C5: MATRIX TRANSFORMATION

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M2 Chap5

LEARNING OUTCOMES I
• Define the terms: objects, image and isometry • Apply the refelction, rotation and dilation matrices to transform points in the plane • Construct a single matric corresponding to a combination of two or more transformations
LEARNING OUTCOMES II
• Find the image of a given line using an inverse matrix • Identify degenerate transformations • Determine which straight lines are mapped to a point under a given degenerate transformation

OBJECT & IMAGE

- Original points (x, y) are called object points.
- New points (x', y') are called image points.

image $\begin{bmatrix} x' \\ y' \end{bmatrix} = T \begin{bmatrix} x \\ y \end{bmatrix}$ object

2x2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- Transformations are also known as mappings

GENERAL LINEAR TRANSFORMATIONS

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

to find T, transfom $(1,0)$ and $(0,1)$

$(1,0)$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$

$(0,1)$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$

REFLECTION IN THE x axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

REFLECTION IN THE y axis

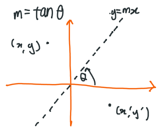
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

REFLECTION IN THE LINE $y=x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

REFLECTION IN THE LINE $y=mx$

$m = \tan\theta$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

ROTATION ABOUT THE ORIGIN

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

* if θ is negative then the rotation is clockwise.

DILATION / CONTRACTION BY A FACTOR k PARALLEL TO THE x axis

- x is stretched by y stays the same

$x' = kx$

$y' = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- k is called the dilation/contraction factor
- dilation when $k>1$ ~ stretch
- contraction when $0<k<1$ ~ shrink

DILATION/CONTRACTION BY A FACTOR k PARALLEL TO THE y axis

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

DILATION/CONTRACTION PARALLEL TO BOTH x and y axes