

仅供参考!仅供参考!仅供参考!

习题1.1

1. $A \cup B = \{0, 1, 2, 3, 5, 6, 7, 8, 9\}$, $AB = \emptyset$,
 $\bar{A} = \{4, 5, 6, 7, 8, 9\}$, $\bar{B} = \{0, 1, 2, 3, 4\}$.
2. $\bar{A} = \{x \in \Omega : x < 1 \text{ 或 } x > 5\}$, $A \cup B = [1, 7)$, $B\bar{C} = B = (3, 7)$,
 $\bar{A} \cap \bar{B} \cap \bar{C} = [0, 1) \cup [7, \infty)$, $(A \cup B)C = \emptyset$.
3. (1) \bar{A} = “掷三枚硬币, 至少有一个反面” ;
 (2) \bar{B} = “抽检一批产品, 至多有两个次品” ;
 (3) \bar{C} = “射击三次, 至少命中两次” .

4-7. 略

习题1.2

1. $a + b - c$
2. 0.6
3. 略
4. 当 $A \subset B$ 时, $P(AB)$ 取到最大值 0.4.
 当 $A \cup B = \Omega$ 时, $P(AB)$ 取到最小值 0.1.

5-8. 略

9. 1

习题1.3

1. $\frac{C_{15}^3 \cdot C_{30}^7}{C_{45}^{10}} = 0.29$
2. $\frac{13^4}{C_{52}^4} = 0.1055$ 或 $\frac{13^4 \times 4! \times 48!}{52!} = 0.1055$.
3. $\frac{4 \times C_{13}^4}{C_{52}^4} = 0.01056$ 或 $\frac{4 \times P_{13}^4 \times 48!}{52!} = 0.01056$.
4. 当 $k = 0, 1, 2$ 时, 所求概率依次为 0.6935, 0.2972 和 0.0093.
5. $\frac{C_{20}^2 P_{365}^{19}}{365^{20}} = 0.323$.
6. $\frac{C_{13}^6 C_{13}^4 C_{13}^2 C_{13}^1}{C_{52}^{13}} = 0.00196$ 或 $\frac{C_{13}^6 P_{13}^6 C_7^4 P_{13}^4 C_3^2 P_{13}^2 C_1^1 P_{13}^1}{P_{52}^{13}} = 0.00196$.
7. $\frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{6^6} = \frac{5}{324} = 0.015$.

$$8. \frac{C_7^{26!}}{6^7} = \frac{35}{648} = 0.054.$$

习题1.4

$$1. \frac{2}{3}$$

$$2. \frac{1}{3}$$

$$3. \frac{7}{12}$$

$$4. \frac{2}{3}$$

$$5. \frac{r}{r+w} \cdot \frac{w}{r+w-1}$$

$$6. 0.146$$

$$7. \frac{13}{35}$$

$$8. 4$$

$$9. \frac{1}{N}$$

习题1.5

1-2. 略.

3. (1)独立. (2)不独立.

$$4. 0.976$$

$$5. 1 - (1 - 3/10)^{10}, C_{10}^3(3/10)^3(1 - 3/10)^7$$

6. 略.

$$7. 1 - (1 - 3\%)(1 - 5\%)$$

$$8. (1) \frac{47}{250}, (2) \frac{33}{47}$$

习题2.1

$$1. P(X < 1) = F(1 - 0), P(|X - 1| \leq 2) = F(3) - F(-1 - 0), P(X^2 > 3) = 1 - F(\sqrt{3}) + F(-\sqrt{3} - 0), \\ P(\sqrt{1+X} \geq 2) = P(X \geq 3) = 1 - F(3 - 0).$$

$$2. (1) P(X < 3) = F(3 - 0) = 11/12$$

$$(2) P(1 \leq X < 3) = F(3 - 0) - F(1 - 0) = 11/12 - 1/2 = 5/12$$

$$(3) P(X > 1/2) = 1 - P(X \leq 1/2) = 1 - F(1/2) = 1 - 1/4 = 3/4$$

$$(4) P(X = 3) = F(3) - F(3 - 0) = 1 - 11/12 = 1/12.$$

3.

$$F_{X^+}(x) = \begin{cases} F_X(x), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{X^-}(x) = \begin{cases} 1 - F_X(-x - 0), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{|X|}(x) = \begin{cases} F_X(x) - F_X(-x - 0), & x \geq 0; \\ 0, & x < 0. \end{cases}$$

$$F_{aX+b}(x) = \begin{cases} F_X((x-b)/a), & a > 0; \\ 1 - F_X((x-b)/a - 0), & a < 0; \\ 1, & a = 0, x \geq b \\ 0, & a = 0, x < b. \end{cases}$$

4.

$$F(x) = \begin{cases} 1, & x \geq 1; \\ 1/2, & 0 \leq x < 1; \\ 0, & x < 0. \end{cases}$$

5. 是.

