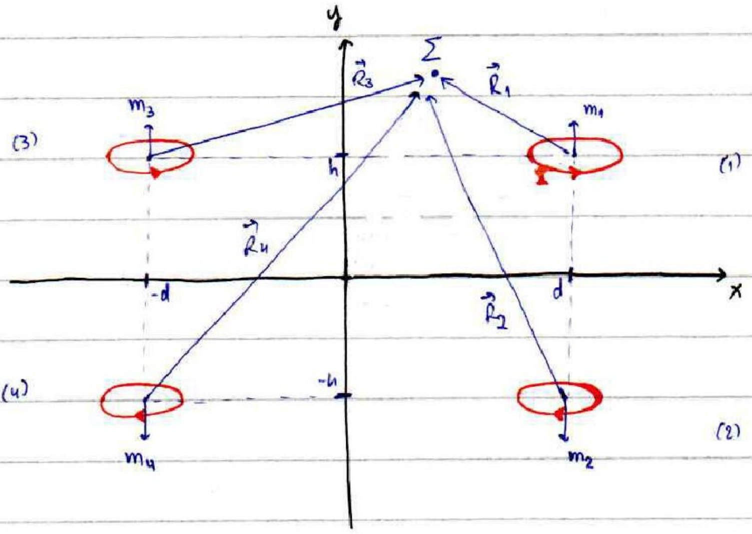
ΗΜΜΥ ΕΜΠ | ΗΜ ΠΕΔΙΑ Β' | ΗΒΗ ΧΑΤΖΗ
5ο ΕΞΑΜΗΝΟ | 3η ΣΕΙΡΑ ΑΣΚΗΣΕΩΝ | [redacted]

Άσκηση 8



(α) Θεωρούμε το ισοδύναμο κατοπτρικό πρόβλημα όπου όλος ο χώρος έχει μαγνητική διαπερατότητα μ_0 και έχουμε τρεις κατοπτρικούς βρόχους συμμετρικά ως προς τους άξονες και την αρχή των αξόνων με ρεύμα I φοράς όπως φαίνεται στο σχήμα.
Με τον τρόπο αυτό το δυναμικό στους άξονες x, y για $x,y>0$ διατηρείται μηδέν

Από επαλληλία είναι
$$\vec{A}(x,y,z) = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4 = \frac{\mu_0}{4\pi}\left(\frac{\vec{m}_1\times\hat{i}_{R_1}}{R_1^2} + \frac{\vec{m}_2\times\hat{i}_{R_2}}{R_2^2} + \frac{\vec{m}_3\times\hat{i}_{R_3}}{R_3^2} + \frac{\vec{m}_4\times\hat{i}_{R_4}}{R_4^2}\right) =$$
$$= \frac{\mu_0}{4\pi}\left(\frac{\vec{m}_1\times\vec{R}_1}{R_1^3} + \frac{\vec{m}_2\times\vec{R}_2}{R_2^3} + \frac{\vec{m}_3\times\vec{R}_3}{R_3^3} + \frac{\vec{m}_4\times\vec{R}_4}{R_4^3}\right)$$

$$\vec{R}_1 = (x-d)\hat{i}_x + (y-h)\hat{i}_y + z\hat{i}_z, \quad R_1 = \sqrt{(x-d)^2+(y-h)^2+z^2}, \quad \vec{m}_1 = I\pi a^2\,\hat{i}_y$$
$$\vec{R}_2 = (x-d)\hat{i}_x + (y+h)\hat{i}_y + z\hat{i}_z, \quad R_2 = \sqrt{(x-d)^2+(y+h)^2+z^2}, \quad \vec{m}_2 = -I\pi a^2\,\hat{i}_y$$
$$\vec{R}_3 = (x+d)\hat{i}_x + (y-h)\hat{i}_y + z\hat{i}_z, \quad R_3 = \sqrt{(x+d)^2+(y-h)^2+z^2}, \quad \vec{m}_3 = I\pi a^2\,\hat{i}_y$$
$$\vec{R}_4 = (x+d)\hat{i}_x + (y+h)\hat{i}_y + z\hat{i}_z, \quad R_4 = \sqrt{(x+d)^2+(y+h)^2+z^2}, \quad \vec{m}_4 = -I\pi a^2\,\hat{i}_y$$

Τελικά
$$\vec{A}(x,y,z) = \frac{\mu_0 I\pi a^2}{4\pi}\left[\left(\frac{z}{R_1^3} - \frac{z}{R_2^3} + \frac{z}{R_3^3} - \frac{z}{R_4^3}\right)\hat{i}_x + \left(-\frac{x-d}{R_1^3} + \frac{x-d}{R_2^3} - \frac{x+d}{R_3^3} + \frac{x+d}{R_4^3}\right)\hat{i}_z\right] \quad \text{για } x,y>0$$

(β) $\vec{H} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A} = \frac{1}{\mu_0}\left(\vec{\nabla}\times\vec{A}_1 + \vec{\nabla}\times\vec{A}_2 + \vec{\nabla}\times\vec{A}_3 + \vec{\nabla}\times\vec{A}_4\right)$

$$\vec{H}_1 = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A}_1 = \frac{I\pi a^2}{4\pi}\left[\frac{3(y-h)(x-d)}{R_1^5}\hat{i}_x + \left(\frac{3(y-h)^2}{R_1^5} - \frac{1}{R_1^3}\right)\hat{i}_y + \frac{3(y-h)z}{R_1^5}\hat{i}_z\right]$$

$$\vec{H}_2 = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A}_2 = -\frac{I\pi a^2}{4\pi}\left[\frac{3(y+h)(x-d)}{R_2^5}\hat{i}_x + \left(\frac{3(y+h)^2}{R_2^5} - \frac{1}{R_2^3}\right)\hat{i}_y + \frac{3(y+h)z}{R_2^5}\hat{i}_z\right]$$

$$\vec{H}_3 = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A}_3 = \frac{I\pi a^2}{4\pi}\left[\frac{3(y-h)(x+d)}{R_3^5}\hat{i}_x + \left(\frac{3(y-h)^2}{R_3^5} - \frac{1}{R_3^3}\right)\hat{i}_y + \frac{3(y-h)z}{R_3^5}\hat{i}_z\right]$$

$$\vec{H}_4 = \frac{1}{\mu_0}\vec{\nabla}\times\vec{A}_4 = \frac{-I\pi a^2}{4\pi}\left[\frac{3(y+h)(x+d)}{R_4^5}\hat{i}_x + \left(\frac{3(y+h)^2}{R_4^5} - \frac{1}{R_4^3}\right)\hat{i}_y + \frac{3(y+h)z}{R_4^5}\hat{i}_z\right]$$

$\vec{H}(x,y,z) = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4$ για $x, y \geq 0$

(γ) Η επιφανειακή πυκνότητα υπολογίζεται από τη συνοριακή συνθήκη $\vec{K} = \hat{i}_n \times (\vec{H}_1 - \vec{H}_2)$

Για την επιφάνεια $x=0$: $\hat{i}_x\times\vec{H}\big|_{x=0} = \frac{I\pi a^2}{4\pi}\left[\hat{i}_z\left(\frac{3(y-h)^2}{R_{10}^5} - \frac{1}{R_{10}^3}\right) - \hat{i}_y\frac{3(y-h)z}{R_{10}^5} - \hat{i}_z\left(\frac{3(y+h)^2}{R_{20}^5} - \frac{1}{R_{20}^3}\right) + \hat{i}_y\frac{3(y+h)z}{R_{20}^5}\right.$

$$\left. + \hat{i}_z\left(\frac{3(y-h)^2}{R_{30}^5} - \frac{1}{R_{30}^3}\right) - \hat{i}_y\frac{3(y-h)z}{R_{30}^5} - \hat{i}_z\left(\frac{3(y+h)^2}{R_{40}^5} - \frac{1}{R_{40}^3}\right) + \hat{i}_y\frac{3(y+h)z}{R_{40}^5}\right]$$

όπου $R_{10} = R_{30} = \sqrt{d^2+(y-h)^2+z^2}$ και $R_{20} = R_{40} = \sqrt{d^2+(y+h)^2+z^2}$

Για την επιφάνεια $y=0$: $\hat{i}_y\times\vec{H}\big|_{y=0} = \frac{I\pi a^2}{4\pi}\left[\hat{i}_z\frac{3h(x-d)}{R_{100}^5} - \hat{i}_x\frac{3hz}{R_{100}^5} + \hat{i}_z\frac{3h(x-d)}{R_{200}^5} - \hat{i}_x\frac{3hz}{R_{200}^5}\right.$

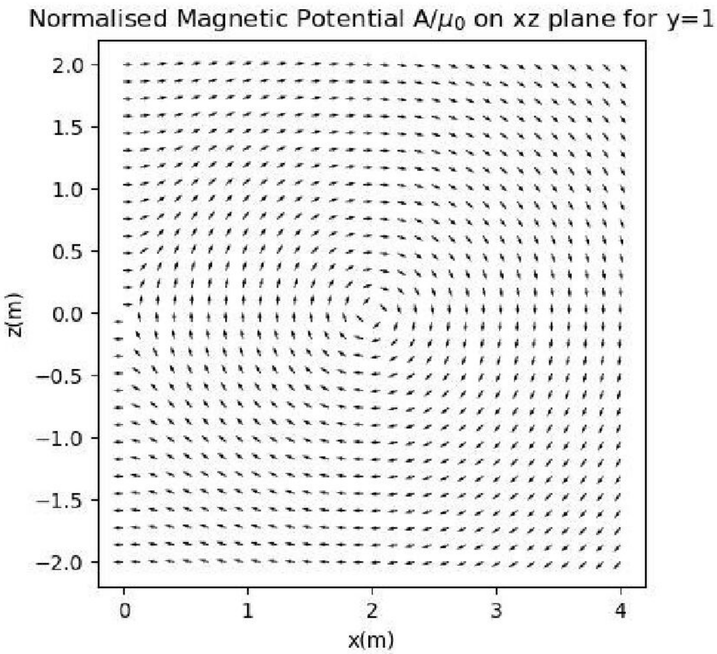
$$\left. + \hat{i}_z\frac{3h(x+d)}{R_{300}^5} - \hat{i}_x\frac{3hz}{R_{300}^5} + \hat{i}_z\frac{3h(x+d)}{R_{400}^5} - \hat{i}_x\frac{3hz}{R_{400}^5}\right]$$

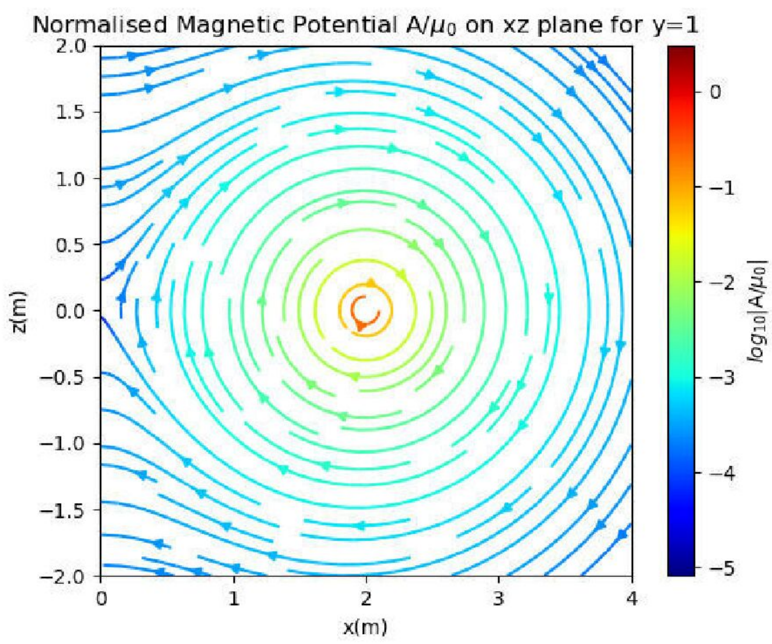
όπου $R_{100} = R_{200} = \sqrt{(x-d)^2+h^2+z^2}$ και $R_{300} = R_{400} = \sqrt{(x+d)^2+h^2+z^2}$

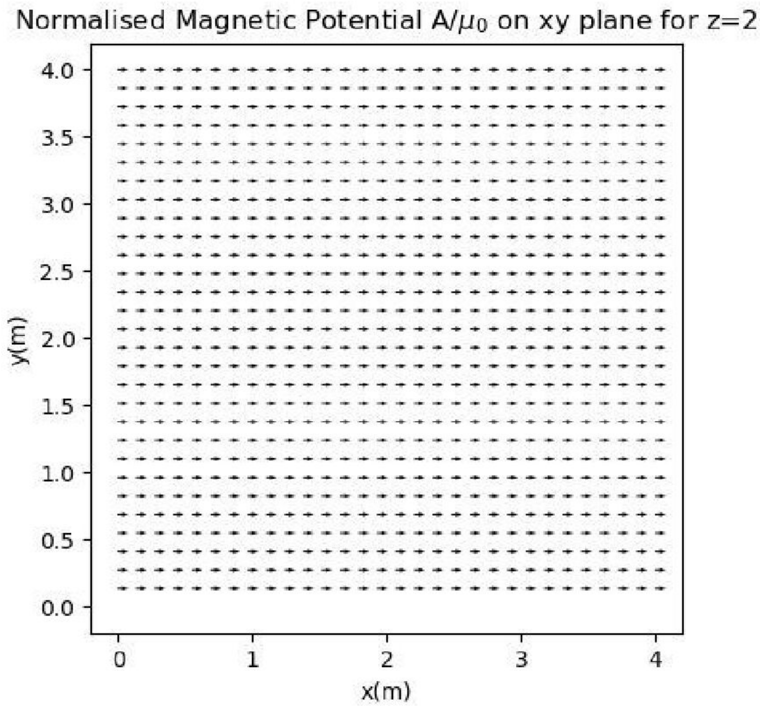
Γραφικές παραστάσεις – Άσκηση 8

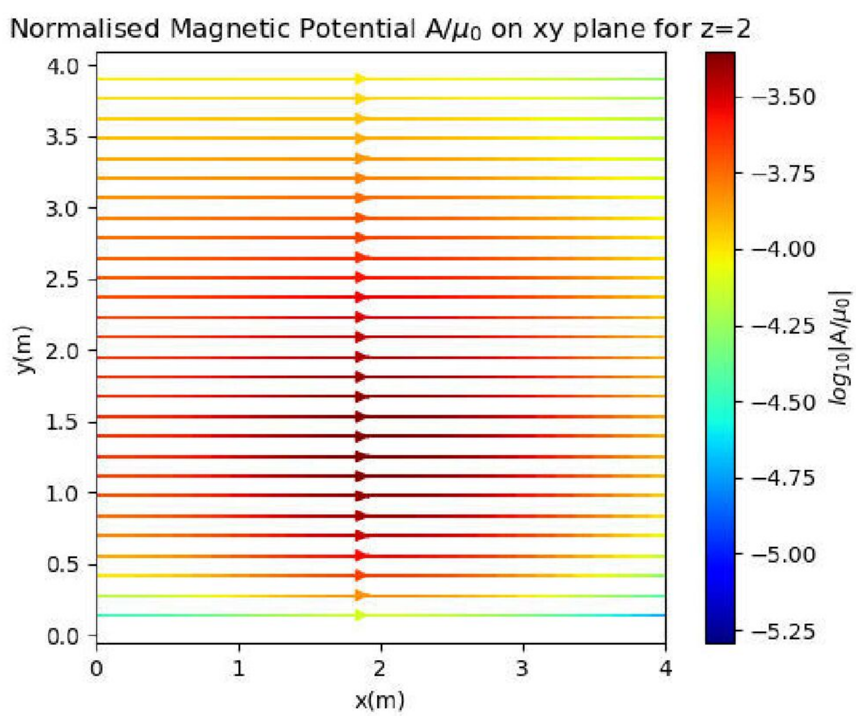
έγιναν σε Python 3.9

(δ)

