delta x}{a} \right)^2 + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{\Delta x}{2h} \right)$ (από διαφάνειες)

Το δυναμικό το βρίσκουμε με επαλληλία:

$$\Phi(x,y,z) = \sum_{j=1}^{N} \frac{\lambda_j \Delta x}{4\pi\varepsilon_1} \left(\frac{1}{R_j} + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{1}{R_j'} \right), \quad z > 0$$

όπου $R_j = \left[(x - x_j)^2 + y^2 + (z-h)^2 \right]^{1/2}$

και $R_j' = \left[(x - x_j)^2 + y^2 + (z+h)^2 \right]^{1/2}$

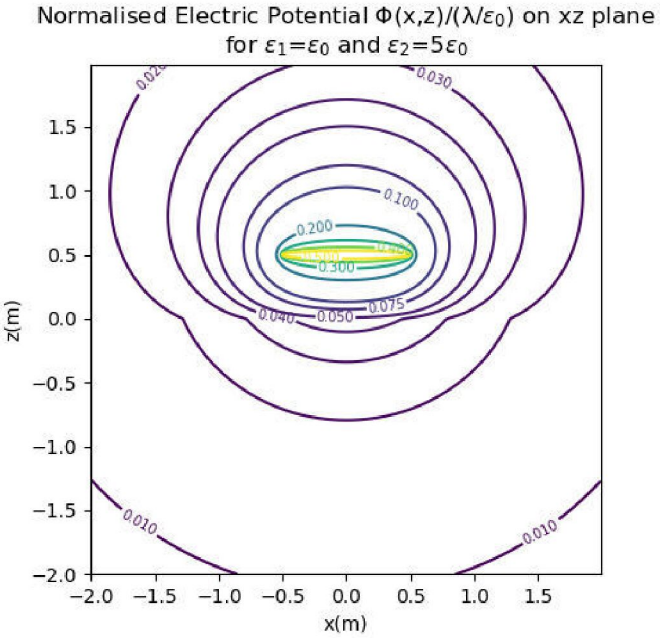
$$\sum_{j=1}^{N} \frac{\lambda_j \Delta x}{4\pi\varepsilon_2} \frac{2\varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{1}{R_j}, \quad z < 0$$

