A Study of Modeling Application about Wolf Population Dynamics at Jellystone National Park

SYSEN 533 Deterministic Models and Simulation

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SPRING 2017

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1.0 Introduction:

The purpose of our model is to determine the feasibility of reintroducing grey wolves to the fictional Jellystone National Park, home of the infamous Yogi Bear. Our model is based upon research performed prior to the reintroduction of wolves at the very real Yellowstone National Park. The food chain is a delicate eco-system that adjusts to any minor adjustments in order to leverage the survivability for the majority of predators or prey. Our primary goal, aside from increased biodiversity of the Park, is to decrease the population of elk. The elk population is estimated at 31,217 and we are hoping to have the elk population decrease, but by no more than 30% over the next 30 years. Additionally, the coyote population is estimated at 2,017, and we are hoping to not decrease that population by more than 50%, while still introducing the grey wolf species to the park. Neither the elk nor the coyote population should increase over the long term over the course of next 30 years (2017-2047). The model and simulation will help us determine what ramifications the reintroduction of the grey wolf will have on our delicate ecosystem.

2.0 Project Statement:

For this project this system will simulate the reintroduction of the grey wolf in Jellystone National Park. First, it simulates over the next thirty years without wolves, and demonstrates how the elk will thrive in that environment, albeit with a grazing limitation as an upper bound. Additionally, we will investigate the coyote population in relation to the elk. Next, we simulate having an initial wolf count of one to twenty-five to determine what the optimal beginning wolf population will be. These subsystems will determine the number wolves require to balance within the ecosystem. Finally, the simulation will look at the populations of the three species over the long term to see the impact of reintroducing wolves to the ecosystem.

3.0 Background:

3.1 Variables

Below in Table 1, shows the variables used for the equations as well as variables for the software.

Variable	Description	Initial Conditions	Notes
E	Estimated elk population (in thousands)	31.217 (sometimes a random number within 10% of 31.217)	
C	Estimated coyote population (in thousands)	2.017 (sometimes a random number with 10% of 2.017)	
W	Estimated wolf population (in thousands)	Ranges between 0 and 14	
acreage	Number of acres elk may potentially graze on (in thousands)	19.68	Constant Value
G	Value between 0 and 1 representing the fraction of potentially grazeable acreage that is actually grazeable.	.8 (sometimes a random number with 10% of .8)	
U	Value between 0 and 1 representing the fraction of potentially grazeable acreage that is not actually grazeable.	1-G	Always equal to 1-G
Rg	Intrinsic rate of growth of grazeable grass	0.27	constant
Ru	Intrinsic rate of growth of not grazeable grass	0.35	constant
qG	Competition coefficient of grazeable grass.	1.07	constant

qU	Competition coefficient of not grazeable grass	0.6	constant
B	Ratio between upper and lower bounds on qG and qU	0.3	constant
graze	Acre per year that an average elk requires for grazing	.492	

Table 1: Variable Descriptions

3.2 Equations

The first two equations give us the change in land that is grazeable by the elk population, and land that is not grazeable by the elk population (Cooper & Huffaker 1997, p. 62-69).

$$dG/dt = Rg * G * (1 - G - qU * U * (B + (G/graze))/(1 + G/graze))$$

Equation 1: Change in grazeable land

$$dW/dt = Ru * U * (1 - U - qG * G * (1 + G/graze)/(B + G/graze))$$

Equation 2: Change in not grazeable land

The following three equations give us the change in populations of our respective animals: elk, coyotes, and wolves (Knickerbocker n.d., p 1).

$$If\ ((19.68G)/graze) > E : \quad dE/dt = E(0.004 - 0.003C - 0.85W)$$
$$If\ ((19.68G)/graze) <= E : \quad dE/dt = E(0.003C - 0.85W)$$

Equation 3: Change in elk population

$$dC/dt = C(-.006 + .001E)$$

Equation 4: Change in coyote population

$$dW/dt = W(-.12 + .005E)$$

Equation 5: Change in wolf population

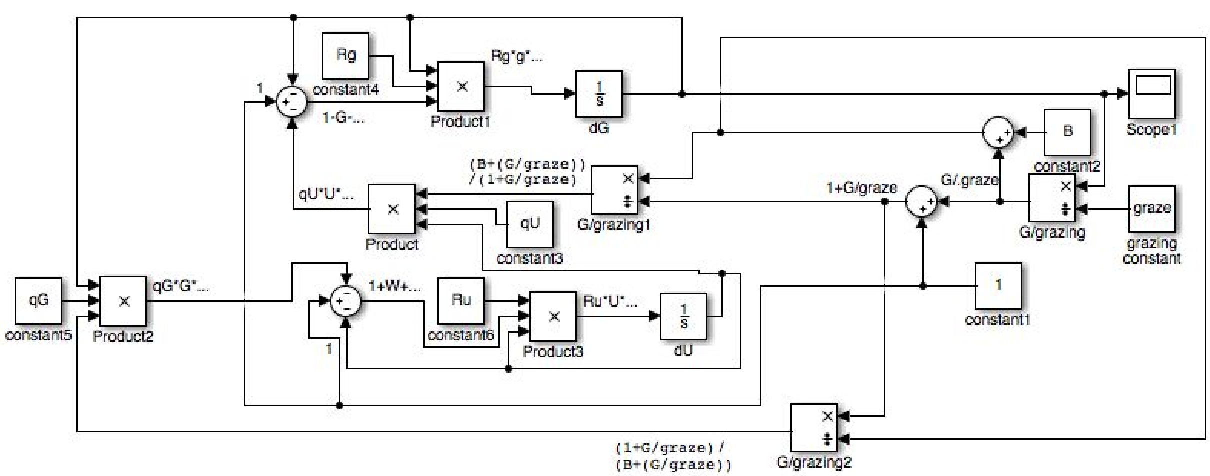
Finally, we created our grazing constant. We estimated that of our population of elk, 70% of the population are cows, 13% of the population are bulls, and 17% of the population are calves (Liebenthal 2015). Utilizing grazing rates where a mature cow uses 0.5 acres of grazeable land a year, a bull uses 0.7 acres of grazeable land a year, and a calf uses 0.3 acres of grazable land a year. Doing this we end up with a grazing constant of 0.492.

