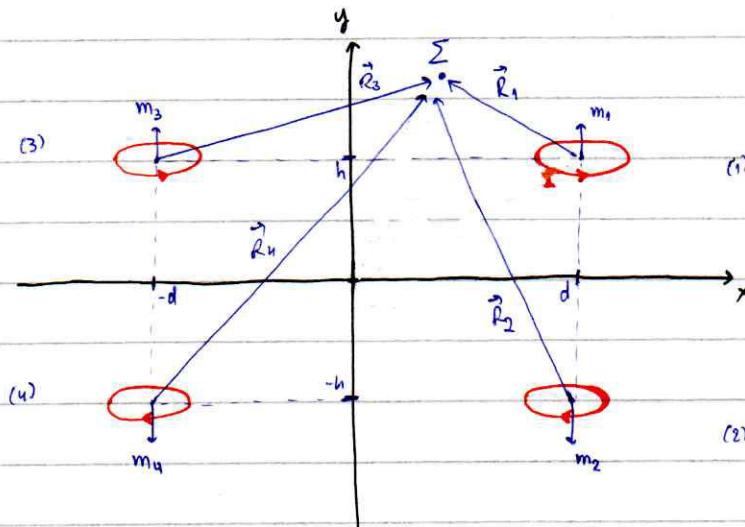


Άσκηση 8

(a) Θεωρείτε το παρόντα κατοπτρικό πρόβλημα όπου όποιος ο χώρος έχει μεγάλημένη διαπερασία με και έχει τρία κατοπτρικά βράχου στην περιοχή ως προς τις αίφοres και για από την αριστερά με τρία I φεράς όπως φαίνεται στο σχήμα.

Με τον γύρον αυτό το διανυμένο στους αίφοres x, y για $x, y > 0$ διαγράφεται με δύινα

$$\text{Άριθμητή είναι } \vec{A}(x, y, z) = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4 = \frac{\mu_0}{4\pi} \left(\frac{\vec{m}_1 \times \hat{R}_1}{R_1^2} + \frac{\vec{m}_2 \times \hat{R}_2}{R_2^2} + \frac{\vec{m}_3 \times \hat{R}_3}{R_3^2} + \frac{\vec{m}_4 \times \hat{R}_4}{R_4^2} \right) =$$

$$= \frac{\mu_0}{4\pi} \left(\frac{\vec{m}_1 \times \vec{R}_1}{R_1^3} + \frac{\vec{m}_2 \times \vec{R}_2}{R_2^3} + \frac{\vec{m}_3 \times \vec{R}_3}{R_3^3} + \frac{\vec{m}_4 \times \vec{R}_4}{R_4^3} \right)$$

$$\vec{R}_1 = (x-d)\hat{i}_x + (y-h)\hat{i}_y + z\hat{i}_z, \quad R_1 = \sqrt{(x-d)^2 + (y-h)^2 + z^2}, \quad \vec{m}_1 = I\pi a^2 \hat{i}_y$$

$$\vec{R}_2 = (x-d)\hat{i}_x + (y+h)\hat{i}_y + z\hat{i}_z, \quad R_2 = \sqrt{(x-d)^2 + (y+h)^2 + z^2}, \quad \vec{m}_2 = -I\pi a^2 \hat{i}_y$$

$$\vec{R}_3 = (x+d)\hat{i}_x + (y-h)\hat{i}_y + z\hat{i}_z, \quad R_3 = \sqrt{(x+d)^2 + (y-h)^2 + z^2}, \quad \vec{m}_3 = I\pi a^2 \hat{i}_y$$

$$\vec{R}_4 = (x+d)\hat{i}_x + (y+h)\hat{i}_y + z\hat{i}_z, \quad R_4 = \sqrt{(x+d)^2 + (y+h)^2 + z^2}, \quad \vec{m}_4 = -I\pi a^2 \hat{i}_y$$

$$\text{Τελικά } \vec{A}(x, y, z) = \frac{\mu_0 I \pi a^2}{4\pi} \left[\left(\frac{z}{R_1^3} - \frac{z}{R_2^3} + \frac{z}{R_3^3} - \frac{z}{R_4^3} \right) \hat{i}_x + \left(-\frac{x-d}{R_1^3} + \frac{x-d}{R_2^3} - \frac{x+d}{R_3^3} + \frac{x+d}{R_4^3} \right) \hat{i}_z \right] \quad \text{για } x, y > 0$$

$$(6) \quad \vec{H} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A} = \frac{1}{\mu_0} \left(\vec{\nabla} \times \vec{A}_1 + \vec{\nabla} \times \vec{A}_2 + \vec{\nabla} \times \vec{A}_3 + \vec{\nabla} \times \vec{A}_4 \right)$$

$$\vec{H}_1 = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}_1 = \frac{I n a^2}{4\pi} \left[\frac{3(y-h)(x-d)}{R_1^5} \hat{i}_x + \left(\frac{3(y-h)^2}{R_1^5} - \frac{1}{R_1^3} \right) \hat{i}_y + \frac{3(y-h)z}{R_1^5} \hat{i}_z \right]$$

$$\vec{H}_2 = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}_2 = -\frac{I n a^2}{4\pi} \left[\frac{3(y+h)(x-d)}{R_2^5} \hat{i}_x + \left(\frac{3(y+h)^2}{R_2^5} - \frac{1}{R_2^3} \right) \hat{i}_y + \frac{3(y+h)z}{R_2^5} \hat{i}_z \right]$$

$$\vec{H}_3 = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}_3 = \frac{I n a^2}{4\pi} \left[\frac{3(y-h)(x+d)}{R_3^5} \hat{i}_x + \left(\frac{3(y-h)^2}{R_3^5} - \frac{1}{R_3^3} \right) \hat{i}_y + \frac{3(y-h)z}{R_3^5} \hat{i}_z \right]$$

$$\vec{H}_4 = \frac{1}{\mu_0} \vec{\nabla} \times \vec{A}_4 = -\frac{I n a^2}{4\pi} \left[\frac{3(y+h)(x+d)}{R_4^5} \hat{i}_x + \left(\frac{3(y+h)^2}{R_4^5} - \frac{1}{R_4^3} \right) \hat{i}_y + \frac{3(y+h)z}{R_4^5} \hat{i}_z \right]$$

$$\vec{H}(x, y, z) = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 \quad \text{para } x, y > 0$$

(7) H enipávvaan nuorinna mukopjäisen ajo ja se onvoi pannu onnittijan $\vec{k} = \vec{i}_n \times (\vec{H}_1 - \vec{H}_2)$

$$\text{Jaa myös enipávua } x=0: \quad \vec{i}_x \times \vec{H}|_{x=0} = \frac{I n a^2}{4\pi} \left[\vec{i}_z \left(\frac{3(y-h)^2}{R_{10}^5} - \frac{1}{R_{10}^3} \right) - \vec{i}_y \frac{3(y-h)z}{R_{10}^5} - \vec{i}_z \left(\frac{3(y+h)^2}{R_{20}^5} - \frac{1}{R_{20}^3} \right) + \vec{i}_y \frac{3(y+h)z}{R_{20}^5} \right. \\ \left. + \vec{i}_z \left(\frac{3(y-h)^2}{R_{30}^5} - \frac{1}{R_{30}^3} \right) - \vec{i}_y \frac{3(y-h)z}{R_{30}^5} - \vec{i}_z \left(\frac{3(y+h)^2}{R_{40}^5} - \frac{1}{R_{40}^3} \right) + \vec{i}_y \frac{3(y+h)z}{R_{40}^5} \right]$$

$$\text{oinou } R_{10} = R_{30} = \sqrt{d^2 + (y-h)^2 + z^2} \quad \text{jaan } R_{20} = R_{40} = \sqrt{d^2 + (y+h)^2 + z^2}$$

$$\text{Jaa myös enipávua } y=0: \quad \vec{i}_y \times \vec{H}|_{y=0} = \frac{I n a^2}{4\pi} \left[\vec{i}_z \frac{3h(x-d)}{R_{100}^5} - \vec{i}_x \frac{3hz}{R_{100}^5} + \vec{i}_z \frac{3h(x-d)}{R_{200}^5} - \vec{i}_x \frac{3hz}{R_{200}^5} \right. \\ \left. + \vec{i}_z \frac{3h(x+d)}{R_{300}^5} - \vec{i}_x \frac{3hz}{R_{300}^5} + \vec{i}_z \frac{3h(x+d)}{R_{400}^5} - \vec{i}_x \frac{3hz}{R_{400}^5} \right]$$

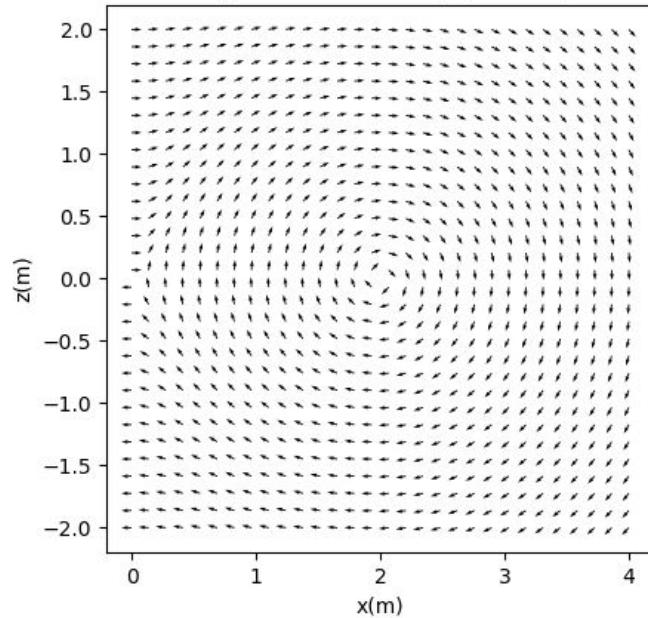
$$\text{oinou } R_{100} = R_{200} = \sqrt{(x-d)^2 + h^2 + z^2} \quad \text{jaan } R_{300} = R_{400} = \sqrt{(x+d)^2 + h^2 + z^2}$$

Γραφικές παραστάσεις – Άσκηση 8

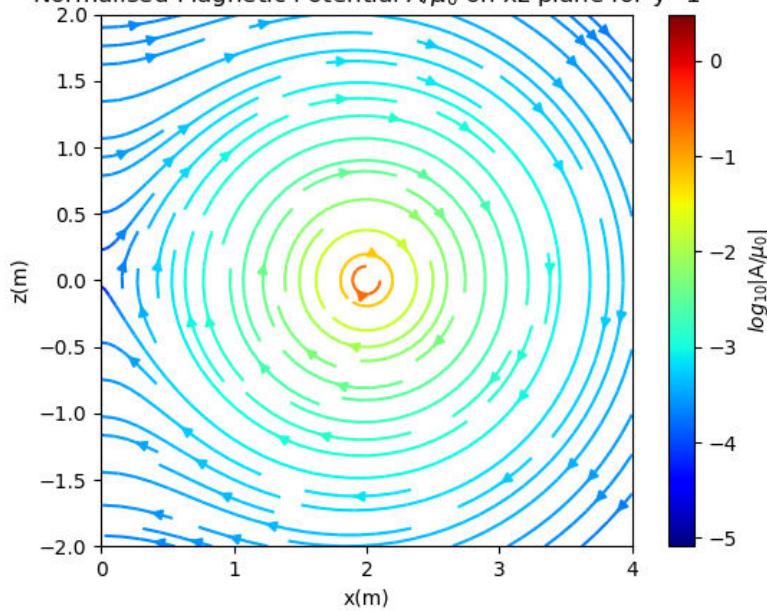
έγιναν σε Python 3.9

(δ)

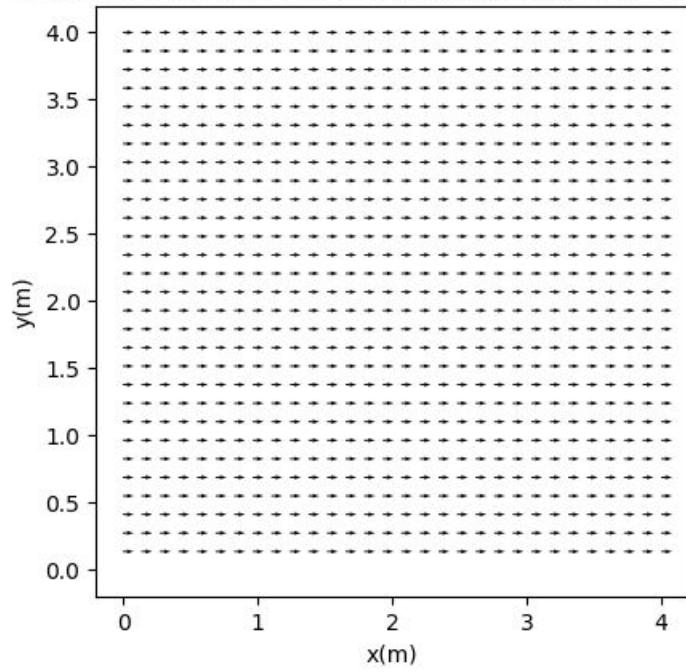
Normalised Magnetic Potential A/μ_0 on xz plane for $y=1$