6-8. 略.

习题2.2

$$1. \begin{array}{c|ccc} X & 3 & 4 & 5 \\ \hline P & 1/10 & 3/10 & 3/5 \end{array}$$

2. $1/2$

3.

$$F(x) = \begin{cases} 0, & x < a, \\ \frac{k-a+1}{b-a+1}, & k \leq x < k+1, \quad k = a, \dots, b-1. \\ 1, & x \geq b. \end{cases}$$

4. 1

5. $1/3$

6.

$$F(x) = \begin{cases} 0, & x < 1, \\ 2/5, & 1 \leq x < 2, \\ 7/10, & 2 \leq x < 3, \\ 9/10, & 3 \leq x < 4, \\ 1, & x \geq 4. \end{cases}$$

$$7. \begin{array}{c|cccc} X & -1 & 0 & 1 & 3 \\ \hline P & 0.2 & 0.4 & 0.3 & 0.1 \end{array}$$

8. 对.

习题2.3

$$1. P(X < 6) = \sum_{k=0}^5 p_k = \sum_{k=0}^5 C_{15}^k 0.5^k (1-0.5)^{15-k} =$$

$$2. 63/64.$$

$$3. 1/2.$$

4. 略.

$$5. e^{-\lambda}.$$

习题2.4

$$1. (1)1/2, (2)1, (3)1/\pi.$$

$$2. 1/2, (\sqrt{2}+1)e^{-\sqrt{2}}.$$

3.

$$F(x) = \begin{cases} 1 - 1/2e^{-x}, & x \geq 0, \\ 1/2e^x, & x < 0. \end{cases}$$

4.

$$F(x) = \begin{cases} 0, & x \leq 0, \\ x^2/2, & 0 < x \leq 1, \\ -x^2/2 + 2x - 1, & 1 < x \leq 2, \\ 1, & x > 2. \end{cases}$$

$$P(1/2 < X < 3/2) = 3/4.$$

$$5. P(X \leq 3) = 1 - e^{-9}, P(X > 1) = e^{-3}.$$

习题2.5

1. 0.8414
2. 0.2857, 0.7745, 10.
3. -0.817.
4. 2.96
5. $\ln 2$
6. 略.

习题3.1

1. 略.
2. (1) $1 - F(a, \infty) - F(\infty, b) + F(a, b)$
 (2) $F(c, \infty) - F(a - 0, \infty) - F(c, d - 0) + F(a - 0, d - 0)$
 (3) $F(a, \infty) - F(a, b - 0)$
 (4) $F(a, \infty) - F(a - 0, \infty) - F(a, b) + F(a - 0, b)$

3. (1) $1/16$, (2) $9/16$, (3) $15/16$.

4.

X \ Y	Y		
	1	2	3
1	4/9	2/9	0
2	1/9	1/9	1/9

5. (1) $1/2$, (2) 0, (3) $\frac{3}{4}$.
6. $\frac{255}{256}$.
7. (1) $5/4$, (2) $\frac{13}{16}$, (3) $1 - \frac{\sqrt{2}}{2}$.

习题3.2

1. $c = 1/40$.
 X 的可能取值有-2, -1, 0, 1, 2, 且

$$P(X = i) = \sum_{j=-2}^2 p_{ij} = \sum_{j=-2}^2 c|i + j| = (6 + i^2)/40, \quad i = -2, -1, 0, 1, 2.$$

2. X 与 Y 的边缘分布列分别为

X	0	1
P	7/8	1/8

Y	0	1	2
P	9/16	3/16	1/4

3. X 与 Y 的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 1, & 0 \leq x \leq 1, \\ 0, & x < 0 \text{ 或 } x > 1. \end{cases} \quad p_Y(y) = \begin{cases} 1/2, & 0 \leq y \leq 2, \\ 0, & y < 0 \text{ 或 } y > 2. \end{cases}$$

4. X 与 Y 的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 3x^2, & 0 < x < 1, \\ 0, & x \leq 0 \text{ 或 } x \geq 1. \end{cases} \quad p_Y(y) = \begin{cases} 3/2(1-y^2), & 0 < y < 1, \\ 0, & y \leq 0 \text{ 或 } y \geq 1. \end{cases}$$

5. X 的边际概率密度函数为 $p_X(x) = \frac{1}{2}e^{-|x|}$.

6. Y 的边际概率密度函数为 $p_Y(y) = \frac{1}{\sqrt{\pi}}e^{-y^2}$. 所求概率 $P(Y > \sqrt{2}) = 1 - \Phi(2)$

7-8. 略

习题3.3

1. $a = 1/18, b = 1/6, c = 2/9$.

