[ES7] COMPOSITION OF FUNCTIONS

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1. Consider the functions

$$f : (4, \infty) \longrightarrow \mathbf{R} \quad \text{where } f(x) = \sqrt{x}$$

and

$$g : \mathbf{R}^- \longrightarrow \mathbf{R} \quad \text{where } g(x) = x^2.$$

(a) Find the domain of $f \circ g$.

⚠must show $\boxed{x \in dom(g) \ \text{ and}/\cap \ g(x) \in dom(f)}$

$x \in \mathbb{R}^-$ and $x^2 \in (4, \infty)$

$x < 0$

$x^2 > 4$

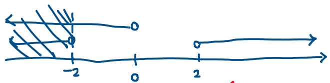
$x^2 - 4 > 0$

$(x+2)(x-2) > 0$

Let $(x+2)(x-2) = 0$

$x = -2 \quad x = 2$

$x < -2 \cup x > 2$



$dom(f \circ g) = (-\infty, -2)$ //

(b) Find the rule for $f(g(x))$.

$f(g(x)) = f(x^2)$

$= \sqrt{x^2}$

$= |x|$ //

2. Consider the functions f and g as given below:

$$f : [2, \infty) \longrightarrow \mathbf{R} \quad \text{where} \quad f(x) = \sqrt{x - 1}$$

and

$$g : [0, 4] \longrightarrow \mathbf{R} \quad \text{where} \quad g(x) = x^2.$$

(a) ① Give the rule for $f(g(x))$ and ② find all values of x for which $f(g(x))$ exists.

$f(g(x)) = f(x^2)$

$= \sqrt{x^2 - 1}$ //

$dom(f(g(x))) = \{x \in dom(g) \cap g(x) \in dom(f)\}$

$= \{x \in [0,4] \cap x^2 \in [2, \infty)\}$

$= 0 \leq x \leq 4$ and $x^2 \geq 2$

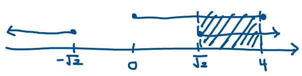
$x^2 - 2 \geq 0$

$(x+\sqrt{2})(x-\sqrt{2}) \geq 0$

Let $(x+\sqrt{2})(x-\sqrt{2}) = 0$

$x = -\sqrt{2} \quad x = \sqrt{2}$

$x \leq -\sqrt{2} \cup x \geq \sqrt{2}$



$x \in [\sqrt{2}, 4]$ //

(b) Give the rule for $g(f(x))$ and find all values of x for

(b) Give the rule for $g(f(x))$ and find all values of x for which $g(f(x))$ exists.

3. Consider the functions

$$f : [1, \infty) \longrightarrow \mathbf{R} \quad \text{where } f(x) = \frac{1}{x} + 1$$

and

$$g : \mathbf{R} \setminus \left\{\frac{1}{2}\right\} \longrightarrow \mathbf{R} \quad \text{where } g(x) = \frac{1}{2x - 1}.$$

(a) Find the rule for $f(g(x))$.

(b) Find the domain of $f \circ g$.

4. Consider the functions

$$f : [4, \infty) \longrightarrow \mathbf{R} \quad \text{where } f(x) = \log_{10} x$$

and

$$h : (0, 2) \longrightarrow \mathbf{R} \quad \text{where } h(x) = 3 + x.$$

(a) Find the rule for $h(f(x))$.

(b) Find the values of x for which $h(f(x))$ is defined.

5. Consider the functions

$$f : [1, \infty) \longrightarrow \mathbf{R} \quad \text{where } f(x) = \sqrt{x}$$

and

$$g : \mathbf{R} \longrightarrow \mathbf{R} \quad \text{where } g(x) = x^2.$$

(a) Find the rule for $f \circ g$.

(b) Find $\mathrm{dom}(f \circ g)$.

6. Which of the following functions has an inverse **function**?

(a) $f : \mathbf{R} \longrightarrow \mathbf{R}$ where $f(x) = x^2 - 2$

(b) $f : [0, \infty) \longrightarrow \mathbf{R}$ where $f(x) = x^2 + 1$

(c) $f : \mathbf{R} \longrightarrow \mathbf{R}$ where $f(x) = x^3$

(d) $f : [-1, \infty) \longrightarrow \mathbf{R}$ where $f(x) = (x+3)^2$

7. For each of the following functions f, find the largest value of b so that f has an inverse **function**.

(a) $f : [-1, b] \longrightarrow \mathbf{R}$ where $f(x) = 4 + x^2$

(b) $f : [-3, b] \longrightarrow \mathbf{R}$ where $f(x) = \sqrt{9 - x^2}$

(c) $f : (-\infty, b] \longrightarrow \mathbf{R}$ where $f(x) = x(x - 10)$

8. Consider the function

$$f : (-\infty, b] \longrightarrow \mathbf{R} \quad \text{where } f(x) = x^2 + 1.$$

(a) Find the largest value of b so that f has an inverse function.

Using this value of b,

(b) state the domain and range of f.

(c) state the domain and range of f^{-1}.

(d) find the rule for f^{-1}.

(e) On the same set of axes, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

9. Find f^{-1} if f is the function defined by

$$f : [-2, 0] \longrightarrow \mathbf{R} \text{ where } f(x) = 4 - x^2.$$

10. Consider the function

$$f : (-\infty, b] \longrightarrow \mathbf{R} \text{ where } f(x) = x^2 + 2x.$$

(a) Find the largest value of b so that f has an inverse function.

Using this value of b,

(b) state the domain and range of f.

(c) state the domain and range of f^{-1}.

(d) find f^{-1}.

11. Consider the function

$$f : S \longrightarrow \mathbf{R} \text{ where } f(x) = 2x + 2.$$

If f has inverse function given by

$$f^{-1} : [0, \infty) \longrightarrow \mathbf{R} \text{ where } f^{-1}(x) = \frac{1}{2}\, x - 1\ ,$$

then find the set S.

12. Consider the function given by $f(x) = x^2$.

(a) Sketch the graph of $y = f(x)$.

(b) Find the domain and range of f.

(c) Does f have an inverse function?

(c) Does f have an inverse function?

13. Consider the function given by $f(x) = x^3$.
(a) Sketch the graph of $y = f(x)$.

(b) Find the domain and range of f.

(c) Does f have an inverse function?

14. Consider the function given by $f(x) = \sqrt{4 - x^2}$.
(a) Sketch the graph of $y = f(x)$.

(b) Find the domain and range of f.

(c) Does f have an inverse function?

15. Let $f : \mathbf{R} \setminus \{2\} \longrightarrow \mathbf{R}$ where $f(x) = \dfrac{3}{x-2}$.

(a) Find the domain and range of f.

(b) Find the domain and range of f^{-1}.

(c) Find the rule for f^{-1}.

16. (a) Let $f : (-\infty, -2) \longrightarrow \mathbf{R}$ where $f(x) = \dfrac{1}{(x+2)^2}$.

i. Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

ii. Completely determine f^{-1}. That is, find the domain, range and rule for f^{-1}.

(b) Let $g : (-2, \infty) \longrightarrow \mathbf{R}$ where $g(x) = \dfrac{1}{(x+2)^2}$.

i. Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$.

ii. Completely determine g^{-1}. That is, find the domain, range and rule for g^{-1}.