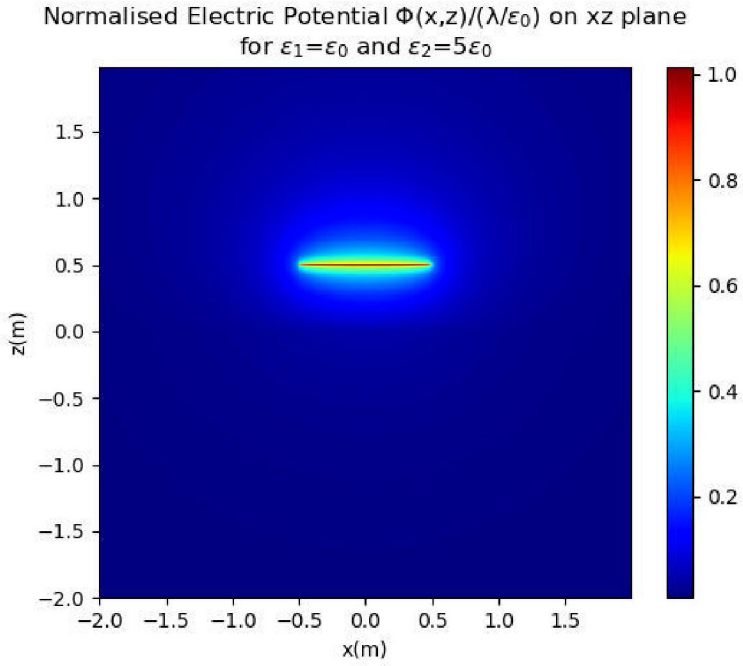
Γραφικές Παραστάσεις

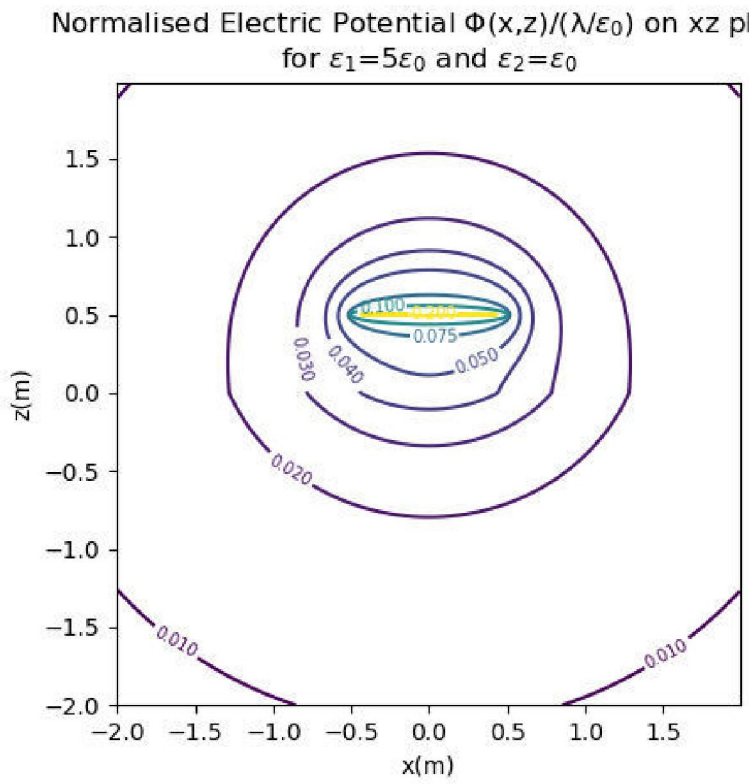
έγιναν σε Python 3.9

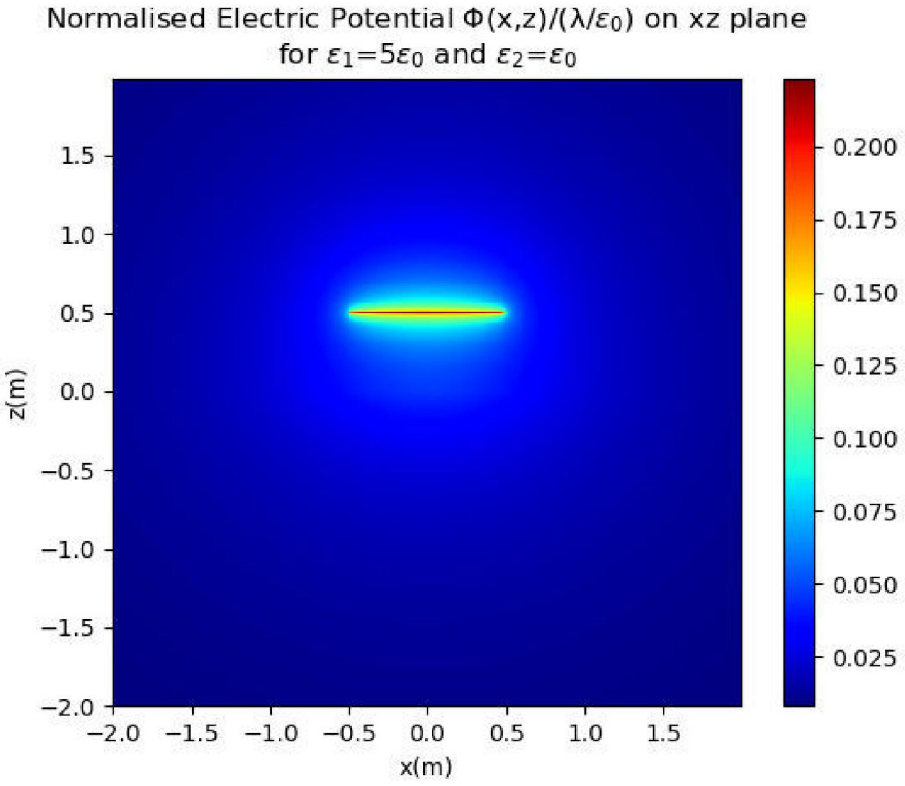
Άσκηση 7

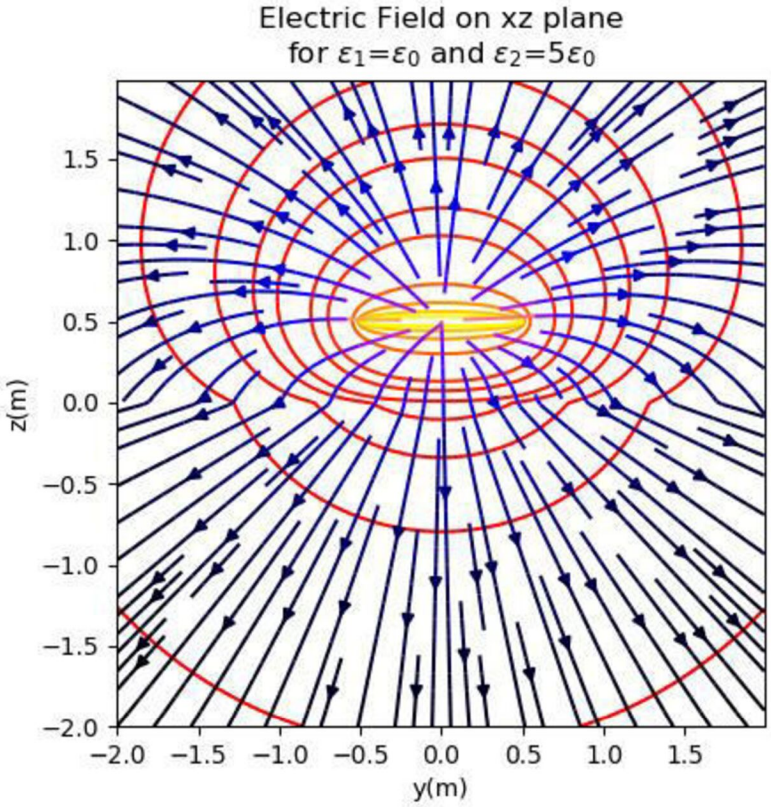
(δ)