2. 不独立.

3. X 与 Y 的边际概率密度函数分别为

$$p_X(x) = \begin{cases} 4x^3, & 0 < x < 1, \\ 0, & x \leq 0 \text{ 或 } x \geq 1. \end{cases} \quad p_Y(y) = \begin{cases} 12y^2(1-y), & 0 < y < 1, \\ 0, & y \leq 0 \text{ 或 } y \geq 1. \end{cases}$$

X 与 Y 不独立.

4. 独立.

5. 不独立.

6. X 与 Y 的边际概率密度函数分别为

$$p_X(x) = \begin{cases} \frac{1}{2}, & |x| < 1, \\ 0, & |x| \geq 1. \end{cases} \quad p_Y(y) = \begin{cases} \frac{1}{2}, & |y| < 1, \\ 0, & |y| \geq 1. \end{cases}$$

X 与 Y 不独立.

7. (1) (X, Y) 的联合概率密度函数为

$$p(x, y) = \begin{cases} e^{-y}, & 0 < x < 1, y > 0, \\ 0, & \text{其他.} \end{cases}$$

(2) $e^{-1}, (3) 1 - e^{-1}$.

习题3.4

$$1. Y \text{ 的概率密度函数为 } p_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln y - \mu)^2}{2\sigma^2}\right) y^{-1}, & y > 0, \\ 0, & y \leq 0. \end{cases}$$

2. 略.

$$3. (U, V) \sim N\left(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2; \mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2; \frac{\sigma_1^2 - \sigma_2^2}{\sigma_1^2 + \sigma_2^2}\right)$$

$$4. \text{ 当 } \lambda_1 = \lambda_2 = \lambda \text{ 时, } Z = X + Y \text{ 的概率密度函数为 } p_Z(z) = \begin{cases} \lambda^2 z e^{-\lambda z}, & z > 0, \\ 0, & z \leq 0. \end{cases}$$

$$\text{当 } \lambda_1 \neq \lambda_2 \text{ 时, } Z = X + Y \text{ 的概率密度函数为 } p_Z(z) = \begin{cases} \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} (e^{-\lambda_2 z} - e^{-\lambda_1 z}), & z > 0, \\ 0, & z \leq 0. \end{cases}$$

$$T = \max(X, Y) \text{ 的分布函数为 } F_T(t) = \begin{cases} (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t}), & t > 0, \\ 0, & t \leq 0. \end{cases}$$

$$S = \min(X, Y) \text{ 的分布函数为 } F_S(s) = \begin{cases} (1 - e^{-(\lambda_1 + \lambda_2)s}), & s > 0, \\ 0, & s \leq 0. \end{cases} \text{ 即 } S = \min(X, Y) \text{ 服从指数分布 } Exp(\lambda_1 + \lambda_2).$$

$$5. (1) T = X - Y \text{ 的概率密度函数 } p_T(t) = \begin{cases} 2(t+1), & -1 < t < 0, \\ 0, & \text{其他}. \end{cases}$$

$$(2) \text{ 所求概率 } P\left(Y - X \leq \frac{1}{2}\right) = 3/4.$$

6. $Y \sim N(0, 1)$.

7. 负二项分布 $Nb(r, p)$.

8. 略.

9. $1/2$.

10. $Y + 1$ 服从几何分布 $Ge(1 - e^{-\lambda})$.

习题4.1

1. 略

$$2. \lambda^3 + 3\lambda^2 + \lambda.$$

$$3. \frac{1}{p}.$$

$$4. \frac{1}{4}, e - 1, \frac{1}{12}.$$

$$5. \frac{\Gamma(n + \alpha)}{\lambda^n \Gamma(\alpha)}.$$

6. 4.

7. $\frac{2}{3}, \frac{1}{3}$.

8. $\frac{e^{-\lambda}}{1 - e^{-\lambda}}$.

