

# Canonical Autoregressive Generation



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Paper

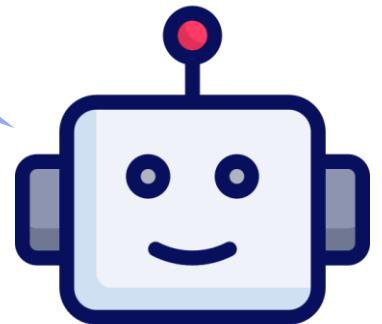
LLMs are trained on canonical token sequences

"I like western movies"  
"She lived in the western suburbs"  
"BC is in western Canada"

Training data

The training data gets tokenized deterministically by the tokenizer

I like western movies  
She lived in the western suburbs  
BC is in western Canada



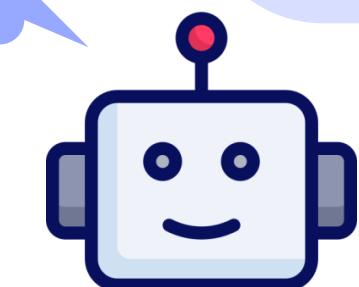
LLMs can generate non-canonical token sequences



Where in Canada is Vancouver?

Vancouver is in the western province

Canonical: Vancouver is in the western province



## Problems with non-canonical token sequences



They can bypass safety filters leading to harmful responses  
"Adversarial Tokenization" Geh et al., 2025



They allow token misreporting by LLM providers  
"Is Your LLM Overcharging You? Tokenization, Transparency, and Incentives" Artola Velasco et al., 2025



String perplexity is computationally hard  
"Where is the signal in tokenization space?" Geh et al., EMNLP 2024  
"You should evaluate your language model on marginal likelihood over tokenisations" Cao & Rimell, EMNLP 2021  
"Should you marginalize over possible tokenizations?" Chirkova et al., ACL 2023

## How to ensure LLMs can only generate canonical token sequences?

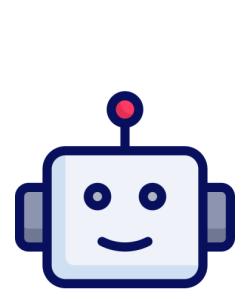
### Theorem

If token sequence  $s$  is non-canonical, then for any token  $t$  the sequence  $s | t$  is also non-canonical



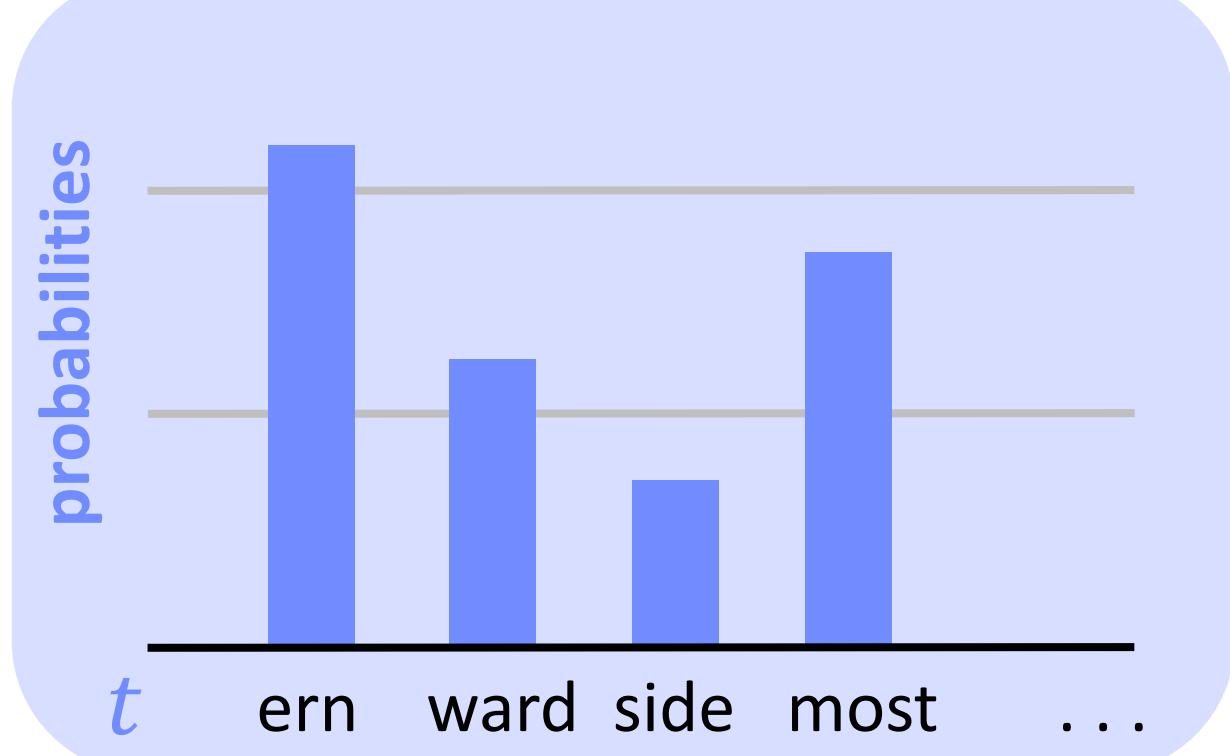
At every step of generation the (partial) token sequence generated so far must be canonical