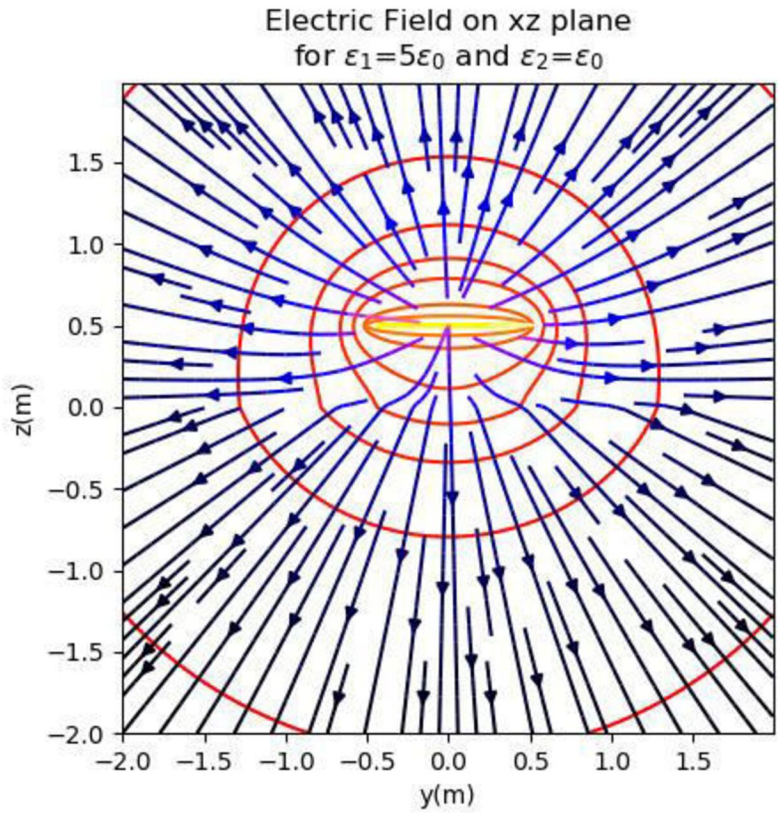




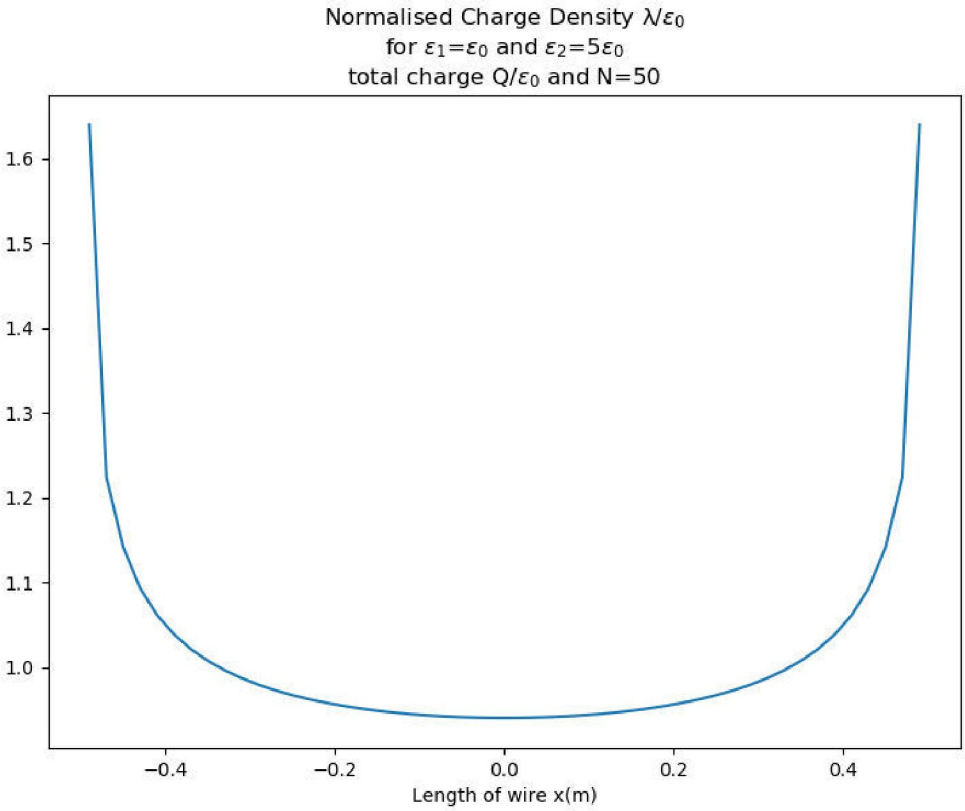

Normalised Electric Potential Φ(x,z)/(λ/ε0) on xz plane
for ε1=5ε0 and ε2=ε0
z(m)
x(m)
0.100
0.075
0.050
0.040
0.030
0.020
0.010
0.010

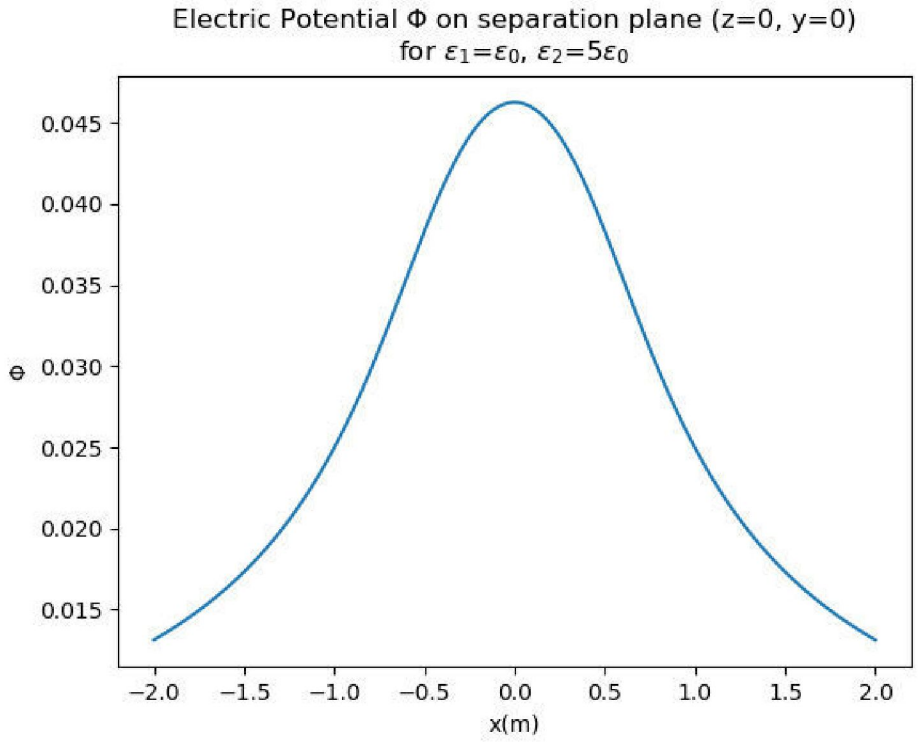

Normalised Electric Potential Φ(x,z)/(λ/ε0) on xz plane
for ε1=5ε0 and ε2=ε0
z(m)
x(m)
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025


Electric Field on xz plane
for $\varepsilon_1=\varepsilon_0$ and $\varepsilon_2=5\varepsilon_0$
z(m)
y(m)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5


Electric Field on xz plane
for $\varepsilon_1=5\varepsilon_0$ and $\varepsilon_2=\varepsilon_0$
z(m)
y(m)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.0 −1.5 −1.0 −0.5 0.0 0.5 1.0 1.5

(ε)


Normalised Charge Density λ/ε0
for ε1=ε0 and ε2=5ε0
total charge Q/ε0 and N=50
1.6
1.5
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4
Length of wire x(m)


Electric Potential Φ on separation plane (z=0, y=0)
for ε1=ε0, ε2=5ε0
0.045
0.040
0.035
0.030
0.025
0.020
0.015
Φ
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
x(m)