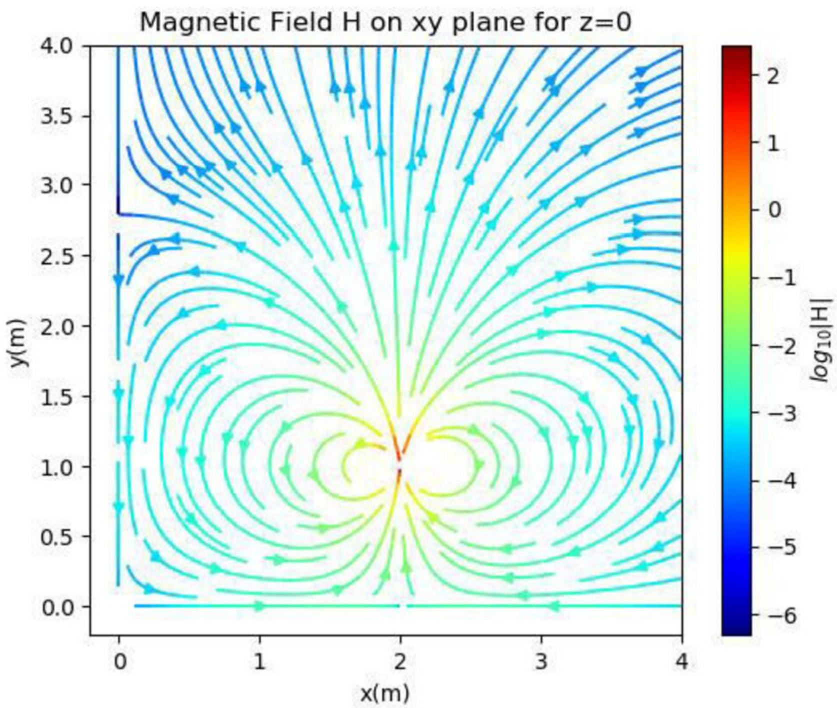




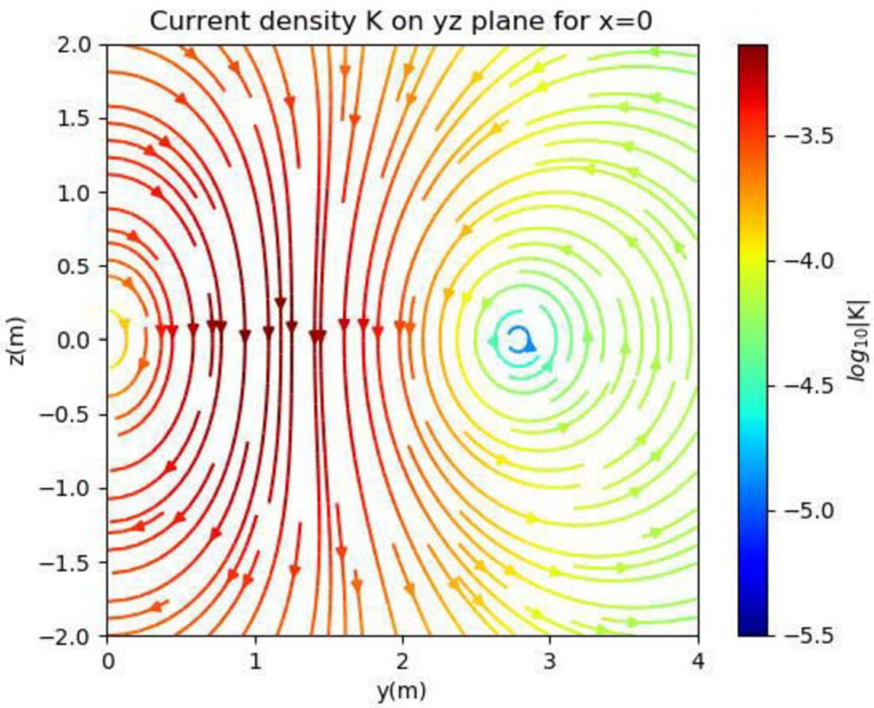

Normalised Magnetic Potential A/μ_0 on xy plane for z=2
y(m)
x(m)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4

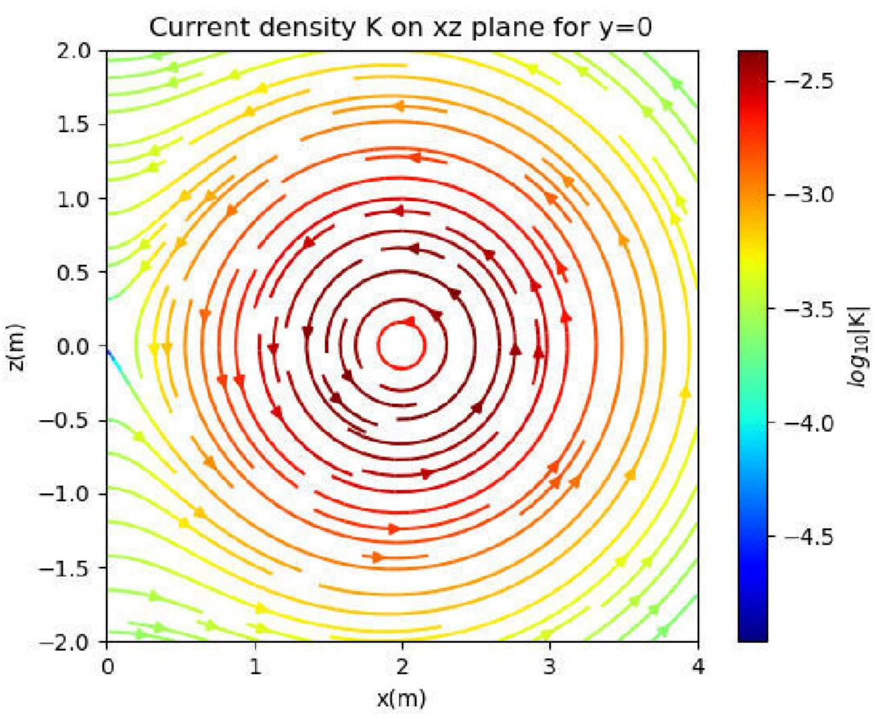

Normalised Magnetic Potential A/μ_0 on xy plane for z=2
y(m)
x(m)
$log_{10}|A/\mu_0|$
−3.50
−3.75
−4.00
−4.25
−4.50
−4.75
−5.00
−5.25

(ε)


Magnetic Field H on xy plane for z=0
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y(m)
0
1
2
3
4
x(m)
2
1
0
−1
−2
−3
−4
−5
−6
log_{10}|H|

(στ)


Current density K on yz plane for x=0
z(m)
y(m)
$log_{10}|K|$


Current density K on xz plane for y=0
z(m)
x(m)
$log_{10}|K|$

Κώδικας – Άσκηση 8

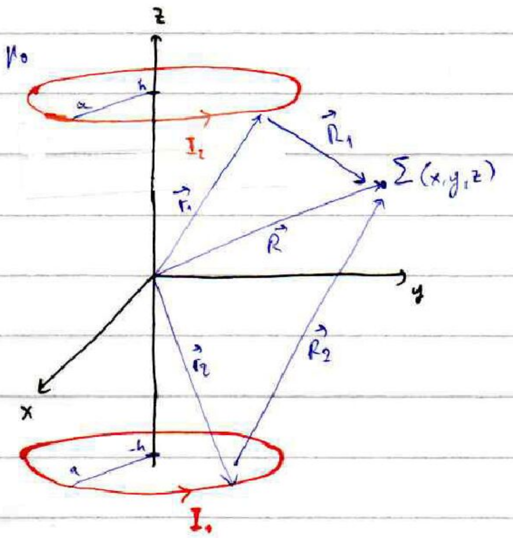
σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
d=2
h=1
a=0.1
I=1

#Axes
N=100

xmin=0
xmax=4
xx=np.linspace(xmin,xmax,N)

ymin=0
ymax=4
yy=np.linspace(ymin,ymax,N)

zmin=-2
zmax=2
zz=np.linspace(zmin,zmax,N)

X,Z=np.meshgrid(xx,zz)
XX,Y=np.meshgrid(xx,yy)
YY,ZZ=np.meshgrid(yy,zz)

#functions for distances
def R1(x,y,z):
    return np.sqrt((x-d)**2+(y-h)**2+z**2)

def R2(x,y,z):
    return np.sqrt((x-d)**2+(y+h)**2+z**2)

def R3(x,y,z):
    return np.sqrt((x+d)**2+(y-h)**2+z**2)

def R4(x,y,z):
    return np.sqrt((x+d)**2+(y+h)**2+z**2)
```

```
#functions for potential
def Ax(x,y,z):
    res=1/R1(x,y,z)**3-1/R2(x,y,z)**3+1/R3(x,y,z)**3-1/R4(x,y,z)**3
    return z*I*(a**2)*res/4

Ay=np.zeros((N,N))

def Az(x,y,z):
    res=-(x-d)/R1(x,y,z)**3+(x-d)/R2(x,y,z)**3-(x+d)/R3(x,y,z)**3+(x+d)/R4(x,y,z)**3
    return I*(a**2)*res/4

#functions for magnetic field
def Hx(x,y,z):
    res=(y-h)*(x-d)/R1(x,y,z)**5-(y+h)*(x-d)/R2(x,y,z)**5+(y-h)*(x+d)/R3(x,y,z)**5-(y+h)*(x+d)/R4(x,y,z)**5
    return 3*I*(a**2)*res/4

def Hy(x,y,z):
    Hy1=1/(R1(x,y,z)**3)*(3*(y-h)**2/R1(x,y,z)**2-1)
    Hy2=1/(R2(x,y,z)**3)*(-3*(y+h)**2/R2(x,y,z)**2+1)
    Hy3=1/(R3(x,y,z)**3)*(3*(y-h)**2/R3(x,y,z)**2-1)
    Hy4=1/(R4(x,y,z)**3)*(-3*(y+h)**2/R4(x,y,z)**2+1)
    return I*(a**2)/4*(Hy1+Hy2+Hy3+Hy4)

#functions for current density on yz plane
def Ky_yz(y,z):
    r1=R1(0,y,z)
    r2=R2(0,y,z)
    r3=R3(0,y,z)
    r4=R4(0,y,z)
    return I*(a**2)/4*3*(-(y-h)*z/r1**5+(y+h)*z/r2**5-(y-h)*z/r3**5+(y+h)*z/r4**5)

def Kz_yz(y,z):
    r1=R1(0,y,z)
    r2=R2(0,y,z)
    r3=R3(0,y,z)
    r4=R4(0,y,z)
    term1=3*(y-h)**2/r1**5-1/r1**3
    term2=-3*(y+h)**2/r2**5+1/r2**3
    term3=3*(y-h)**2/r3**5-1/r3**3
    term4=-3*(y+h)**2/r4**5+1/r4**3
    return I*(a**2)/4*(term1+term2+term3+term4)


#functions for current density on xz plane
def Kx_xz(x,z):
    r1=R1(x,0,z)
    r2=R2(x,0,z)
    r3=R3(x,0,z)
    r4=R4(x,0,z)
    return -I*(a**2)/4*3*h*z*(1/r1**5+1/r2**5+1/r3**5+1/r4**5)

def Kz_xz(x,z):
    r1=R1(x,0,z)
    r2=R2(x,0,z)
    r3=R3(x,0,z)
    r4=R4(x,0,z)
    return I*(a**2)/4*3*(h*(x-d)/r1**5+h*(x-d)/r2**5+h*(x+d)/r3**5+h*(x+d)/r4**5)
```

```
#PLOTS
#streamplot of magnetic potential on xz plane
fig1, ax1 = plt.subplots()
p1=ax1.streamplot(X,Z,Ax(X,1,Z),Az(X,1,Z),color=np.log10(np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)),cmap=cm.jet)
ax1.set_aspect('equal','box')
c1=fig1.colorbar(p1.lines)
c1.set_label('$log_{10}$|A/$μ_0$|')
ax1.set_title('Normalised Magnetic Potential A/$μ_0$ on xz plane for y=1')
ax1.set_xlabel('x(m)')
ax1.set_ylabel('z(m)')