$$graze = 0.5(cows/total) + 0.7(bulls/total) + 0.3(calves/total)$$

Equation 6: Grazing Constant

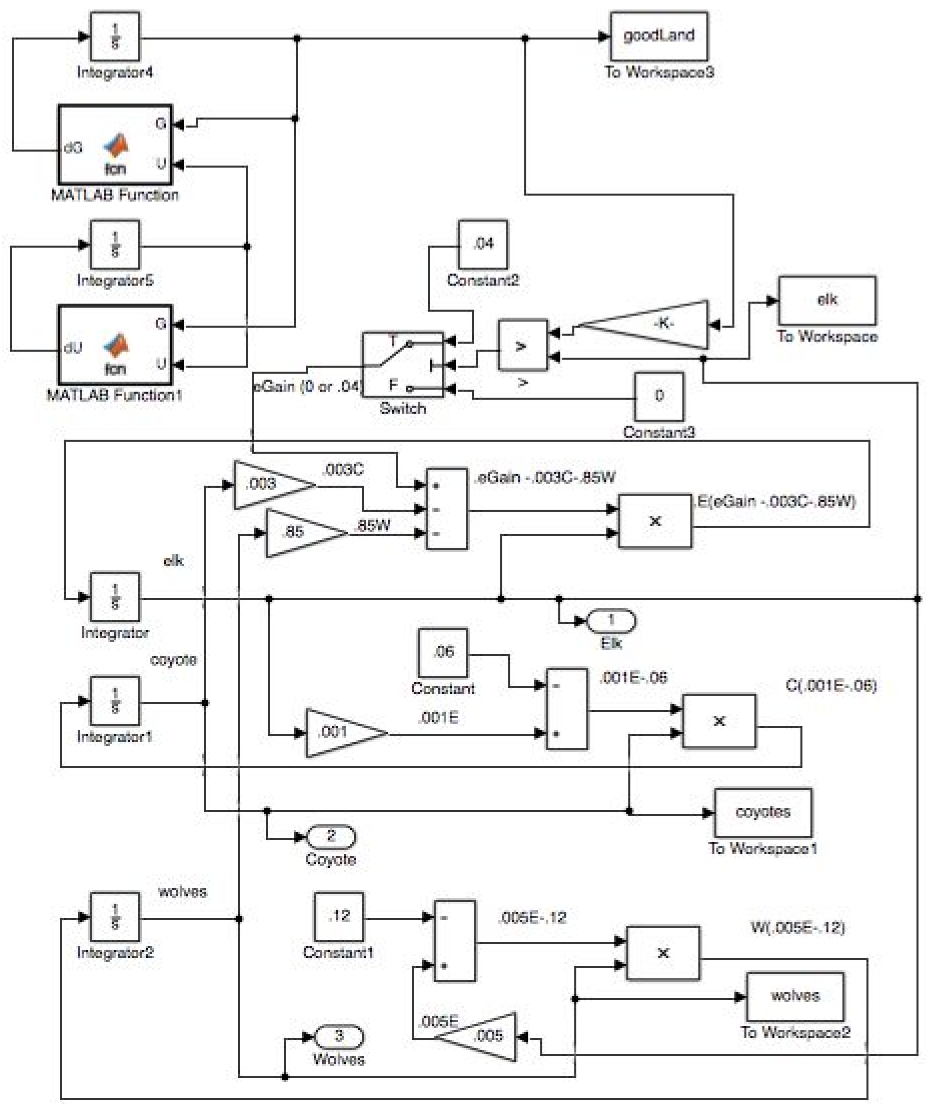
3.3 Model Overview:

The first model is just the model for the elk grazeable land equations. Note we do not use this actual model in the simulation, this is merely a representation of the grass portion of the model. This is represented by two “Matlab function” blocks in the real model.



Model 1: Sample Grazeable vs. Non Grazeable Land

The second model is the model that the simulation actually executes. This model determines the animal populations over a certain number of years. Note that the two “MATLAB Function” blocks represent Model 1 above. Also note, the switch that is in place determines the which part of Equation 3 that will be used during execution of the model.



Model 2: Jellystone Animal Populations

4.0 <u>Simulations:</u>

Before we investigate introducing wolves into Jellystone National Park, let's get a baseline for what we expect the elk and coyote populations to look like in 30 years. To do this we will run 25 simulations with several initial parameters randomized to be within 10% of our defined initial conditions. We see in Figure 1 that the elk populations all increase and top out around 40,000. However, the coyote population goes down a substantial amount. In fact the coyote population decreases by almost 50% which is what we were hoping would be the maximum decrease **after** an introduction of wolves.

