

EXERCISE SHEET 19: STATISTICS

$$\textcircled{1} a) T = 0 \times 6 + 1 \times 13 + 2 \times 12 + 3 \times 9 + 4 \times 5 + 5 \times 2 + 6 \times 0 + 7 \times 1 = 48$$

$$\textcircled{3} a) \text{Mean} = 1653 \div 12 = 137.75$$

$$b) \text{Mean} = \frac{81}{3} = 27$$

$$b) \text{Standard deviation} = 25.7580 \approx 25.76 \text{ cm}$$

$$\textcircled{1} a) T = 6 + 13 + 12 + 9 + 3 + 2 + 0 + 1 = 48$$

$$c) \text{Variance} = 663.4772127 \approx 663.48 \text{ cm}^2$$

$$b) (0 \times 6) + (1 \times 13) + (2 \times 12) + (3 \times 9) + (4 \times 5) + (5 \times 2) + (6 \times 0) + (7 \times 1) \div 48 = 210.41$$

$$\text{Mean} \approx 2.10$$

$$d) (132 + 133) \div 2 = 132.5$$

$$\textcircled{4} e) \text{Mean} = 1072 \div 12 = 89.3333 \approx 89.33$$

$$c) \text{Mode} = 1$$

$$d) \text{Median} = 2$$

$$\frac{48}{2} = 24$$

$$24\text{th number} = 2$$

$$25\text{th number} = 2$$

$$\frac{242}{2} = 2$$

$$b) \text{Standard deviation} = 7.9353 \approx 7.94$$

$$c) \text{Variance} = 62.9696 \approx 62.97$$

$$\textcircled{2} a) \text{Mean} = 6345 \div 8 = 793.125 \approx 793.13 \text{ km}$$

$$d) \text{Range} = 105 - 79 = 26$$

$$ii) \text{Variance} = \frac{1}{n(n-1)} \left[n \sum x_i^2 - \left(\sum x_i \right)^2 \right]$$

$$= \frac{1}{8(8-1)} \left[8 \times 11227659 - (6345)^2 \right]$$

$$= 885040.125$$

$$\approx 885040.13 \text{ km}^2$$

$$e) \text{Median} = (89 + 88) \div 2 = 88.5$$

x	x^2
112	12544
567	321489
2746	7540516
1618	2617924
155	24025
54	2916
311	96721
782	611524
6349	11227659

$$f) \text{Lower quartile} = (82 + 86) \div 2 = 84$$

$$\text{Upper} = (91 + 96) \div 2 = 93.5$$

$$\textcircled{5} a) \text{Mean} = 844 \div 14 = 60.2857 \approx 60.29$$

$$b) \text{Standard deviation} = 4.0213 \approx 4.02$$

$$ii) \text{Standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{885040.125}$$

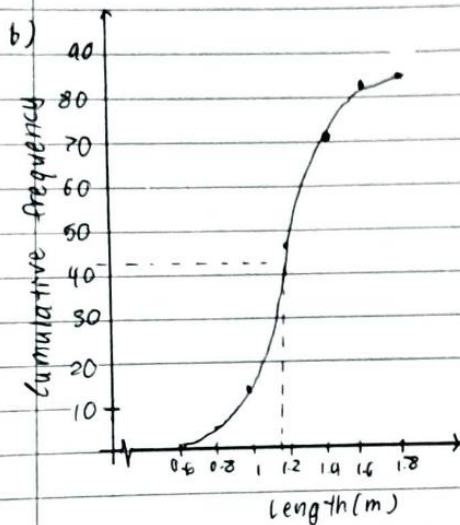
$$= 940.7657$$

$$\approx 940.77 \text{ km}$$

$$c) \text{Variance} = 24.2497 \approx 24.22$$

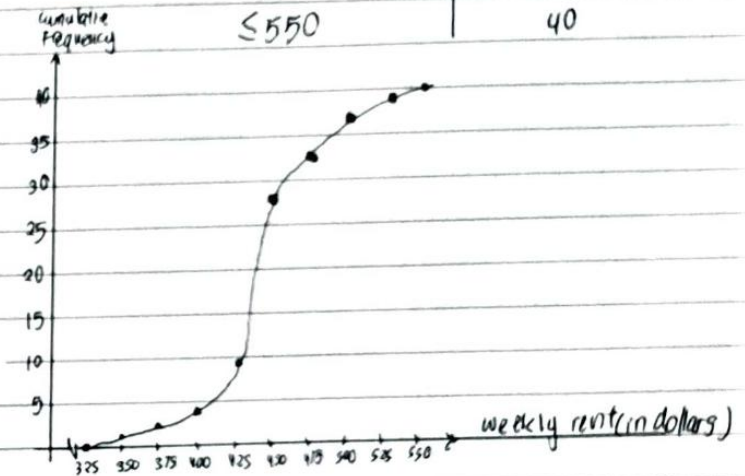
⑥ a)

length(m)	Cumulative frequency
< 0.6	0
< 0.8	3
< 1	13
< 1.2	48
< 1.4	48.71
< 1.6	83
≤ 1.8	89



d)