#quiver plot of magnetic potential on xz plane
#use N=30
Ax_norm=Ax(X,1,Z)/np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)
Az_norm=Az(X,1,Z)/np.sqrt(Ax(X,1,Z)**2+Az(X,1,Z)**2)

fig2, ax2 = plt.subplots()
plt.quiver(X,Z,Ax_norm,Az_norm)
ax2.set_aspect('equal','box')
ax2.set_title('Normalised Magnetic Potential A/$μ_0$ on xz plane for y=1')
ax2.set_xlabel('x(m)')
ax2.set_ylabel('z(m)')

#streamplot of magnetic potential on xy plane
fig3, ax3 = plt.subplots()
p3=ax3.streamplot(XX,Y,Ax(XX,Y,2),Ay,color=np.log10(np.sqrt(Ax(XX,Y,2)**2+Ay**2)),cmap=cm.jet)
ax3.set_aspect('equal','box')
c3=fig3.colorbar(p3.lines)
c3.set_label('$log_{10}$|A/$μ_0$|')
ax3.set_title('Normalised Magnetic Potential A/$μ_0$ on xy plane for z=2')
ax3.set_xlabel('x(m)')
ax3.set_ylabel('y(m)')

#quiver plot of magnetic potential on xy plane
#use N=30
Ax_norm2=Ax(XX,Y,2)/np.sqrt(Ax(XX,Y,2)**2+Ay**2)
Ay_norm=Ay/np.sqrt(Ax(XX,Y,2)**2+Ay**2)

fig4, ax4 = plt.subplots()
plt.quiver(XX,Y,Ax_norm2,Ay)
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Magnetic Potential A/$μ_0$ on xy plane for z=2')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('y(m)')


#streamplot of magnetic field on xy plane
fig5, ax5 = plt.subplots()
p5=ax5.streamplot(XX,Y,Hx(XX,Y,0),Hy(XX,Y,0),color=np.log10(np.sqrt(Hx(XX,Y,0)**2+Hy(XX,Y,0)**2)),cmap=cm.jet,density=1.2)
ax5.set_aspect('equal','box')
c5=fig5.colorbar(p5.lines)
c5.set_label('$log_{10}$|H|')
ax5.set_title('Magnetic Field H on xy plane for z=0')
ax5.set_xlabel('x(m)')
ax5.set_ylabel('y(m)')

#streamplot of current density on yz plane
fig6, ax6 = plt.subplots()
p6=ax6.streamplot(YY,ZZ,Ky_yz(YY,ZZ),Kz_yz(YY,ZZ),color=np.log10(np.sqrt(Ky_yz(YY,ZZ)**2+Kz_yz(YY,ZZ)**2)),cmap=cm.jet,density=1.2)
ax6.set_aspect('equal','box')
c6=fig6.colorbar(p6.lines)
c6.set_label('$log_{10}$|K|')
ax6.set_title('Current density K on yz plane for x=0')
ax6.set_xlabel('y(m)')
ax6.set_ylabel('z(m)')

#streamplot of current density on xz plane
fig7, ax7 = plt.subplots()
p7=ax7.streamplot(X,Z,Kx_xz(X,Z),Kz_xz(X,Z),color=np.log10(np.sqrt(Kx_xz(X,Z)**2+Kz_xz(X,Z)**2)),cmap=cm.jet,density=1.2)
ax7.set_aspect('equal','box')
c7=fig7.colorbar(p7.lines)
c7.set_label('$log_{10}$|K|')
ax7.set_title('Current density K on xz plane for y=0')
ax7.set_xlabel('x(m)')
ax7.set_ylabel('z(m)')