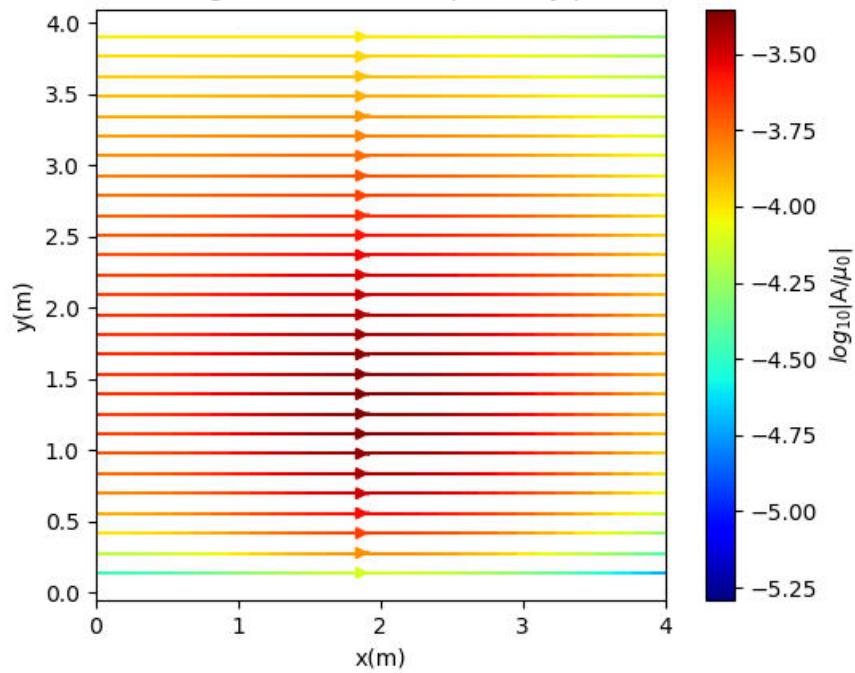
Normalised Magnetic Potential A/μ_0 on xz plane for $y=1$



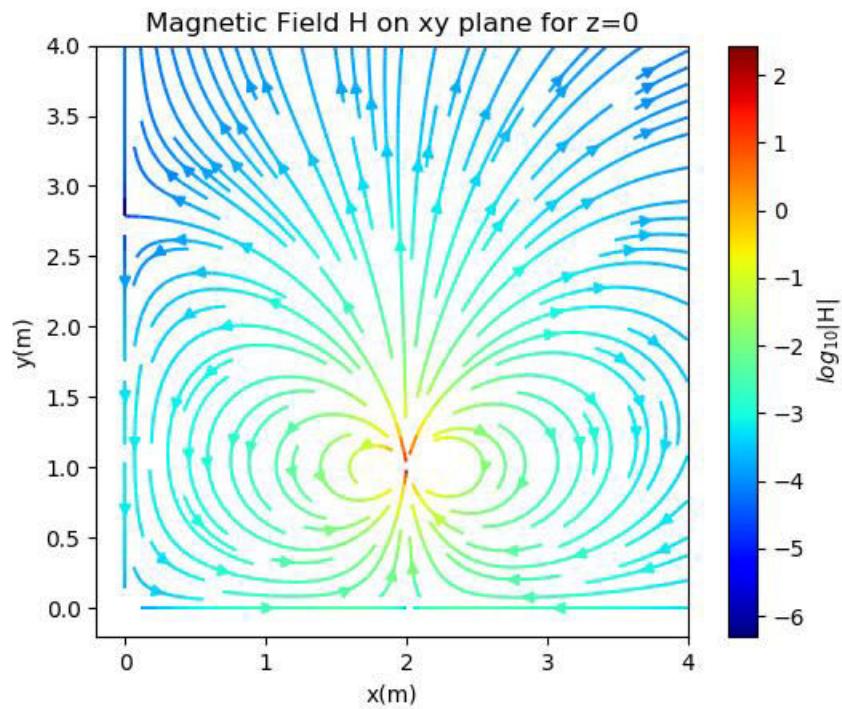
Normalised Magnetic Potential A/μ_0 on xy plane for $z=2$



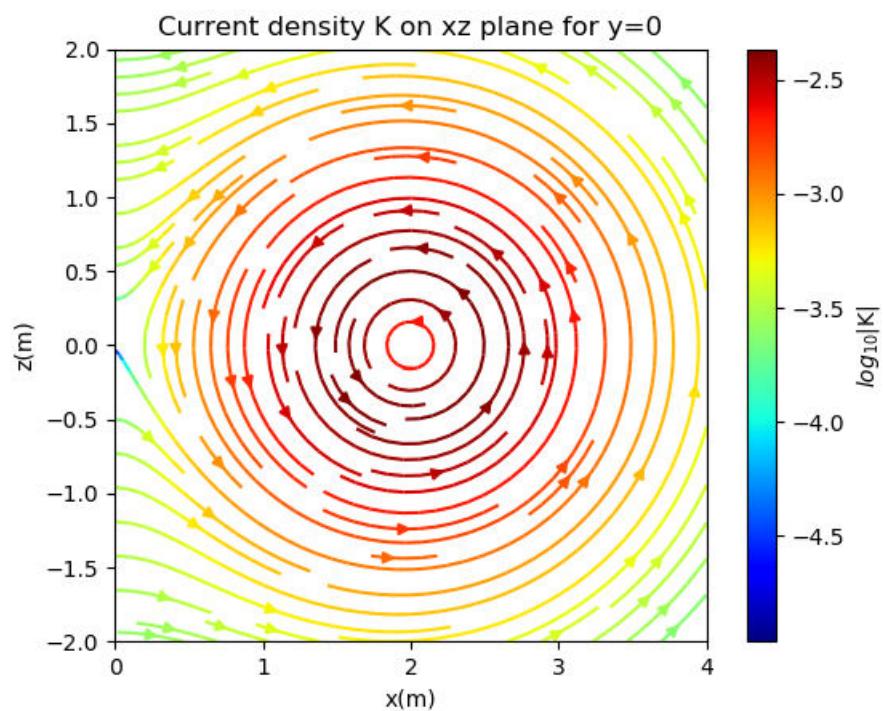
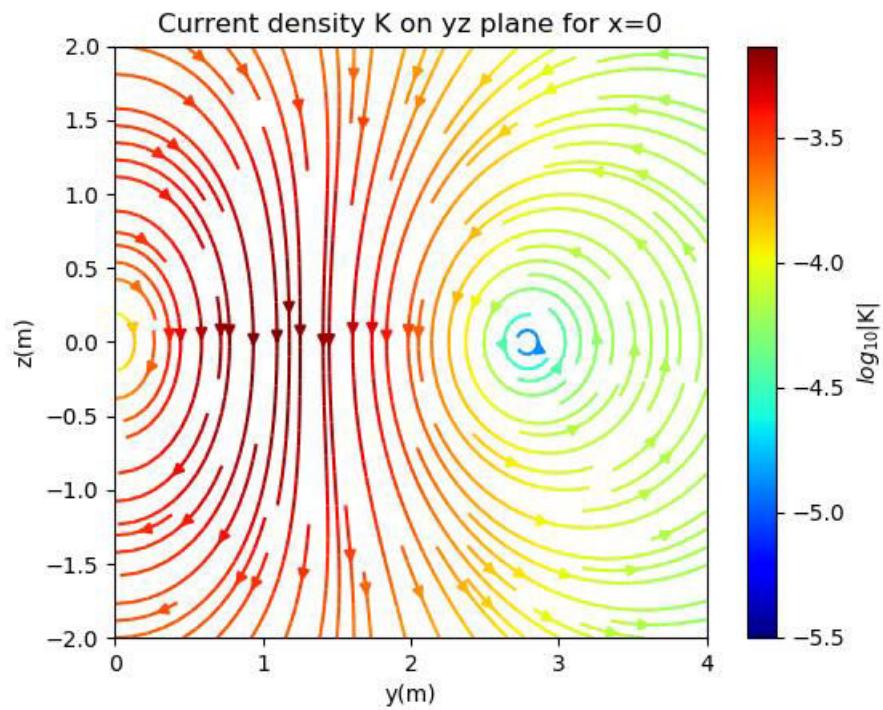
Normalised Magnetic Potential A/μ_0 on xy plane for $z=2$



(ε)



$(\sigma\tau)$



Κώδικας – Ασκηση 8

σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
d=2
h=1
a=0.1
I=1

#Axes
N=100

xmin=0
xmax=4
xx=np.linspace(xmin,xmax,N)

ymin=0
ymax=4
yy=np.linspace(ymin,ymax,N)

zmin=-2
zmax=2
zz=np.linspace(zmin,zmax,N)

X,Z=np.meshgrid(xx,zz)
XX,Y=np.meshgrid(xx,yy)
YY,ZZ=np.meshgrid(yy,zz)

#functions for distances
def R1(x,y,z):
    return np.sqrt((x-d)**2+(y-h)**2+z**2)

def R2(x,y,z):
    return np.sqrt((x-d)**2+(y+h)**2+z**2)

def R3(x,y,z):
    return np.sqrt((x+d)**2+(y-h)**2+z**2)

def R4(x,y,z):
    return np.sqrt((x+d)**2+(y+h)**2+z**2)
```

```

#functions for potential
def Ax(x,y,z):
    res=1/R1(x,y,z)**3-1/R2(x,y,z)**3+1/R3(x,y,z)**3-1/R4(x,y,z)**3
    return z*I*(a**2)*res/4

Ay=np.zeros((N,N))

def Az(x,y,z):
    res=-(x-d)/R1(x,y,z)**3+(x-d)/R2(x,y,z)**3-(x+d)/R3(x,y,z)**3+(x+d)/R4(x,y,z)**3
    return I*(a**2)*res/4

#functions for magnetic field
def Hx(x,y,z):
    res=(y-h)*(x-d)/R1(x,y,z)**5-(y+h)*(x-d)/R2(x,y,z)**5+(y-h)*(x+d)/R3(x,y,z)**5-(y+h)*(x+d)/R4(x,y,z)**5
    return 3*I*(a**2)*res/4

def Hy(x,y,z):
    Hy1=1/(R1(x,y,z)**3)*(3*(y-h)**2/R1(x,y,z)**2-1)
    Hy2=1/(R2(x,y,z)**3)*(-3*(y+h)**2/R2(x,y,z)**2+1)
    Hy3=1/(R3(x,y,z)**3)*(3*(y-h)**2/R3(x,y,z)**2-1)
    Hy4=1/(R4(x,y,z)**3)*(-3*(y+h)**2/R4(x,y,z)**2+1)
    return I*(a**2)/4*(Hy1+Hy2+Hy3+Hy4)

#functions for current density on yz plane
def Ky_yz(y,z):
    r1=R1(0,y,z)
    r2=R2(0,y,z)
    r3=R3(0,y,z)
    r4=R4(0,y,z)
    return I*(a**2)/4*3*(-(y-h)*z/r1**5+(y+h)*z/r2**5-(y-h)*z/r3**5+(y+h)*z/r4**5)

def Kz_yz(y,z):
    r1=R1(0,y,z)
    r2=R2(0,y,z)
    r3=R3(0,y,z)
    r4=R4(0,y,z)
    term1=3*(y-h)**2/r1**5-1/r1**3
    term2=-3*(y+h)**2/r2**5+1/r2**3
    term3=3*(y-h)**2/r3**5-1/r3**3
    term4=-3*(y+h)**2/r4**5+1/r4**3
    return I*(a**2)/4*(term1+term2+term3+term4)

#functions for current density on xz plane
def Kx_xz(x,z):
    r1=R1(x,0,z)
    r2=R2(x,0,z)
    r3=R3(x,0,z)
    r4=R4(x,0,z)
    return -I*(a**2)/4*3*h*z*(1/r1**5+1/r2**5+1/r3**5+1/r4**5)

def Kz_xz(x,z):
    r1=R1(x,0,z)
    r2=R2(x,0,z)
    r3=R3(x,0,z)
    r4=R4(x,0,z)
    return I*(a**2)/4*3*(h*(x-d)/r1**5+h*(x-d)/r2**5+h*(x+d)/r3**5+h*(x+d)/r4**5)

```

```

#PLOTS
#streamplot of magnetic potential on xz plane
fig1, ax1 = plt.subplots()
p1=ax1.streamplot(X,Z,Ax(X,1,Z),Az(X,1,Z),color=np.log10(np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)),cmap=cm.jet)
ax1.set_aspect('equal','box')
c1=fig1.colorbar(p1.lines)
c1.set_label('$log_{10}|A/\mu_0|$')
ax1.set_title('Normalised Magnetic Potential A/$\mu_0$ on xz plane for y=1')
ax1.set_xlabel('x(m)')
ax1.set_ylabel('z(m)')

#quiver plot of magnetic potential on xz plane
#use N=30
Ax_norm=Ax(X,1,Z)/np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)
Az_norm=Az(X,1,Z)/np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)

fig2, ax2 = plt.subplots()
plt.quiver(X,Z,Ax_norm,Az_norm)
ax2.set_aspect('equal','box')
ax2.set_title('Normalised Magnetic Potential A/$\mu_0$ on xz plane for y=1')
ax2.set_xlabel('x(m)')
ax2.set_ylabel('z(m)')