Κώδικας σε Python 3.9

Άσκηση 7

(δ)

```
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
L=0.5
h=0.5
levels=np.array([0.01, 0.02, 0.03, 0.04, 0.05, 0.075, 0.1, 0.2, 0.3, 0.4, 0.5])

#Axes
xmin=-2
xmax=2
zmin=-2
zmax=2

N=403
dx=(xmax-xmin)/N
dz=(zmax-zmin)/N

xx=np.arange(xmin,xmax,dx)
zz=np.arange(zmin,zmax,dz)

X,Z=np.meshgrid(xx,zz)

#functions to calculate potential
def phi_term1(x,z):
    R1=np.sqrt((x-L)**2+(z-h)**2)
    R2=np.sqrt((x+L)**2+(z-h)**2)
    return np.log((x-L+R1)/(x+L+R2))

def phi_term2(x,z):
    R3=np.sqrt((x-L)**2+(z+h)**2)
    R4=np.sqrt((x+L)**2+(z+h)**2)
    return np.log((x-L+R3)/(x+L+R4))

def potential1(x,z):
    condlist=[z>=0,z<0]
    choicelist=[phi_term1(x,z)/(4*math.pi)-2*phi_term2(x,z)/(12*math.pi),phi_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

def potential2(x,z):
    condlist=[z>=0,z<0]
    choicelist=[phi_term1(x,z)/(20*math.pi)+2*phi_term2(x,z)/(60*math.pi),phi_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

phi1=potential1(X,Z)
phi2=potential2(X,Z)
```

```
#functions to calculate electric field
def electric_field_term1(x,z):
    R1=np.sqrt((x-L)**2+(z-h)**2)
    R2=np.sqrt((x+L)**2+(z-h)**2)
    return (x-L)/(R1*((z-h)**2))-(x+L)/(R2*((z-h)**2))

def electric_field_term2(x,z):
    R3=np.sqrt((x-L)**2+(z+h)**2)
    R4=np.sqrt((x+L)**2+(z+h)**2)
    return (x-L)/(R3*((z+h)**2))-(x+L)/(R4*((z+h)**2))

def Ex_func1(x,z):
    condlist=[z>=0,z<0]
    choicelist=[x*electric_field_term1(x,z)/(4*math.pi)-2*x*electric_field_term2(x,z)/
(12*math.pi),x*electric_field_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

def Ex_func2(x,z):
    condlist=[z>=0,z<0]
    choicelist=[x*electric_field_term1(x,z)/(20*math.pi)+2*x*electric_field_term2(x,z)/
(60*math.pi),x*electric_field_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

def Ez_func1(x,z):
    condlist=[z>=0,z<0]
    choicelist=[(z-h)*electric_field_term1(x,z)/(4*math.pi)-2*(z+h)*electric_field_term2(x,z)/(12*math.pi),
(z-h)*electric_field_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

def Ez_func2(x,z):
    condlist=[z>=0,z<0]
    choicelist=[(z-h)*electric_field_term1(x,z)/(20*math.pi)+2*(z+h)*electric_field_term2(x,z)/(60*math.pi),
(z-h)*electric_field_term1(x,z)/(12*math.pi)]
    return np.select(condlist,choicelist)

Ex1=Ex_func1(X,Z)
Ex2=Ex_func2(X,Z)
Ez1=Ez_func1(X,Z)
Ez2=Ez_func2(X,Z)
```

```
#PLOTS

#contour lines of potential
fig1, ax1 = plt.subplots()
CS1 = ax1.contour(X,Z,-phi1,levels)
ax1.clabel(CS1, inline=True, fontsize=7)
ax1.set_aspect('equal','box')
ax1.set_title('Normalised Electric Potential ϕ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=$ε_0$ and
$ε_2$=5$ε_0$')
ax1.set_xlabel('x(m)')
ax1.set_ylabel('z(m)')

fig2, ax2 = plt.subplots()
CS2 = ax2.contour(X,Z,-phi2,levels)
ax2.clabel(CS2, inline=True, fontsize=7)
ax2.set_aspect('equal','box')
ax2.set_title('Normalised Electric Potential ϕ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=5$ε_0$ and
$ε_2$=$ε_0$')
ax2.set_xlabel('x(m)')
ax2.set_ylabel('z(m)')

#surface plot of potential
fig3, ax3 = plt.subplots()
s3=ax3.pcolormesh(X,Z,-phi1,cmap=cm.jet)
ax3.set_aspect('equal','box')
ax3.set_title('Normalised Electric Potential ϕ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=$ε_0$ and
$ε_2$=5$ε_0$')
cb3=fig3.colorbar(s3)
ax3.set_xlabel('x(m)')
ax3.set_ylabel('z(m)')

fig4, ax4 = plt.subplots()
s4=ax4.pcolormesh(X,Z,-phi2,cmap=cm.jet)
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Electric Potential ϕ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=5$ε_0$ and
$ε_2$=$ε_0$')
cb4=fig4.colorbar(s4)
ax4.set_xlabel('x(m)')
ax4.set_ylabel('z(m)')
```

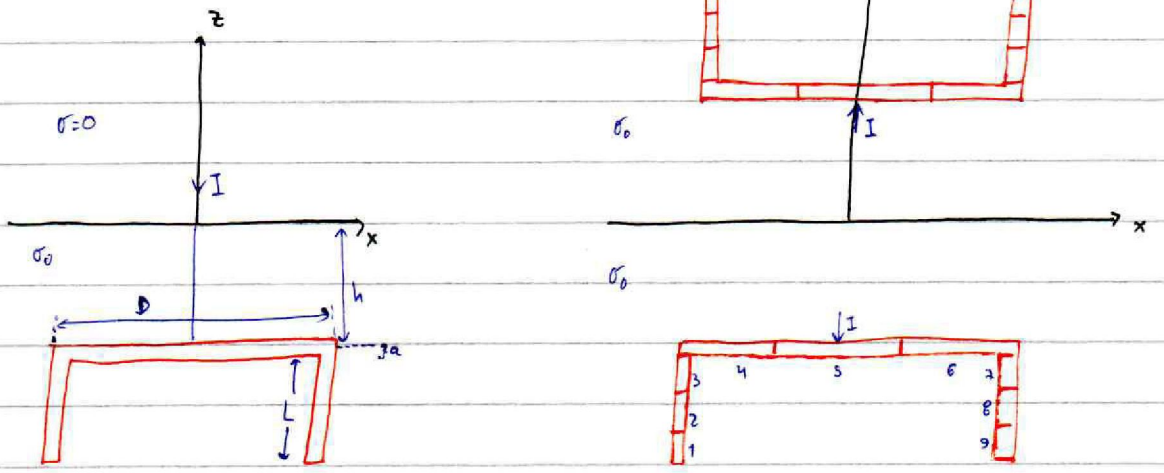
```
#surface plot of potential
fig3, ax3 = plt.subplots()
s3=ax3.pcolormesh(X,Z,-phi1,cmap=cm.jet)
ax3.set_aspect('equal','box')
ax3.set_title('Normalised Electric Potential Φ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=$ε_0$ and
$ε_2$=5$ε_0$')
cb3=fig3.colorbar(s3)
ax3.set_xlabel('x(m)')
ax3.set_ylabel('z(m)')

fig4, ax4 = plt.subplots()
s4=ax4.pcolormesh(X,Z,-phi2,cmap=cm.jet)
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Electric Potential Φ(x,z)/(λ/$ε_0$) on xz plane\nfor $ε_1$=5$ε_0$ and
$ε_2$=$ε_0$')
cb4=fig4.colorbar(s4)
ax4.set_xlabel('x(m)')
ax4.set_ylabel('z(m)')


#electric field streamplot
fig5,ax5=plt.subplots()
CS5=ax5.contour(X,Z,-phi1,levels,cmap=cm.autumn)
sl5=ax5.streamplot(X,Z,-Ex1,-Ez1,color=np.log10(np.sqrt(Ex1**2+Ez1**2)),cmap=cm.gnuplot2,density=1.2)
ax5.set_aspect('equal','box')
ax5.set_title('Electric Field on xz plane\nfor $ε_1$=$ε_0$ and $ε_2$=5$ε_0$')
ax5.set_xlabel('y(m)')
ax5.set_ylabel('z(m)')

fig6,ax6=plt.subplots()
CS6=ax6.contour(X,Z,-phi2,levels,cmap=cm.autumn)
sl6=ax6.streamplot(X,Z,-Ex2,-Ez2,color=np.log10(np.sqrt(Ex1**2+Ez1**2)),cmap=cm.gnuplot2,density=1.2)
ax6.set_aspect('equal','box')
ax6.set_title('Electric Field on xz plane\nfor $ε_1$=5$ε_0$ and $ε_2$=$ε_0$')
ax6.set_xlabel('y(m)')
ax6.set_ylabel('z(m)')

plt.show()
```