plt.show()
```

Άσκηση 9



(a) $$\vec{A}(x,y,z) = \vec{A}_1 + \vec{A}_2 = \frac{\mu_0}{4\pi}\int_{C_1} \frac{I_1(\ell_1)\,d\vec{\ell}_1}{R_1} + \frac{\mu_0}{4\pi}\int_{C_2}\frac{I_2(\ell_2)\,d\vec{\ell}_2}{R_2} =$$

$$= \hat{i}_\varphi \frac{\mu_0 I_1}{4\pi}\int_0^{2\pi} \frac{a\cos\varphi_1\, d\varphi_1}{R_1} + \hat{i}_\varphi \frac{\mu_0 I_2}{4\pi}\int_0^{2\pi}\frac{a\cos\varphi_2\, d\varphi_2}{R_2}$$

όπου $\vec{R} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$

$\vec{r}_1 = a\cos\varphi_1 \hat{i}_x + a\sin\varphi_1 \hat{i}_y - h\hat{i}_z$

$\vec{R}_1 = \vec{R} - \vec{r}_1 = (x - a\cos\varphi_1)\hat{i}_x + (y - a\sin\varphi_1)\hat{i}_y + (z+h)\hat{i}_z \Rightarrow R_1 = \sqrt{x^2 + y^2 + (z+h)^2 + a^2 - 2a(x\cos\varphi_1 + y\sin\varphi_1)}$

$\vec{r}_2 = a\cos\varphi_2 \hat{i}_x + a\sin\varphi_2 \hat{i}_y - h\hat{i}_z$

$\vec{R}_2 = \vec{R} - \vec{r}_2 = (x - a\cos\varphi_2)\hat{i}_x + (y - a\sin\varphi_2)\hat{i}_y + (z-h)\hat{i}_z \Rightarrow R = \sqrt{x^2 + y^2 + (z-h)^2 + a^2 - 2a(x\cos\varphi_2 + y\sin\varphi_2)}$

(β) Επαλληλία: $\vec{H} = \vec{H}_1 + \vec{H}_2 = \frac{\vec{B}_1}{\mu_0} + \frac{\vec{B}_2}{\mu_0}$

Νόμος Biot-Savart: $\vec{H}_1 = \frac{1}{4\pi}\int_{C_1} \frac{I_1 \, d\vec{l}_1 \times \hat{i}_{R_1}}{R_1^2} = \frac{I_1}{4\pi}\int_{C_1} \frac{d\vec{l}_1 \times \vec{R}_1}{R_1^3}$

όπου $\vec{R}_1 = (x - a\cos\varphi_1)\hat{i}_x + (y - a\sin\varphi_1)\hat{i}_y + (z+h)\hat{i}_z$

$R_1 = \sqrt{x^2+y^2+(z+h)^2+a^2-2a(x\cos\varphi_1+y\sin\varphi_1)}$

$d\vec{l}_1 = a\,d\varphi_1\,\hat{i}_{\varphi_1}$

$\hat{i}_{\varphi_1} = -\sin\varphi_1\,\hat{i}_x + \cos\varphi_1\,\hat{i}_y$

Άρα $$d\vec{l}_1 \times \vec{\ell}_1 = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -a\sin\varphi_1 d\varphi_1 & a\cos\varphi_1 d\varphi_1 & 0 \\ x-a\cos\varphi_1 & y-a\sin\varphi & z+h \end{vmatrix} = \hat{i}_x (z+h)a\cos\varphi_1 d\varphi_1 + \hat{i}_y (z+h)a\sin\varphi_1 d\varphi_1 - \hat{i}_z a(x\cos\varphi_1 + y\sin\varphi_1 - a)d\varphi_1$$

Ακολουθώντας όμοια διαδικασία για το $\vec{H}_2$ έχουμε τελικά

$$\vec{H}(x,y,z) = \frac{I_1}{4\pi}\int_{\varphi_1=0}^{2\pi} \frac{\left[\hat{i}_x(z+h)\cos\varphi_1 + \hat{i}_y(z+h)\sin\varphi_1 - \hat{i}_z(x\cos\varphi_1 + y\sin\varphi_1 - a)\right]a\,d\varphi_1}{\left[x^2+y^2+(z+h)^2+a^2-2a(x\cos\varphi_1+y\sin\varphi_1)\right]^{3/2}} +$$

$$+ \frac{I_2}{4\pi}\int_{\varphi_2=0}^{2\pi} \frac{\left[\hat{i}_x(z-h)\cos\varphi_2 + \hat{i}_y(z-h)\sin\varphi_2 - \hat{i}_z(x\cos\varphi_2 + y\sin\varphi_2 - a)\right]a\,d\varphi_2}{\left[x^2+y^2+(z-h)^2+a^2-2a(x\cos\varphi_2+y\sin\varphi_2)\right]^{3/2}}$$

(γ) $\vec{H}_z(0,0,z) = \frac{I_1}{4\pi}\int_{\varphi_1=0}^{2\pi} \frac{\hat{i}_z a^2 d\varphi_1}{R_1^3(0,0,z)} + \frac{I_2}{4\pi}\int_{\varphi_2=0}^{2\pi} \frac{\hat{i}_z a^2 d\varphi_2}{R_2^3(0,0,z)} = \left(\frac{I_1}{2}\frac{a^2}{[a^2+(z+h)^2]^{3/2}} + \frac{I_2}{2}\frac{a^2}{[a^2+(z-h)^2]^{3/2}}\right)\hat{i}_z$

Σειρά Taylor:

$$H_z(z) = H_z(0) + H_z'(0)\,z + \frac{H_z''(0)}{2}z^2 + \frac{H_z'''(0)}{3!}z^3 + \frac{H_z^{(4)}(0)}{4!}z^4$$

όπου $H_z(0) = \frac{(I_1+I_2)}{2}\frac{a^2}{[a^2+h^2]^{3/2}}\,\hat{i}_z$