#streamplot of magnetic potential on xy plane
fig3, ax3 = plt.subplots()
p3=ax3.streamplot(XX,Y,Ax(XX,Y,2),Ay,color=np.log10(np.sqrt(Ax(XX,Y,2)**2+Ay**2)),cmap=cm.jet)
ax3.set_aspect('equal','box')
c3=fig3.colorbar(p3.lines)
c3.set_label('$log_{10}|A/\mu_0|$')
ax3.set_title('Normalised Magnetic Potential A/$\mu_0$ on xy plane for z=2')
ax3.set_xlabel('x(m)')
ax3.set_ylabel('y(m)')

#quiver plot of magnetic potential on xy plane
#use N=30
Ax_norm2=Ax(XX,Y,2)/np.sqrt(Ax(XX,Y,2)**2+Ay**2)
Ay_norm=Ay/np.sqrt(Ax(XX,Y,2)**2+Ay**2)

fig4, ax4 = plt.subplots()
plt.quiver(XX,Y,Ax_norm2,Ay)
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Magnetic Potential A/$\mu_0$ on xy plane for z=2')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('y(m)')

#streamplot of magnetic field on xy plane
fig5, ax5 = plt.subplots()
p5=ax5.streamplot(XX,Y,Hx(XX,Y,0),Hy(XX,Y,0),color=np.log10(np.sqrt(Hx(XX,Y,0)**2+Hy(XX,Y,0)**2)),cmap=cm.jet,density=1.2)
ax5.set_aspect('equal','box')
c5=fig5.colorbar(p5.lines)
c5.set_label('$log_{10}|H|$')
ax5.set_title('Magnetic Field H on xy plane for z=0')
ax5.set_xlabel('x(m)')
ax5.set_ylabel('y(m)')

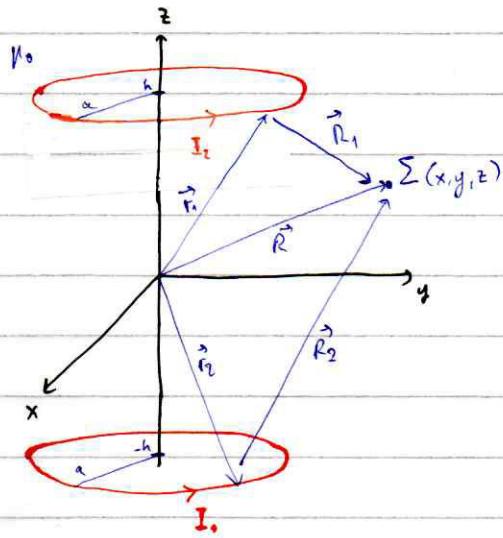
#streamplot of current density on yz plane
fig6, ax6 = plt.subplots()
p6=ax6.streamplot(YY,ZZ,Ky_yz(YY,ZZ),Kz_yz(YY,ZZ),color=np.log10(np.sqrt(Ky_yz(YY,ZZ)**2+Kz_yz(YY,ZZ)**2)),cmap=cm.jet,density=1.2)
ax6.set_aspect('equal','box')
c6=fig6.colorbar(p6.lines)
c6.set_label('$log_{10}|K|$')
ax6.set_title('Current density K on yz plane for x=0')
ax6.set_xlabel('y(m)')
ax6.set_ylabel('z(m)')

#streamplot of current density on xz plane
fig7, ax7 = plt.subplots()
p7=ax7.streamplot(X,Z,Kx_xz(X,Z),Kz_xz(X,Z),color=np.log10(np.sqrt(Kx_xz(X,Z)**2+Kz_xz(X,Z)**2)),cmap=cm.jet,density=1.2)
ax7.set_aspect('equal','box')
c7=fig7.colorbar(p7.lines)
c7.set_label('$log_{10}|K|$')
ax7.set_title('Current density K on xz plane for y=0')
ax7.set_xlabel('x(m)')
ax7.set_ylabel('z(m)')