(ε)

```
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
L=0.5
h=0.5
a=0.0025
e0=8.85*(10**(-12))

#Axes
N=50
xrmin=-L
xrmax=L
Dxr=(xrmax-xrmin)/(N+1)
Xr=np.linspace(xrmin,xrmax,N+1)

N2=203
xmin=-2
xmax=2
Dx=(xmax-xmin)/N2
X=np.linspace(xmin,xmax,N2)

#initialising arrays
xx=np.zeros(N)
AA=np.zeros((N,N))

for i in range (0,N):
    xx[i]=0.5*(Xr[i]+Xr[i+1])


#calculating A matrix
for i in range(0,N):
    for j in range(0,N):
        if i==j:
            AA[i][j]=Dxr*(np.log((Dxr/2+np.sqrt(a**2+(Dxr/2)**2))/(-Dxr/2+np.sqrt(a**2+(Dxr/2)**2)))-2/
(3*2*h))/(4*math.pi)
        else:
            R1=abs(xx[j]-xx[i])
            R2=np.sqrt((xx[j]-xx[i])**2+(2*h)**2)
            AA[i][j]=Dxr*(Dxr/R1 -2*Dxr/(3*R2))/(4*math.pi)

#charge density
l0=np.dot(np.linalg.inv(AA),np.ones(N))
l=l0/(Dxr*sum(l0))


#function for phi
def potential(x):
    res=0
    for j in range(0,N):
        Rj=np.sqrt((x-xx[j])**2+h**2)
        res=res+l[j]*Dxr*5/(12*Rj*5*math.pi)
    return res

phi=potential(X)

#plots
fig1, ax1 = plt.subplots()
test=ax1.plot(xx,l)
ax1.set_aspect('equal','box')
ax1.set_title('Normalised Charge Density λ/$ε_0$\nfor $ε_1$=$ε_0$ and $ε_2$=5$ε_0$\ntotal charge Q/$ε_0$ and N=50')
ax1.set_xlabel('Length of wire x(m)')

fig2, ax2 = plt.subplots()
test=ax2.plot(X,phi)
ax2.set_title('Electric Potential Φ on separation plane (z=0, y=0)\nfor $ε_1$=$ε_0$, $ε_2$=5$ε_0$')
ax2.set_ylabel('Φ')
ax2.set_xlabel('x(m)')

plt.show()
```

Άσκηση 8

(a)



Παίρνουμε το καταστρώμα για $z>0$ όπου δίχτυαν επίσης ρεύμα I και είναι παντού $\sigma=\sigma_0$

Χωρίζουμε τον αγωγό σε $N=9$ τμήματα μήκους $\Delta l_j = \begin{cases} L/3 & \text{για } j=1,2,3,7,8,9 \\ D/3 & \text{για } j=4,5,6 \end{cases}$

όπου το j-οστό έχει σταθερό ρεύμα I_j

Τον θεωρούμε ισοδυναμικό αφού $a \ll L, D$ (από αριθμητικές τιμές)

Συνολικό ρεύμα $I = \sum_{j=1}^{N} I_j \Delta l_j$

Θέτω x_j, z_j τις συντεταγμένες της αρχής κάθε κομματιού και $\hat{x}_j = \frac{1}{2}(x_j + x_{j-1})$, $\hat{z}_j = \frac{1}{2}(z_j + z_{j-1})$ τις συντεταγμένες του κέντρου κάθε κομματιού.

Το δυναμικό στο κέντρο κάθε κομματιού είναι $\Phi_i = \sum_{j=1}^{N} VDF_{ij} I_j$

Ο πίνακας VDF ορίζεται ως εξής:

Για $i \neq j$ $VDF_{ij} = \frac{1}{4\pi\sigma_0} \Delta l_j \left(\frac{1}{R_{ij}} + \frac{1}{R_{ij'}} \right)$

όπου $R_{ij} = \sqrt{(\hat{z}_i - \hat{z}_j)^2 + (\hat{x}_i - \hat{x}_j)^2}$ και $R_{ij'} = \sqrt{(\hat{z}_i - \hat{z}_j)^2 + (\hat{x}_i - \hat{x}_{j'})^2}$

Για $i=j$ $VDF_{ii} = \frac{1}{4\pi\sigma_0} \left[\ln \frac{\Delta l_i/2 + \sqrt{a^2 + (\Delta l_i/2)^2}}{-\Delta l_i/2 + \sqrt{a^2 + (\Delta l_i/2)^2}} + \frac{\Delta l_i}{R_{ii'}} \right]$

όπου $R_{ii'} = |\hat{z}_i - \hat{z}_{i'}|$

Άρα $[\Phi] = [VDF][I] \overset{\text{ισοδυναμία}}{\Rightarrow} [I] = [VDF]^{-1}[1]\Phi$ $\Big\}$ $[I] = \frac{\Phi}{\Phi_0}[I_0]$

Θεωρούμε γνωστό δυναμικό Φ_0 και βρίσκουμε $[I_0] = [VDF]^{-1}[1]\Phi_0$ $\Big\}$

Συνολικό ρεύμα $I = \frac{\Phi}{\Phi_0}\sum_{j=1}^{N} I_{0j}\Delta l_j \Rightarrow \Phi = \Phi_0 \frac{I}{\sum_{j=1}^{N} I_{0j}\Delta l_j}$

$[I] = \frac{I}{\sum_{j=1}^{N} I_{0j}\Delta l_j}[I_0] = \frac{I}{\sum_{j=1}^{N} I_{0j}\Delta l_j}\Phi_0[VDF]^{-1}[1]$ Θέτουμε $\Phi_0 = 1$ για ευκολία

(β) Η αντίσταση γείωσης είναι $R_g = \frac{\Phi}{I} = \frac{\Phi_0}{\sum_{j=1}^{N} I_{0j}\Delta l_j}$

Το δυναμικό για $z \le 0$ είναι από επαλληλία: $\Phi(x,y,z) = \sum_{j=1}^{N} \frac{I_j \Delta l_j}{4\pi\sigma_0}\left(\frac{1}{R_j} + \frac{1}{R_{j'}}\right)$, $z \le 0$

όπου $R_j = \sqrt{(x-\hat{x}_j)^2 + y^2 + (z-\hat{z}_j)^2}$ και $R_{j'} = \sqrt{(x-\hat{x}_{j'})^2 + y^2 + (z-\hat{z}_{j'})^2}$

Στην αρχή των αξόνων είναι $\Phi(0,0,0) = \sum_{j=1}^{N} \frac{I_j \Delta l_j}{4\pi\sigma_0}\left(\frac{1}{\sqrt{\hat{x}_j^2 + \hat{z}_j^2}} + \frac{1}{\sqrt{\hat{x}_{j'}^2 + \hat{z}_{j'}^2}}\right)$