习题4.2

1. $\frac{2-p}{p^2} - \left(\frac{1}{p}\right)^2$.

2. $\frac{\alpha}{\lambda^2}$.

3. $nm, 2mn(m+n-4)$.

4. $9/113$.

5. $e^{1/2}, e^2 - e$.

6. $8/9$.

7. $\sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2, \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$.

8. $1, 2, 3, 1, 2, 5$

9. 略.

10. $0, 2$.

习题4.3

1. 略.

2. $\frac{1}{36}$

3. 0.

4. 0.

5. 0. X 与 Y 不独立.

6. $\frac{1}{9}, \frac{1}{3}$.

7. $\frac{\sigma_1^2 - \sigma_2^2}{\sqrt{(\sigma_1^2 + \sigma_2^2)^2 - 4\rho^2\sigma_1^2\sigma_2^2}}$.

8. $2, \frac{\sqrt{2}}{2}$.

习题4.4

$$1. \mu_k = \frac{1}{k+1}, \quad \nu_k = \begin{cases} 0, & k \text{ 为奇数,} \\ \frac{1}{(k+1)2^k}, & k \text{ 为偶数.} \end{cases}$$

$$2. \beta_s = 0, \quad \beta_k = -\frac{6}{5}, \quad C_v = \frac{\sqrt{3}}{3}.$$

$$3. \alpha$$

$$4. 13.92.$$

$$5. \chi_{0.05}^2(8) = 2.7326, \quad \chi_{0.95}^2(10) = 18.3070, \quad \chi_{0.975}^2(12) = 23.3367.$$

习题4.5

$$1. \text{ 略}$$

$$2. 0.9966$$

$$3. 0.6443.$$

$$4. 0.0287.$$

$$5. \text{ 由}$$

$$\frac{100 - 0.5 - n \cdot 0.99}{\sqrt{n \cdot 0.99 \cdot 0.01}} \leq 0.5199,$$

解得 $n \geq ?$.

$$6-7. \text{ 略.}$$

习题5.1

$$1-2. \text{ 略}$$

$$3.$$

$$P(X_1 = k_1, X_2 = k_2, \dots, X_m = k_m) = \prod_{i=1}^m \left[\binom{n}{k_i} p^{k_i} (1-p)^{n-k_i} \right] = \prod_{i=1}^m \binom{n}{k_i} p^{\sum_{i=1}^m k_i} (1-p)^{nm - \sum_{i=1}^m k_i}$$

$$k_i = 0, 1, \dots, n, \quad i = 1, 2, \dots, m.$$

$$4.$$

$$P(X_1 = k_1, X_2 = k_2, \dots, X_n = k_n) = \frac{\lambda^{\sum_{i=1}^n k_i}}{\prod_{i=1}^n k_i!} e^{-n\lambda}, \quad k_i \in \mathbb{Z}^+, \quad i = 1, 2, \dots, n.$$

$$5.$$

$$P(X_1 = k_1, X_2 = k_2, \dots, X_n = k_n) = \begin{cases} \frac{1}{\theta^n}, & k_i \in (0, \theta), \quad i = 1, 2, \dots, n \\ 0. & \text{其他} \end{cases}$$

$$6.$$

$$P(X_1 = k_1, X_2 = k_2, \dots, X_n = k_n) = \begin{cases} n - e^{-\lambda \sum_{i=1}^n k_i}, & k_i \geq 0, \quad i = 1, 2, \dots, n \\ 0. & \text{其他} \end{cases}$$

7.

$$F(x) = \begin{cases} 0 & x \in (-\infty, -0.5) \\ 0.1 & x \in [-0.5, -0.2) \\ 0.2 & x \in [-0.2, 0.2) \\ 0.4 & x \in [0.2, 0.5) \\ 0.7 & x \in [0.5, 0.7) \\ 0.9 & x \in [0.7, 1.5) \\ 1 & x \in [1.5, \infty) \end{cases}$$

习题5.2

1. T_1, T_2, T_4 是统计量, T_3 不是统计量.

2. (1) $\bar{X}=4.6, S^2=5.3$.

(2) $\bar{X}=104.6, S^2=5.3$.