plt.show()

```

Araçyon 9



$$(a) \vec{A}(x, y, z) = \vec{A}_1 + \vec{A}_2 = \frac{\mu_0}{4\pi} \int_{C_1} \frac{I_1(l_1) d\vec{l}_1}{R_1} + \frac{\mu_0}{4\pi} \int_{C_2} \frac{I_2(l_2) d\vec{l}_2}{R_2} =$$

$$= \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \frac{a \cos \varphi_1 d\varphi_1}{R_1} + \frac{\mu_0 I_2}{4\pi} \int_0^{2\pi} \frac{a \cos \varphi_2 d\varphi_2}{R_2}$$

önü $\vec{R} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$

$$\vec{r}_1 = a \cos \varphi_1 \hat{i}_x + a \sin \varphi_1 \hat{i}_y - h\hat{i}_z$$

$$\vec{r}_1 = \vec{R} - \vec{r}_1 = (x - a \cos \varphi_1) \hat{i}_x + (y - a \sin \varphi_1) \hat{i}_y + (z - h) \hat{i}_z \Rightarrow R_1 = \sqrt{x^2 + y^2 + (z-h)^2 + a^2 - 2a(x \cos \varphi_1 + y \sin \varphi_1)}$$

$$\vec{r}_2 = a \cos \varphi_2 \hat{i}_x + a \sin \varphi_2 \hat{i}_y - h\hat{i}_z$$

$$\vec{r}_2 = \vec{R} - \vec{r}_2 = (x - a \cos \varphi_2) \hat{i}_x + (y - a \sin \varphi_2) \hat{i}_y + (z - h) \hat{i}_z \Rightarrow R_2 = \sqrt{x^2 + y^2 + (z-h)^2 + a^2 - 2a(x \cos \varphi_2 + y \sin \varphi_2)}$$

$$(8) \text{ Εναλλαγμα: } \vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{\vec{B}_1}{\mu_0} + \frac{\vec{B}_2}{\mu_0}$$

$$\text{Νόμος Biot-Savart: } \vec{H}_1 = \frac{1}{4\pi} \int_{C_1} I_1 \frac{d\vec{l}_1 \times \vec{i}_{R_1}}{R_1^3} = \frac{I_1}{4\pi} \int_{C_1} \frac{d\vec{l}_1 \times \vec{R}_1}{R_1^3}$$

$$\text{δηνou } \vec{R}_1 = (x - a \cos \varphi_1) \hat{i}_x + (y - a \sin \varphi_1) \hat{i}_y + (z + h) \hat{i}_z$$

$$R_1 = \sqrt{x^2 + y^2 + (z+h)^2 + a^2 - 2a(x \cos \varphi_1 + y \sin \varphi_1)}$$

$$d\vec{l}_1 = ad\varphi_1 \hat{i}_{\varphi_1}$$

$$\hat{i}_{\varphi_1} = -\sin \varphi_1 \hat{i}_x + \cos \varphi_1 \hat{i}_y$$

$$\text{Απo } d\vec{l}_1 \times \vec{R}_1 = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -a \sin \varphi_1 d\varphi_1 & a \cos \varphi_1 d\varphi_1 & 0 \\ x - a \cos \varphi_1 & y - a \sin \varphi_1 & z + h \end{vmatrix} = i_x (z+h) a \cos \varphi_1 d\varphi_1 + i_y (z+h) a \sin \varphi_1 d\varphi_1 - i_z a (x \cos \varphi_1 + y \sin \varphi_1 - a) d\varphi_1$$

Αυτούσιων αριθμ. διαίρεση για το \vec{H}_2 έχουμε τελικά

$$\begin{aligned} \vec{H}(x, y, z) = & \frac{I_1}{4\pi} \int_{\varphi_1=0}^{2\pi} \frac{[i_x(z+h) \cos \varphi_1 + i_y(z+h) \sin \varphi_1 - i_z(x \cos \varphi_1 + y \sin \varphi_1 - a)] ad\varphi_1}{[x^2 + y^2 + (z+h)^2 + a^2 - 2a(x \cos \varphi_1 + y \sin \varphi_1)]^{3/2}} + \\ & + \frac{I_2}{4\pi} \int_{\varphi_2=0}^{2\pi} \frac{[i_x(z-h) \cos \varphi_2 + i_y(z-h) \sin \varphi_2 - i_z(x \cos \varphi_2 + y \sin \varphi_2 - a)] ad\varphi_2}{[x^2 + y^2 + (z-h)^2 + a^2 - 2a(x \cos \varphi_2 + y \sin \varphi_2)]^{3/2}} \end{aligned}$$

$$(7) \quad \vec{H}_z(0,0,z) = \frac{I_1}{4\pi} \int_{q_1=0}^{2\pi} \frac{i_z a^2 dq_1}{R_1^3(0,0,z)} + \frac{I_2}{4\pi} \int_{q_2=0}^{2\pi} \frac{i_z a^2 dq_2}{R_2^3(0,0,z)} = \left(\frac{I_1}{2} \frac{a^2}{[a^2 + (z+h)^2]^{3/2}} + \frac{I_2}{2} \frac{a^2}{[a^2 + (z-h)^2]^{3/2}} \right) \hat{z}$$

Selvä Taylor:

$$H_z(z) = H_z(0) + H'_z(0) z + \frac{H''_z(0)}{2} z^2 + \frac{H'''_z(0)}{3!} z^3 + \frac{H^{(4)}_z(0)}{4!} z^4$$

$$\text{ò növ } H_z(0) = \frac{I_1 + I_2}{2} \frac{a^2}{[a^2 + h^2]^{3/2}} \hat{z}$$

$$H'_z(0) = \frac{I_1 - I_2}{2} \frac{3a^2 h}{[a^2 + h^2]^{5/2}} \hat{z}$$

$$H''_z(0) = \frac{I_1 + I_2}{2} \left(\frac{15a^2 h^2}{[a^2 + h^2]^{7/2}} - \frac{3a^2}{[a^2 + h^2]^{5/2}} \right) \hat{z}$$

$$H'''_z(0) = \frac{I_1 - I_2}{2} \left(\frac{3a^2 \cdot 8h}{[a^2 + h^2]^{7/2}} - \frac{21a^2 h (4h^2 - a^2)}{[a^2 + h^2]^{9/2}} \right) \hat{z}$$

$$H^{(4)}_z(0) = \frac{I_1 + I_2}{2} \left(\frac{-15a^2 (12h^2 - 3a^2)}{[a^2 + h^2]^{9/2}} + \frac{135a^2 h^2 (4h^2 - 3a^2)}{[a^2 + h^2]^{11/2}} \right) \hat{z}$$

Parasympohts öv, av $I_1 = I_2$ ei näpäjwja se pikkud vahad löövur O

$$(7) \quad X_{\text{perekonnia}} \text{ on } L_{12} = L_{21} = \mu_0 a \left[\frac{2-k^2}{k} K(k) - \frac{2}{k} E(k) \right]$$

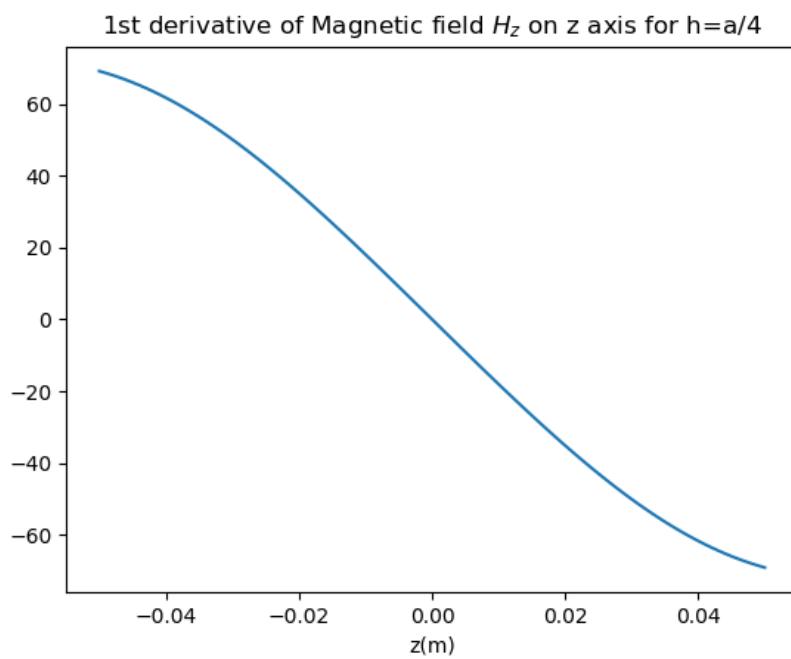
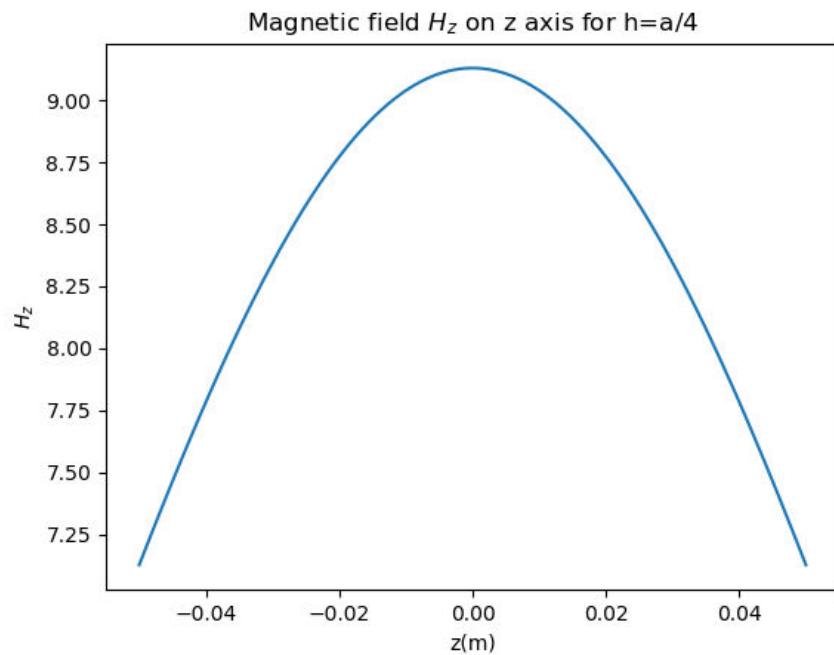
$$\text{m } K(k) = \int_0^{\pi/2} \frac{dw}{\sqrt{1-k^2 \sin^2 w}} \quad \text{m } E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 w} dw$$

$$\text{ò növ } k = \frac{2a}{\sqrt{h^2 + a^2}}$$

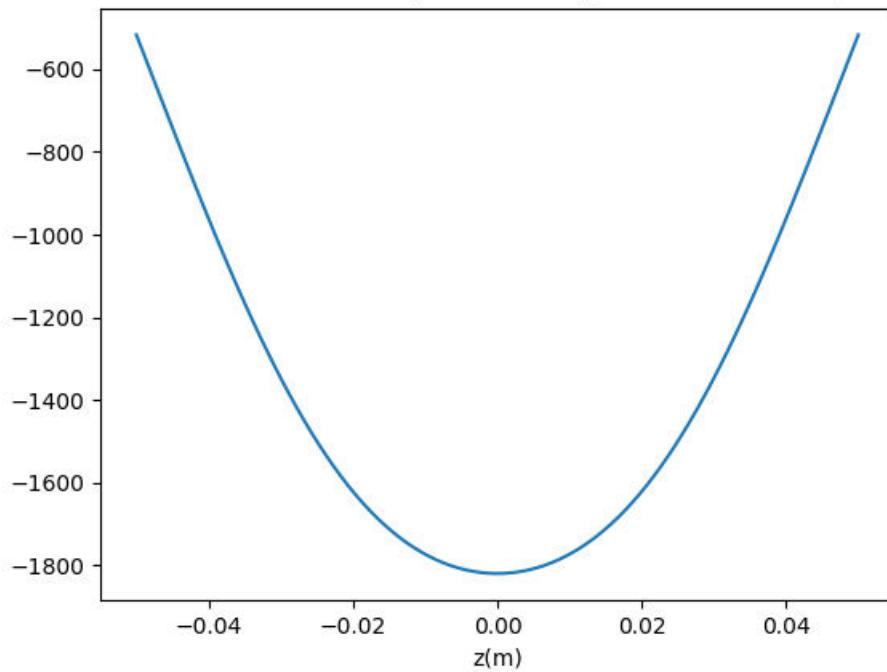
Γραφικές παραστάσεις – Άσκηση 9

έγιναν σε Python 3.9

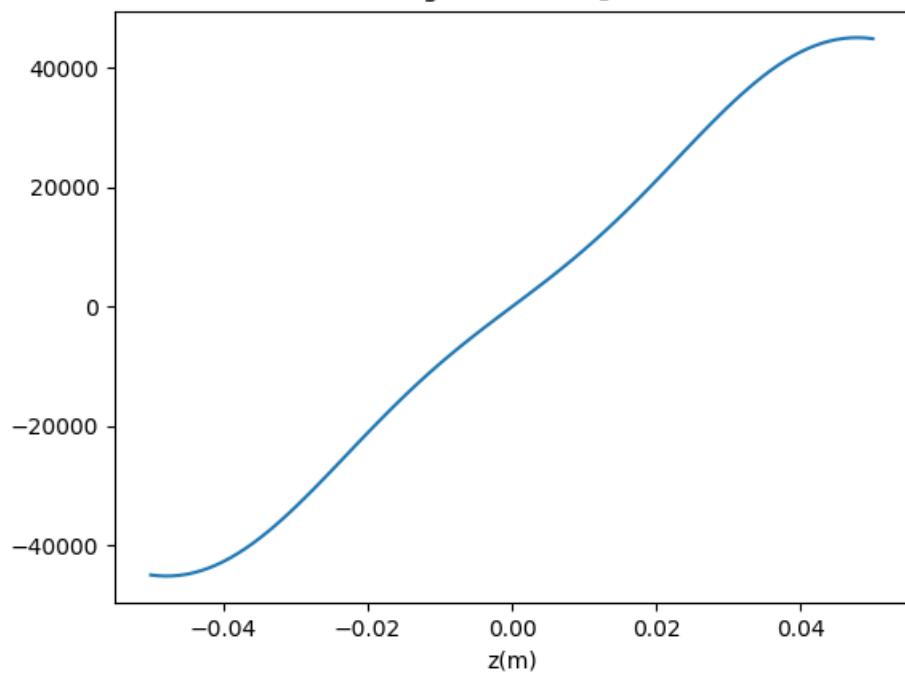
(δ)



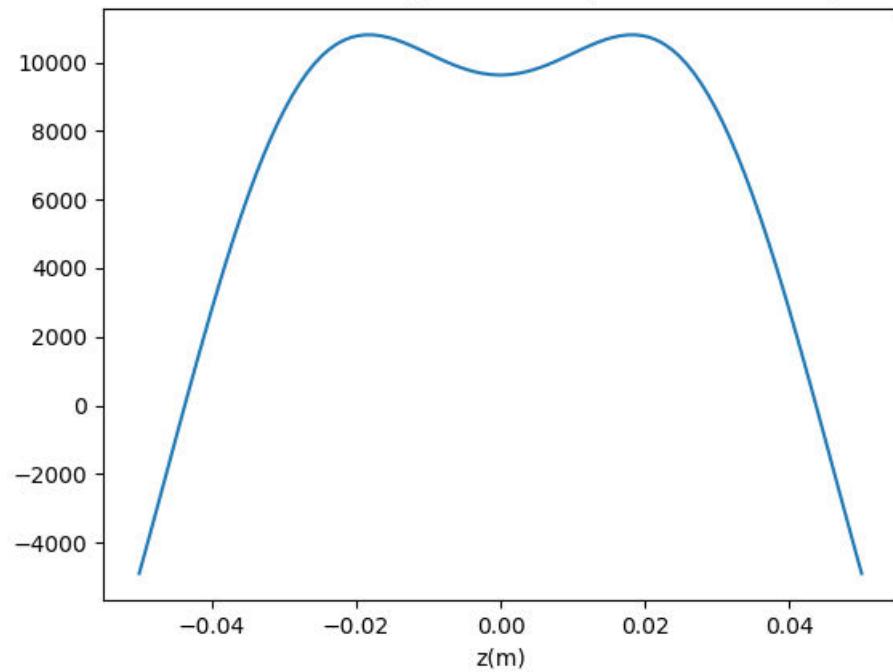
2nd derivative of Magnetic field H_z on z axis for $h=a/4$

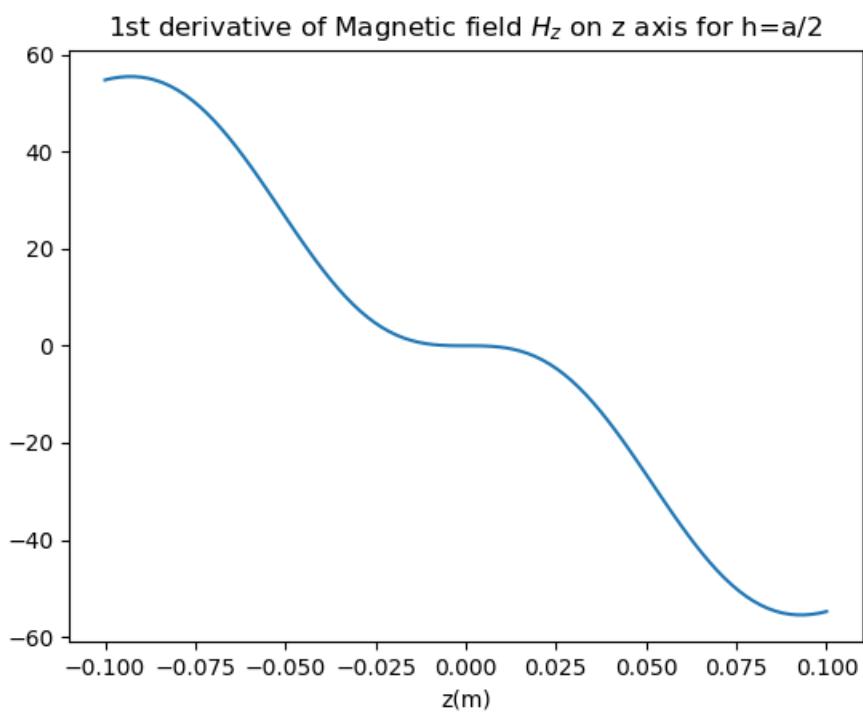
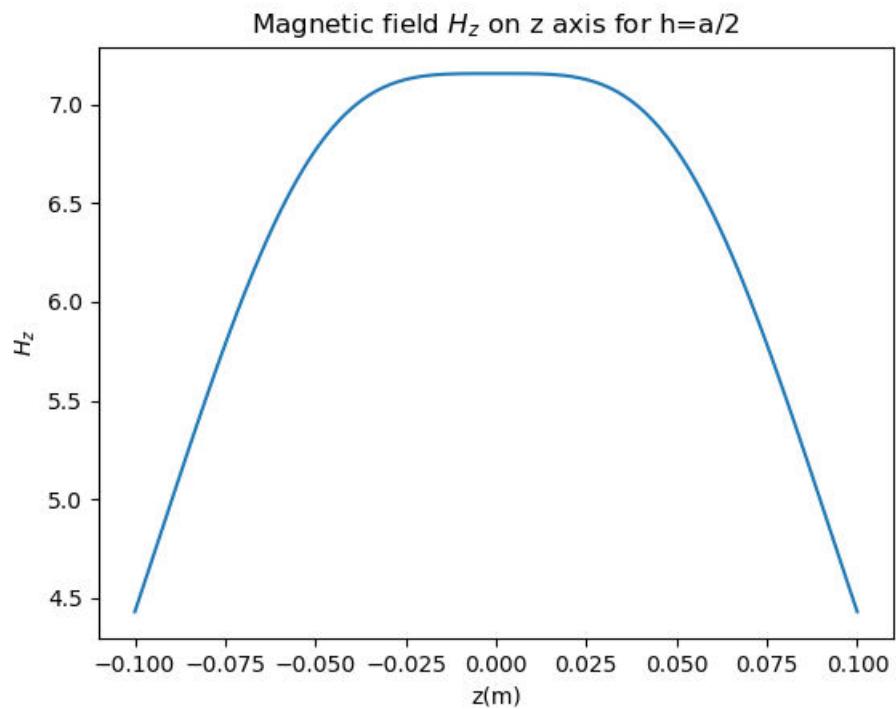


3rd derivative of Magnetic field H_z on z axis for $h=a/4$

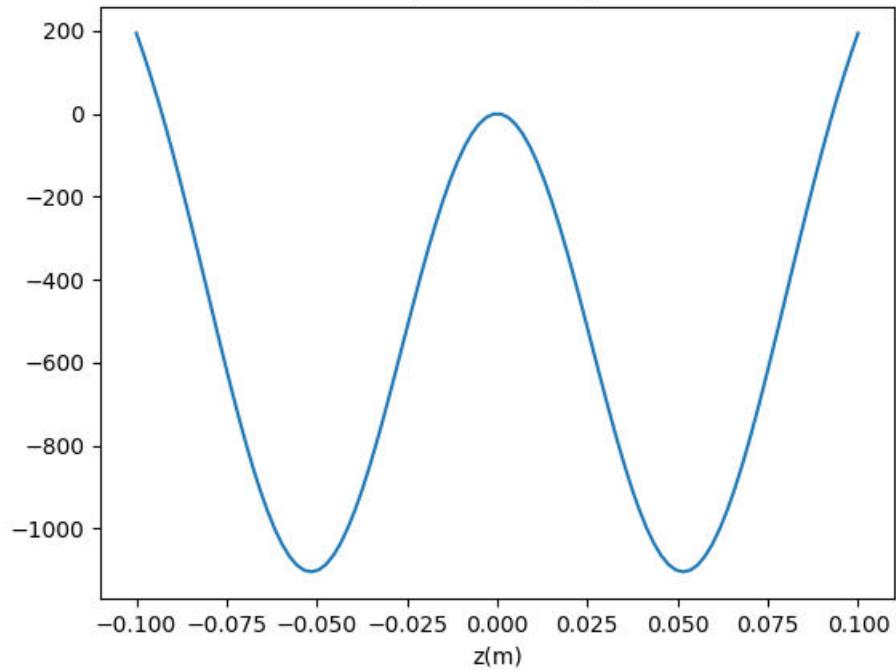


4th derivative of Magnetic field H_z on z axis for $h=a/4$

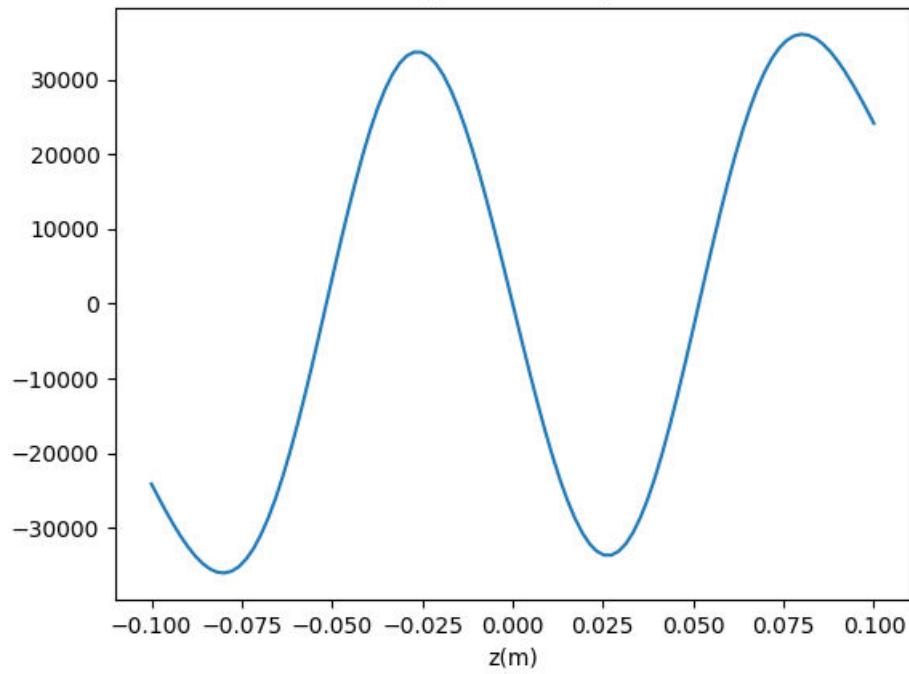




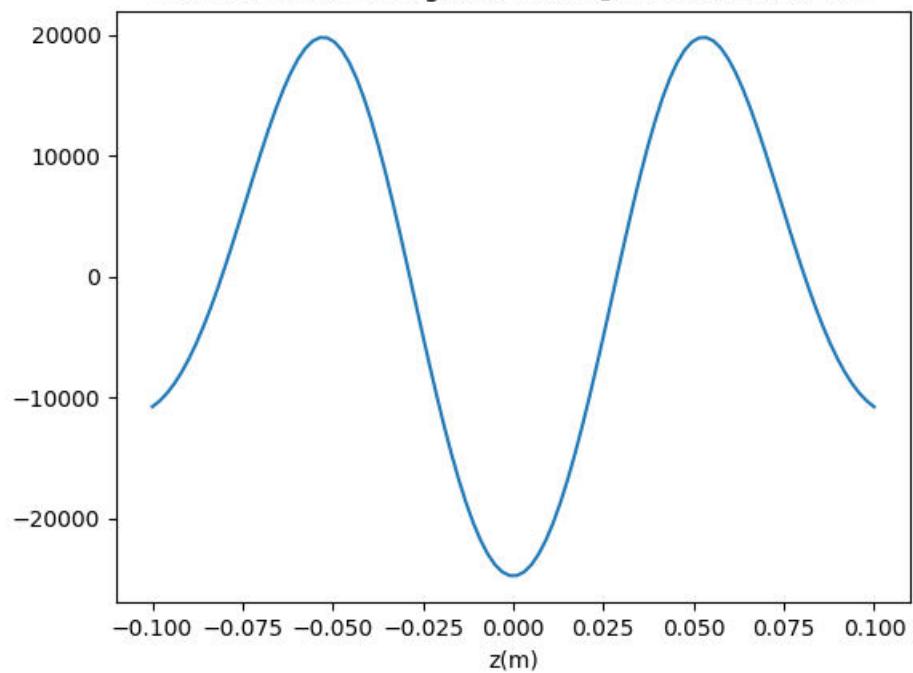
2nd derivative of Magnetic field H_z on z axis for $h=a/2$

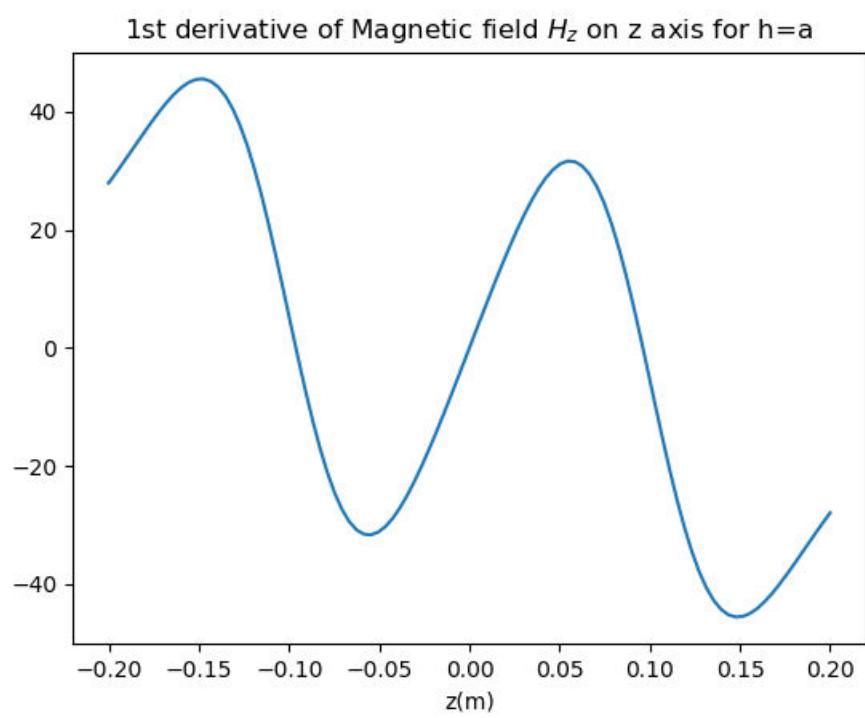
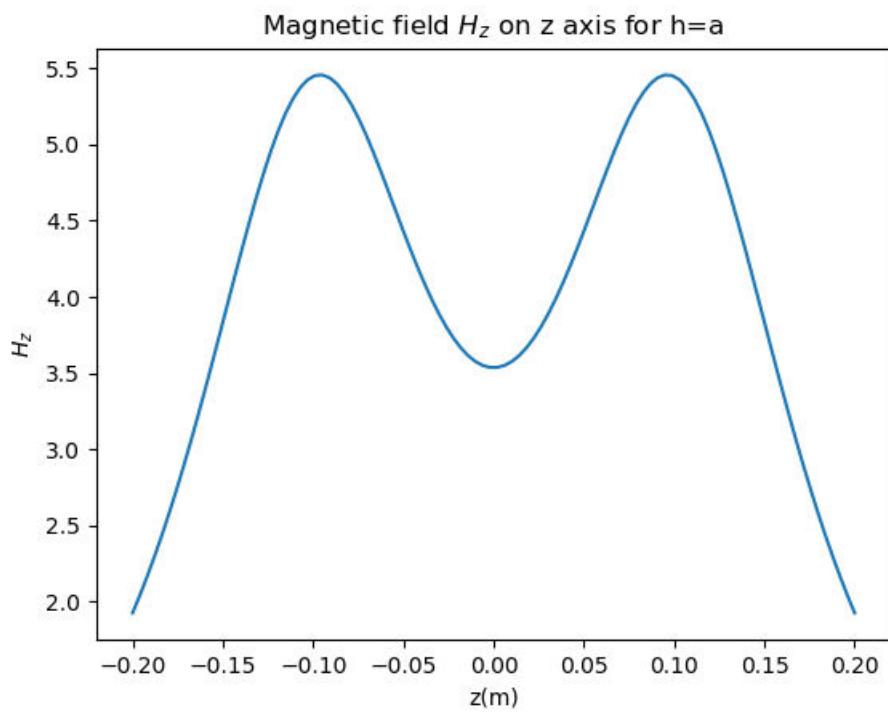


3rd derivative of Magnetic field H_z on z axis for $h=a/2$

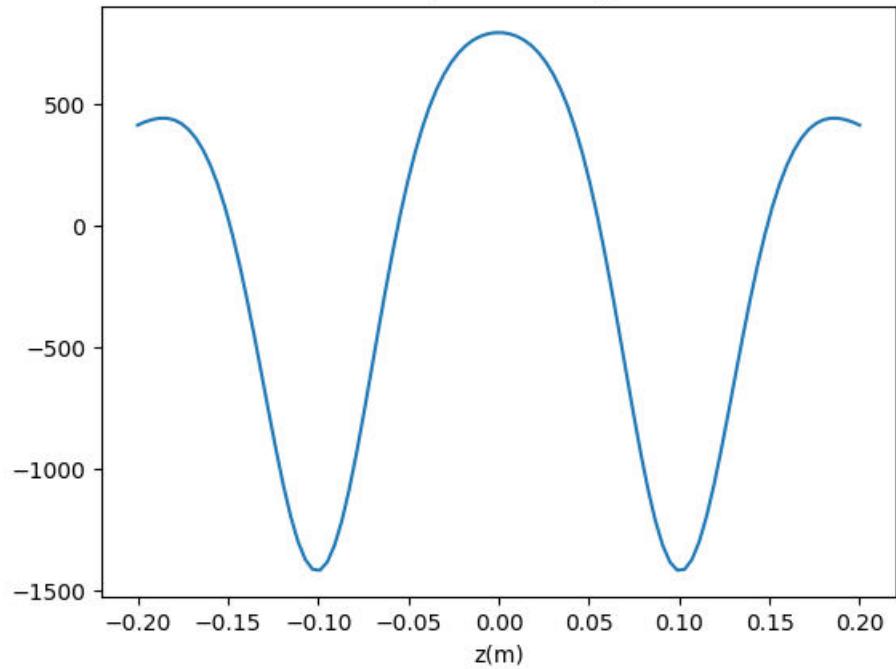


4th derivative of Magnetic field H_z on z axis for $h=a/2$

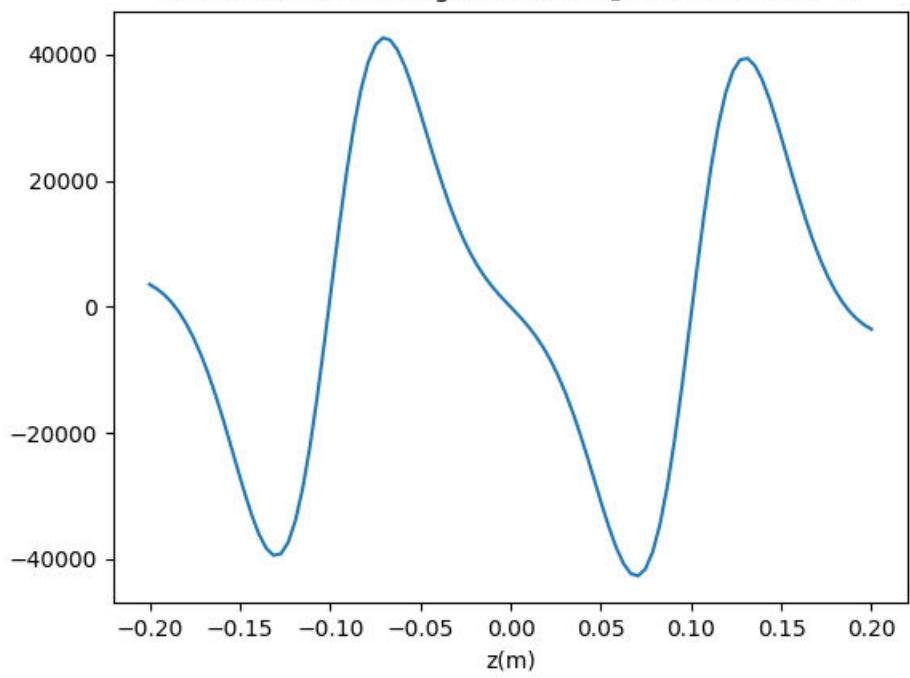




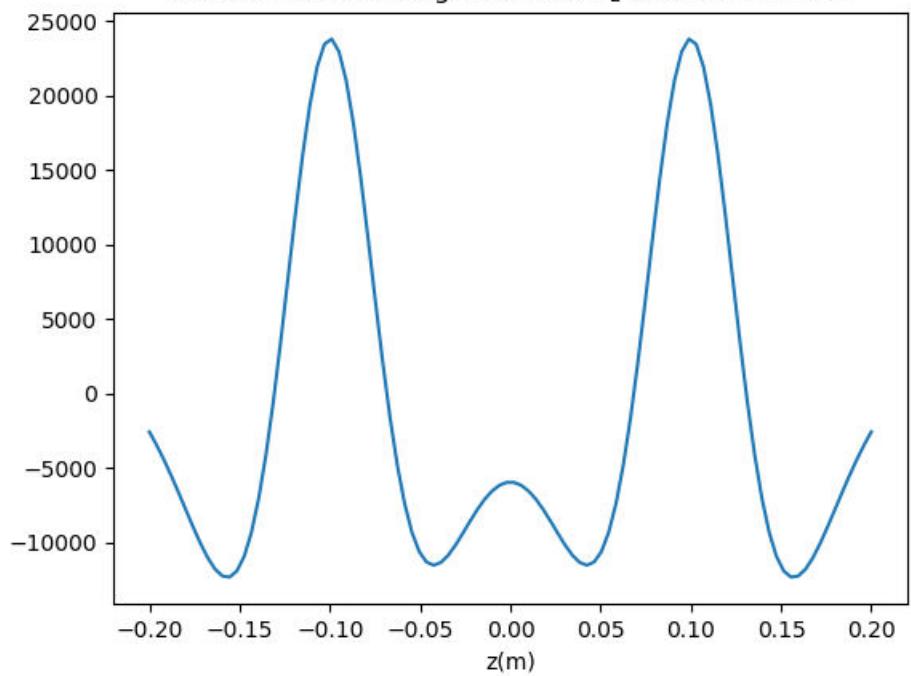
2nd derivative of Magnetic field H_z on z axis for $h=a$



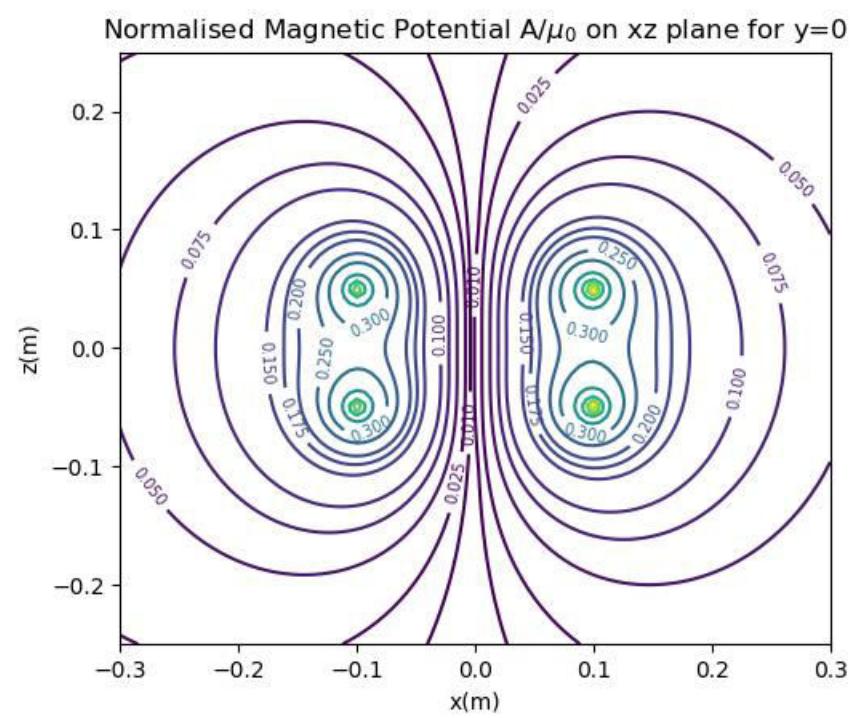
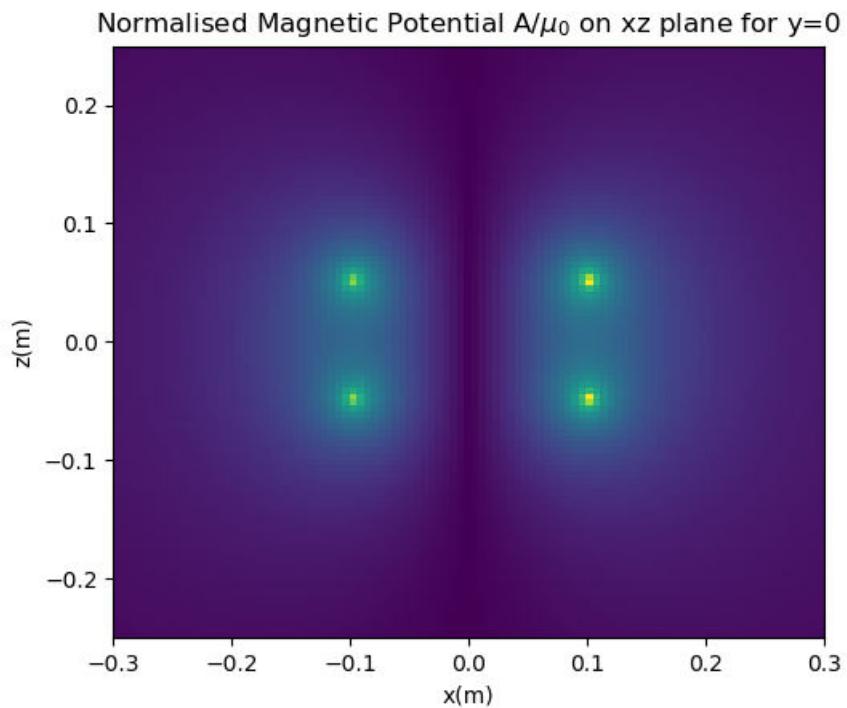
3rd derivative of Magnetic field H_z on z axis for $h=a$



4th derivative of Magnetic field H_z on z axis for $h=a$

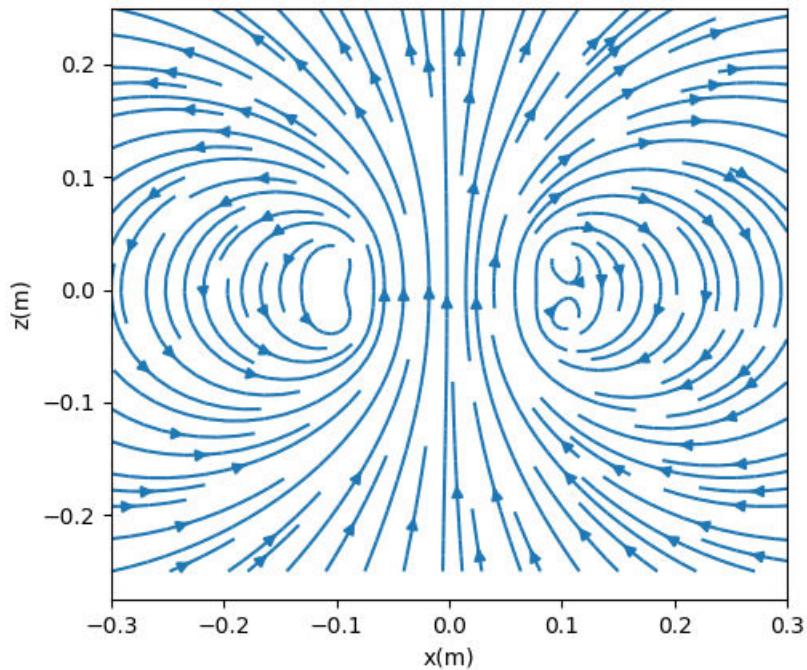


(ε)

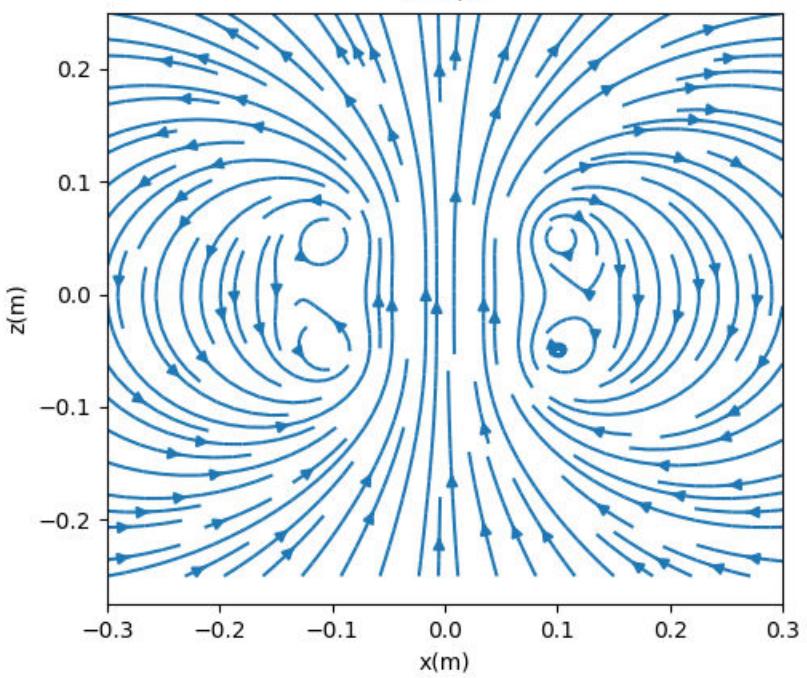


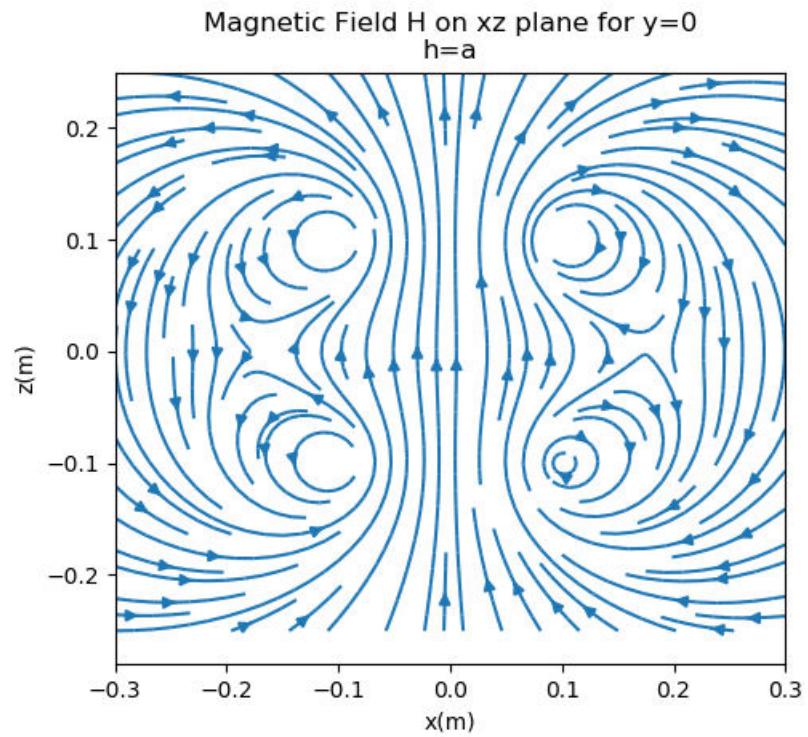
$(\sigma\tau)$

Magnetic Field H on xz plane for $y=0$
 $h=a/4$

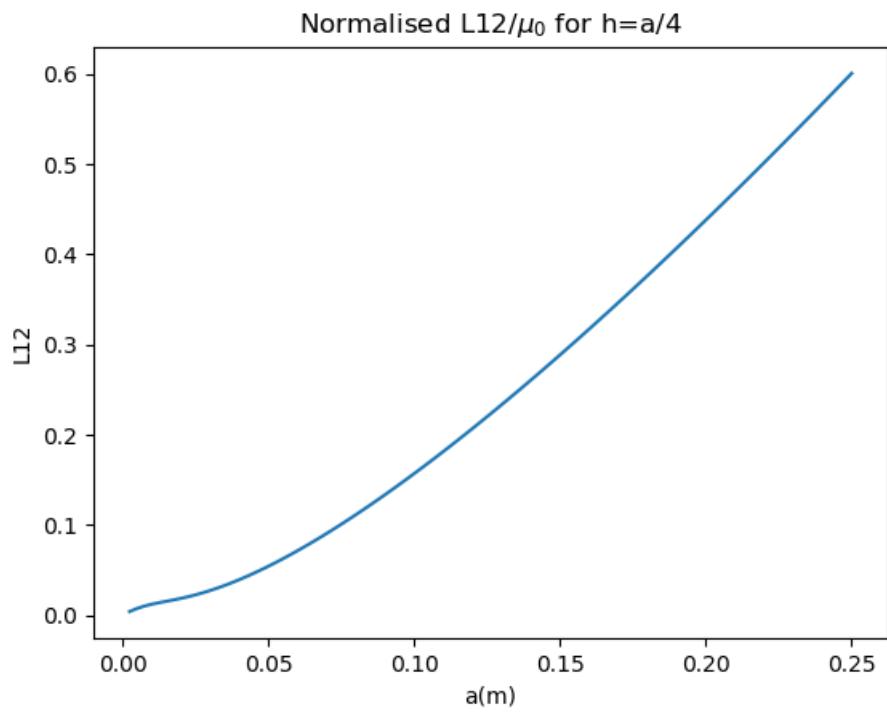


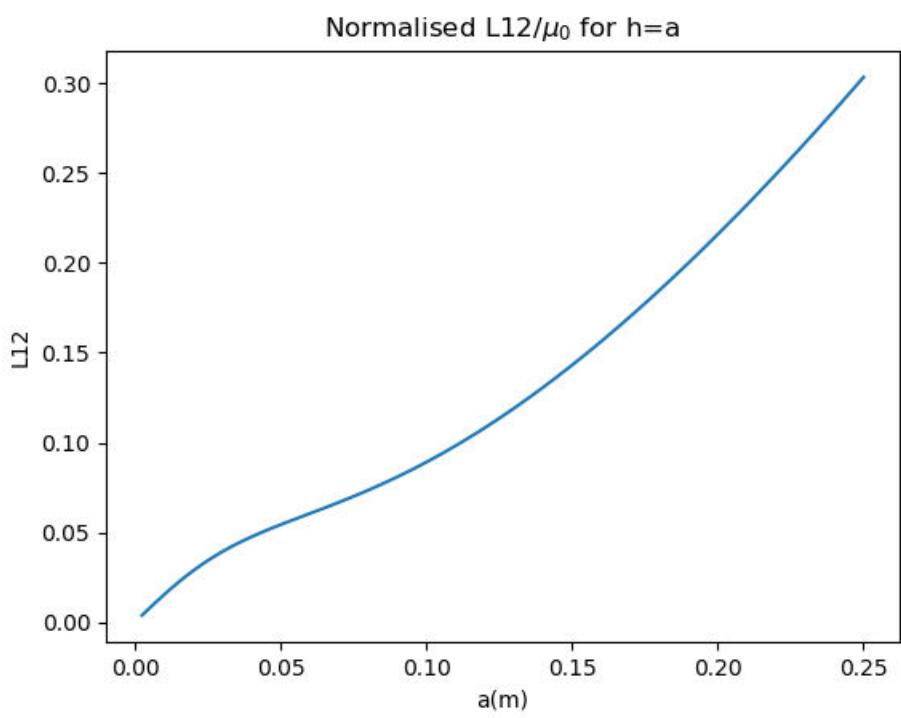
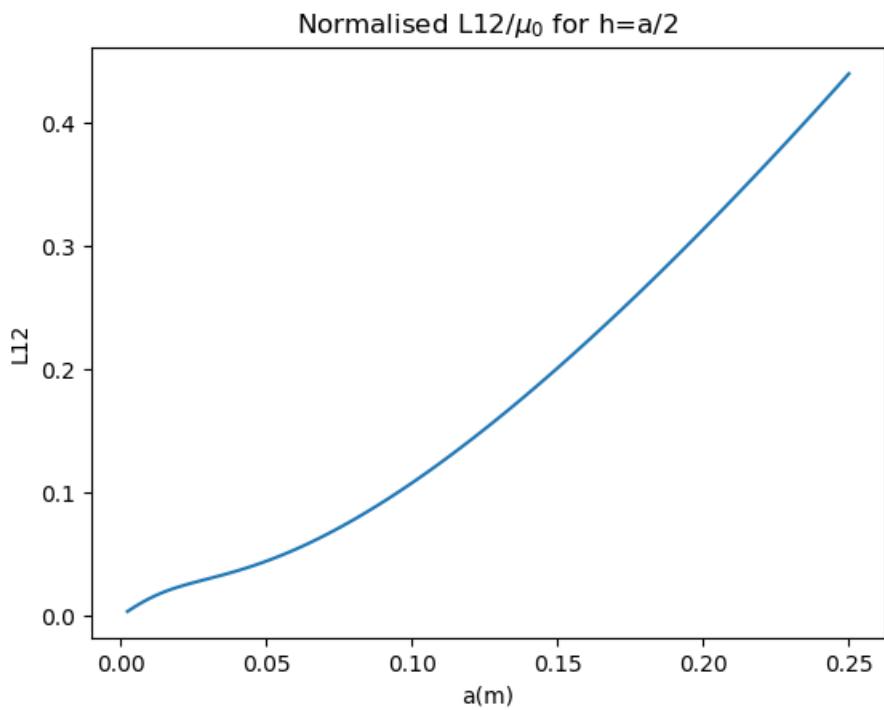
Magnetic Field H on xz plane for $y=0$
 $h=a/2$





(ζ)





Κώδικας – Ασκηση 9

σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math
from scipy.special import ellipk
from scipy.special import ellipe

#constants
a=0.1
h=a/2
I1=1
I2=1
levels=[0.01,0.025,0.05,0.075,0.1,0.15,0.175,0.2,0.25,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1,1.25,1.5,2,3,5]

#Axes
N=100

xmin=-3*a
xmax=3*a
xx=np.linspace(xmin,xmax,N)

zmin=-(h+2*a)
zmax=h+2*a
zz=np.linspace(zmin,zmax,N)

X,Z=np.meshgrid(xx,zz)

amax=0.25
aa=np.linspace(0,amax,N)

#function for Hz for x,y=0
def Hz0(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    return a**2/2*(I1/R1**3+I2/R2**3)

#first derivative of Hz
def Hz1(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=-3*a**2*(z+h)/R1**5
    term2=-3*a**2*(z-h)/R2**5
    return I1/2*term1+I2/2*term2

#second derivative of Hz
def Hz2(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=3*a**2*(4*z**2+8*h*z+4*h**2-a**2)/R1**7
    term2=3*a**2*(4*z**2-8*h*z+4*h**2-a**2)/R2**7
    return I1/2*term1+I2/2*term2

#third derivative of Hz
def Hz3(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=-15*a**2*(z+h)*(4*z**2+8*h*z+4*h**2-3*a**2)/R1**9
    term2=-15*a**2*(z-h)*(4*z**2-8*h*z+4*h**2-3*a**2)/R2**9
    return I1/2*term1+I2/2*term2

#fourth derivative of Hz
def Hz4(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=45*a**2*(8*z**4+32*h*z**3+(48*h**2-12*a**2)*z**2-(24*a**2*h-32*h**3)*z+8*h**4-12*a**2*h**2+a**4)/R1**9
    term2=45*a**2*(8*z**4-32*h*z**3+(48*h**2-12*a**2)*z**2+(24*a**2*h-32*h**3)*z+8*h**4-12*a**2*h**2+a**4)/R2**9
    return I1/2*term1+I2/2*term2
```

```

#function for Potential
def A_phi(x,y,z):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=a*np.cos(i)*dphi/R1
        term2+=a*np.cos(i)*dphi/R2
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)

#functions for Magnetic Field
def Hx(x,y,z,h):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=(z+h)*a*np.cos(i)*dphi/R1**3
        term2+=(z-h)*a*np.cos(i)*dphi/R2**3
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)

def Hz(x,y,z,h):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=(-x*np.cos(i)-y*np.sin(i)+a)*a*dphi/R1**3
        term2+=(-x*np.cos(i)-y*np.sin(i)+a)*a*dphi/R2**3
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)

#function for L12
def L12(a,h):
    k=4*a**2/(h**2+4*a**2)
    return a*((2-k**2)/k*ellip(k)-2/k*ellipe(k))