Αριθμητικά αποτελέσματα και γραφικές παραστάσεις

έγιναν σε Python 3.9

Άσκηση 8

(α)

Έχω αριθμήσει τα κομμάτια ως εξής: πρώτα η αριστερή ράβδος από κάτω προς τα πάνω (κομμάτια 1-3), μετά η μεσαία ράβδος από τα αριστερά προς τα δεξιά (κομμάτια 4-6) και τέλος η δεξιά ράβδος από πάνω προς τα κάτω (κομμάτια 7-9)

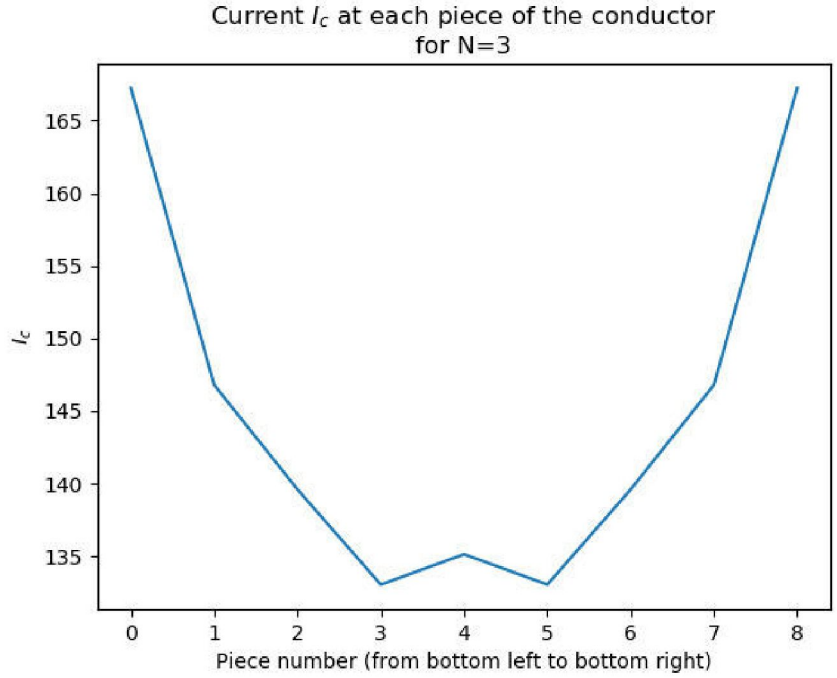
Πίνακας VDF αντιστάσεων:

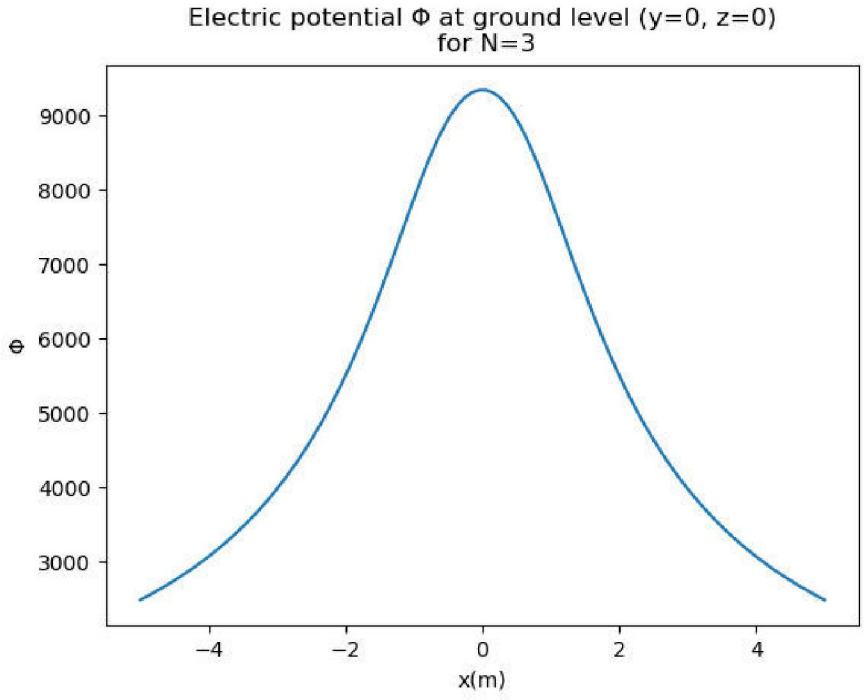
i/j	1	2	3	4	5	6	7	8	9
1	107.846	13.998	7.771	9.630	7.888	6.311	3.820	3.889	3.866
2	13.998	108.100	14.312	14.019	9.528	7.016	4.112	4.057	3.889
3	7.771	14.312	108.498	24.174	11.069	7.596	4.317	4.112	3.820
4	6.356	9.252	15.955	120.455	15.820	9.213	5.013	4.631	4.166
5	5.206	6.288	7.305	15.820	120.455	15.820	7.305	6.288	5.206
6	4.166	4.631	5.013	9.213	15.820	120.455	15.955	9.252	6.356
7	3.820	4.112	4.317	7.596	11.069	24.174	108.498	14.312	7.771
8	3.889	4.057	4.112	7.016	9.528	14.019	14.312	108.100	13.998
9	3.866	3.889	3.820	6.311	7.888	9.630	7.771	13.998	107.846

Κατανομή ρεύματος I_j:

	Αριστερός αγωγός			Μεσαίος αγωγός			Δεξιός αγωγός		
j	1	2	3	4	5	6	7	8	9
I_j	167.243	146.796	139.591	133.040	135.128	133.040	139.591	146.796	167.243

(β)