$$H_z'(0) = \frac{I_1-I_2}{2}\frac{3a^2h}{[a^2+h^2]^{5/2}}\,\hat{i}_z$$

$$H_z''(0) = \frac{I_1+I_2}{2}\left(\frac{15a^2h^2}{[a^2+h^2]^{7/2}} - \frac{3a^2}{[a^2+h^2]^{5/2}}\right)\hat{i}_z$$

$$H_z'''(0) = \frac{I_1-I_2}{2}\left(\frac{3a^2\cdot 8h}{[a^2+h^2]^{7/2}} - \frac{21a^2h\,(4h^2-a^2)}{[a^2+h^2]^{9/2}}\right)\hat{i}_z$$

$$H_z^{(4)}(0) = \frac{I_1+I_2}{2}\left(\frac{-15a^2(12h^2-3a^2)}{[a^2+h^2]^{9/2}} + \frac{135a^2h^2(4h^2-3a^2)}{[a^2+h^2]^{11/2}}\right)\hat{i}_z$$

Παρατηρούμε ότι αν $I_1 = I_2$ οι παράγωγοι περιττού βαθμού βγαίνουν 0

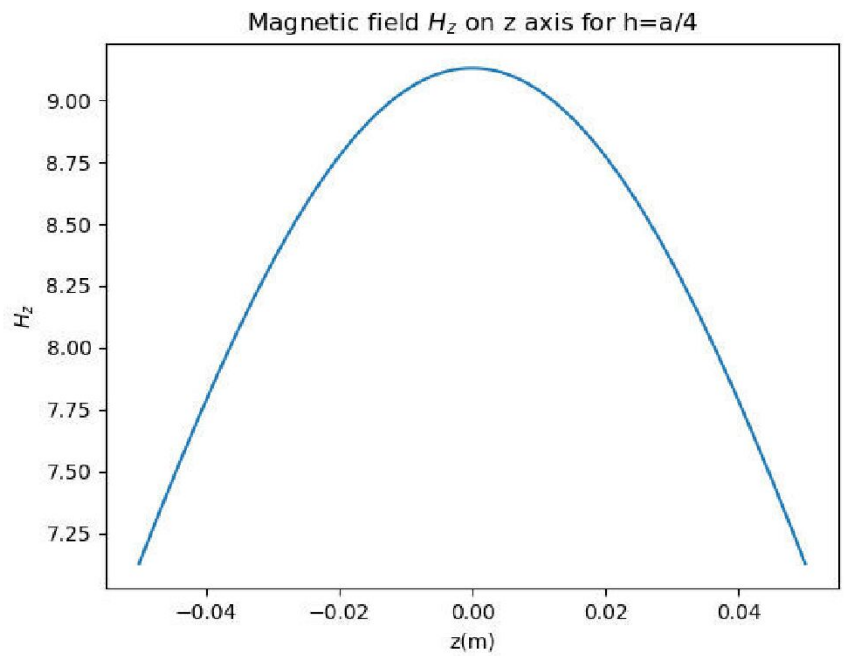
(ζ) Χρησιμοποιήσα τον τύπο $L_{12} = L_{21} = \mu_0 a\left[\frac{2-k^2}{k}K(k) - \frac{2}{k}E(k)\right]$
με $K(k) = \int_0^{2\pi}\frac{dw}{\sqrt{1-k\sin^2 w}}$ και $E(k) = \int_0^{\pi/2}\sqrt{1-k\sin^2 w}\,dw$

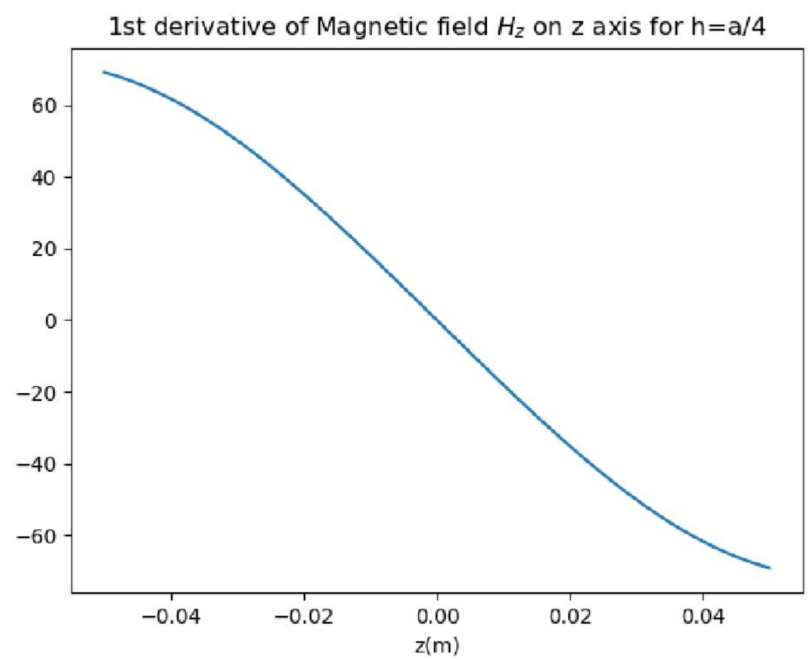
όπου $k = \frac{2a}{\sqrt{h^2+a^2}}$

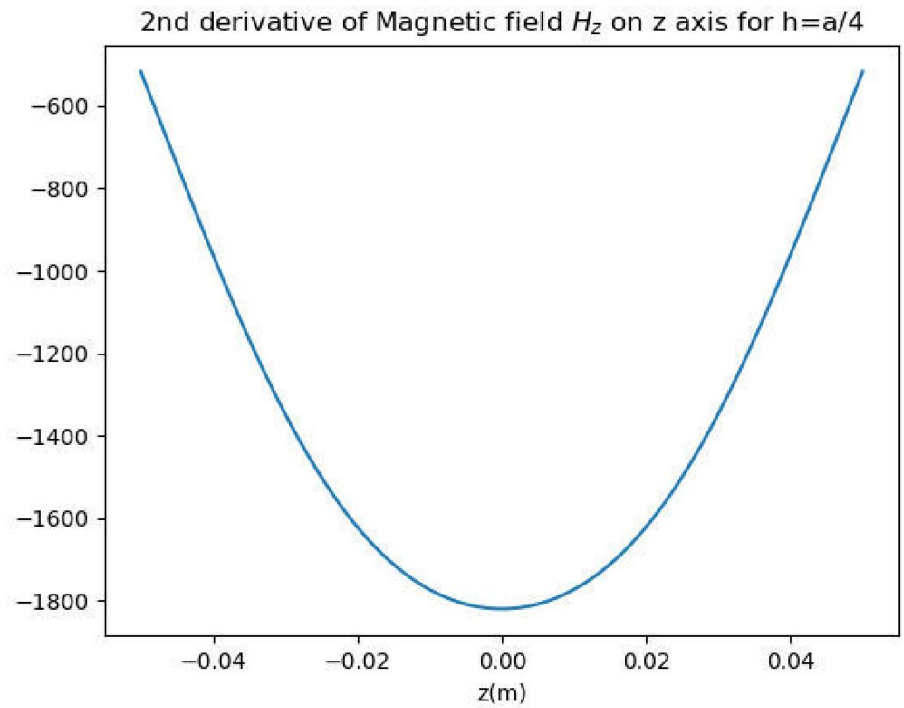
Γραφικές παραστάσεις – Άσκηση 9

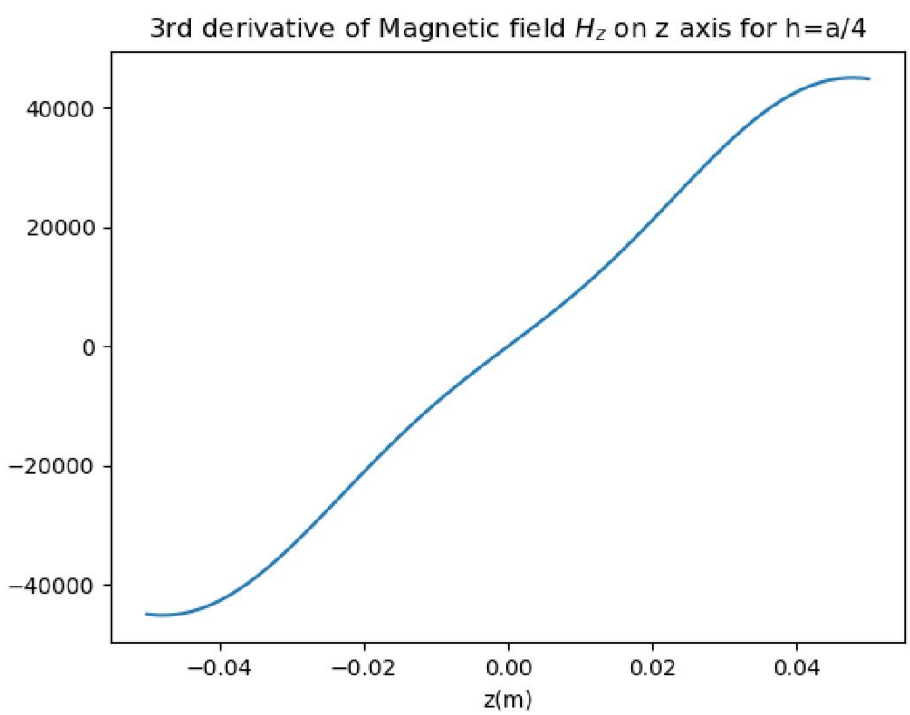
έγιναν σε Python 3.9

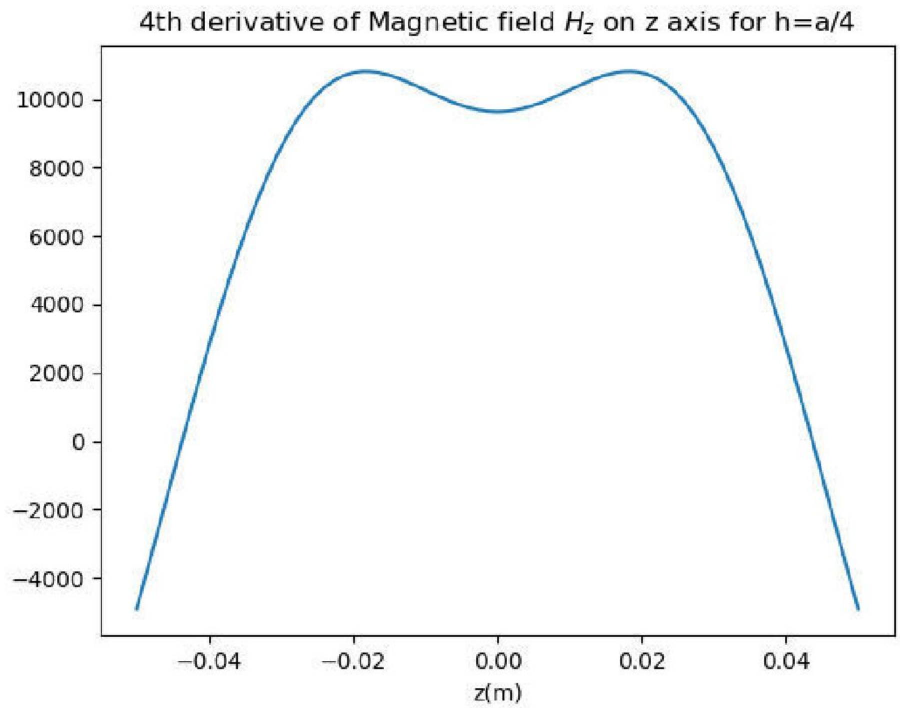
(δ)

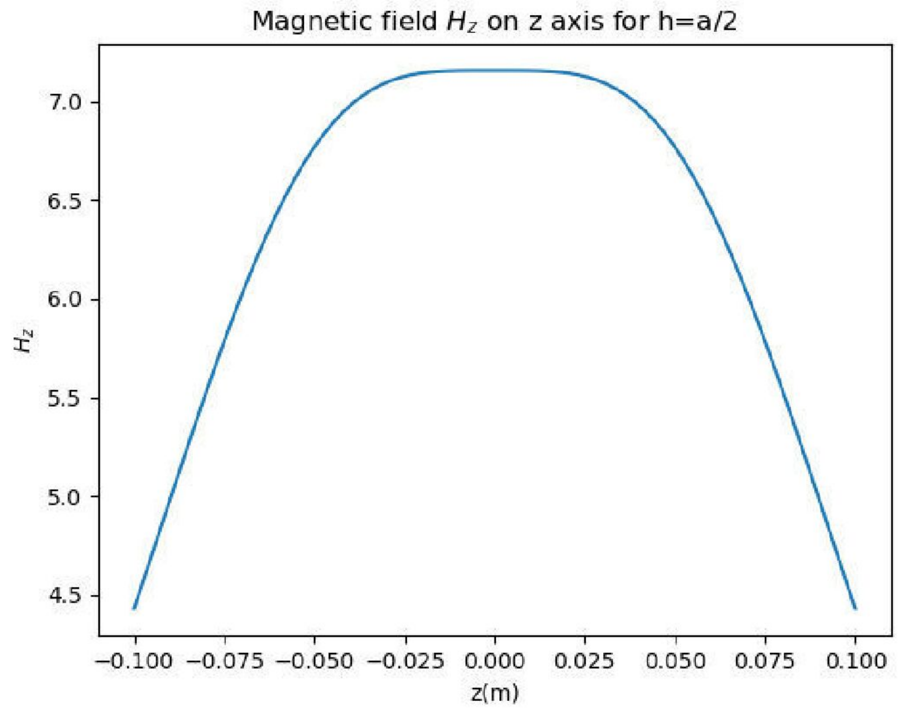


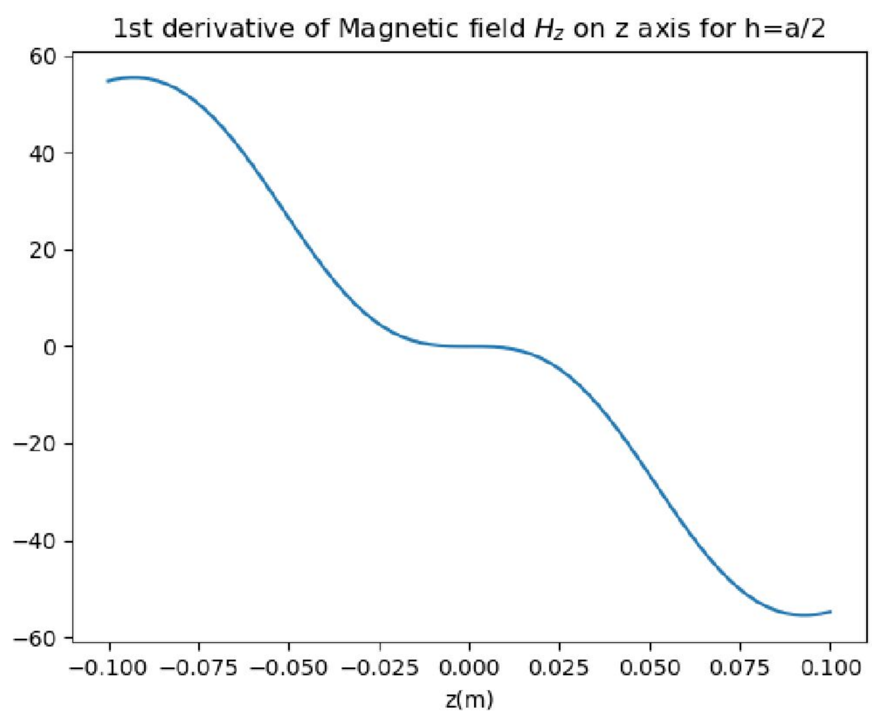


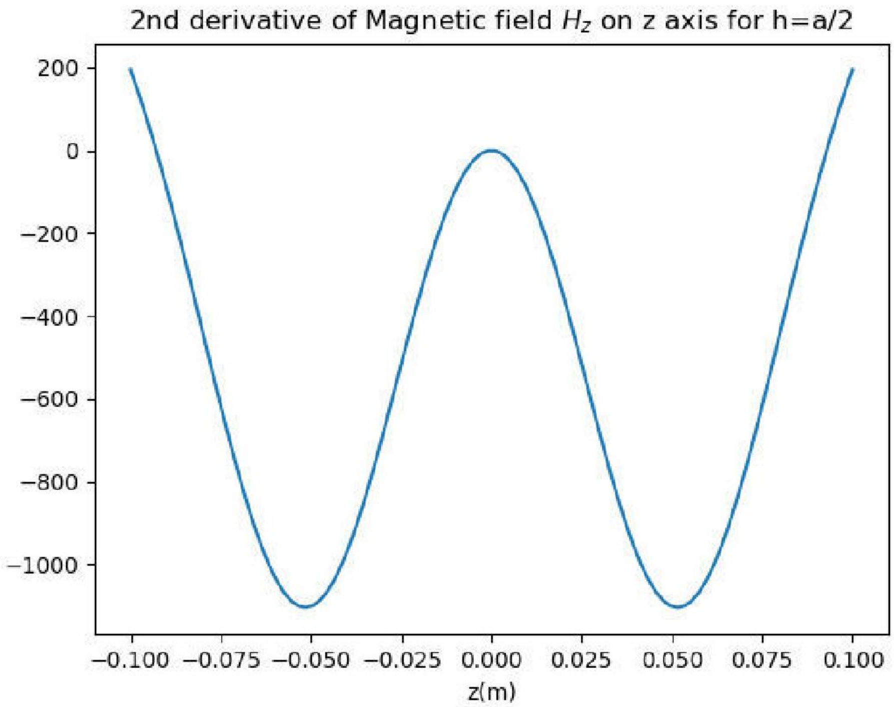

2nd derivative of Magnetic field H_z on z axis for h=a/4
−600
−800
−1000
−1200
−1400
−1600
−1800
−0.04
−0.02
0.00
0.02
0.04
z(m)


3rd derivative of Magnetic field H_z on z axis for h=a/4
40000
20000
0
−20000
−40000
−0.04
−0.02
0.00
0.02
0.04
z(m)

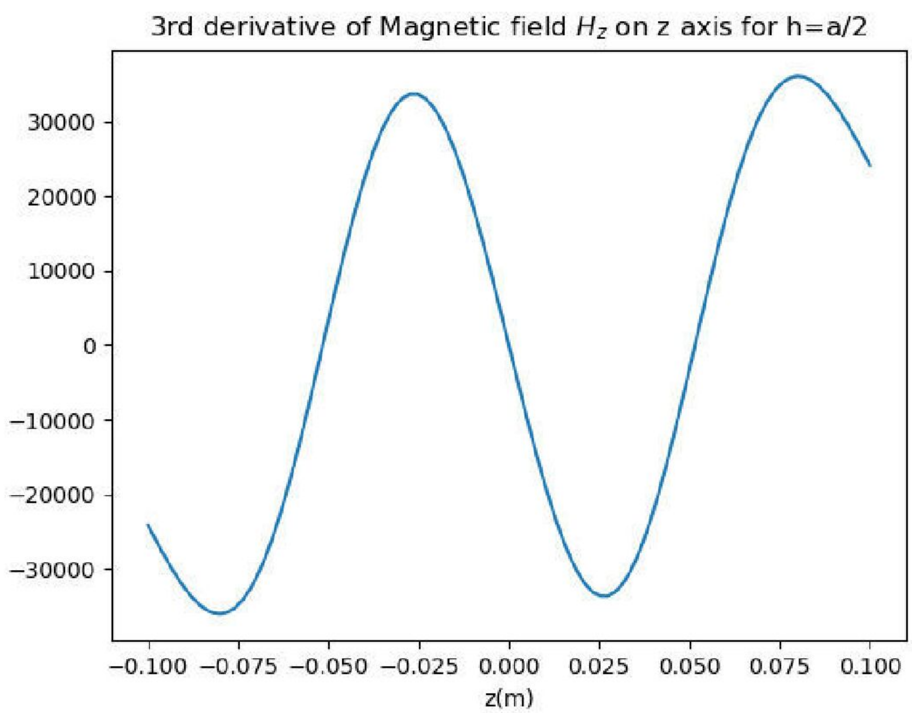

4th derivative of Magnetic field H_z on z axis for h=a/4
10000
8000
6000
4000
2000
0
−2000
−4000
−0.04
−0.02
0.00
0.02
0.04
z(m)


Magnetic field H_z on z axis for h=a/2
H_z
7.0
6.5
6.0
5.5
5.0
4.5
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
z(m)

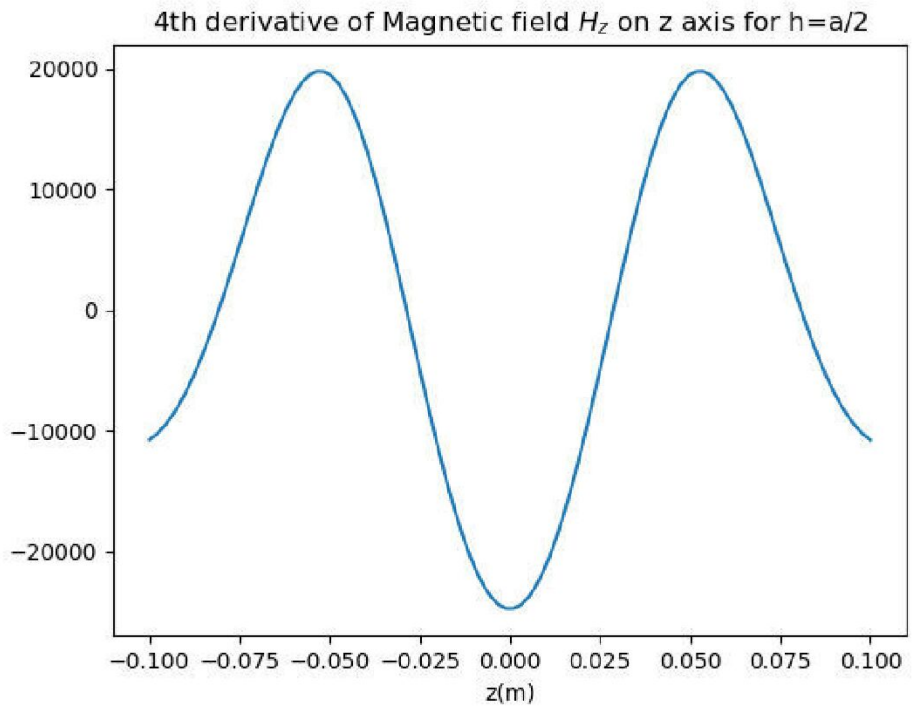

1st derivative of Magnetic field H_z on z axis for h=a/2
60
40
20
0
−20
−40
−60
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
z(m)

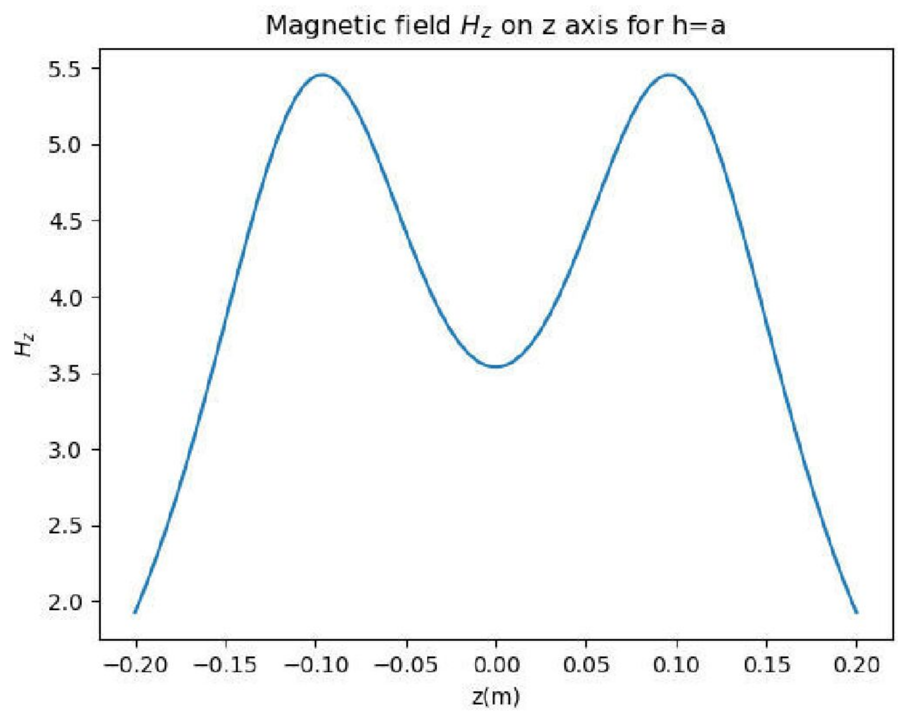


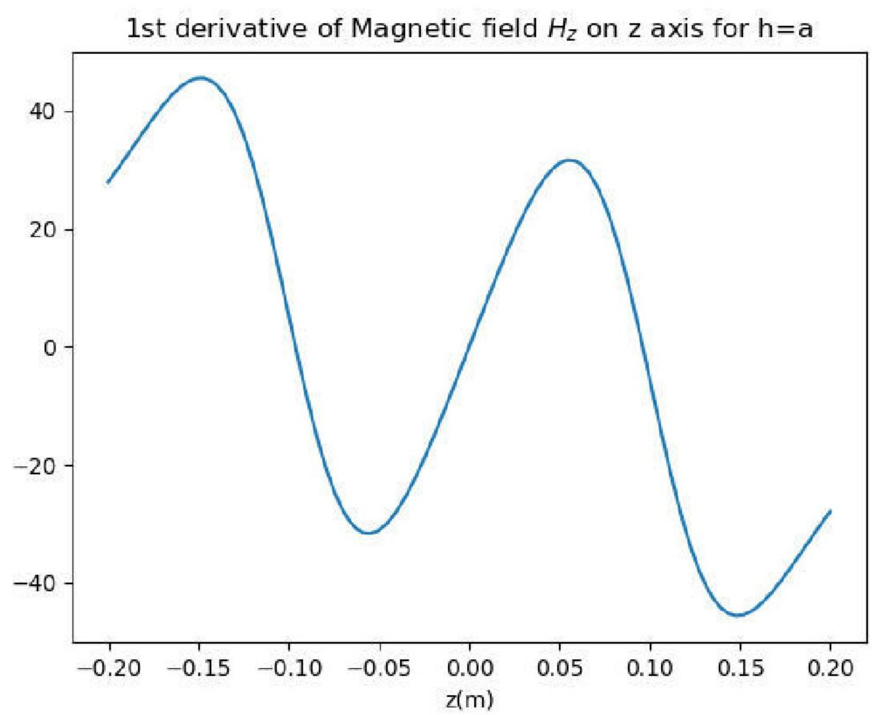
2nd derivative of Magnetic field H_z on z axis for h=a/2
200
0
−200
−400
−600
−800
−1000
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
z(m)

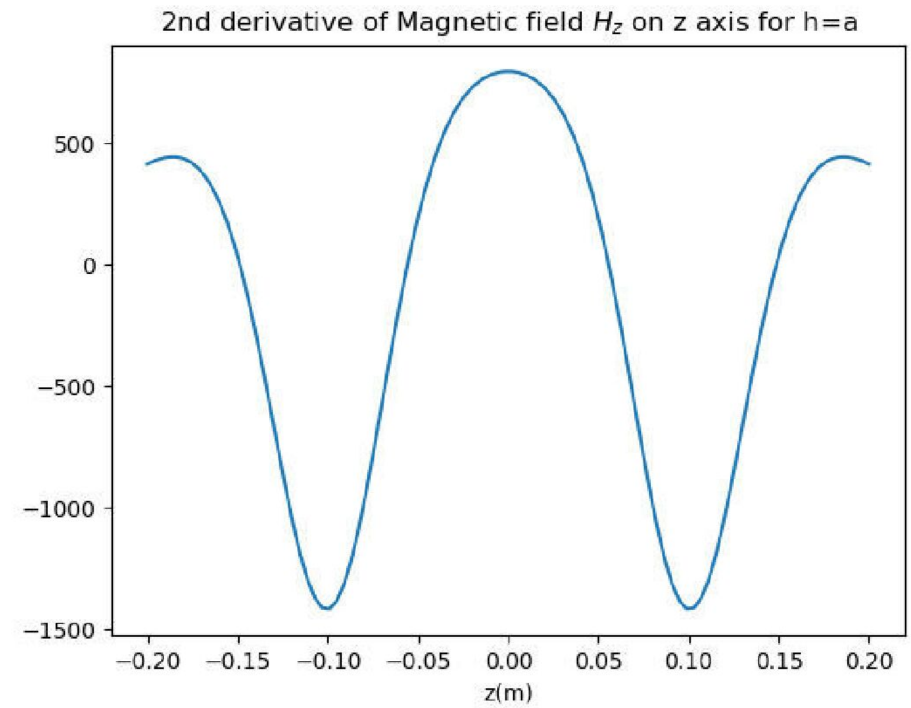


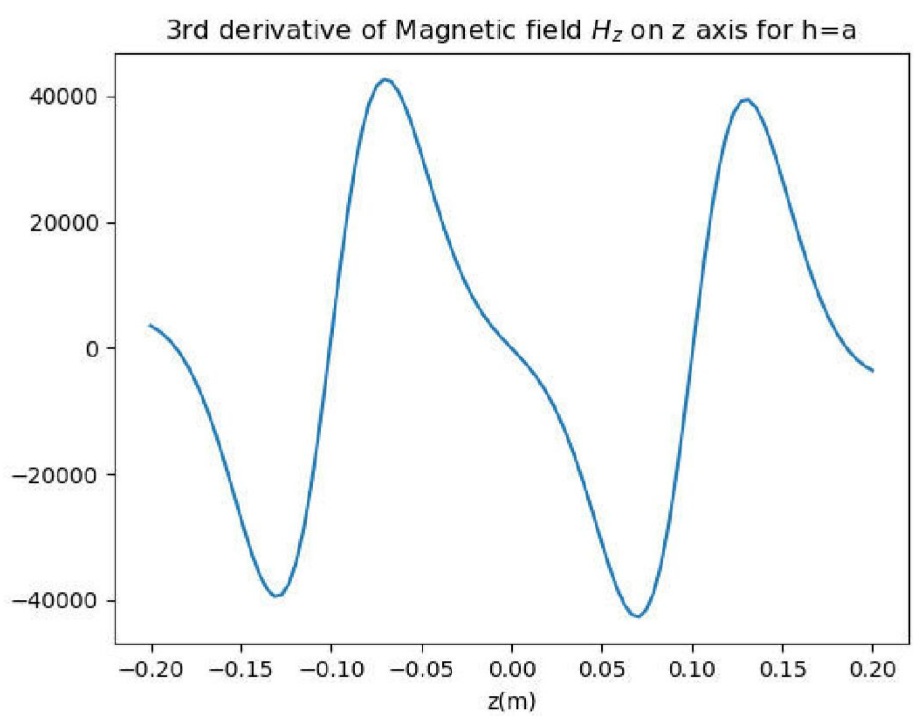
3rd derivative of Magnetic field H_z on z axis for h=a/2
30000
20000
10000
0
−10000
−20000
−30000
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
z(m)

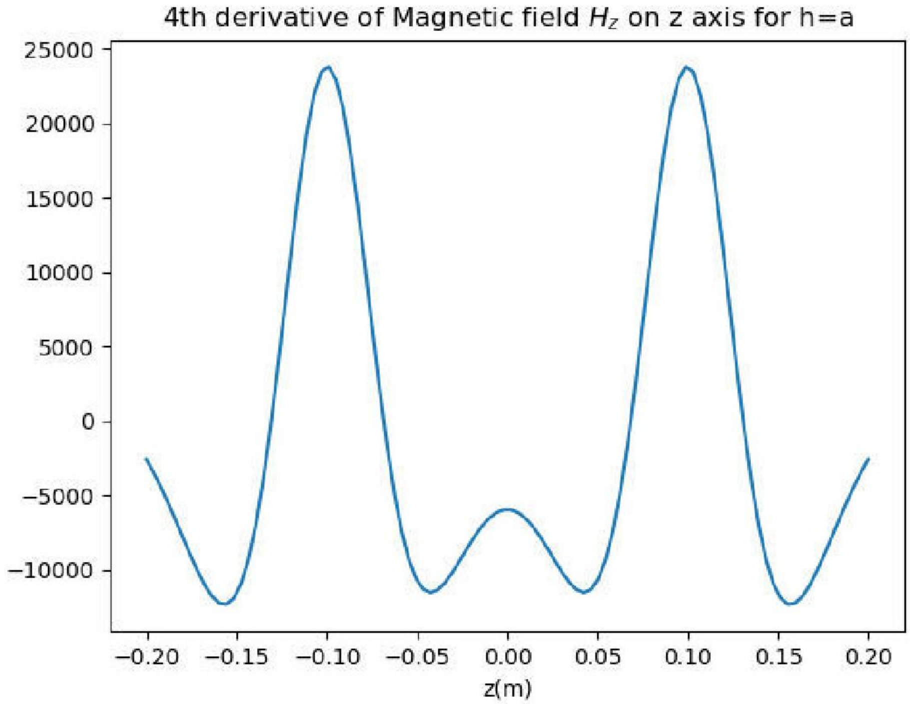

4th derivative of Magnetic field H_z on z axis for h=a/2
20000
10000
0
−10000
−20000
−0.100
−0.075
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
z(m)


Magnetic field H_z on z axis for h=a
H_z
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
−0.20 −0.15 −0.10 −0.05 0.00 0.05 0.10 0.15 0.20
z(m)

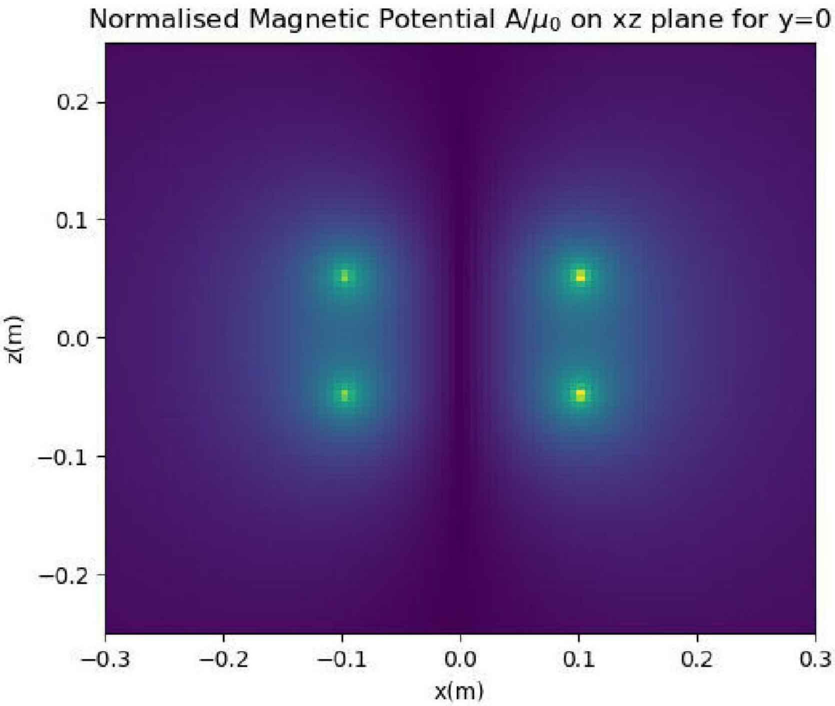

1st derivative of Magnetic field H_z on z axis for h=a
40
20
0
−20
−40
−0.20 −0.15 −0.10 −0.05 0.00 0.05 0.10 0.15 0.20
z(m)

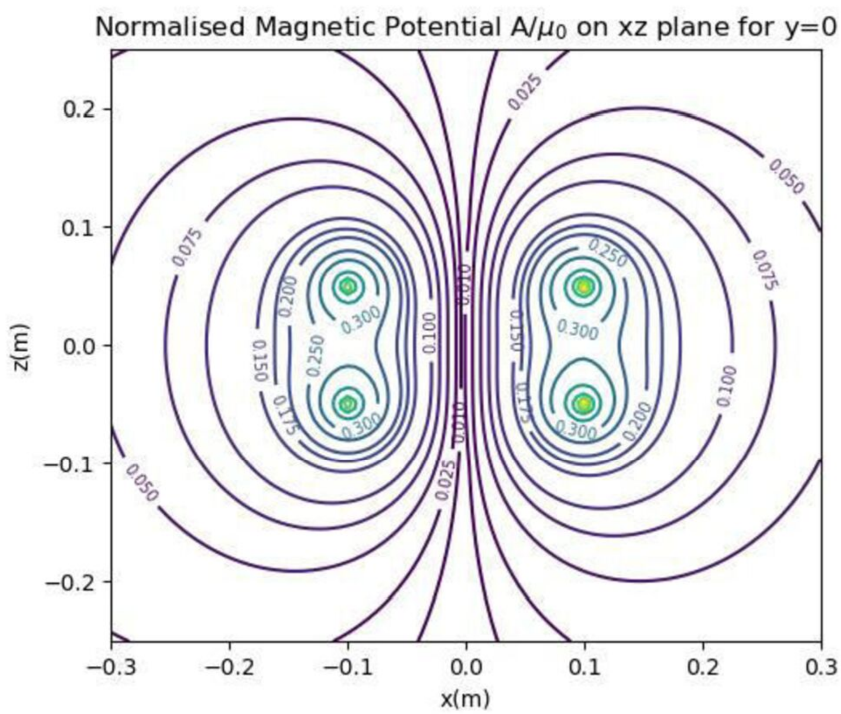

2nd derivative of Magnetic field H_z on z axis for h=a
500
0
−500
−1000
−1500
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