```

```

#PLOTS
#plot of Hz for h=a/4
hh=a/4
z1=np.linspace(-2*hh,2*hh,N)
fig11,ax11=plt.subplots()
p11=ax11.plot(z1,Hz0(z1, hh))
ax11.set_title('Magnetic field $H_z$ on z axis for h=a/4')
ax11.set_xlabel('z(m)')
ax11.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a/4
fig12,ax12=plt.subplots()
p12=ax12.plot(z1,Hz1(z1, hh))
ax12.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax12.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a/4
fig13,ax13=plt.subplots()
p13=ax13.plot(z1,Hz2(z1, hh))
ax13.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax13.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a/4
fig14,ax14=plt.subplots()
p14=ax14.plot(z1,Hz3(z1, hh))
ax14.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax14.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a/4
fig15,ax15=plt.subplots()
p15=ax15.plot(z1,Hz4(z1, hh))
ax15.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax15.set_xlabel('z(m)')

#plot of Hz for h=a/2
hh=a/2
z1=np.linspace(-2*hh,2*hh,N)
fig21,ax21=plt.subplots()
p21=ax21.plot(z1,Hz0(z1, hh))
ax21.set_title('Magnetic field $H_z$ on z axis for h=a/2')
ax21.set_xlabel('z(m)')
ax21.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a/2
fig22,ax22=plt.subplots()
p22=ax22.plot(z1,Hz1(z1, hh))
ax22.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax22.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a/2
fig23,ax23=plt.subplots()
p23=ax23.plot(z1,Hz2(z1, hh))
ax23.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax23.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a/2
fig24,ax24=plt.subplots()
p24=ax24.plot(z1,Hz3(z1, hh))
ax24.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax24.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a/2
fig25,ax25=plt.subplots()
p25=ax25.plot(z1,Hz4(z1, hh))
ax25.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax25.set_xlabel('z(m)')

