

# Mathematics 1

## Sheet 24: Bernoulli Trials

### Binomial Distribution

1. A die is rolled three times. If  $X$  is the number of occasions when the die shows a 4, then give the probability distribution for  $X$ . *That is, give a table containing the values of  $\Pr(X = x)$ , for  $x = 0, 1, 2, 3$ .*
2. The probability that a golfer sinks a putt is  $\frac{2}{3}$ . Find the probability that, of three similar putts, the golfer sinks
  - (a) all three
  - (b) at least two.
3. A shooter has 3 shots at a target. If the probability that he hits the target on any one shot is 0.05, find the probability that he hits the target at most once in his 3 shots.
4. It is known that 85% of the graduates of a certain secretarial college are placed in a job within six months of graduation. If a class has 10 graduates, what is the probability that at least 9 of them will have a job within six months? Give your answer in a sentence, accurate to four decimal places.
5. A shooter has 3 shots at a target. If the probability that he hits the target on any one shot is 0.75, find the probability that he misses the target on every shot.
6. An airline pilot is given the task of dropping a package containing food and medical supplies to a group of people stranded on an island. Because of the mountains, the thick vegetation and the treacherous winds that may carry the parachutes off course, the pilot thinks that the probability that the people will recover the package in usable condition is only  $\frac{1}{3}$ .  
Find the smallest number of drops necessary for the pilot to be at least 95% certain of at least one successful drop.
7. How many times must a fair coin be tossed so that the probability of getting at least one head exceeds 0.9?
8. It is known that in a batch of milk cartons, 30% are underfilled.
  - (a) Consider a sample of 6 milk cartons.
    - i. Find the probability that no more than 2 are underfilled.
    - ii. Find the expected number of underfilled cartons.
  - (b) Find the minimum number of cartons that I need to select to ensure that the probability of getting at least 1 **full** carton is at least 0.9.

## Geometric Distribution

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9. Cuttings of a particular plant have a success rate of 10%. A gardener takes cuttings until he has success in growing a plant.
  - (a) Find, to four decimal places, the probability that he has more than 5 unsuccessful tries before he is successful.
  - (b) Find the expected number of unsuccessful cuttings.
  - (c) Find, to four decimal places, the standard deviation of the number of unsuccessful cuttings.
10. The probability that a punter picks a winner in any one race is  $\frac{1}{6}$ . Find the probability that he fails to pick the winner on exactly 5 races, before he finally picks a winner. Write your answer accurate to four decimal places.
11. The probability that a student passes the written test for a driving test is  $\frac{7}{10}$ .
  - (a) Find the probability that a student passes the test on the third try (and fails the first two tries).
  - (b) Find the probability that a student passes the test before the fourth try (and fails any earlier tries).
12. The probability that a newly-born baby is a girl is 0.48. Let  $X$  denote the number of girls born before a boy is born, on a particular day in a large hospital.
  - (a) Find  $\Pr(X = 0)$ .
  - (b) Find  $\Pr(X \geq 1)$ .
  - (c) Find, to four decimal places,  $\Pr(2 < X < 8)$ .
13. Two fair dice are rolled until a sum of 11 occurs.
  - (a) What, to four decimal places, is the probability that exactly 6 rolls are required (including the roll which gives the sum of 11)?
  - (b) What is the probability that at least 2 rolls are required (including the roll which gives the sum of 11)?
14. Suppose that 40% of people in a population have blood type  $A$ . Find the probability that when some randomly selected people are tested, the first person to have blood type  $A$  is found on the fourth test.
15. The probability that a punter picks a winner in any one race is  $\frac{1}{8}$ . Let  $X$  be the number of times he fails to pick a winner, before he finally picks a winner.
  - (a) Find the expected value of  $X$ .
  - (b) What is the variance of  $X$ ?