z(m)


3rd derivative of Magnetic field H_z on z axis for h=a
40000
20000
0
−20000
−40000
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
z(m)

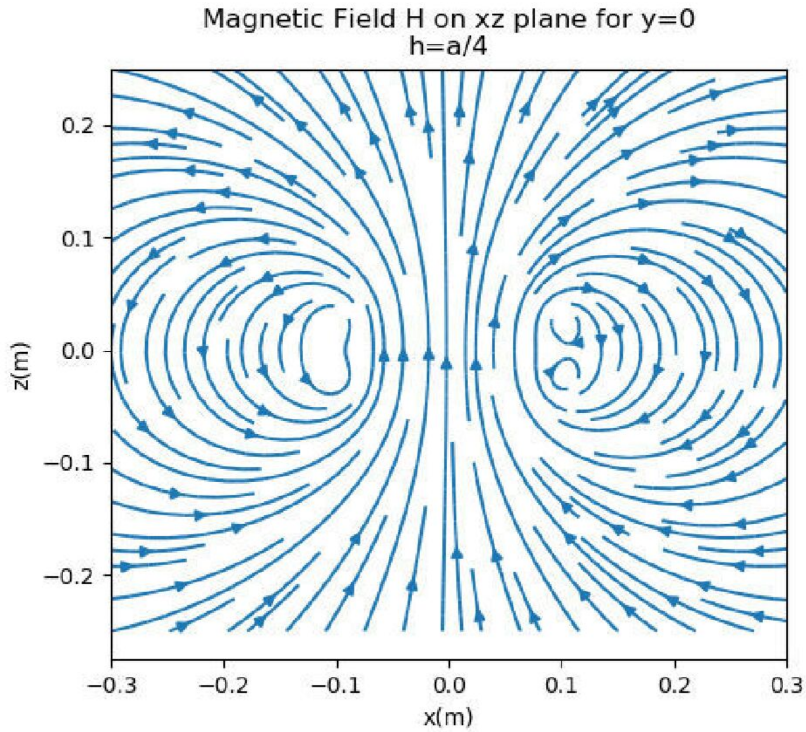

4th derivative of Magnetic field H_z on z axis for h=a
25000
20000
15000
10000
5000
0
−5000
−10000
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
z(m)

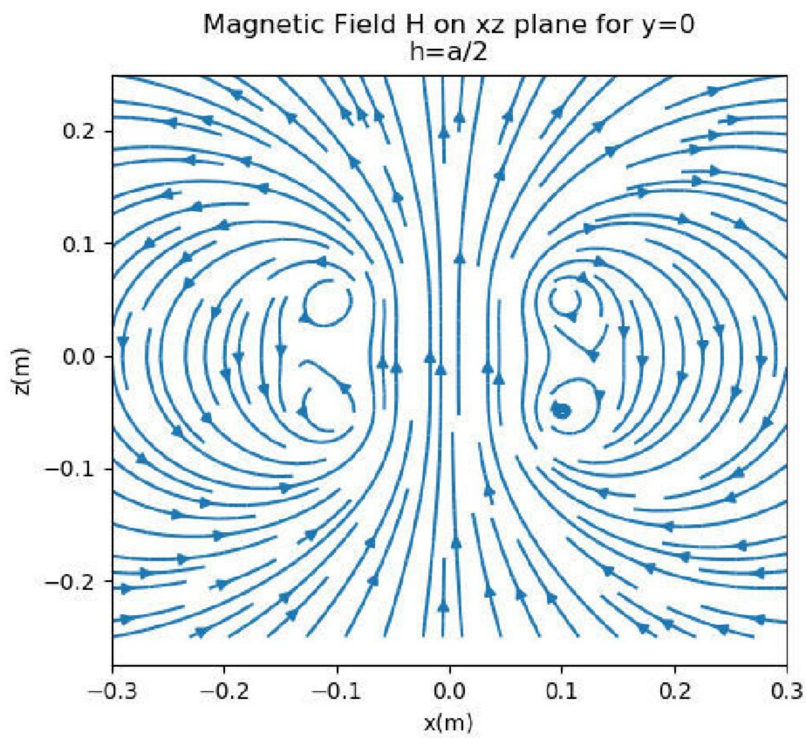
(ε)

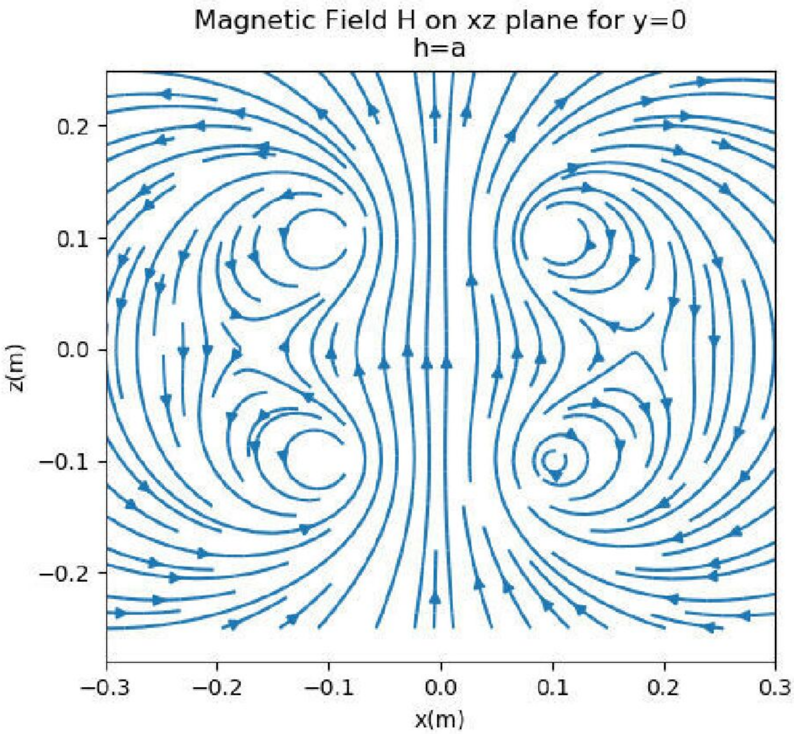

Normalised Magnetic Potential A/μ_0 on xz plane for y=0
0.2
0.1
0.0
−0.1
−0.2
z(m)
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
x(m)


Normalised Magnetic Potential A/μ_0 on xz plane for y=0
0.025
0.050
0.075
0.100
0.150
0.175
0.200
0.250
0.300
0.010
z(m)
x(m)

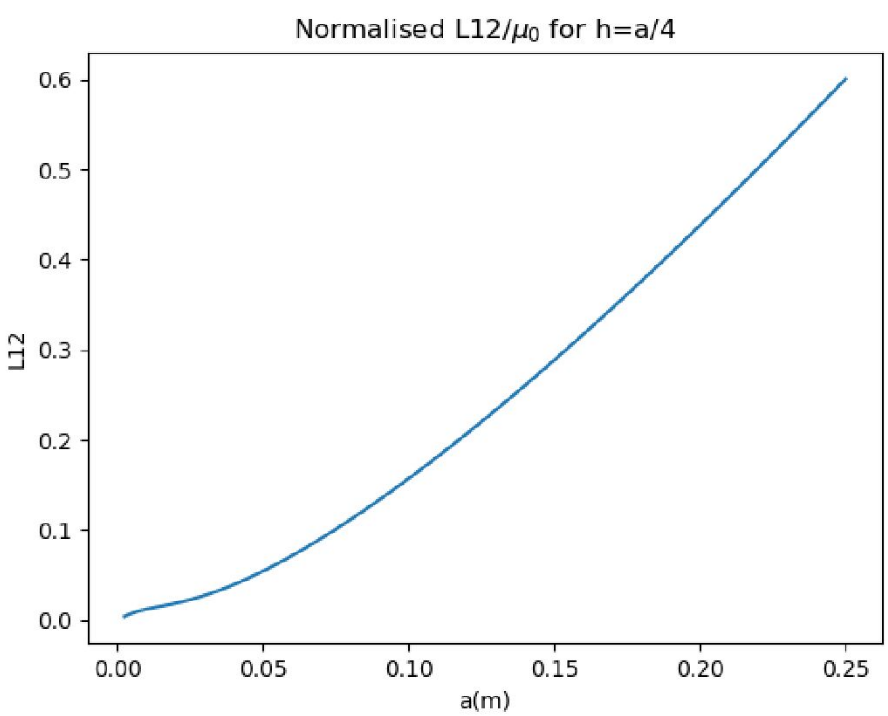
(στ)

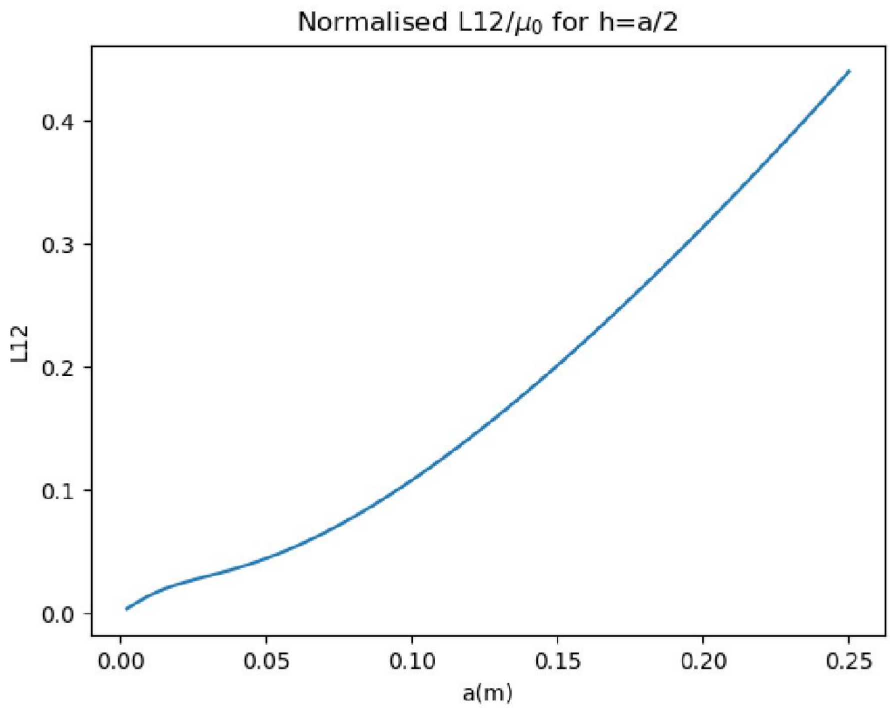

Magnetic Field H on xz plane for y=0
h=a/4
z(m)
x(m)
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3

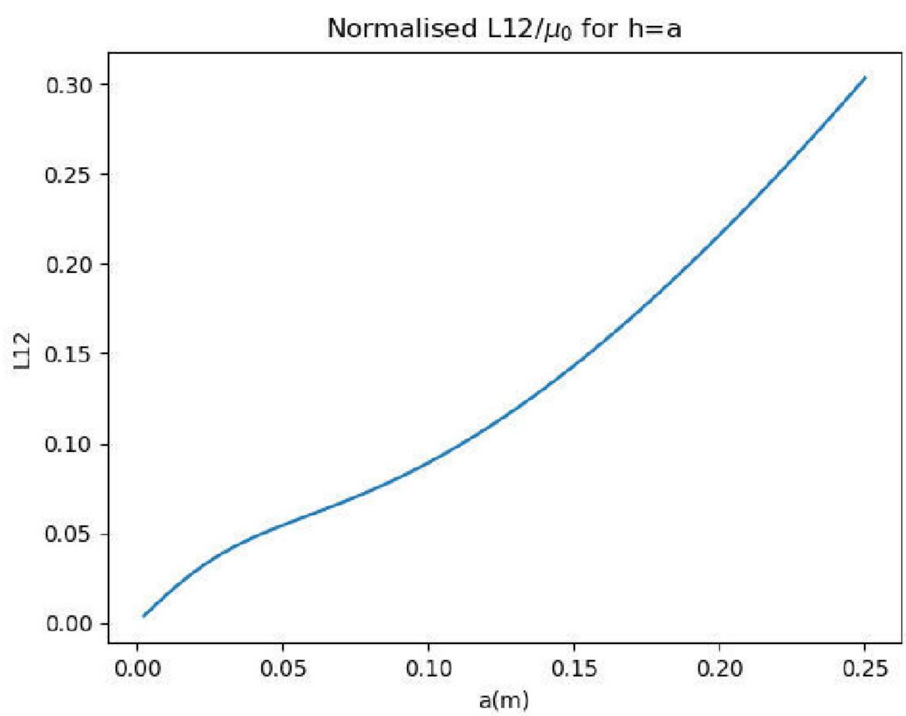

Magnetic Field H on xz plane for y=0
h=a/2
z(m)
x(m)
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3


Magnetic Field H on xz plane for y=0
h=a
z(m)
x(m)
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3

(ζ)


Normalised L12/μ_0 for h=a/4
L12
a(m)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.05
0.10
0.15
0.20
0.25


Normalised L12/μ_0 for h=a/2
L12
0.0
0.1
0.2
0.3
0.4
0.00
0.05
0.10
0.15
0.20
0.25
a(m)


Normalised L12/μ_0 for h=a
L12
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.00
0.05
0.10
0.15
0.20
0.25
a(m)

Κώδικας – Άσκηση 9

σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math
from scipy.special import ellipk
from scipy.special import ellipe

#constants
a=0.1
h=a/2
I1=1
I2=1
levels=[0.01,0.025,0.05,0.075,0.1,0.15,0.175,0.2,0.25,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1,1.25,1.5,2,3,5]

#Axes
N=100

xmin=-3*a
xmax=3*a
xx=np.linspace(xmin,xmax,N)

zmin=-(h+2*a)
zmax=h+2*a
zz=np.linspace(zmin,zmax,N)

X,Z=np.meshgrid(xx,zz)

amax=0.25
aa=np.linspace(0,amax,N)

#function for Hz for x,y=0
def Hz0(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    return a**2/2*(I1/R1**3+I2/R2**3)

#first derivative of Hz
def Hz1(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=-3*a**2*(z+h)/R1**5
    term2=-3*a**2*(z-h)/R2**5
    return I1/2*term1+I2/2*term2


#second derivative of Hz
def Hz2(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=3*a**2*(4*z**2+8*h*z+4*h**2-a**2)/R1**7
    term2=3*a**2*(4*z**2-8*h*z+4*h**2-a**2)/R2**7
    return I1/2*term1+I2/2*term2

#third derivative of Hz
def Hz3(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=-15*a**2*(z+h)*(4*z**2+8*h*z+4*h**2-3*a**2)/R1**9
    term2=-15*a**2*(z-h)*(4*z**2-8*h*z+4*h**2-3*a**2)/R2**9
    return I1/2*term1+I2/2*term2

#fourth derivative of Hz
def Hz4(z,h):
    R1=np.sqrt(a**2+(z+h)**2)
    R2=np.sqrt(a**2+(z-h)**2)
    term1=45*a**2*(8*z**4+32*h*z**3+(48*h**2-12*a**2)*z**2-(24*a**2*h-32*h**3)*z+8*h**4-12*a**2*h**2+a**4)/R1**9
    term2=45*a**2*(8*z**4-32*h*z**3+(48*h**2-12*a**2)*z**2+(24*a**2*h-32*h**3)*z+8*h**4-12*a**2*h**2+a**4)/R2**9
    return I1/2*term1+I2/2*term2
```

