

**2.4.2.**由归一化条件得  $\int_{-\infty}^{+\infty} ce^{-\sqrt{x}}dx = 1$ , 解得  $c = \frac{1}{2}$

$$F(x) = \int_{-\infty}^x p(x)dx = \int_0^x p(x)dx = 1 - (\sqrt{x} + 1)e^{-\sqrt{x}}$$

$$P(X > 2) = 1 - F(2) = (\sqrt{2} + 1)e^{-\sqrt{2}}$$

**2.4.3.**当  $x < 0$  时,  $p(x) = \frac{1}{2}e^x$ ,  $F(x) = \int_{-\infty}^x p(x)dx = \frac{1}{2}e^x$

当  $x \geq 0$  时,  $p(x) = \frac{1}{2}e^{-x}$ ,  $F(x) = \frac{1}{2} + \int_0^x p(x)dx = 1 - \frac{1}{2}e^{-x}$

$$F(x) = \begin{cases} \frac{1}{2}e^x, & x < 0 \\ 1 - \frac{1}{2}e^{-x}, & x \geq 0 \end{cases}$$

$$\mathbf{2.4.4.} F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{1}{2}x^2, & 0 < x \leq 1 \\ -\frac{1}{2}x^2 + 2x - 1, & 1 < x \leq 2 \\ 1, & x > 2 \end{cases}$$

$$P(\frac{1}{2} < X < \frac{3}{2}) = F(\frac{3}{2}) - F(\frac{1}{2}) = \frac{3}{4}$$

**2.5.1.**由题意得,  $X \sim N(1, 0.5)$

$$\text{则 } P(0 \leq X \leq 2) = \Phi(\frac{2-1}{\frac{\sqrt{2}}{2}}) - \Phi(\frac{0-1}{\frac{\sqrt{2}}{2}}) = 2\Phi(\sqrt{2}) - 1 = 0.8414$$

$$\mathbf{2.5.2.} P(6 < X \leq 9) = \Phi(\frac{9-10}{2}) - \Phi(\frac{6-10}{2}) = 0.2857$$

$$P(7 \leq X < 12) = \Phi(\frac{12-10}{2}) - \Phi(\frac{7-10}{2}) = 0.7745$$

$$c = 10$$

$$\mathbf{2.5.4.} P(-1 < X < 3) = \Phi(\frac{3-1}{\sigma}) - \Phi(\frac{-1-1}{\sigma}) = 2\Phi(\frac{2}{\sigma}) - 1 = 0.5$$

$$\text{所以 } \Phi(\frac{2}{\sigma}) = 0.75$$

查表得  $\sigma = 2.96$

$$\mathbf{2.5.5.} F(x) = \begin{cases} 1 - e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\text{又 } P(X > 1) = 2P(X > 2) \text{ 得 } e^{-\lambda} = 2e^{-2\lambda}, \text{ 解得 } \lambda = \ln 2$$

$$\mathbf{3.1.2.} (1) 1 - F(a, \infty) - F(\infty, b) + F(a, b)$$

$$(2) F(c, \infty) - F(a - 0, \infty) - F(c, d - 0) + F(a - 0, d - 0)$$

$$(3) F(a, \infty) - F(a, b - 0)$$

$$(4) F(a, \infty) - F(a - 0, \infty) - F(a, b) + F(a - 0, b)$$

$$\mathbf{3.1.3.} (1) \text{由归一化条件得, } c = \frac{1}{16}$$

$$(2) P(X = Y) = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$$

$$(3) P(X \leq Y) = 1 - P(X > Y) = 1 - \frac{1}{16} = \frac{15}{16}$$

### 3.1.4.

X/Y	1	2	3
1	$\frac{4}{9}$	$\frac{2}{9}$	0
2	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

**3.1.7.**(1)由归一化条件得  $\iint_{0 \leq y \leq 1-x^2} c(x^2 + y) = c \int_{-1}^1 dx \int_0^{1-x^2} (x^2 + y) dy = 1$

解得  $c = \frac{5}{4}$

(2)由题意得  $P(Y \leq X + 1) = \frac{5}{4} \int_0^1 dx \int_0^{1-x^2} (x^2 + y) dy + \frac{5}{4} \int_{-1}^0 dx \int_0^{x+1} (x^2 + y) dy = \frac{13}{16}$

(3)  $\begin{cases} x^2 + y^2 = 1 \\ y = x^2 \end{cases}$  解得  $x_1 = \sqrt{\frac{\sqrt{5}-1}{2}}, x_2 = -\sqrt{\frac{\sqrt{5}-1}{2}}$

$P(Y \leq X^2) = \frac{5}{4} (\int_{-1}^{x_2} dx \int_0^{1-x^2} (x^2 + y) dy + \int_{x_1}^1 dx \int_0^{1-x^2} (x^2 + y) dy + \int_{x_2}^{x_1} dx \int_0^{x^2} (x^2 + y) dy) = \frac{2-\sqrt{2}}{2}$