EXERCISE SHEET 24

How many combination — how many success — how many fail

$$Pr(X=x) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$$

probability of success — probability of fail

①

x	$Pr(X=x)$
0	${}^{3}C_{3} \times \left(\frac{1}{6}\right)^{0}\left(\frac{5}{6}\right)^{3} = \frac{125}{216}$
1	${}^{3}C_{1} \times \left(\frac{1}{6}\right) \times \left(\frac{5}{6}\right)^{2} = \frac{75}{216} = \frac{25}{72}$
2	${}^{3}C_{2} \times \left(\frac{1}{6}\right)^{2} \times \left(\frac{5}{6}\right) = \frac{15}{216} = \frac{5}{72}$
3	${}^{3}C_{3} \times \left(\frac{1}{6}\right)^{3} \times \left(\frac{5}{6}\right)^{0} = \frac{1}{216}$

② a) $Pr(3\ sink) = {}^{3}C_{3} \times \left(\frac{2}{3}\right)^{3} \times \left(\frac{1}{3}\right)^{0}$

$= \frac{8}{27}$ //

b) $Pr(at\ least\ 2) = Pr(2\ sink) + Pr(3\ sink)$

$= {}^{3}C_{2} \times \left(\frac{2}{3}\right)^{2} \times \left(\frac{1}{3}\right) + \frac{8}{27}$

$= \frac{20}{27}$ //

③ $Pr(at\ most\ 1\ hit) = Pr(0\ hit) + Pr(1\ hit)$

$= {}^{3}C_{3} \times (0.05)^{0} \times (0.95)^{3} + {}^{3}C_{1} \times (0.05)^{1} \times (0.95)^{2}$

$= 0.99\ 275$ //

⑤ $Pr(3\ misses) = {}^{3}C_{3} \times (1-0.75)^{3}$

$= 0.015625$ //

⑦ X = number of getting head

$Pr(X \geq 1) > 0.9$

$Pr(X = 0) \leq 0.1$

$(0.5)^n \leq 0.1$

BTAE:

$n = 3 : (0.5)^3 = 0.125$

$n = 4 : (0.5)^4 = 0.0625$

The coin has to be tossed at least 4 times

⑨ X = number of unsuccessful tries

a) $Pr(X > 5) = (1 - 0.1)^6$

$= 0.5314$ <4dp>

b) $E(X) = \frac{1 - 0.1}{0.1}$

$= 9$

c) $Var(X) = \frac{1 - 0.1}{(0.1)^2}$

$= 90$

$\sqrt{90} = 9.4868$ <4dp>

EXERCISE SHEET 24

① <Selected Question>

② <Selected Question>

③ <Selected Question>

④ Let X = students have job within 6 months

$$\Pr(X \geq 9) = {}^{10}C_9(0.85)^9(0.15)^1 + {}^{10}C_{10}(0.85)^{10}(0.15)^0$$
$$= 0.5442998238$$
$$\approx 0.5443 \text{ <4 dp>} //$$

⑤ <Selected question>

⑥ Let X = successful drop

$$\Pr(X \geq 1) \geq 0.95$$
$$1 - {}^nC_0\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \geq 0.95$$
$$\left(\frac{2}{3}\right)^n \leq 0.05$$

BTAE:

n	$\left(\frac{2}{3}\right)^n$
6	0.08779149...
7	0.0585276...
8	0.03901844...

$\therefore$ At least 8 drops //

⑦ <Selected question>

⑧ Let X = milk cartons underfilled

a) i) $Pr(X \leq 2) = {}^6C_0(0.3)^0(0.7)^6 + {}^6C_1(0.3)^1(0.7)^5 + {}^6C_2(0.3)^3(0.7)^4$

$= 0.7443$ <4 dp> //

ii) $E(X) = 6(0.3)$

$= 1.8$ //

b) $Pr(X \geq 1) \geq 0.9$

$1 - C(0.7)(0.3) \geq 0.9$

$(0.3) \leq 0.1$

$n \ln|0.3| \leq \ln|0.1|$

$n \geq 1.9125$ <4dp> //

$\therefore$ minimum is 2 //

⑩ Let X = pick loser

$Pr(X=5) = \left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)$

$= 0.0670$ <4dp> //

⑪ a) Let X = number of tries before pass

$Pr(X=2) = \left(1-\frac{7}{10}\right)^2\left(\frac{7}{10}\right)$

$= \frac{63}{1000}$ //

b) $Pr(X<3) = 1 - \left(1-\frac{7}{10}\right)^3$

$= 0.973$ <3dp> //

⑫ a) Let X = number of girls born before boy

$Pr(X=0) = (0.48)^0(0.52)$

$= 0.52$ //

b) $Pr(X \geq 1) = 0.48$ //

c) $Pr(2<X<8) = (0.48)^3 - (0.48)^8$

$= 0.1078$ <4dp> //

(13) a) Let X = number of rolls before sum 11

$Pr(X=5) = \left(\frac{17}{18}\right)^6 \left(\frac{1}{18}\right)$

$= 0.0417$ <4dp> //

b) $Pr(X \geq 11) = \frac{17}{18}$ //

(14) Let X = number of selected people that not type A before type A found

$Pr(X=3) = (0.6)^3 (0.4)$

$= 0.08640$ <4dp> //

(15) a) $E(X) = \frac{1-p}{p}$

$= \frac{\frac{7}{8}}{\frac{1}{8}}$

$= 7$

b) $Var(X) = \frac{1-p}{p^2}$

$= \frac{7}{8} \times (8)^2$

$= 56$

(16) a) Let X = number of success in first 4 trials

$Pr(X=1) = {}^4C_1 \ (0.8)^1 \ (0.2)^3$

$= 0.0256$ <4dp> //

b) Let Z = number of failures before success

$Pr(Z=2) = (1-0.8)^2 \ (0.8)$

$= 0.032$ <3dp> //

(17) Let X = number of radio valve that are defective.

$Pr(X=3) = {}^{8}C_{3}\,(0.12)^{3}(0.88)^{5}$

$= 0.05107$ <5dp> //

(18) Let X = number of defective valves

$Pr(X>1) = 1-\left(\frac{{}^{3}C_{0} \times {}^{5}C_{4}}{{}^{8}C_{4}}\right)-\left(\frac{{}^{3}C_{1} \times {}^{5}C_{3}}{{}^{8}C_{4}}\right)$

$= 0.500$ <3dp> //

(19) Let X = number of shots before hit target

$Pr(X>6) = (1-0.6)^{6}$

$= 0.004096$ <6dp> //

(20) Let x = number of germinated seeds

$Pr(X \geq 18) = {}^{20}C_{18}(0.9)^{18}(0.1)^{2} + {}^{20}C_{19}(0.9)^{19}(0.1)^{1} + {}^{20}C_{20}(0.9)^{20}(0.1)^{0}$

$= 0.6769$ <4dp> //

(21) Let X = number of selected white cubes Let Y = number selected black cubes

$Pr(X=3\ Y=3) = \frac{{}^{5}C_{3} \times {}^{3}C_{0}}{{}^{8}C_{3}} + \frac{{}^{3}C_{3} \times {}^{5}C_{0}}{{}^{8}C_{3}}$

$= \frac{11}{56}$ //

(22) a) Let x = number of rolls before sum 10

$Pr(X=5) = \left(1-\left(\frac{1}{6}\times\frac{1}{6}\right)\times 3\right)^{5}\left(\frac{1}{12}\right)$

$= 0.0539$ <4dp> //

b) $Pr(X \geq 1) = \left(1-\frac{1}{12}\right)^{1}$

$= \frac{11}{12}$ //