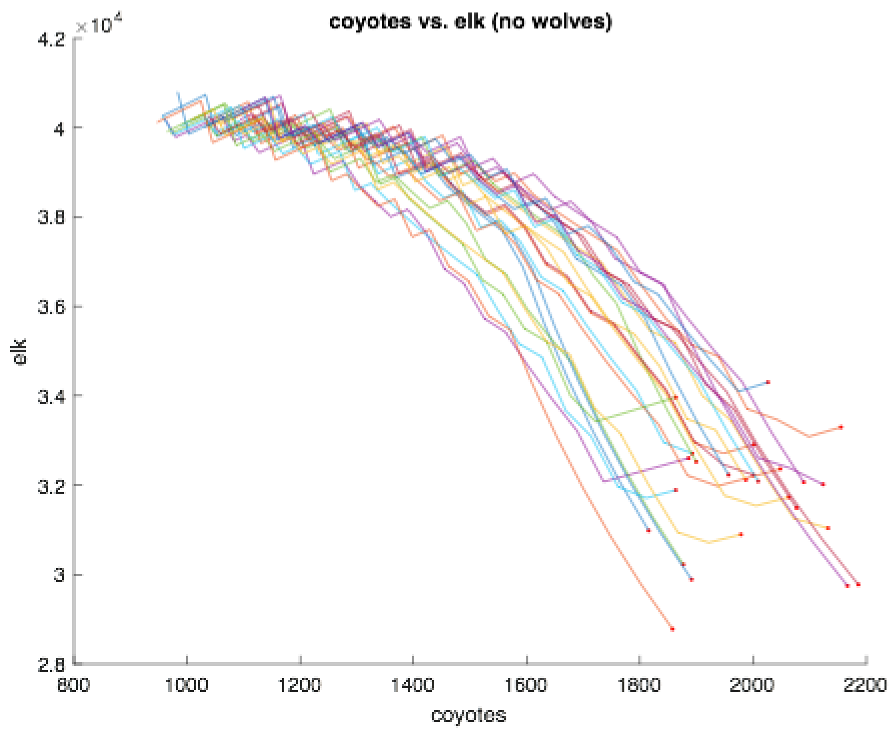


Figure 1: Coyotes vs. elk (without wolves) over 30 years

Our primary goal for wolf reintroduction into the park was to decrease the elk population by no more than 30%. Because a secondary goal was to decrease the coyote population by no more than 50%, and we have almost achieved that with an intial wolf count of 0, we will determine the minimum number of wolves that will decrease the elk population over a 30 year period. It just so happens that that answer is 14. Figures 2 through 4 show the populations over 30 years when we introduce 14 wolves in 2017. We see the wolf population increase, the elk population decrease by less than 1%, and the coyote population decrease by almost 51%.