```

```

#plot of Hz for h=a
hh=a
z1=np.linspace(-2*hh,2*hh,N)
fig31,ax31=plt.subplots()
p31=ax31.plot(z1,Hz0(z1, hh))
ax31.set_title('Magnetic field $H_z$ on z axis for h=a')
ax31.set_xlabel('z(m)')
ax31.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a
fig32,ax32=plt.subplots()
p32=ax32.plot(z1,Hz1(z1, hh))
ax32.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a')
ax32.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a
fig33,ax33=plt.subplots()
p33=ax33.plot(z1,Hz2(z1, hh))
ax33.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a')
ax33.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a
fig34,ax34=plt.subplots()
p34=ax34.plot(z1,Hz3(z1, hh))
ax34.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a')
ax34.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a
fig35,ax35=plt.subplots()
p35=ax35.plot(z1,Hz4(z1, hh))
ax35.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a')
ax35.set_xlabel('z(m)')

#surface plot of potential
fig4,ax4=plt.subplots()
p4=ax4.pcolormesh(X,Z,abs(A_phi(X,0,Z)))
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Magnetic Potential A/$\mu_0$ on xz plane for y=0')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('z(m)')

#contour lines of potential
fig5,ax5=plt.subplots()
p5=ax5.contour(X,Z,abs(A_phi(X,0,Z)),levels)
ax5.clabel(p5, inline=True, fontsize=7)
ax5.set_aspect('equal','box')
ax5.set_title('Normalised Magnetic Potential A/$\mu_0$ on xz plane for y=0')
ax5.set_xlabel('x(m)')
ax5.set_ylabel('z(m)')

#streamplot of magnetic field for h=a/4
fig61, ax61 = plt.subplots()
p61=ax61.streamplot(X,Z,Hx(X,0,Z,a/4),Hz(X,0,Z,a/4),density=1.2)
ax61.set_aspect('equal','box')
ax61.set_title('Magnetic Field H on xz plane for y=0\nh=a/4')
ax61.set_xlabel('x(m)')
ax61.set_ylabel('z(m)')

#streamplot of magnetic field for h=a/2
fig62, ax62 = plt.subplots()
p62=ax62.streamplot(X,Z,Hx(X,0,Z,a/2),Hz(X,0,Z,a/2),density=1.2)
ax62.set_aspect('equal','box')
ax62.set_title('Magnetic Field H on xz plane for y=0\nh=a/2')
ax62.set_xlabel('x(m)')
ax62.set_ylabel('z(m)')

#streamplot of magnetic field for h=a
fig63, ax63 = plt.subplots()
p63=ax63.streamplot(X,Z,Hx(X,0,Z,a),Hz(X,0,Z,a),density=1.2)
ax63.set_aspect('equal','box')
ax63.set_title('Magnetic Field H on xz plane for y=0\nh=a')
ax63.set_xlabel('x(m)')
ax63.set_ylabel('z(m)')