Weekly rent (in dollars)	Cumulative frequency
< 325	0
< 350	1
< 375	2
< 400	4
< 425	9
< 450	28
< 475	33
< 500	37
< 525	39
≤ 550	40



i) Approximately 1.17 metres

i) $\frac{28}{40} \times 100\% = 70\%$

ii) $\frac{38}{40} \times 100\% = 95\%$

⑦ a)

Weekly Rent (in dollars)	Frequency
325 -	1
350 -	1
375 -	2
400 -	5
425 -	19
450 -	5
475 -	4
500 -	2
525 - 550	1
Total	40

⑧ a) $f(x) = 6x - \frac{1}{5x^2}$

b) $f'(x) = 3 + 2e^x$

c) $f'(x) = e^x (\ln x + \frac{1}{x})$

d) $f'(x) = 2x \tan x + x^2 \sec^2 x$

e) $f'(x) = e^x (\ln x + \frac{1}{x})$

f) $f'(x) = x e^{3x} (2 + 3x)$

g) $f'(x) = \sqrt{1+3x} + \frac{1}{x} \times \frac{1}{2} (1+3x)^{-\frac{1}{2}} \times 3$
 $= \sqrt{1+3x} + \frac{3}{2\sqrt{1+3x}}$

h) $f'(x) = \frac{\ln x - \frac{1}{x}}{(\ln x)^2}$

i) $f'(x) = \frac{\ln x - 1}{\ln x} e^x - e^x \cos x$
 $= \frac{\ln x - 1 - \cos x}{\ln x} e^x$

j) $f'(x) = \frac{\cos x (\pi + 1) - \sin x}{(\pi + 1)^2}$

k) $f'(x) = \frac{e^{x+1} (x+1)^2}{(x+1)^3}$

l) $f'(x) = \frac{2(2x+1) - 2(x^2+1)}{(2x+1)^2}$

$= \frac{2x^2 + 2x + 2}{(2x+1)^2}$

m) $f'(x) = 7(1-2x^2)^6 (-4x)$

$= -28x(1-2x^2)^6$

$$n) f'(x) = \frac{1}{2} (8x + x^4)^{-\frac{1}{2}} (8 + 4x^3) \\ = \frac{4 + 2x^3}{\sqrt{8x + x^4}}$$

$$o) f'(x) = \frac{20x^3 + 5x^{-1}}{(8x+1)^{\frac{5}{2}}}$$

$$p) f'(x) = (3 + 2e^x) e^{5x+2e^x}$$

$$q) f'(x) = e^{-x^2+1} - xe^{-x^2+1}(-2x) \\ = e^{-x^2+1} (1 - 2x^2)$$

$$r) f'(x) = \frac{1}{12x^2+11x} (24x+11)$$

$$s) f'(x) = \frac{1}{2} \ln(x^2-1) \\ = \frac{1}{2} \times \frac{1}{x^2-1} \times 2x \\ = \frac{x}{x^2-1}$$

$$t) f'(x) = \sec^2 x e^{\tan x}$$

$$u) f'(x) = \frac{1}{\sin x} (\cos x) \\ = \cot x$$

$$v) f'(x) = -\sin x + x \cos x - \frac{x}{x}$$

$$w) f'(x) = \cos(e^x + 1)(e^x)$$

$$x) f'(x) = 3 \left(\frac{1}{\sqrt{1-(3x)^2}} \right) \\ = \frac{3}{\sqrt{1-9x^2}}$$

$$y) f'(x) = 2x \cos^{-1} x - \frac{x^2}{\sqrt{1-x^2}}$$

$$z) f'(x) = \frac{2x}{1+x^4}$$

$$9a) y = xe^y = x + 1$$

$$\frac{d}{dx} \left[(1) \frac{dy}{dx} + 1 \cdot e^y + xe^y \left(\frac{dy}{dx} \right) \right] = \frac{d}{dx} (x+1)$$

$$\frac{dy}{dx} + e^y + xe^y \left(\frac{dy}{dx} \right) = 1$$

$$\frac{dy}{dx} (1 + xe^y) = 1 - e^y$$

$$\frac{dy}{dx} = \frac{1 - e^y}{1 + xe^y}$$

$$b) xy^3 - y^2 = x^2$$

$$\frac{d}{dx} [y^3 + x3y^2 \left(\frac{dy}{dx} \right) - 2y \left(\frac{dy}{dx} \right)] = \frac{d}{dx} (x^2)$$

$$y^3 + x3y^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} (3xy^2 - 2y) = 2x - y^3$$

$$\frac{dy}{dx} = \frac{2x - y^3}{3xy^2 - 2y}$$