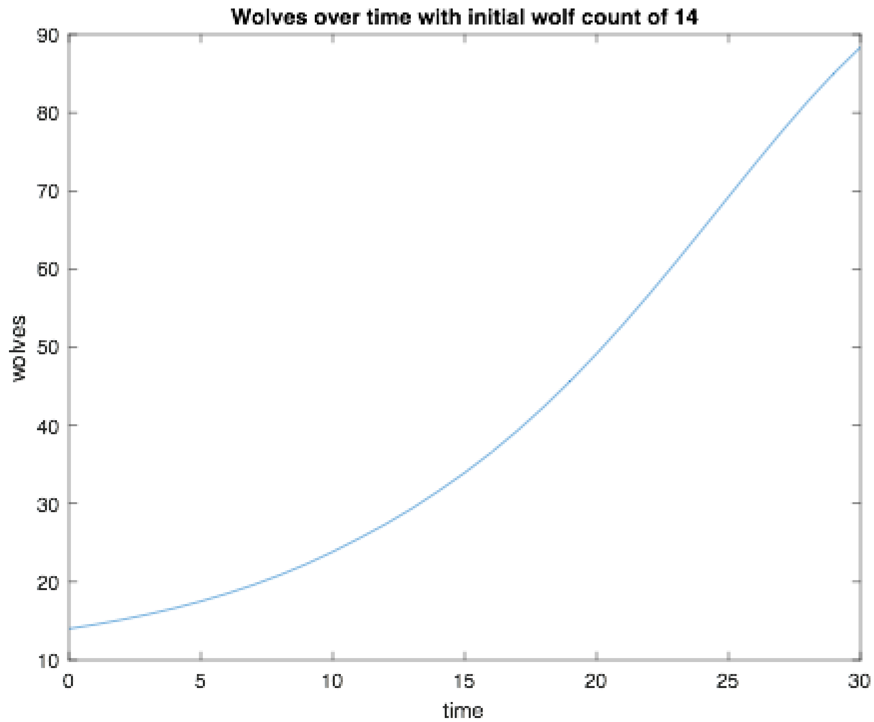


Figure 2: Wolf population over 30 years after wolf reintroduction

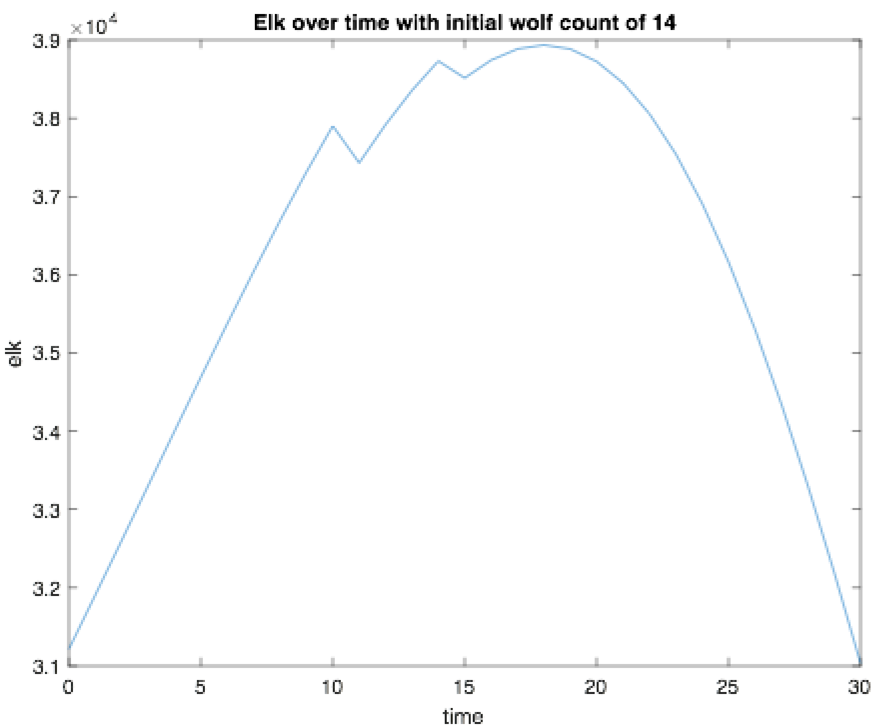


Figure 3: Elk population over 30 years after wolf reintroduction

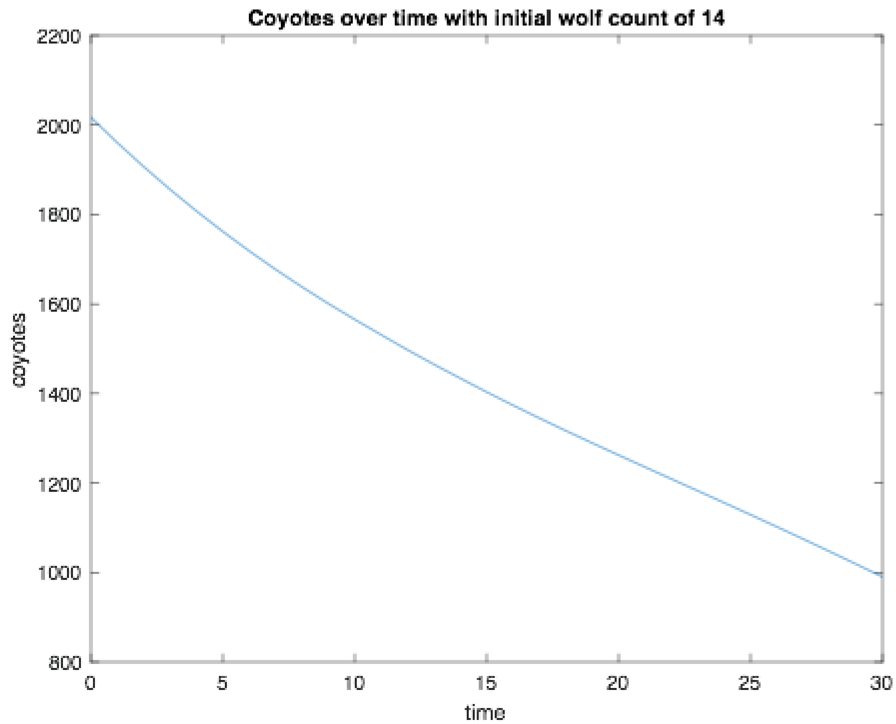


Figure 4: Coyote population over 30 years after wolf reintroduction

Our primary goal of reducing the elk population seems to be achievable by adding 14 wolves to the park. But because the coyote population decreased by more than what was desired, we decided it would be good to investigate what happens beyond thirty years. What would the park biodiversity dynamics be like in a century? Figures 5-7 show the population of the respective animals over the next century.

