12 INDEFINITE INTEGRALS AND FURTHER APPLICATIONS OF INTEGRATION

EXERCISES FOR SECTION 12.1

Find the following integrals

1 a) $\int_0^3 x\,dx = \left[\frac{1}{2}x^2\right]_0^3$

$= \frac{1}{2}(3)^2 - \frac{1}{2}(0)^2$

$= \frac{9}{2}$

b) $\int_0^3 1\,dx = \left[x\right]_0^3$

$= 3$

c) $\int_0^3 0\,dx = 0$

d) $\int_0^3 -3x^2\,dx = \left[-x^3\right]_0^3$

$= -27$

e) $\int_0^3 2x^3\,dx = \left[\frac{1}{2}x^4\right]_0^3$

$= \frac{81}{2}$

2 a) $\int \frac{1}{x}\,dx = \int \frac{1}{(1)x+(0)}\,dx$ <use rule 7>

$= \ln|x| + C$

b) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx$

$= -\frac{1}{x} + C$

c) $\int \frac{1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}\,dx$

$= 2\sqrt{x} + C$

3 a) $\int (x+x^2)\,dx = \frac{1}{2}x^2 + \frac{1}{3}x^3 + C$

b) $\int (3x+\sqrt{x})\,dx = \frac{3}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}} + C$

4 a) $\int \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right)dx = \ln|x| - 2\sqrt{x} + C$

b) $\int (7x^4 + \sin(2x))\,dx = \frac{7}{5}x^5 - \frac{1}{2}\cos(2x) + C$

5 a) $\int \frac{e^{7x}-e^x}{e^{2x}}\,dx = \int \frac{\cancel{e^{7x}}^{\,5}}{\cancel{e^{2x}}} - \frac{\cancel{e^{x}}}{e^{\cancel{2}x}}\,dx$

$= \int e^{5x} - e^{-x}\,dx$

$= \frac{1}{5}e^{5x} + e^{-x} + C$

b) $\int \frac{x^2+5x}{x^2}\,dx = \int \frac{x^2}{x^2} + \frac{5\cancel{x}}{x^{\cancel{2}}}\,dx$

$= \int 1 + \frac{5}{x}\,dx$

$= x + 5\ln|x| + C$

6 a) $\int \sqrt{3x+1}\,dx = \int (3x+1)^{\frac{1}{2}}\,dx$

$= \frac{1}{3\left(\frac{3}{2}\right)}(3x+1)^{\frac{3}{2}}$

$= \frac{2}{9}(3x+1)^{\frac{3}{2}}$

b) $\int \frac{1}{7x+2}\,dx = \frac{1}{7}\ln|7x+2| + C$

7 a) $\int \frac{1}{9+x^2}\,dx = \int \frac{1}{3^2+x^2}\,dx$

$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$

EXERCISES FOR SECTION 12.1 (CONTINUED)

⑦ b) $\int \frac{7}{\sqrt{4-x^2}}\,dx = \int \frac{7}{\sqrt{2^2-x^2}}\,dx$

$= 7\sin^{-1}\left(\frac{2}{x}\right) + C$ //

EXERCISES FOR SECTION 12.2

① Find the average value of the following $f^{\underline{n}}$s over the indicated intervals

a) $f(x) = 3x+7$ over the interval $[0, 9]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$
$$= \frac{1}{9-0}\int_0^9 3x+7\,dx$$
$$= \frac{1}{9}\left[\frac{3}{2}x^2+7x\right]_0^9$$
$$= \frac{41}{2}$$ //

b) $f(x) = \sqrt{x}$ over the interval $[1,16]$

$$f_{ave} = \frac{1}{16-1}\int_1^{16} x^{\frac{1}{2}}\,dx$$
$$= \frac{1}{15}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^{16}$$
$$= \frac{14}{15}$$ //

c) $f(x)=x^3$ over the interval $[-2, 2]$

$$f_{ave} = \frac{1}{2--2}\int_{-2}^{2} x^3\,dx$$
$$= \frac{1}{4}\left[\frac{1}{4}x^4\right]_{-2}^{2}$$
$$= 0$$ //

d) $f(x) = \sin x$ over the interval $[0, 2\pi]$

$$f_{ave} = \frac{1}{2\pi-0}\int_0^{2\pi} \sin x\,dx$$
$$= \frac{1}{2\pi}\left[-\cos x\right]_0^{2\pi}$$
$$= 0$$

② An object moves along the x-axis with velocity $v(t)=8t-3$. Suppose that the object's initial position (at $t=0$) is at $x=2$

a) Find $a(t)$

$$a(t) = \frac{d}{dt}(v(t))$$
$$= \frac{d}{dt}(8t-3)$$
$$= 8$$ //

b) Find $x(t)$

$$x(t) = \int v(t)\,dt$$
$$= \int 8t-3\,dt$$
$$= 4t^2-3t + C$$

when $t=0$, $x=2$:

$2 = 4(0)^2-3(0)+C$

$C=2$

$x(t)=4t^2-3t+2$ //

c) Find the pos$^{\underline{n}}$ of the object at $t=4$

$x(4)=4(4)^2-3(4)+2$

$= 54$

③ An object moves along the x-axis with acceleration $a(t)=-10$. Suppose that the object's initial pos$^{\underline{n}}$ (at time $t=0$) is at $x=100$, and the object's initial velocity is $v=4$.

a) Find $v(t)$

$$v(t)=\int a(t)\,dt$$
$$=\int -10\,dt$$
$$=-10t+C$$

at $t=0$, $v=4$:

$4=-10(0)+C$

$C=4$

$v(t)=-10t+4$ //

b) Find $x(t)$

$x(t) = \int v(t)\,dt$

$= \int -10t + 4\,dt$

$= -5t^2 + 4t + C$

at $t=0$, $x = 100$

$100 = -5(0)^2 + 4(0) + C$

$C = 100$

$x(t) = -5t^2 + 4t + 100$ //

c) Find $pos^{\underline{n}}$ at $t=3$

$x(t) = -5t^2 + 4t + 100$

$x = -5(3)^2 + 4(3) + 100$

$= 67$ //

d) Find time at which $v(t) = 0$

$-10t + 4 = 0$

$t = 0.4$ //