4.1.1.$EX^r=\int_0^\infty x^r p(x)dx$

$EX^r=\int_0^\infty rx^{r-1}P(X>x)dx=\int_0^\infty rx^{r-1}(1-P(X\le x))dx$

由分部积分公式得$EX^r=x^r(1-P(X\le x))+\int_0^\infty x^r p(x)dx$

$=\int_0^\infty x^r p(x)dx$

原命题得证

4.1.2$EX^3=\lambda E(X+1)^2=\lambda E(X^2+2X+1)$

$=\lambda(\lambda E(X+1)+E(2X)+E(1))=\lambda^3+3\lambda^2+\lambda$

4.1.4.$EX^3=\int_0^1 x^3dx=\frac{1}{4}$

$Ee^X=\int_0^1 e^x dx=e-1$

$E(X-\frac{1}{2})^2=\int_0^1(x-\frac{1}{2})^2dx=\frac{1}{12}$

4.1.8.$EY=\Sigma_{k\in N_+}k\int_k^{k+1}\lambda e^{-\lambda x}dx=(1-e^{-\lambda})\Sigma_{k\in N_+}ke^{-\lambda k}$

$=\frac{e^{-\lambda}}{1-e^{-\lambda}}$

4.2.2.$EX=\frac{\alpha}{\lambda}$

$VarX=E(X-\frac{\alpha}{\lambda})^2=EX^2-2\frac{\alpha}{\lambda}EX+\frac{\alpha}{\lambda}=\frac{\alpha}{\lambda^2}$

4.2.6.$E(X+Y)=\iint_{|x|<1,|y|<1}(x+y)(\frac{1+xy}{4})dxdy=0$

$Var(X+Y)=E(X+Y)^2=\frac{8}{9}$

4.2.8$p(y)=\int_0^y e^{-y}dx=ye^{-y},p(x)=\int_x^\infty e^{-y}dy=e^{-x}$

$EX=\int_0^\infty xp(x)dx=1,EY=\int_0^\infty yp(y)dy=2$

$E(XY)=\iint_{0<x<y}xyp(x,y)dxdy=3$

$VarX=E(X-EX)^2=1,VarY=E(Y-EY)^2=2$

$Var(XY)=E(XY-E(XY))^2=5$

4.2.9.(1)$EX=\int_0^\infty xp(x)dx$，显然对$\forall p(x)$有$p(x)\ge 0$

又$\forall x\ge 0$，所以$\forall xp(x)\ge 0$，故$EX\ge 0$

(2)$E(X-c)^2-VarX=-2cEX+c^2+E^2X=(EX-c)^2\ge 0$

因此$E(X-c)^2\ge VarX$

(3)当X在$[a,b]$均匀分布时，$VarX=\frac{(b-a)^2}{12}$

当X为只取a和b的离散型随机变量

当$p(X=a)=0.5,p(X=b)=0.5$时，$VarX$有最大值$\frac{(b-a)^2}{4}$

其他任意分布的随机变量，其方差均小于均匀分布时的方差

综上所述，$VarX\le(\frac{b-a}{2})^2$

4.3.1. $\rho = Cov(X,Y) = E(XY) - EXEY = E(XY)$

不妨设 $X > Y$

则 $E^2(X^2) + E^2(XY) \leq 2E(X^2)$

$E^2(X^2) - 2E(X^2) + 1 \leq 1 - E^2(XY)$

$E(X^2) - 1 \leq sqrt1 - \sqrt{E(XY)} + 1$

即 $Emax(X^2, Y^2) \leq 1 + \sqrt{1-\rho^2}$

4.3.2. $max(X,Y) = \frac{X+Y+|X-Y|}{2}, min(X,Y) = \frac{X+Y-|X-Y|}{2}$

$EX = EY = \frac{1}{2}$

$Cov(max(X,Y), min(X,Y)) =$

$E(max(X,Y)min(X,Y)) - Emax(X,Y)Emin(X,Y) = \frac{1}{36}$

4.3.8. $p(y) = \int_0^y e^{-y}dx = ye^{-y}, p(x) = \int_x^\infty e^{-y}dy = e^{-x}$

$Cov(X,Y) = E(XY) = EXEY = 2$

$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{VarXVarY}} = \frac{\sqrt{2}}{2}$