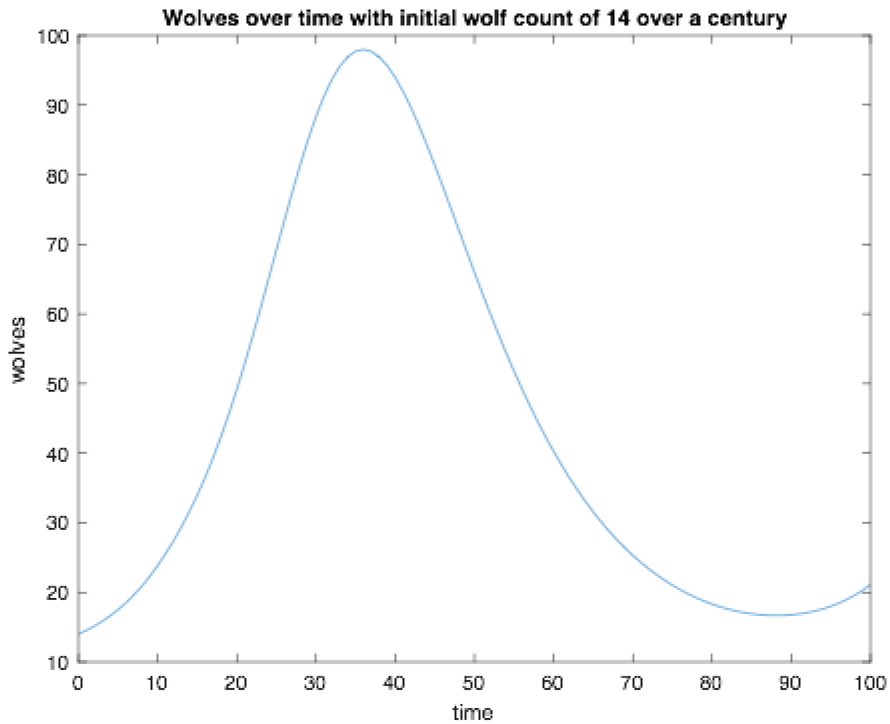


Figure 5: Wolf population over 100 years

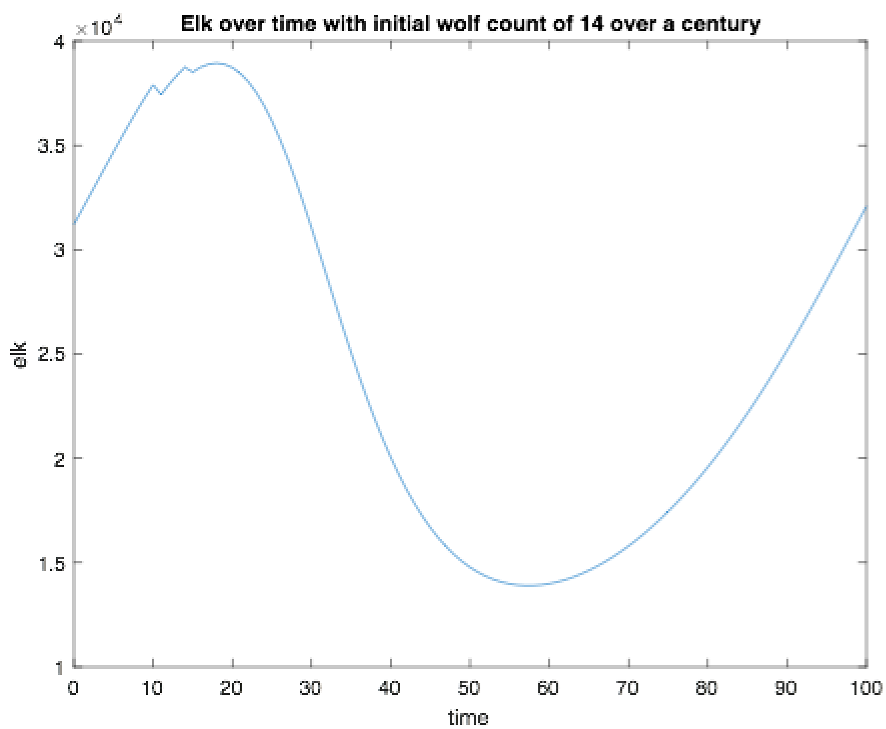


Figure 6: Elk population over 100 years

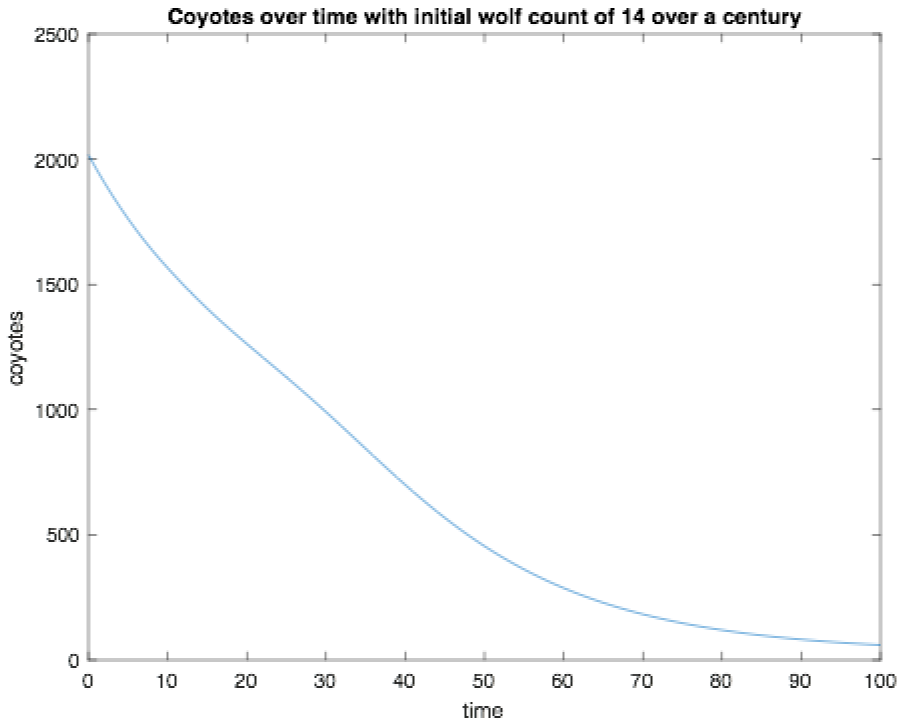


Figure 7: Coyote population over 100 years

Interestingly, we see that the elk and wolf populations seem to fluctuate against one another, which makes sense from the equations that we have used. Unfortunately, it doesn't look like the coyote population recovers and seems to continue to decline throughout the century. Let's take a look at if these patterns continue over a thousand years (shown in Figures 8-10).