```

```

#plot of L12 for h=a/4
fig71,ax71=plt.subplots()
p71=ax71.plot(aa,L12(aa,a/4))
ax71.set_title('Normalised L12/$\mu_0$ for h=a/4')
ax71.set_xlabel('a(m)')
ax71.set_ylabel('L12')

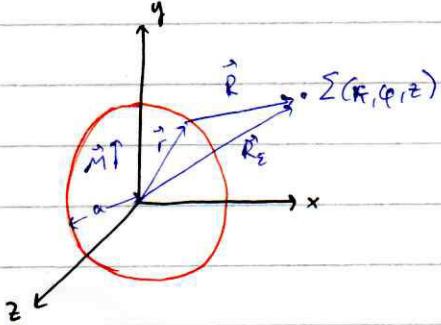
#plot of L12 for h=a/2
fig72,ax72=plt.subplots()
p72=ax72.plot(aa,L12(aa,a/2))
ax72.set_title('Normalised L12/$\mu_0$ for h=a/2')
ax72.set_xlabel('a(m)')
ax72.set_ylabel('L12')

#plot of L12 for h=a
fig73,ax73=plt.subplots()
p73=ax73.plot(aa,L12(aa,a))
ax73.set_title('Normalised L12/$\mu_0$ for h=a')
ax73.set_xlabel('a(m)')
ax73.set_ylabel('L12')

plt.show()

