

1. 设  $A = "X \text{ 为奇数}"$ ,  $B = "X < 8"$

$$P(AB) = P("X = 3") + P("X = 5") + P("X = 7") = \frac{2+4+6}{6^2} = \frac{1}{3}$$

$$P(A) = P("X \in \{3, 5, 7, 9, 11\}") = P(AB) + P("X = 9") + P("X = 11") \\ = \frac{1}{3} + \frac{4+2}{6^2} = \frac{1}{2}$$

$$\text{则 } P(B|A) = \frac{P(AB)}{P(A)} = \frac{2}{3}$$

答: 在  $X$  为奇数的条件下,  $X < 8$  的概率为  $\frac{2}{3}$ .

$$3. P(\bar{A}|\bar{B}) = \frac{P(\bar{A}\bar{B})}{P(\bar{B})} = \frac{P(\overline{A+B})}{1-P(B)} = \frac{(1-P(A+B))}{\frac{2}{3}}$$

$$= \frac{3}{2}(1 - (P(A) + P(B) - P(AB))) = \frac{3}{2}(\frac{1}{3} + P(AB)) = \frac{3}{2}(\frac{1}{3} + P(A|B)P(B)) \\ = \frac{3}{2}(\frac{1}{3} + \frac{1}{6} \times \frac{1}{3}) = \frac{7}{12}.$$

$$4. P("第一只为红球, 第二只为白球") = \frac{r}{r+w} \times \frac{w}{r+w-1} = \frac{rw}{(r+w)(r+w-1)}$$

答: “第一只为红球, 第二只为白球” 概率为  $\frac{rw}{(r+w)(r+w-1)}$ .

6. 设  $A = "第三次比赛时取出的3个球都是新球"$ ,  $B_i = "第二次比赛取出了i个新球"$

$$\text{则 } P(A) = P(A|B_0)P(B_0) + P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3) \\ = \frac{C_9^3}{C_{12}^3} \times \frac{C_3^3}{C_{12}^3} + \frac{C_8^3}{C_{12}^3} \times \frac{C_3^2 C_9^1}{C_{12}^3} + \frac{C_7^3}{C_{12}^3} \times \frac{C_3^1 C_9^2}{C_{12}^3} + \frac{C_6^3}{C_{12}^3} \times \frac{C_9^3}{C_{12}^3} \\ = \frac{441}{3025} \approx 0.146$$

8.

设  $A = "从选择的罐子中任取两个球, 发现两个都是黑球"$ ,  $B = "选到4个白球、6个黑球的罐子"$ ,

$C = "选到5个白球、5个黑球的罐子"$ ,  $D = "有5个白球和3个黑球留在选出的罐子中"$

$$P(A) = P(A|B)P(B) + P(A|C)P(C) = \frac{C_6^2}{C_{10}^2} \times \frac{n}{n+1} + \frac{C_5^2}{C_{10}^2} \times \frac{1}{n+1}$$

$$P(AD) = P(AD|B)P(B) + P(AD|C)P(C) = P(AD|C)P(C) = P(A|C)P(C) = \frac{C_5^2}{C_{10}^2} \times \frac{1}{n+1}$$

$$\text{则 } \frac{1}{7} = P(D|A) = \frac{P(AD)}{P(A)} = \frac{\frac{C_5^2}{C_{10}^2} \times \frac{1}{n+1}}{\frac{C_6^2}{C_{10}^2} \times \frac{n}{n+1} + \frac{C_5^2}{C_{10}^2} \times \frac{1}{n+1}} = \frac{C_5^2 \times 1}{C_6^2 \times n + C_5^2 \times 1} = \frac{2}{3n+2}$$

解得  $n = 4$ .