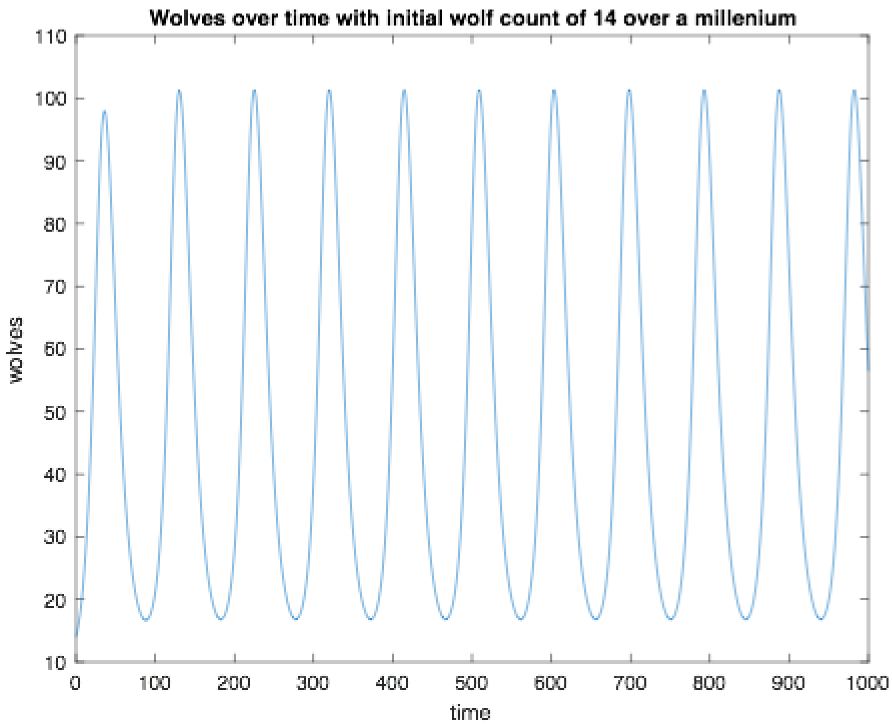


Figure 8: Wolf population over 1,000 years

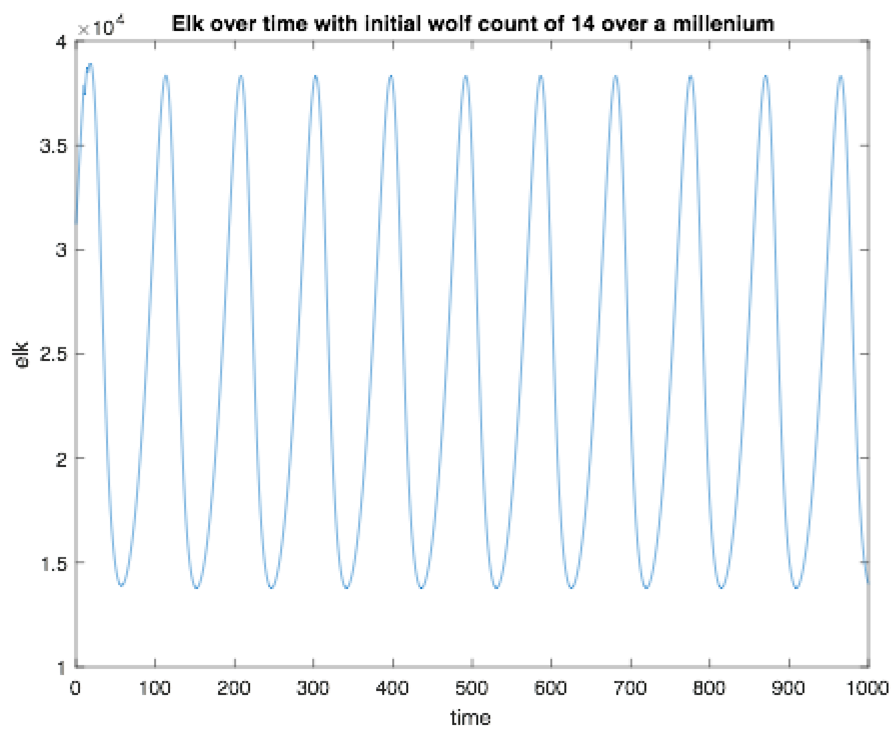


Figure 9: Elk population over 1,000 years

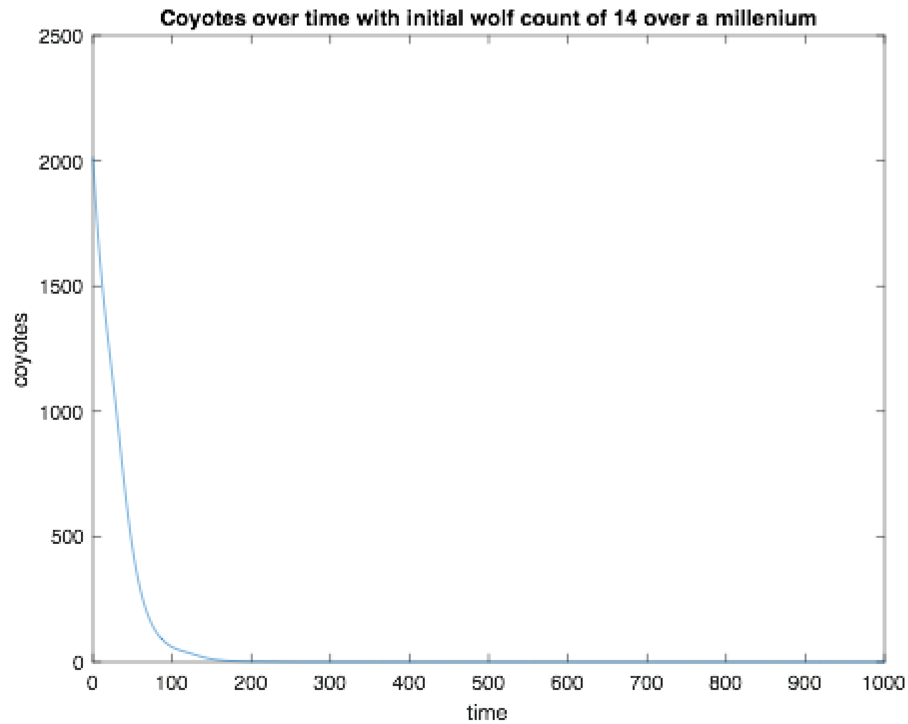


Figure 10: Coyote population over 1,000 years

Unfortunately it looks like the coyote population is doomed to extinction if we introduce wolves into the park. It is estimated that this will occur in 2240. So we may not wish to introduce wolves to the park if it comes at the expense of our coyote population. However, it seemed as if the coyote population was going to be cut almost in half in just a 30 year time earlier, without wolf reintroduction. Let’s investigate how the coyote population will do over a longer term without wolves as a competitor by running 25 simulations with randomized initial conditions, as before, in Figure 11.