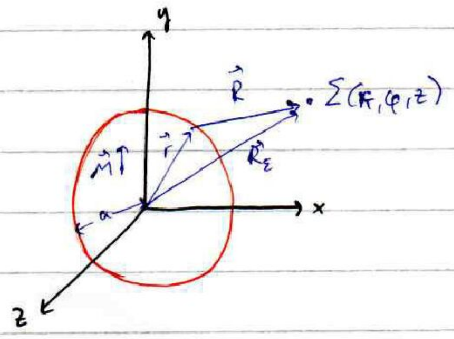
```python
#function for Potential
def A_phi(x,y,z):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=a*np.cos(i)*dphi/R1
        term2+=a*np.cos(i)*dphi/R2
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)

#functions for Magnetic Field
def Hx(x,y,z,h):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=(z+h)*a*np.cos(i)*dphi/R1**3
        term2+=(z-h)*a*np.cos(i)*dphi/R2**3
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)

def Hz(x,y,z,h):
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    term1=0
    term2=0
    for i in phi:
        R1=np.sqrt(x**2+y**2+(z+h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        R2=np.sqrt(x**2+y**2+(z-h)**2+a**2-2*a*(x*np.cos(i)+y*np.sin(i)))
        term1+=(-x*np.cos(i)-y*np.sin(i)+a)*a*dphi/R1**3
        term2+=(-x*np.cos(i)-y*np.sin(i)+a)*a*dphi/R2**3
    return I1*term1/(4*math.pi)+I2*term2/(4*math.pi)


#function for L12
def L12(a,h):
    k=4*a**2/(h**2+4*a**2)
    return a*((2-k**2)/k*ellipk(k)-2/k*ellipe(k))
```

```
#PLOTS
#plot of Hz for h=a/4
hh=a/4
z1=np.linspace(-2*hh,2*hh,N)
fig11,ax11=plt.subplots()
p11=ax11.plot(z1,Hz0(z1,hh))
ax11.set_title('Magnetic field $H_z$ on z axis for h=a/4')
ax11.set_xlabel('z(m)')
ax11.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a/4
fig12,ax12=plt.subplots()
p12=ax12.plot(z1,Hz1(z1,hh))
ax12.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax12.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a/4
fig13,ax13=plt.subplots()
p13=ax13.plot(z1,Hz2(z1,hh))
ax13.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax13.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a/4
fig14,ax14=plt.subplots()
p14=ax14.plot(z1,Hz3(z1,hh))
ax14.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax14.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a/4
fig15,ax15=plt.subplots()
p15=ax15.plot(z1,Hz4(z1,hh))
ax15.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a/4')
ax15.set_xlabel('z(m)')


#plot of Hz for h=a/2
hh=a/2
z1=np.linspace(-2*hh,2*hh,N)
fig21,ax21=plt.subplots()
p21=ax21.plot(z1,Hz0(z1,hh))
ax21.set_title('Magnetic field $H_z$ on z axis for h=a/2')
ax21.set_xlabel('z(m)')
ax21.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a/2
fig22,ax22=plt.subplots()
p22=ax22.plot(z1,Hz1(z1,hh))
ax22.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax22.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a/2
fig23,ax23=plt.subplots()
p23=ax23.plot(z1,Hz2(z1,hh))
ax23.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax23.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a/4
fig24,ax24=plt.subplots()
p24=ax24.plot(z1,Hz3(z1,hh))
ax24.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax24.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a/2
fig25,ax25=plt.subplots()
p25=ax25.plot(z1,Hz4(z1,hh))
ax25.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a/2')
ax25.set_xlabel('z(m)')
```

```python
#plot of Hz for h=a
hh=a
z1=np.linspace(-2*hh,2*hh,N)
fig31,ax31=plt.subplots()
p31=ax31.plot(z1,Hz0(z1,hh))
ax31.set_title('Magnetic field $H_z$ on z axis for h=a')
ax31.set_xlabel('z(m)')
ax31.set_ylabel('$H_z$')

#plot of 1st derivative of Hz for h=a
fig32,ax32=plt.subplots()
p32=ax32.plot(z1,Hz1(z1,hh))
ax32.set_title('1st derivative of Magnetic field $H_z$ on z axis for h=a')
ax32.set_xlabel('z(m)')

#plot of 2nd derivative of Hz for h=a
fig33,ax33=plt.subplots()
p33=ax33.plot(z1,Hz2(z1,hh))
ax33.set_title('2nd derivative of Magnetic field $H_z$ on z axis for h=a')
ax33.set_xlabel('z(m)')

#plot of 3rd derivative of Hz for h=a
fig34,ax34=plt.subplots()
p34=ax34.plot(z1,Hz3(z1,hh))
ax34.set_title('3rd derivative of Magnetic field $H_z$ on z axis for h=a')
ax34.set_xlabel('z(m)')

#plot of 4th derivative of Hz for h=a
fig35,ax35=plt.subplots()
p35=ax35.plot(z1,Hz4(z1,hh))
ax35.set_title('4th derivative of Magnetic field $H_z$ on z axis for h=a')
ax35.set_xlabel('z(m)')


#surface plot of potential
fig4,ax4=plt.subplots()
p4=ax4.pcolormesh(X,Z,abs(A_phi(X,0,Z)))
ax4.set_aspect('equal','box')
ax4.set_title('Normalised Magnetic Potential A/$μ_0$ on xz plane for y=0')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('z(m)')

#contour lines of potential
fig5,ax5=plt.subplots()
p5=ax5.contour(X,Z,abs(A_phi(X,0,Z)),levels)
ax5.clabel(p5, inline=True, fontsize=7)
ax5.set_aspect('equal','box')
ax5.set_title('Normalised Magnetic Potential A/$μ_0$ on xz plane for y=0')
ax5.set_xlabel('x(m)')
ax5.set_ylabel('z(m)')

#streamplot of magnetic field for h=a/4
fig61, ax61 = plt.subplots()
p61=ax61.streamplot(X,Z,Hx(X,0,Z,a/4),Hz(X,0,Z,a/4),density=1.2)
ax61.set_aspect('equal','box')
ax61.set_title('Magnetic Field H on xz plane for y=0\nh=a/4')
ax61.set_xlabel('x(m)')
ax61.set_ylabel('z(m)')

#streamplot of magnetic field for h=a/2
fig62, ax62 = plt.subplots()
p62=ax62.streamplot(X,Z,Hx(X,0,Z,a/2),Hz(X,0,Z,a/2),density=1.2)
ax62.set_aspect('equal','box')
ax62.set_title('Magnetic Field H on xz plane for y=0\nh=a/2')
ax62.set_xlabel('x(m)')
ax62.set_ylabel('z(m)')

#streamplot of magnetic field for h=a
fig63, ax63 = plt.subplots()
p63=ax63.streamplot(X,Z,Hx(X,0,Z,a),Hz(X,0,Z,a),density=1.2)
ax63.set_aspect('equal','box')
ax63.set_title('Magnetic Field H on xz plane for y=0\nh=a')
ax63.set_xlabel('x(m)')
ax63.set_ylabel('z(m)')
```

```
#plot of L12 for h=a/4
fig71,ax71=plt.subplots()
p71=ax71.plot(aa,L12(aa,a/4))
ax71.set_title('Normalised L12/$μ_0$ for h=a/4')
ax71.set_xlabel('a(m)')
ax71.set_ylabel('L12')

#plot of L12 for h=a/2
fig72,ax72=plt.subplots()
p72=ax72.plot(aa,L12(aa,a/2))
ax72.set_title('Normalised L12/$μ_0$ for h=a/2')
ax72.set_xlabel('a(m)')
ax72.set_ylabel('L12')

#plot of L12 for h=a
fig73,ax73=plt.subplots()
p73=ax73.plot(aa,L12(aa,a))
ax73.set_title('Normalised L12/$μ_0$ for h=a')
ax73.set_xlabel('a(m)')
ax73.set_ylabel('L12')

plt.show()
```

