C4: MATRICES

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M2 Chap4

LEARNING OUTCOMES 1

- Define the terms: matrix, order, square matrix, identity matrix and inverse matrix
- Perform scalar multiplication of a matrix
- Identify when two martices are equal and when two martices may be added or multiplied
- Perform addition and multiplication of compatible matrices
- Calculate the determinant and inverse of a 2X2 matrix
- Define the terms: singular matrix and non-singular matrix
- Solve equations for an unknown matrix using the matrix inverse

DEFINITION

A matrix is a rectangular array of elements arranged in rows and columns

eg.

$$\begin{array}{cc} \begin{bmatrix} 20 & 3 \\ 25 & 12 \\ 30 & 20 \end{bmatrix} & \begin{array}{l} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{array} \\ \begin{array}{cc} \text{column 1} & \text{column 2} \end{array} & \end{array}$$

SCALAR MULTIPLICATION

To multiply a matrix A by a scalar (constant) k, multiply each element of A by k

eg.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then: $kA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

EXAMPLE 6.1

Given $k=2$ and $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \\ 2 & 0 \end{bmatrix}$

$$kA = \begin{bmatrix} 4 & 2 \\ 6 & -2 \\ 4 & 0 \end{bmatrix}$$

ORDER

A matrix which has m rows and n columns is said to have order $\underset{\text{rows}}{m} \times \underset{\text{columns}}{n}$

* If $m=n$ then the matrix is a square matrix

EXAMPLE 6.2

Give the orders of the following matrices

$$\begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 2 & 3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Order: 3×2 $\qquad$ 2×3 $\qquad$ 2×2 (square matrix)

GENERAL FORM OF A MATRIX

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots & a_{mn} \end{bmatrix}$$

element ~ small letter

$a_{\underset{\text{row \#}}{m}\,\underset{\text{column \#}}{n}}$

EXAMPLE 6.3

Given the matrix $A = \begin{bmatrix} 3 & 1 & -4 \\ 0 & 2 & 6 \end{bmatrix}$

write down the elements a_{21}, a_{12} and a_{23}

$a_{21} = 0$

$a_{12} = 1$

$a_{23} = 6$