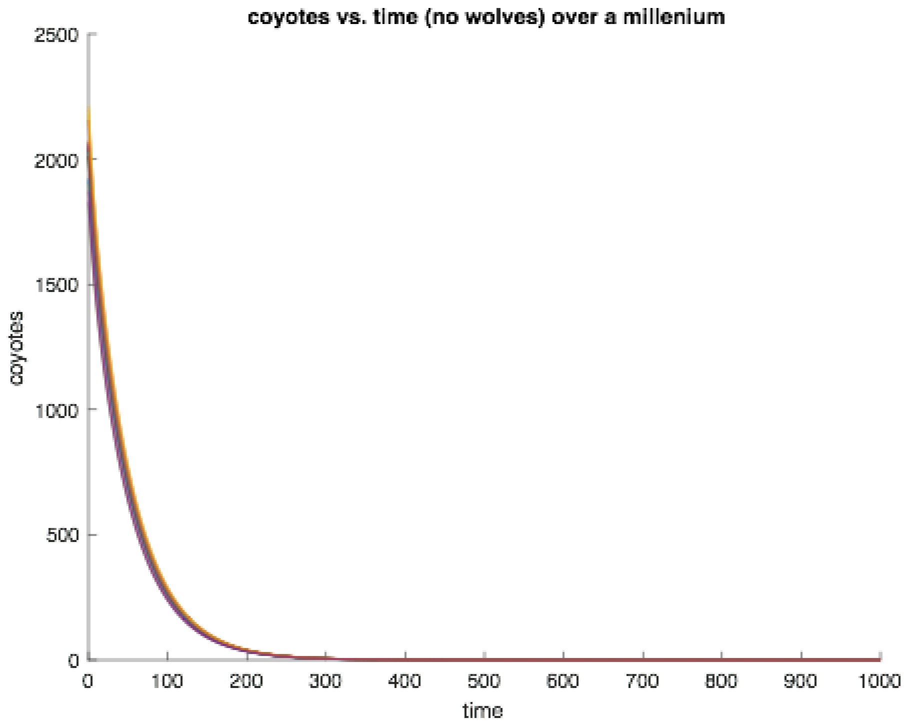


Figure 11: Coyote population over 1,000 years (without wolves)

It appears as if the park's coyote population is doomed to extinction either way. Running 25 simulations, on average, the coyote population went extinct in the year 2403. So without the wolf population we bought the coyotes another 163 years, but still they go extinct. In fact if we were to introduce only 1 wolf in 2017 to the park, coyotes would still go extinct in the year 2245, buying them only an additional 5 years over the initial wolf count being 14.

5.0 Conclusions:

The introduction of grey wolves into Jellystone National Park will help to limit the populations of elk. The relationship between elk and wolves is cyclical. Both species will experience population highs and lows. Based on our model the extinction of coyotes from Jellystone is imminent and the introduction of grey wolves will only impact the year of the coyote extinction.

It's possible that there is an issue with the model that we have for the coyote population, as no matter what we seem to do, they always seem doomed to extinction. It is possible that the model is not accounting for an alternative food source, or perhaps the model for Yellowstone National Park that we have applied is not truly valid for Jellystone. In fact, the only time the coyotes ever seem to be able to stay afloat, is when the elk population reaches 60,000 (which is beyond the capacity for Jellystone Park). It just so happens that this is the initial elk population of the Yellowstone model that we applied to our simulation was 60,000.

If decreasing the elk population is indeed the most important thing that we need to do at Jellystone, then reintroducing wolves would probably be a good idea. However, if we wish to try to keep the coyote population, we should probably determine what the best way is to save that species, before we introduce wolves. Then again, perhaps we should leave the park to itself by following the advice of Capt. Kathryn Janeway of the Starship Voyager when she said "Who are we to swoop in, play god, and then continue on our way without the slightest consideration of the long-term effects of our actions?"

If you wish to download or use the model (and previous versions of it), script, or any other aspects of our project please visit: https://github.com/Qvist30/jellystone.

Works Cited:

Arychuk, Cathie Erichsen. "Setting Stocking Rates for Pastures." Bison Producers of Alberta, June 2000. Web. 21 Apr. 2017. <http://bisoncentre.com/resources/resource-library/advanced-bison-information-producers/pasture-management/setting-stocking-rates-pastures/>.

Kevin Cooper, Ray Huffaker, The long-term bioeconomic impacts of grazing on plant succession in a rangeland ecosystem, Ecological Modelling, Volume 97, Issue 1, 1997, Pages 59-73, ISSN 0304-3800, http://dx.doi.org/10.1016/S0304-3800(96)00072-5. <http://www.sciencedirect.com/science/article/pii/S0304380096000725>.

Knickerbocker, CJ. "A Modeling Application: Wolf Population Dynamics." (N.d.): 1-3. A Modeling Application. Web. 21 Apr. 2017. <http://www.cengage.com/math/book_content/0495108243_zill/projects_archive/debv4e/AModelingApplication2.pdf>.

Liebenthal, Steve. "Wildlife Crews Find Robust Elk Populations." Idaho Fish and Game, 19 Feb. 2015. Web. 21 Apr. 2017. <https://idfg.idaho.gov/press/wildlife-crews-find-robust-elk-populations>.