Άσκηση 10



$$\vec{R}_\Sigma = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{r} = a\cos\varphi'\,\hat{i}_x + a\sin\varphi'\,\hat{i}_y + z'\hat{i}_z$$

$$\vec{R} = \vec{R}_\Sigma - \vec{r} = (x - a\cos\varphi')\hat{i}_x + (y - a\sin\varphi')\hat{i}_y + (z - z')\hat{i}_z$$

$$R = \left[(x - a\cos\varphi')^2 + (y - a\sin\varphi')^2 + (z - z')^2\right]^{1/2} \quad \text{(καρτεσιανές)}$$

$$R = \left[r_T^2 + a^2 - 2r_T a\cos(\varphi - \varphi') + (z - z')^2\right]^{1/2} \quad \text{(κυλινδρικές)}$$

(α) $\vec{J}_m = \nabla \times \vec{M} = 0$

$$\vec{K}_m = -\hat{i}_{r_T} \times \vec{M} = -\left[\cos\varphi'\,\hat{i}_x + \sin\varphi'\,\hat{i}_y\right] \times M_0\hat{i}_y = -\hat{i}_z M_0\cos\varphi'$$

$$\vec{A} = \frac{\mu_0}{4\pi}\iint_S \frac{\vec{K}_m\, dS}{R}$$

Χρησιμοποιούμε τη λύση για άπειρο (~~κατά μήκος~~ στην κατεύθυνση του z άξονα) ευθύγραμμο αγωγό θεωρώντας επαλληλία τέτοιων αγωγών στην επιφάνεια του κυλίνδρου με ρεύμα $I = -M_0\cos\varphi'\, a\, d\varphi'$. Σημείο αναφοράς ο άξονας z άρα $r_{T_0} = a$

$$\vec{A} = \hat{i}_z \int_{\varphi'=0}^{2\pi} \frac{-\mu_0}{2\pi} M_0\cos\varphi'\, a \ln\frac{a}{\left[r_T^2 + a^2 - 2r_T a\cos(\varphi - \varphi')\right]^{1/2}}\, d\varphi' \Rightarrow \vec{A}(r_T, \varphi, z) = -\hat{i}_z \frac{\mu_0 M_0 a}{2\pi}\int_{\varphi'=0}^{2\pi} \cos\varphi' \ln\frac{a}{\left[r_T^2 + a^2 - 2r_T a\cos(\varphi - \varphi')\right]^{1/2}}\, d\varphi'$$

(β) Νόμος Biot-Savart: $\vec{B} = \frac{\mu_0}{4\pi}\iint_S \frac{\vec{K}_m \times \hat{i}_R}{R^2}\, dS = \frac{\mu_0}{4\pi}\int_{\varphi'=0}^{2\pi}\int_{z'=-\infty}^{+\infty} \frac{\vec{K}_m \times \vec{R}}{R^3}\, dz\, a\, d\varphi$

Όπως πριν θεωρούμε επαλληλία αγωγών με $I = -M_0\cos\varphi'\, a\, d\varphi'$

Κατεύθυνση $\frac{-\hat{i}_z \times \vec{R}}{R} = \frac{\hat{i}_x(y - a\sin\varphi') - \hat{i}_y(x - a\cos\varphi')}{\left[(x - a\cos\varphi')^2 + (y - a\sin\varphi')^2 + \cancel{(z-z')^2}\right]^{1/2}}$ λόγω συμμετρίας θεωρώ $z = z'$

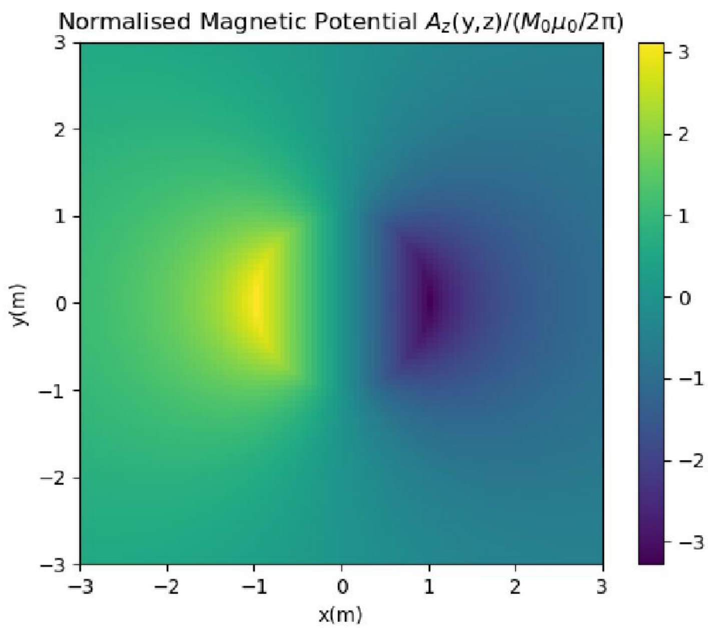
$$\vec{B}(x, y, z) = \frac{\mu_0 M_0}{2\pi}\, a \int_{\varphi'=0}^{2\pi} \frac{\left[\hat{i}_x(y - a\sin\varphi') - \hat{i}_y(x - a\cos\varphi')\right]\cos\varphi'\, d\varphi'}{(x - a\cos\varphi')^2 + (y - a\sin\varphi')^2}$$

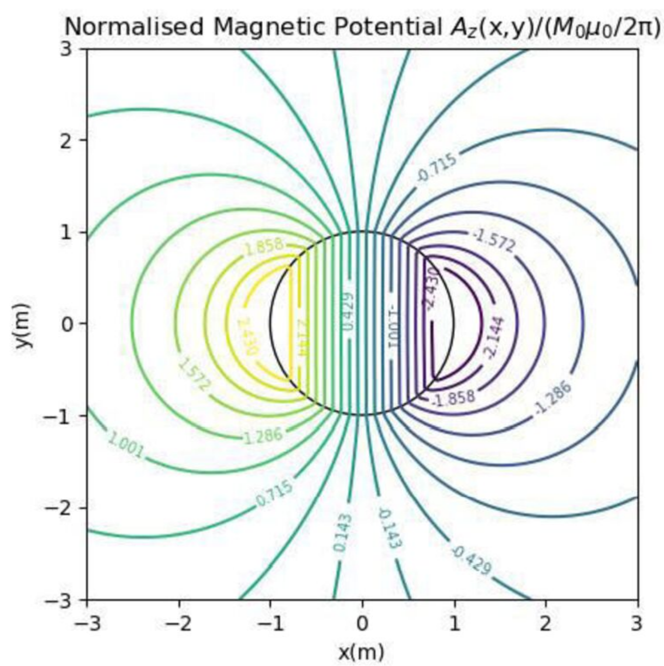
$$\vec{H}(x, y, z) = \begin{cases} \dfrac{\vec{B}}{\mu_0} - M_0\hat{i}_y, & \sqrt{x^2 + y^2} < a \quad \text{(εντός του μαγνήτη)} \\[2ex] \dfrac{\vec{B}}{\mu_0}, & \sqrt{x^2 + y^2} > a \quad \text{(εκτός του μαγνήτη)} \end{cases}$$

Γραφικές παραστάσεις – Άσκηση 10

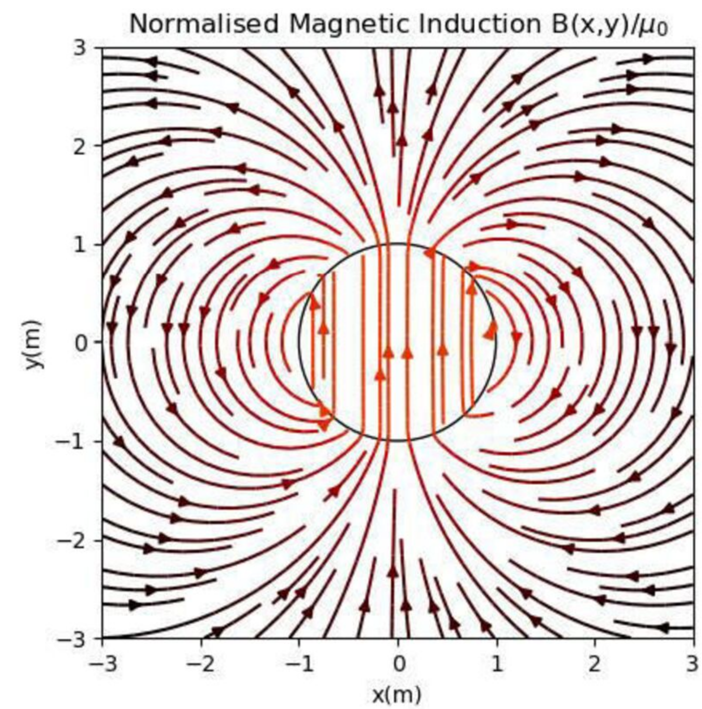
έγιναν σε Python 3.9

(γ)

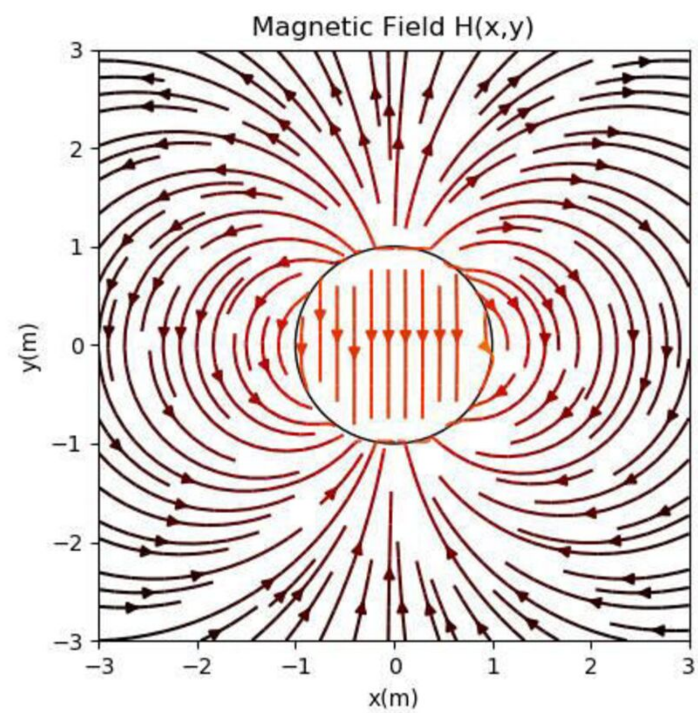




(δ)



Normalised Magnetic Induction B(x,y)/μ_0
y(m)
x(m)
3
2
1
0
−1
−2
−3



Magnetic Field H(x,y)
y(m)
x(m)
3
2
1
0
−1
−2
−3

Κώδικας – Άσκηση 10

σε Python 3.9

```
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import numpy as np
import math

#constants
a=1
M0=1
levels=2.7*np.linspace(-0.9,0.9,num=18)

#Axes
N=100

xmin=-3*a
xmax=3*a
xx=np.linspace(xmin,xmax,N)

ymin=-3*a
ymax=3*a
yy=np.linspace(ymin,ymax,N)

X,Y=np.meshgrid(xx,yy)

#function for Az
def Az(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*np.log(a/np.sqrt(r**2+a**2-2*r*a*np.cos(theta-i)))
    return -a*res

#functions for B
def Bx(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*(y-a*np.sin(i))/(r**2+a**2-2*r*a*np.cos(theta-i))
    return a*M0*res/(2*math.pi)
```

```python
def By(x,y):
    r=np.sqrt(x**2+y**2)
    theta=np.arctan2(y,x)
    phimin=0
    phimax=2*math.pi
    Nphi=200
    dphi=(phimax-phimin)/Nphi
    phi=np.linspace(phimin,phimax,Nphi)
    res=0
    for i in phi:
        res+=dphi*np.cos(i)*(x-a*np.cos(i))/(r**2+a**2-2*r*a*np.cos(theta-i))
    return -a*M0*res/(2*math.pi)

#functions for H
def Hx(x,y):
    return Bx(x,y)

def Hy(x,y):
    r=np.sqrt(x**2+y**2)
    condlist=[r<a,r>a]
    choicelist=[By(x,y)-M0,By(x,y)]
    return np.select(condlist,choicelist)

#PLOTS
#surface plot of Az
fig1,ax1=plt.subplots()
p1=ax1.pcolormesh(X,Y,Az(X,Y))
ax1.set_aspect('equal','box')
ax1.set_title('Normalised Magnetic Potential $A_z$(y,z)/($M_0μ_0$/2π)')
cb1=fig1.colorbar(p1)
ax1.set_xlabel('x(m)')
ax1.set_ylabel('y(m)')

#contour lines of Az
fig2,ax2=plt.subplots()
p2=ax2.contour(X,Y,Az(X,Y),levels)
ax2.clabel(p2, inline=True, fontsize=7)
c2=plt.Circle((0,0),a,fill=False)
ax2.add_artist(c2)
ax2.set_aspect('equal','box')
ax2.set_title('Normalised Magnetic Potential $A_z$(x,y)/($M_0μ_0$/2π)')
ax2.set_xlabel('x(m)')
ax2.set_ylabel('y(m)')


#streamplot of B
fig3,ax3=plt.subplots()
p3=ax3.streamplot(X,Y,Bx(X,Y),By(X,Y),density=1.2,color=np.log10(np.sqrt(Bx(X,Y)**2+By(X,Y)**2)),cmap=cm.gist_heat)
c3=plt.Circle((0,0),a,fill=False)
ax3.add_artist(c3)
ax3.set_aspect('equal','box')
ax3.set_title('Normalised Magnetic Induction B(x,y)/$μ_0$')
ax3.set_xlabel('x(m)')
ax3.set_ylabel('y(m)')

#streamplot of H
fig4,ax4=plt.subplots()
p4=ax4.streamplot(X,Y,Hx(X,Y),Hy(X,Y),density=1.2,color=np.log10(np.sqrt(Hx(X,Y)**2+Hy(X,Y)**2)),cmap=cm.gist_heat)
c4=plt.Circle((0,0),a,fill=False)
ax4.add_artist(c4)
ax4.set_aspect('equal','box')
ax4.set_title('Magnetic Field H(x,y)')
ax4.set_xlabel('x(m)')
ax4.set_ylabel('y(m)')

plt.show()
```