## Mixed Distributions

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*To calculate the answers for the remaining exercises, you will first need to determine whether they are*

- *binomial (from Chapter 21.1),*
- *geometric (from Chapter 21.2),*

*or possibly*

- *hypergeometric (from Chapter 20.4)*  
*(in which case those particular exercises do NOT involve Bernoulli Trials).*

[Touch here to see a video about identifying distributions.](#)

16. Consider a sequence of independent trials, each having probability of success equal to 0.8.
- (a) Let  $X$  denote the number of successes in the first 4 trials. Find  $\Pr(X = 1)$ .
- (b) Let  $Y$  denote the number of failures before the first success. Find  $\Pr(Y = 2)$ .
17. Twelve percent of a certain type of radio valve are known to be defective. In a batch of 8 valves, find the probability that there are 3 defectives. Give your answer accurate to five decimal places.

18. A batch containing 8 valves is known to contain 3 defectives. If a random sample of 4 valves is taken without replacement, what is the probability that the sample contains more than one defective?
19. The probability that a shooter hits a target is 0.6 for each shot. If the shooter fires until she hits the target, find the probability that she fires at least 7 shots.
20. Bugs Bunny wants to grow some carrot plants, so he plants 20 carrot-seeds. If each seed has a probability of 0.9 of germinating, find the probability that Bugs Bunny gets at least 18 carrot plants. Give your answer in a sentence, accurate to four decimal places.
21. A carton contains 8 cubes, 5 of which are white and the remainder black. A random sample of 3 cubes is taken, without replacement, from the carton. Find the probability that the 3 cubes are of the same colour.

22. Two fair dice are rolled until a sum of 10 occurs.

- (a) What, to four decimal places, is the probability that exactly 6 rolls are needed (including the roll which gives the sum of 10)?
- (b) What is the probability that at least 2 rolls are needed (including the roll which gives the sum of 10)?

### Answers:

1.

$x$	$\Pr(X = x)$
0	$\frac{125}{216}$
1	$\frac{75}{216}$
2	$\frac{15}{216}$
3	$\frac{1}{216}$

2. (a) The probability of sinking all three putts is  $\frac{8}{27}$ .  
 (b) The probability of sinking at least two putts is  $\frac{20}{27}$ .
3. The probability of hitting the target at most once is 0.99 275.
4. The probability that at least 9 of the graduates will be employed within six months is 0.5443 (4 d.p.).
5. The probability of missing the target on each of the 3 shots is 0.015 625.
6. The smallest number of drops needed is 8.
7. The coin must be tossed at least 4 times.
8. (a) i. The probability that no more than 2 of the cartons are underfilled is 0.74 431.  
 ii. The expected number of underfilled cartons is 1.8.  
 (b) The minimum number of cartons needed is 2.
9. (a) The probability of having more than 5 unsuccessful tries is 0.5314 (4 d.p.).  
 (b) The expected number of unsuccessful cuttings (before having a success) is 9.  
 (c) The standard deviation is 9.4868 (4 d.p.).
10. The probability that he loses on exactly 5 races before picking a winner is 0.0670 (4 d.p.).

11. (a) The probability of passing on the third try is 0.063.  
(b) The probability of passing on the first, second or third try is 0.973.
12. (a) 0.52            (b) 0.48            (c) 0.1078 (4 d.p.)
13. (a) The probability that exactly six rolls are needed is 0.0417 (4 d.p.)  
(b) The probability that at least two rolls are needed is  $\frac{17}{18}$ .
14. The probability that the first person to have blood type  $A$  is found on the fourth test is 0.0864
15. (a) The expected value is 7.  
(b) The variance is 56.
16. (a) 0.0256            (b) 0.032
17. The probability that there are 3 defectives in the batch is 0.05 107 (5 d.p.).
18. The probability that the sample has more than one defective is  $\frac{1}{2}$ .
19. The probability that she fires at least seven shots is 0.004 096.
20. The probability that he gets at least 18 carrot plants is 0.6769 (4 d.p.).
21. The probability that the 3 cubes are the same colour is  $\frac{11}{56}$ .
22. (a) The probability that exactly 6 rolls are needed is 0.0539 (4 d.p.).  
(b) The probability that at least 2 rolls are needed is  $\frac{11}{12}$ .