Appendix: Script for Model Execution

```
% executeModel.m
% Written by Tom Kennedy and Matthew Ellis
% Executes the Jellystone_v6 model.

close all
acreage=19.68; % *1000
graze = .492;

figure
hold on
set_param('jellystone_v6', 'StopTime', '30')

initialWolfCount = 0;

%Shows Executes the simulation over 30 years without wolves, shows that
%the elk population is continuing to approach 40,000.  Uses random numbers
% for G0 (grazing ground, which should be able to go from .72 to .88, for
%C0 it can be 1,815 to 2,219, for E0 it can go from 25,873 to 37,461.
for i=1:25
   G0=.8+(rand-.5)*.16;
   W0=0;
   C0=2.017+(rand-.5)*.404;
   E0=31.217+(rand-.5)*6.244;
   plot([C0]*1000,[E0]*1000,'.r')
   xlabel('coyotes')
   ylabel('elk')
   title('coyotes vs. elk (no wolves)')
   [t,x,y]=sim('jellystone_v6');
   E=x(:,1);
   C=x(:,2);
   W=x(:,3);
   plot(C*1000,E*1000);
end

figure
hold on
%Determines what the number of wolves would be that we would need to
%introduce into the model for the elk population to decrease over 30 years.
```

```
for i=1:25
    G0=.8;
    W0=i/1000;
    C0=2.017;
    E0=31.217;
    plot([C0]*1000,[E0]*1000,'.r')
    xlabel('coyotes')
    ylabel('elk')
    title('coyotes vs. elk (with wolves)')
    [t,x,y]=sim('jellystone_v6');
    E=x(:,1);
    C=x(:,2);
    W=x(:,3);
    plot(C*1000,E*1000);
    disp(['Coyote percentage:',num2str(C(end)/C0),'Elk percentage:',num2str(E(end)/E0)]);
    if(E(end)<E0)
        initialWolfCount = W0;
        disp(['Initial Wolf Count =',num2str(W0*1000), 'elk count=',num2str(E(end)*1000), 'coyote count=',
num2str(C(end)*1000)]);
        break
    end
end
figure
plot(W*1000,E*1000);
title(['Wolves vs. Elk with initial Wolf count of ', num2str(initialWolfCount*1000)]);
ylabel('elk');
xlabel('wolves');
figure
plot(t,W*1000);
title(['Wolves over time with initial wolf count of ', num2str(initialWolfCount*1000)]);
ylabel('wolves');
xlabel('time');

figure
plot(t,E*1000);
title(['Elk over time with initial wolf count of ', num2str(initialWolfCount*1000)]);
ylabel('elk');
xlabel('time');

figure
plot(t,C*1000);
title(['Coyotes over time with initial wolf count of ', num2str(initialWolfCount*1000)]);
ylabel('coyotes');
```

```
xlabel('time');

figure
hold on

%Plots using previously defined random numbers with the initial
%introduction wolf count.
for i=1:25
  G0=.8+(rand-.5)*.16;
  W0=initialWolfCount;
  C0=2.017+(rand-.5)*.404;
  E0=31.217+(rand-.5)*3.122;
  plot([C0]*1000,[E0]*1000,'.r')
  xlabel('coyotes')
  ylabel('elk')
  title(['coyotes vs. elk with initial wolf count of ', num2str(initialWolfCount*1000)])
  [t,x,y]=sim('jellystone_v6');
  E=x(:,1);
  C=x(:,2);
  W=x(:,3);
  plot(C*1000,E*1000);
end

%Executes the model over 100 years.
for i=1:1
  G0=.8;
  W0=initialWolfCount;
  C0=2.017;
  E0=31.217;

  set_param('jellystone_v6', 'StopTime', '100')
  [t,x,y]=sim('jellystone_v6');
    E=x(:,1);
  C=x(:,2);
  W=x(:,3);
end
figure
plot(W*1000,E*1000);
title(['Wolves vs. Elk with initial Wolf count of ', num2str(initialWolfCount*1000), ' over a century']);
ylabel('elk');
xlabel('wolves');
figure
plot(t,W*1000);
```

```
title(['Wolves over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a century']);
ylabel('wolves');
xlabel('time');

figure
plot(t,E*1000);
title(['Elk over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a century']);
ylabel('elk');
xlabel('time');

figure
plot(t,C*1000);
title(['Coyotes over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a century']);
ylabel('coyotes');
xlabel('time');

%Executes the model over 1,000 years.  The coyotes go extinct :(
% Perhaps we should try to demonstrate that coyotes will go extinct anyway?
% or maybe they won't.
for i=1:1
  G0=.8;
  W0=initialWolfCount;
  C0=2.017;
  E0=31.217;
  set_param('jellystone_v6', 'StopTime', '1000')
  [t,x,y]=sim('jellystone_v6');
  E=x(:,1);
  C=x(:,2);
  W=x(:,3);
  for j=1:size(C)
    if(C(j) < .001)
      disp(['After adding wolves, coytoes go extinct in year ', num2str(2016+j)]);
      break;
    end
  end
end
figure
plot(W*1000,E*1000);
title(['Wolves vs. Elk with initial Wolf count of ', num2str(initialWolfCount*1000), ' over a millenium']);
ylabel('elk');
xlabel('wolves');
figure
plot(t,W*1000);
```

```
title(['Wolves over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a millenium']);
ylabel('wolves');
xlabel('time');

figure
plot(t,E*1000);
title(['Elk over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a millenium']);
ylabel('elk');
xlabel('time');

figure
plot(t,C*1000);
title(['Coyotes over time with initial wolf count of ', num2str(initialWolfCount*1000), ' over a millenium']);
ylabel('coyotes');
xlabel('time');

figure;
hold on;

%Shows whether or not coyotes go extinct if we don't introduce any wolves.
for i=1:25
  G0=.8+(rand-.5)*.16;
  W0=0;
  C0=2.017+(rand-.5)*.404;
  E0=31.217+(rand-.5)*6.244;
  xlabel('time')
  ylabel('coyotes')
  title('coyotes vs. time (no wolves) over a millenium')
  [t,x,y]=sim('jellystone_v6');
  C=x(:,2);
  plot(t,C*1000);
  for j=1:size(C)
    if(C(j) < .001)
      averageExtinctionYear(i) = j+2016;
      break;
    end
  end
end
disp(['Without wolves, coyotes will go extinct around the year ', num2str(mean(averageExtinctionYear))]);
%Do we buy anything by introducing less wolves?
for i=1:1
  G0=.8;
  W0=.001;
```

```
    C0=2.017;
    E0=31.217;
    set_param('jellystone_v6', 'StopTime', '1000')
    [t,x,y]=sim('jellystone_v6');
    E=x(:,1);
    C=x(:,2);
    W=x(:,3);
    for j=1:size(C)
        if(C(j) < .001)
            disp(['After adding only 1 wolf, coytoes go extinct in year ', num2str(2016+j)]);
            break;
        end
    end
end
```