2.1

由题意得 $A \in \mathcal{R}^{m\times n}$，因此 A 可表示为 $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_m \end{pmatrix}$，其中 $\alpha_1, \alpha_2, \ldots, \alpha_m \in \mathcal{R}^n$.
由 $r(A) = m$ 知 $\alpha_1, \alpha_2, \ldots, \alpha_m$ 线性无关.
假设 $m > n$，即 $m \geq n+1$，则由 $n+1$ 个 n 维向量必线性相关可知 $\alpha_1, \alpha_2, \ldots, \alpha_m$ 线性相关.
这与 $\alpha_1, \alpha_2, \ldots, \alpha_m$ 线性无关相矛盾. 因此 $m \leq n$

2.6

令 $A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 1 & -2 & 0 & -1 \end{pmatrix}$, 由于 $A \to \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & -3 & -2 & -2 \end{pmatrix}$，故 $r(A) = 2$.
$\widetilde{A} = [A; b] = \left(\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 1 & -2 & 0 & 1 & -2 \end{array}\right)$，由于 $\widetilde{A} \to \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & -3 & -2 & -2 & -3 \end{pmatrix}$，故 $r(A, b) = 2$
因此，这一线性方程组有解.
令 $x_3 = d_3, x_4 = d_4$,$B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$，则有 $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = B^{-1}[\begin{pmatrix} 1 \\ -2 \end{pmatrix} - d_3 \begin{pmatrix} 2 \\ 0 \end{pmatrix} - d_4 \begin{pmatrix} 1 \\ -1 \end{pmatrix}] =$
$\begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 - 2d_3 - d_4 \\ d_4 - 2 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3}d_3 - \frac{1}{3}d_4 \\ 1 - \frac{2}{3}d_3 - \frac{2}{3}d_4 \end{pmatrix}$
因此，方程组的通解为 $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -\frac{4}{3}d_3 - \frac{1}{3}d_4 \\ 1 - \frac{2}{3}d_3 - \frac{2}{3}d_4 \\ d_3 \\ d_4 \end{pmatrix}$

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