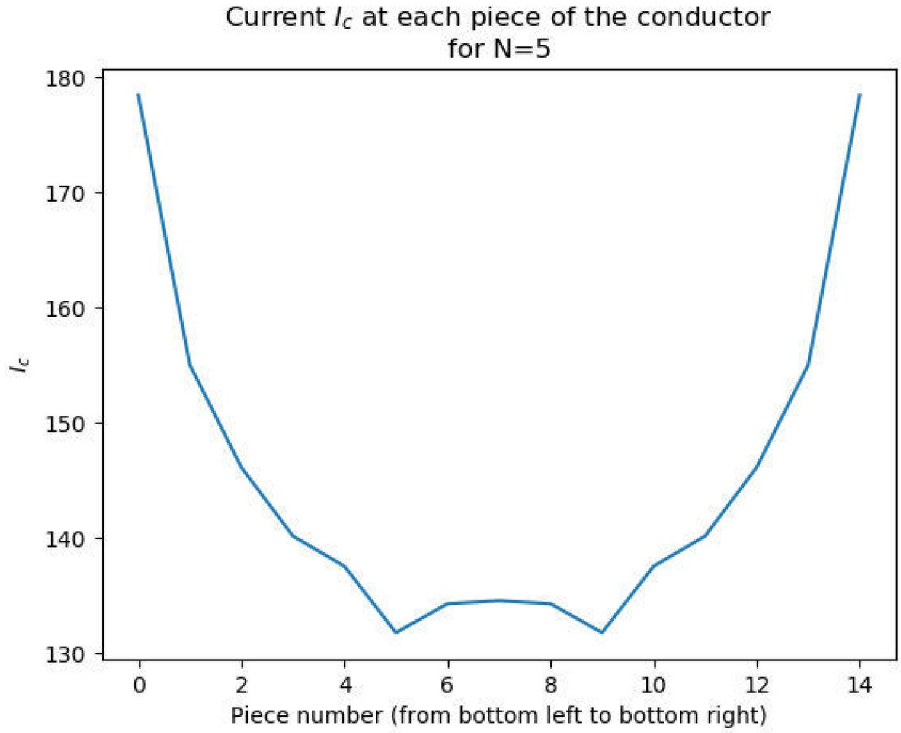


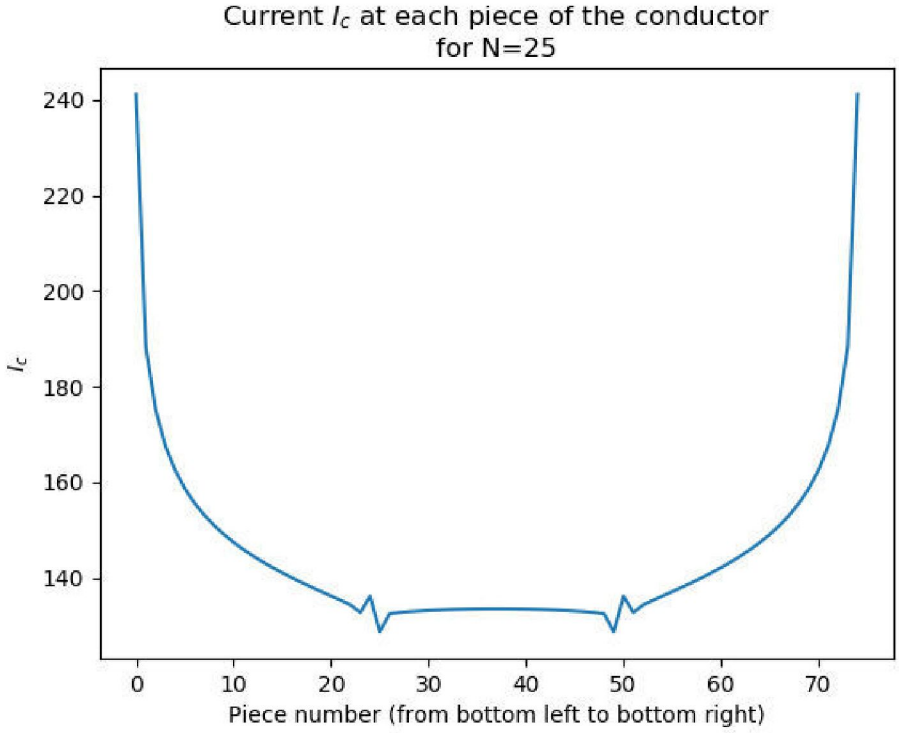


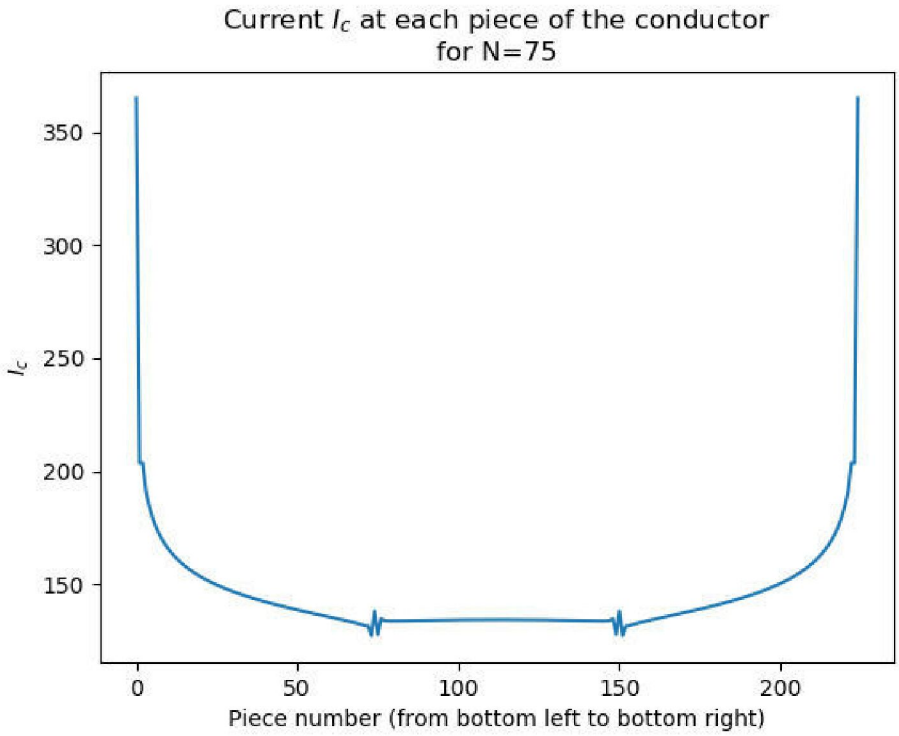
N	Δυναμικό αγωγού Φ	Αντίσταση γείωσης R_g	$\Phi(0,0,0)$
3	26113.464	52.227	9343.564