3. (1)

$$P(X_1 = k_1, X_2 = k_2, \dots, X_n = k_n) = \prod_{i=1}^n p^{k_i} (1-p)^{1-k_i} = p^{\sum_{i=1}^n k_i} (1-p)^{n-\sum_{i=1}^n k_i}$$

$$k_i = 0, 1; i = 1, 2, \dots, n.$$

(2)

$$P\left(\sum_{k=1}^n X_k = m\right) = \binom{n}{m} p^m (1-p)^{n-m}, \quad m = 0, 1, 2, \dots, n.$$

$$(3) p, \frac{p(1-p)}{n}, p(1-p)$$

$$4. \lambda, \frac{\lambda}{n}, \lambda$$

5. 略

$$6. \frac{n\bar{X} + X_{n+1}}{n+1}, \quad \frac{n-1}{n} S^2 + \frac{1}{n+1} (X_{n+1} - \bar{X}_n)^2$$

习题5.3

$$1. \frac{1}{20}, \quad \frac{1}{100}$$

$$2. 0.1$$

$$3. 0.705, \text{ 注意 } \frac{(X+Y)^2}{(X-Y)^2} \sim F(1, 1)$$

$$4. \sqrt{\frac{3}{2}}$$

$$5. 1.3304, -1.3304$$

6. 0.5

7-8. 略

9. $t(n-1)$

10. 0, $\frac{2}{n(n+1)}$

习题6.1

1. $\frac{1}{\bar{x}}$

2. $\left(\frac{\bar{X}}{1-\bar{X}}\right)^2$

3. $\frac{\bar{X}}{m}$

4. $1 - \frac{s_n^2}{\bar{X}}, \left[\frac{\bar{X}^2}{\bar{X} - s_n^2} \right]$, 其中[]表示取整.

5. $3\bar{X}$

6. $2\bar{X}$

7. 74.002, $6 * 10^{-6}$

习题6.2

1. $\frac{1}{\bar{X}}$
2. $\left(\frac{1}{n} \sum_{k=1}^n \ln X_k\right)^{-2}$
3. $\frac{\bar{X}}{m}$
4. $\frac{\sum_{i=1}^2 X_i^2}{2n}$
5. $\frac{-1 + \sqrt{1 + \frac{4 \sum_{i=1}^n X_i^2}{n}}}{2}$
6. $X_{(n)}$
7. 74.002, $6 * 10^{-6}$
8. $\bar{X}, \frac{1}{\bar{X}}$

习题6.3

1. 略
2. 不是
3. $\bar{X}^2 - \frac{\bar{X}}{n}$
4. 当 $n = 1, 2, \dots, 8$ 时, $\hat{\theta}_1$ 较有效.
当 $n \geq 9$ 时, $\hat{\theta}_3$ 与 $\hat{\theta}_2$ 较有效, 且有效性相同, $\hat{\theta}_1$ 有效性较差.
5. $k_1 = \frac{1}{3}, k_2 = \frac{2}{3}$.
6. $\hat{\theta}_2$

习题6.5

1. [2.08, 9.92]
2. [2.6895, 2.7205]
3. $n \geq \left(\frac{2}{L}\right)^2 \sigma^2 u_{1-\alpha/2}^2$
4. [6562.618, 6877.382]
5. [0.0566, 2.4523]
6. [7.4300, 21.0736]

7. $[2.6762, 2.9238]$, $[0.0268, 0.1244]$

8. (1) $[-0.98, 0.98]$, (2) $[e^{-0.48}, e^{1.48}]$

习题7.1

1. 存伪(第二类错误), 拒真(第一类错误)
2. 控制犯第一类错误的概率
3. (1) 0.0037, 0.0367.
(2) 略
(3) 34
4. 0.0016

习题7.2

1. $H_0: \mu = \mu_0 = 2.64$, $H_1: \mu \neq \mu_0$
拒绝 H_0 , 即有显著影响.
2. $H_0: \mu = \mu_0 = 350$, $H_1: \mu \neq \mu_0$
不能拒绝 H_0 , 即同意该厂的说法.
3. $H_0: \mu \leq \mu_0 = 32$, $H_1: \mu > \mu_0$
拒绝 H_0 , 即今年家庭平均每月耗电量有所提高.
4. $H_0: \mu = \mu_0 = 800$, $H_1: \mu \neq \mu_0$
不能拒绝 H_0 , 即可以认为这批产品的平均重量为800克.
5. $H_0: \sigma = \sigma_0 = 0.04$, $H_1: \sigma \neq 0.04\%$
不能拒绝 H_0 , 即总体方差为 $\sigma = 0.04\%$.
6. $H_0: \sigma = \sigma_0 = 1.2$, $H_1: \sigma \neq 1.2$
拒绝原假设 H_0 , 即纱的均匀度显著变劣.