```

Axiom 10



$$\vec{R}_E = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{r} = a \cos \varphi \hat{i}_x + a \sin \varphi \hat{i}_y + z' \hat{i}_z$$

$$\vec{R} = \vec{R}_E - \vec{r} = (x - a \cos \varphi) \hat{i}_x + (y - a \sin \varphi) \hat{i}_y + (z - z') \hat{i}_z$$

$$R = \sqrt{(x - a \cos \varphi)^2 + (y - a \sin \varphi)^2 + (z - z')^2} \quad (\text{hypotenuse})$$

$$R = \sqrt{R_E^2 + a^2 - 2R_E a \cos(\varphi - \varphi') + (z - z')^2} \quad (\text{distance})$$

(a) $\vec{J}_m = \nabla \times \vec{M} = 0$

$$\vec{K}_m = -i_{r_1} \times \vec{M} = -[\cos \varphi' \hat{i}_x + \sin \varphi' \hat{i}_y] \times M_0 \hat{i}_z = -i_z M_0 \cos \varphi'$$

$$\vec{A} = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K}_m d\vec{s}}{R}$$

Χρησιμοποιείται δύναμη γραμμής (~~κατά κατεύθυνση~~ του ζεύγους) αντίστροφης αρμόδιας επιπλέοντος τοπού σε γραμμής που συγχέεται του μεταξύ των περιφερών $I = -M_0 \cos \varphi' a d\varphi'$. Σύμφωνα με αυτόποια ο ζεύγος της άποψης $r_{10} = a$

$$\vec{A} = i_z \int_{\varphi=0}^{2\pi} \frac{-\mu_0}{2\pi} M_0 \cos \varphi' a \ln \frac{a}{[r_1^2 + a^2 - 2r_1 a \cos(\varphi - \varphi')]^{1/2}} d\varphi' \Rightarrow \vec{A}(r_1, \varphi, z) = -i_z \frac{\mu_0 M_0 a}{2\pi} \int_{\varphi=0}^{2\pi} \cos \varphi' \ln \frac{a}{[r_1^2 + a^2 - 2r_1 a \cos(\varphi - \varphi')]^{1/2}} d\varphi'$$

(b) Νόμος Biot-Savart: $\vec{B} = \frac{\mu_0}{4\pi} \iint_S \frac{\vec{K}_m \times \vec{R}}{R^2} d\vec{s} = \frac{\mu_0}{4\pi} \int_{\varphi=0}^{2\pi} \int_{z=-\infty}^{+\infty} \frac{\vec{K}_m \times \vec{R}}{R^3} dz d\varphi'$

Όπως από προηγούμενη επιπλέοντος τοπού $I = -M_0 \cos \varphi' a d\varphi'$

$$\text{Κατεύθυνση } \frac{-i_z \times \vec{R}}{R} = \frac{i_x (y - a \sin \varphi) - i_y (x - a \cos \varphi)}{[(x - a \cos \varphi)^2 + (y - a \sin \varphi)^2 + z^2]^{1/2}} \quad \text{Σύγχρονης θέσης } z = z'$$

$$\vec{B}(x, y, z) = \frac{\mu_0 M_0 a}{2\pi} \int_{\varphi=0}^{2\pi} \frac{[i_x (y - a \sin \varphi) - i_y (x - a \cos \varphi)]}{(x - a \cos \varphi)^2 + (y - a \sin \varphi)^2} \cancel{\cos \varphi' d\varphi'}$$

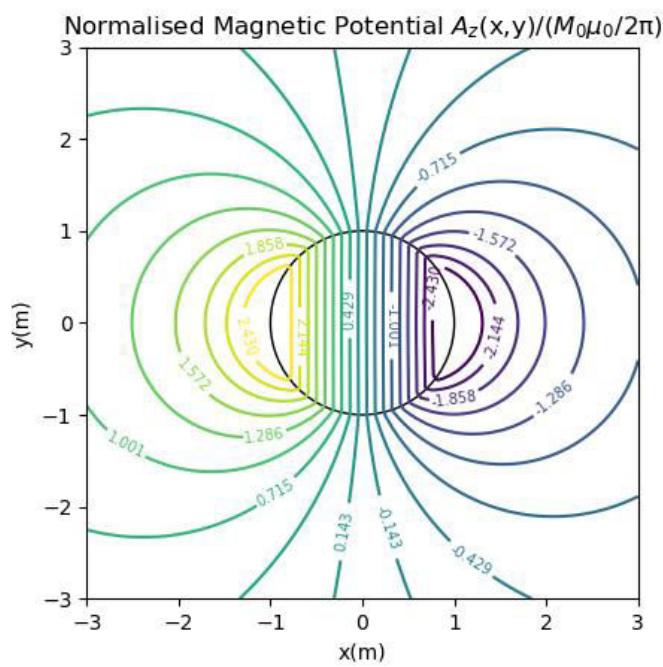
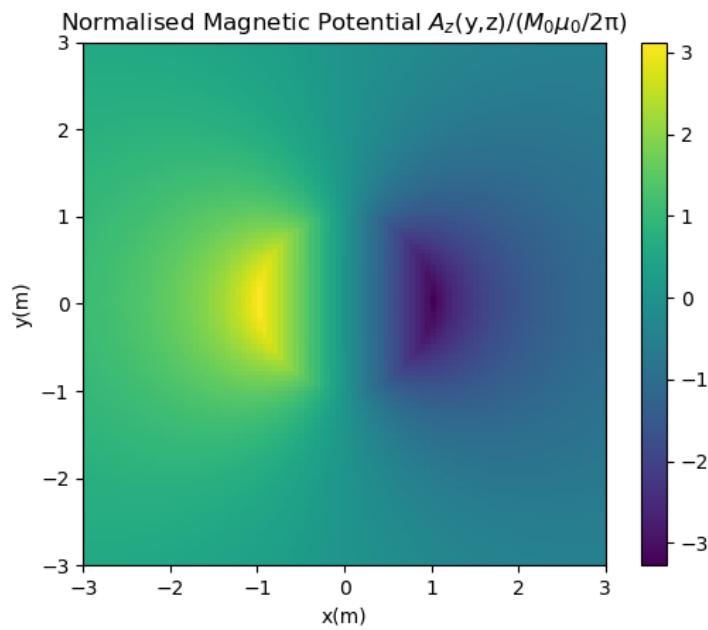
$$\vec{H}(x, y, z) = \begin{cases} \frac{\vec{B}}{\mu_0} - M_0 \hat{i}_y & , \sqrt{x^2 + y^2} < a \\ 0 & , \sqrt{x^2 + y^2} > a \end{cases} \quad (\text{ευρίσκων περιβολή})$$

$$\vec{H}(x, y, z) = \begin{cases} \frac{\vec{B}}{\mu_0} - M_0 \hat{i}_y & , \sqrt{x^2 + y^2} < a \\ \frac{\vec{B}}{\mu_0} & , \sqrt{x^2 + y^2} > a \end{cases} \quad (\text{ευρίσκων περιβολή})$$

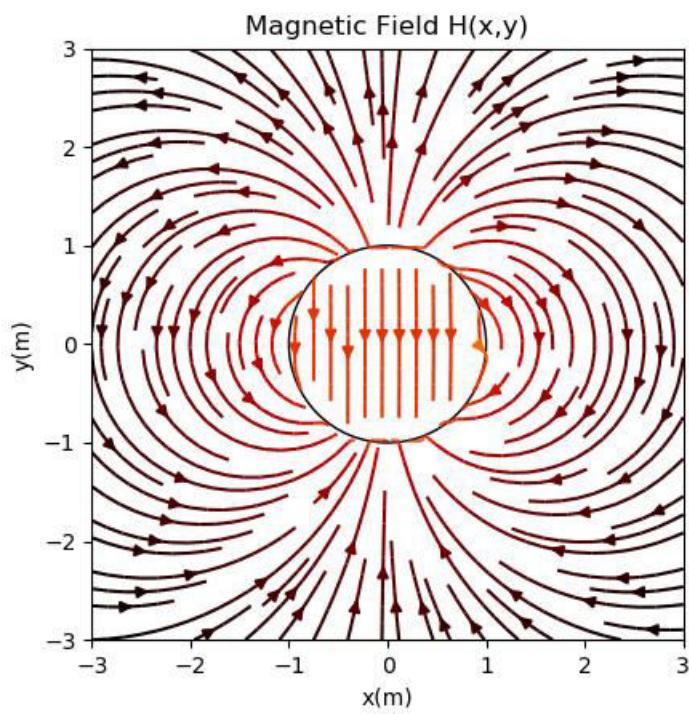
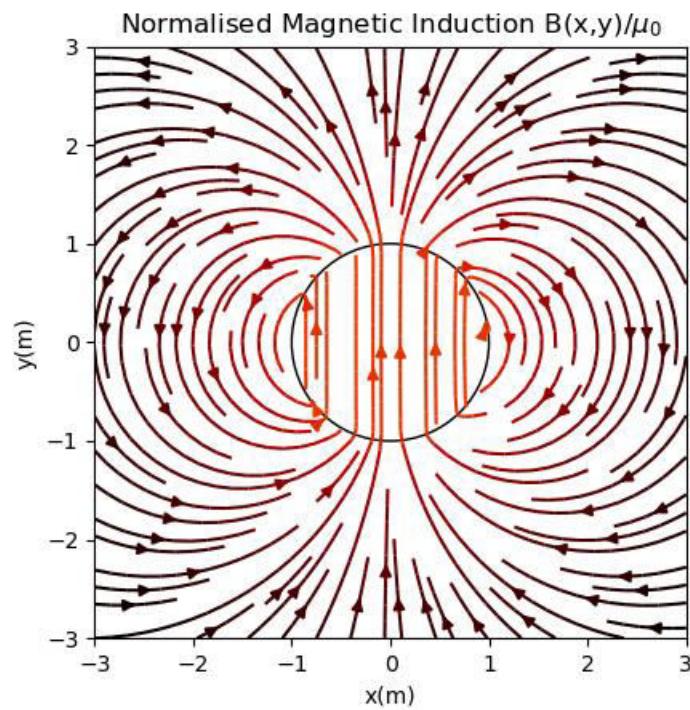
Γραφικές παραστάσεις – Ασκηση 10

έγιναν σε Python 3.9

(γ)



(δ)



Κώδικας – Άσκηση 10

σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
a=1
M0=1
levels=2.7*np.linspace(-0.9,0.9,num=18)

#Axes
N=100

xmin=-3*a
xmax=3*a
xx=np.linspace(xmin,xmax,N)

ymin=-3*a
ymax=3*a
yy=np.linspace(ymin,ymax,N)

X,Y=np.meshgrid(xx,yy)

#function for Az
def Az(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*np.log(a/np.sqrt(r**2+a**2-2*r*a*np.cos(theta-i)))
    return -a*res

#functions for B
def Bx(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*(y-a*np.sin(i))/(r**2+a**2-2*r*a*np.cos(theta-i))
    return a*M0*res/(2*math.pi)
```

```

def By(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*(x-a*np.cos(i))/(r**2+a**2-2*r*a*np.cos(theta-i))
    return -a*M0*res/(2*math.pi)

#functions for H
def Hx(x,y):
    return Bx(x,y)

def Hy(x,y):
    r=np.sqrt(x**2+y**2)
    condlist=[r<a,r>a]
    choicelist=[By(x,y)-M0,By(x,y)]
    return np.select(condlist,choicelist)

#PLOTS
#surface plot of Az
fig1,ax1=plt.subplots()
p1=ax1.pcolormesh(X,Y,Az(X,Y))
ax1.set_aspect('equal','box')
ax1.set_title('Normalised Magnetic Potential $A_z$(y,z)/($M_0\mu_0$/2n)')
cb1=fig1.colorbar(p1)
ax1.set_xlabel('x(m)')
ax1.set_ylabel('y(m)')

#contour lines of Az
fig2,ax2=plt.subplots()
p2=ax2.contour(X,Y,Az(X,Y),levels)
ax2.clabel(p2, inline=True, fontsize=7)
c2=plt.Circle((0,0),a,fill=False)
ax2.add_artist(c2)
ax2.set_aspect('equal','box')
ax2.set_title('Normalised Magnetic Potential $A_z$(x,y)/($M_0\mu_0$/2n)')
ax2.set_xlabel('x(m)')
ax2.set_ylabel('y(m)')

#streamplot of B
fig3,ax3=plt.subplots()
p3=ax3.streamplot(X,Y,Bx(X,Y),By(X,Y),density=1.2,color=np.log10(np.sqrt(Bx(X,Y)**2+By(X,Y)**2)),cmap=cm.gist_heat)
c3=plt.Circle((0,0),a,fill=False)
ax3.add_artist(c3)
ax3.set_aspect('equal','box')
ax3.set_title('Normalised Magnetic Induction B(x,y)/$\mu_0$')
ax3.set_xlabel('x(m)')
ax3.set_ylabel('y(m)')

#streamplot of H
fig4,ax4=plt.subplots()
p4=ax4.streamplot(X,Y,Hx(X,Y),Hy(X,Y),density=1.2,color=np.log10(np.sqrt(Hx(X,Y)**2+Hy(X,Y)**2)),cmap=cm.gist_heat)
c4=plt.Circle((0,0),a,fill=False)
ax4.add_artist(c4)
ax4.set_aspect('equal','box')
ax4.set_title('Magnetic Field H(x,y)')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('y(m)')

plt.show()

```