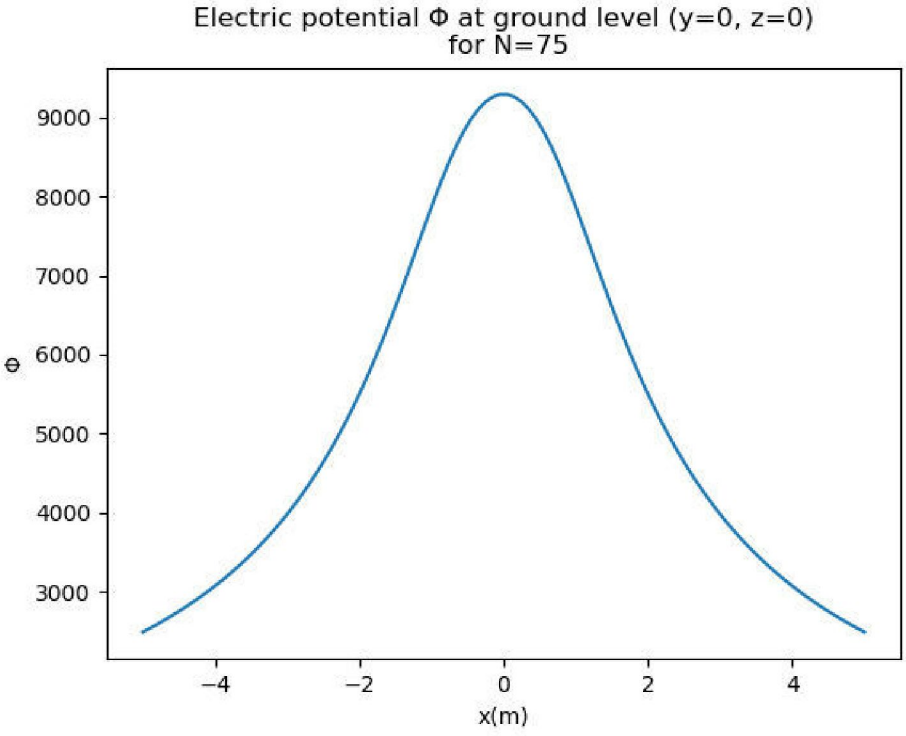
(γ)

N	Δυναμικό αγωγού Φ	Αντίσταση γείωσης R_g	$\Phi(0,0,0)$
5	25998.003	51.996	9317.350
10	25900.028	51.800	9298.684
15	25867.822	51.735	9292.212
20	25856.976	51.714	9288.923
25	25857.395	51.715	9287.261
30	25865.335	51.731	9286.399
35	25878.982	51.758	9286.121
40	25897.280	51.795	9286.269
45	25919.520	51.839	9286.774
50	25945.176	51.890	9287.478
55	25973.823	51.948	9288.420
60	26005.104	52.010	9289.529
65	26038.705	52.077	9290.772
70	26074.349	52.149	9292.121
75	26111.782	52.224	9293.553


Current I_c at each piece of the conductor
for N=5
180
170
160
150
140
130
I_c
0
2
4
6
8
10
12
14
Piece number (from bottom left to bottom right)


Current I_c at each piece of the conductor
for N=25
240
220
200
180
160
140
I_c
0
10
20
30
40
50
60
70
Piece number (from bottom left to bottom right)


Current I_c at each piece of the conductor
for N=75
350
300
250
200
150
I_c
0
50
100
150
200
Piece number (from bottom left to bottom right)


Electric potential Φ at ground level (y=0, z=0)
for N=75
9000
8000
7000
6000
5000
4000
3000
Φ
−4
−2
0
2
4
x(m)

Κώδικας σε Python 3.9

Άσκηση 8

```
import matplotlib
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
a=0.005
L=0.99
h=1
D=1.5
sigma0=1/160
I=500
N=3

#Axes for plot
xmin=-5
xmax=5
Nx=100
dx=(xmax-xmin)/Nx
X=np.linspace(xmin,xmax,Nx)

#length of pieces
Dl1=L/N*np.ones(N)
Dl2=D/N*np.ones(N)
Dl3=L/N*np.ones(N)
Dl=np.concatenate((Dl1,Dl2,Dl3))

#x coordinate of start of pieces (approximate, ignoring a)
x1=np.ones(N)*(-D/2)
x3=np.ones(N)*(D/2)
x2=np.linspace(-D/2,D/2,N+1)
x=np.concatenate((x1,x2,x3))

#x coordinate of center of pieces
xx1=np.zeros(N)
xx2=np.zeros(N)
xx3=np.zeros(N)

for i in range(0,N):
    xx1[i]=0.5*(x[i]+x[i+1])
    xx2[i]=0.5*(x[i+N]+x[i+1+N])
    xx3[i]=0.5*(x[i+2*N]+x[i+1+2*N])
xx=np.concatenate((xx1,xx2,xx3))
```

```
#z coordinate of start of pieces (approximate, ignoring a)
z1=np.linspace(-h-L,-h,N+1)
z3=np.linspace(-h,-h-L,N+1)
z2=np.ones(N-1)*(-h)
z=np.concatenate((z1,z2,z3))

#z coordinate of center of pieces
zz1=np.zeros(N)
zz2=np.zeros(N)
zz3=np.zeros(N)

for i in range(0,N):
    zz1[i]=0.5*(z[i]+z[i+1])
    zz2[i]=0.5*(z[i+N]+z[i+1+N])
    zz3[i]=0.5*(z[i+2*N]+z[i+1+2*N])
zz=np.concatenate((zz1,zz2,zz3))

#VDF matrix
VDF=np.zeros((3*N,3*N))

for i in range(0,3*N):
    for j in range(0,3*N):
        if i==j:
            R1=Dl[i]/2+np.sqrt(a**2+(Dl[i]/2)**2)
            R2=-Dl[i]/2+np.sqrt(a**2+(Dl[i]/2)**2)
            Riim=abs(2*zz[i])
            VDF[i][j]=(np.log(R1/R2)+Dl[i]/Riim)/(4*math.pi*sigma0)
        else:
            R1=np.sqrt((zz[i]-zz[j])**2+(xx[i]-xx[j])**2)
            R2=np.sqrt((zz[i]+zz[j])**2+(xx[i]-xx[j])**2)
            VDF[i][j]=Dl[j]*(1/R1+1/R2)/(4*math.pi*sigma0)

#charge at each piece
I0=np.dot(np.linalg.inv(VDF),np.ones(3*N))
Itot=0
for i in range (0, 3*N):
    Itot=Itot+I0[i]*Dl[i]

Ic=np.multiply(I0,I/Itot)
```

```
#potential of rod and ground resistance
phi_rod=I/Itot
Rg=phi_rod/I

#potential at ground level
def potential(x):
    res=0
    for i in range(0,3*N):
        R=np.sqrt((x-xx[i])**2+zz[i]**2)
        res=res+Ic[i]*Dl[i]*2/R
    return res/(4*math.pi*sigma0)

phi=potential(X)
phi0=potential(0)

print('For N={}:\nPotential at the rod: {}\nGround resistance: {}\nPotential at (0,0,0): {}'.format(N,phi_rod, Rg,
phi0))

#plots
fig1, ax1 = plt.subplots()
potplot=ax1.plot(X,phi)
ax1.set_title('Electric potential Φ at ground level (y=0, z=0)\n for N={}'.format(N))
ax1.set_xlabel('x(m)')
ax1.set_ylabel('Φ')

fig2,ax2=plt.subplots()
iplot=ax2.plot(Ic)
ax2.set_title('Current $I_c$ at each piece of the conductor\nfor N={}'.format(N))
ax2.set_xlabel('Piece number (from bottom left to bottom right)')
ax2.set_ylabel('$I_c$')
plt.show()
```