CHAPTER 8 VECTOR MULTIPLICATION

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M2 chap8

CHAPTER 8: VECTOR MULTIPLICATION

DOT PRODUCT

AKA scalar product

for $\underset{\sim}{a} = a_x\underset{\sim}{i} + a_y\underset{\sim}{j} + a_z\underset{\sim}{k}$

$\underset{\sim}{b} = b_x\underset{\sim}{i} + b_y\underset{\sim}{j} + b_z\underset{\sim}{k}$

$$\boxed{\underset{\sim}{a} \cdot \underset{\sim}{b} = a_x b_x + a_y b_y + a_z b_z}$$

DOT PRODUCT USEFUL PRODUCTS

① $a \cdot a = |a|^2$

cummulative rule ② $a \cdot b = b \cdot a$

distributive rules:

③ $a \cdot (b + c) = a \cdot b + a \cdot c$

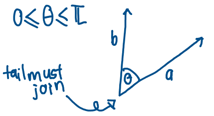
④ $(a + b) \cdot c = a \cdot c + b \cdot c$

⑤ $(a + b) \cdot (c + a) = a \cdot c + a \cdot d + b \cdot c + b \cdot d$

DOT PRODUCT THEOREM

$$\boxed{a \cdot b = |a|\,|b| \cos\theta}$$

$0 \leq \theta \leq \pi$



$0 \leq \theta < \frac{\pi}{2}$	$\cos\theta > 0$	$a \cdot b > 0$
$\theta = \frac{\pi}{2}$	$\cos\theta = 0$	$a \cdot b = 0$
$\frac{\pi}{2} < \theta \leq \pi$	$\cos\theta < 0$	$a \cdot b < 0$

ORTHOGONAL means perpendicular

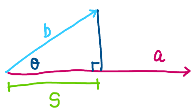
2 vectors are octagonal iff & only if:

$a \cdot b = 0$

ANGLE BTWN 2 VECTORS

$$\boxed{\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|\,|\underset{\sim}{b}|}} \Leftrightarrow \underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos\theta$$

SCALAR PROJECTION



scalar projection of b onto a is S

$\cos\theta = \frac{S}{|b|}$

$S = |\underset{\sim}{b}|\cos\theta = \frac{|\underset{\sim}{a}|\,|\underset{\sim}{b}|\cos\theta}{|\underset{\sim}{a}|} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{a}} = \underset{\sim}{b} \cdot \hat{\underset{\sim}{a}}$

$$\boxed{\therefore S = \underset{\sim}{b} \cdot \hat{\underset{\sim}{a}}}$$