

$$4.4.1. \mu_k = EX^k = \int_0^1 x^k dx = \frac{1}{k+1}$$

$$v_k = E(X - EX)^k = \int_0^1 (x - \frac{1}{2})^k dx = \int_{-\frac{1}{2}}^{\frac{1}{2}} (x - \frac{1}{2})^k d(x - \frac{1}{2})$$

$$= \begin{cases} 0, & k \text{ 为奇数} \\ \frac{1}{(k+1)2^k}, & k \text{ 为偶数} \end{cases}$$

$$4.4.3. F(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 1, & x > 1 \end{cases}$$

显然, $x_\alpha = \alpha$

$$4.4.4. \text{查表得 } \Phi(1.96) = 0.975, \text{ 则有 } \frac{x_{0.975} - 10}{2} = 1.96, \text{ 解得 } x_{0.975} = 13.92$$

4.5.2. 设 X_k 表示第 k 次掷骰子的点数, $k = 1, 2, \dots, 100$, X_1, X_2, \dots, X_{100} 独立同分布

$\mu = 3.5, \sigma \approx 1.708$

$$P(300 \leq \sum_{k=1}^{100} X_k \leq 400) = \Phi\left(\frac{400 - 100 \times 3.5}{1.708 \times 10}\right) - \Phi\left(\frac{300 - 100 \times 3.5}{1.708 \times 10}\right) = 2\Phi(2.93) - 1 = 0.9966$$

4.5.3. 设 X 表示为正面的硬币数, 则有 $X \sim B(900, 0.5)$

$$P(X \geq 495) = 1 - \Phi\left(\frac{495 - 0.5 \times 900}{\sqrt{225}}\right) \approx 1 - \Phi(2.97) \approx 0.0015$$

4.5.4. 设 X 表示为考生答对的题目数, 则 $X \sim B(100, 0.5)$

$$P(X \geq 60) = 1 - \Phi\left(\frac{60 - 0.5 \times 100}{5}\right) \approx 1 - \Phi(1.9) \approx 0.0287$$