## We propose Canonical Sampling

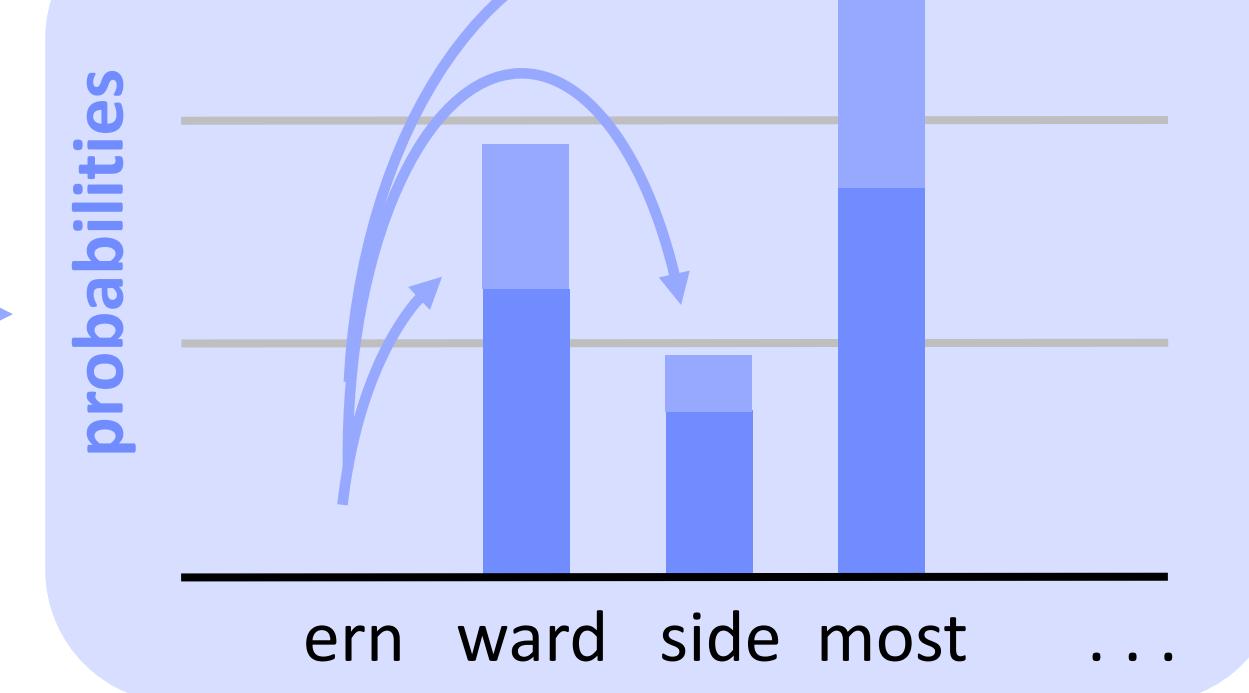


$s$   
Vancouver is in the west

Original next-token distribution  $d_s$



Canonicalized next-token distribution  $\tilde{d}_s(t) = \begin{cases} \frac{d_s(t)}{Z}, & s | t \text{ is canonical} \\ 0, & s | t \text{ is non-canonical} \end{cases}$



### Theorem

Sampling from  $\tilde{d}_s$  leads to output token sequences closer to the true distribution of token sequences (as seen during training)

$s | t$  is canonical X ✓ ✓ ✓

## Efficient Canonical Sampling

## ALGORITHM

Computing the normalization constant  $Z = \sum_{t \in V: s | t \text{ is canonical}} d_s(t)$  requires checking if  $s | t$  is canonical for all tokens  $t$

But we can efficiently sample from  $\tilde{d}_s$  using the Gumbel-Max trick:

$$\tilde{d}_s(t) \sim \operatorname{argmax}_{t \in V: s | t \text{ is canonical}} \{\log(d_s(t)) + u_t\}$$

Sample  $u_t \sim \text{Gumbel}(0,1)$  for every token  $t$  in the vocabulary

For every token  $t$  in decreasing order of  $\log(d_s(t)) + u_t$ :  
If  $s | t$  is canonical then return  $t$

Requires fewer than  $\frac{1}{Z}$  canonicity checks on average