1.5.2.当$P(A)=0$时, $A=\varnothing$, 对于$\forall$事件B, $AB=\varnothing$, 故$P(AB)=0=P(A)P(B)$

当$P(A)=1$时, $A=\Omega$, 对于$\forall$事件B, $AB=B$, 故$P(AB)=P(B)=P(A)P(B)$

因此, 若$P(a)=0$或1, A与任一事件都独立

1.5.4.(1)$P(A_1)=P(A_2)=\frac{3}{10}$, $P(A_1A_2)=\frac{9}{100}$, $P(A_1)P(A_2)=P(A_1A_2)$, 因此A_1与A_2独立

(2)$P(A_1)=\frac{3}{10}$

$P(A_2)=P(A_2|A_1)P(A_1)+P(A_2|\bar{A_1})P(\bar{A_1})=\frac{3}{10}$

$P(A_1A_2)=P(A_2|A_1)P(A_1)=\frac{1}{15}\neq P(A_1)P(A_2)$

因此, A_1与A_2不独立

1.5.7.(1)设事件甲、乙、丙不及格分别为A_1、A_2、A_3，设事件恰有两位同学不及格为B

$P(B)=P(\bar{A_1}A_2A_3)+P(A_1\bar{A_2}A_3)+P(A_1A_2\bar{A_3})=0.1888$

(2)$P(A_2|B)=\frac{P(A_2B)}{P(B)}=\frac{P(\bar{A}_1A_2A_3)+P(A_1A_2\bar{A}_3)}{P(B)}=\frac{33}{47}\approx 0.702$

1.5.8.$P($十次能取到黑球$)=1-P($十次中全为白球$)=1-0.7^{10}$

$P($三次取到黑球$)=C_{10}^3 0.3^3 0.7^7$

2.1.1.$P(X<1)=F(1-0)$, $P(|X-1|\neq 2)=P(-1\leq X\leq 3)=F(3)-F(-1-0)$

$P(X^2>3)=F(X>\sqrt{3}$或$X<-\sqrt{3})=1-F(\sqrt{3})+F(-\sqrt{3}-0)$

$P(\sqrt{1+X}\geq 2)=P(X\geq 3)=1-F(3-0)$

2.1.2.(1)$P(X<3)=\frac{11}{12}$

(2)$P(1\leq X<3)=\frac{11}{12}-\frac{1}{2}=\frac{5}{12}$

(3)$P(X>\frac{1}{2})=1-\frac{1}{4}=\frac{3}{4}$

(4)$P(X=3)=P(X\leq 3)-P(X<3)=\frac{1}{12}$

2.1.3.(1)$F_{X^+}(x)=\begin{cases}0,\ x<0\\ F_X(x),\ x\geq 0\end{cases}$

(2)$F_{X^-}(x)=\begin{cases}0,\ x<0\\ 1-F_X(-x-0),\ x\geq 0\end{cases}$

(3)$F_{|X|}(x)=\begin{cases}0,\ x<0\\ F_X(x)-F_X(-x-0),\ x\geq 0\end{cases}$

(4)$F_{aX+b}(x)=\begin{cases}0,\ a=0,x<b\\ 1,\ a=0,x\geq b\\ 1-F_X(\frac{x-b}{a}-0),\ a<0\\ F_X(\frac{x-b}{a}),\ a>0\end{cases}$

2.1.4.$F(x)=\begin{cases}0,\ x<0\\ \frac{1}{2},\ 0\leq x<1\\ 1,\ x\geq 1\end{cases}$

2.2.2.由题意得, $\sum_{k=0}^{\infty}\frac{c}{2^k}=1$, 由等比数列求和公式得 等号左边 $=\lim_{n\to\infty}c\frac{1-(\frac{1}{2})^n}{1-\frac{1}{2}}=2c$，故$c=\frac{1}{2}$

2.2.3.$F(x)=\begin{cases}0,\ x<a\\ \frac{k-a+1}{b-a+1},\ k=a,a+1,\ldots,b-1\\ 1,\ x\geq b\end{cases}$

2.2.6.$F(x)=\begin{cases}0,\ x\in(-\infty,1)\\ 0.4,\ x\in[1,2)\\ 0.7,\ x\in[2,3)\\ 0.9,\ x\in[3,4)\\ 1,\ x\in[4,+\infty)\end{cases}$

2.2.7.

X	-1	0	1	3
P	0.2	0.4	0.3	0.1

2.3.1.$P(X<6)=\sum_{i=0}^{5}P(X=i)=\sum_{i=0}^{5}C_{15}^{i}0.5^{i}0.5^{(15-i)}=\sum_{i=0}^{5}C_{15}^{i}0.5^{15}$

2.3.2.$P(X=0)=1-P(X>0)=\frac{1}{16}$, 又$P(X=0)=(1-p)^2, 0\leq p\leq 1$

解得$p=\frac{3}{4}$

则$P(Y>0)=1-P(Y=0)=1-(1-p)^3=\frac{63}{64}$

2.3.4.因为$X\sim Ge(P)$, 故$P(X>n)=\sum_{k=n+1}^{\infty}p(1-p)^{k-1}=\lim_{k\to\infty}p(1-p)^n\frac{1-(1-p)^k}{1-(1-p)}=(1-p)^n$

同理可得$P(X>m)=(1-p)^m$, $P(X>m+n\text{且}X>m)=P(X>m+n)=(1-p)^{(m+n)}$

由条件概率公式得$P(X>m+n|X>m)=\frac{P(X>m+n\text{且}X>m)}{P(X>m)}=\frac{(1-p)^{(m+n)}}{(1-p)^m}=(1-p)^n$

所以$P(X>m+n|X>m)=P(X>n)$

2.3.5.$P(X<\frac{2020}{2021})=P(X=0)=e^{-\lambda}$