C11: ANTIDIFFERENTIATION

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11.1 ANTIDIFFERENTIATION

$$\text{if } f'(x) = x^r \text{, then } f(x) = \frac{1}{r+1}x^{r+1} + C$$

$$\text{if } f'(x) = \sin kx \text{, then } f(x) = -\frac{1}{k}\cos kx + C.$$

$$\text{if } f'(x) = (ax+b)^r \text{, then } f(x) = \frac{(ax+b)^{r+1}}{a(r+1)} + C$$

$$\text{if } f'(x) = \frac{1}{ax+b} \text{ then } f(x) = \frac{1}{a}\ln|ax+b| + C$$

EXERCISES [p.7]

Find the most general antiderivative of each of the following expressions:

(a) $\frac{1}{4}x^4 + 8x + C$

(b) $\frac{1}{5}x^5 + 2x^{\frac{3}{2}} - \frac{2}{3}x^3 + x^{-1} + C$

(c) $\cos\left(\frac{3}{2}x\right)$

$\frac{2}{3}\sin\left(\frac{3}{2}x\right) + C$

(d) $\frac{1}{4}e^{4x} + C$

(e) $\frac{2}{x}$ $= 2x^{-1}$

$2\ln|x| + C$

(f) $\frac{1}{2x}$

$\frac{1}{2}\ln|x| + C$

(g) $\frac{1}{3}x^3 + x^2 + C$

(h) $\frac{1}{3}\sin(3x) + C$

(i) $\frac{1}{x^2}$ $= x^{-2}$

$\frac{1}{-3}x^3$

(j) $\sin\left(\frac{1}{2}x\right)$

$-2\cos\left(\frac{1}{2}x\right)$

(k) $\sqrt{x}$ $= x^{\frac{1}{2}}$

$\frac{2}{3}x^{\frac{3}{2}}$

(l) $\sec^2\left(\frac{1}{2}x\right)$

$2\tan\left(\frac{1}{2}x\right)$

(m) $(2x+7)^9$

$\frac{(2x+7)^{10}}{20} + C$

(n) $\sqrt{6x-5}$ $= (6x-5)^{\frac{1}{2}}$

$\frac{(6x-5)^{\frac{3}{2}}}{9} + C$

(o) $(1-3x)^{10}$

$\frac{(1-3x)^{11}}{-33} + C$

(p) $\frac{1}{(4x+5)^{10}}$ $= (4x+5)^{-10}$

$\frac{(4x+5)^{-9}}{-36} + C = \frac{1}{-36(4x+5)} + C$ ✓

(q) $\frac{1}{9x+2}$

$\frac{1}{9}\ln|9x+2| + C$ ✓

(r) $\frac{2}{2x+3}$

$\ln|2x+3| + C$ ✓

(s) $\frac{5}{x^2+25}$ $5(x^2+25)$

11.3 Answers to Chapter 11 Exercises

11.1: (a) $\frac{1}{4}x^4 + 8x + C$ (b) $\frac{1}{5}x^5 + 2x^{\frac{3}{2}} - \frac{2}{3}x^3 - x^{-1} + C$ (c) $\frac{2}{3}\sin\left(\frac{3}{2}x\right) + C$

(d) $\frac{1}{4}e^{4x} + C$ (e) $2\ln|x| + C$ (f) $\frac{1}{2}\ln|x| + C$

(g) $\frac{1}{3}x^3 + x^2 + C$ (h) $\frac{1}{3}\sin 3x + C$ (i) $-\frac{1}{x} + C$

(j) $-2\cos\left(\frac{1}{2}x\right) + C$ (k) $\frac{2}{3}x^{\frac{3}{2}} + C$ (l) $2\tan\left(\frac{1}{2}x\right) + C$

(m) $\frac{(2x+7)^{10}}{20} + C$ (n) $\frac{(6x-5)^{\frac{3}{2}}}{9} + C$ (o) $-\frac{(1-3x)^{11}}{33} + C$

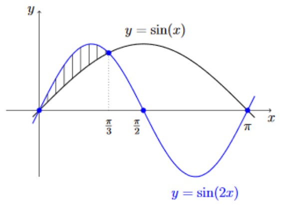
(p) $-\frac{1}{36(4x+5)^9} + C$ (q) $\frac{1}{9}\ln|9x+2| + C$ (r) $\ln|2x+3| + C$

(s) $\tan^{-1}\left(\frac{x}{5}\right) + C$ (t) $\frac{2}{5}\tan^{-1}\left(\frac{x}{5}\right) + C$

11.2: 1. $\ln 2$ 2. $3 - e^{-2}$ 3. $\frac{22}{3}$ 4. $\frac{1}{2}$ 5. $\frac{4}{15}$

6. $\frac{32}{3}$ 7. 36 8. $\frac{125}{6}$ 9. 4 10. $\frac{9}{2}$

11. $\frac{1}{4}$ (See diagram below.) 12. $\frac{1}{2}$



(t) $\dfrac{2}{x^2+25}$

11.2 INTEGRAL CALCULUS

$$\boxed{\int_a^b f(x)\,dx}$$

This is called the **integral** of f from a to b.

FUNDAMENTAL THEOREM OF CALCULUS

Fundamental Theorem of Calculus: Suppose f is continuous on $[a,b]$, and let F be any antiderivative of f. Then

$$\boxed{\int_a^b f(x)\,dx = F(b) - F(a)}$$

Properties of Definite Integrals

The following properties help us to evaluate integrals.

(a) $\displaystyle\int_a^b 0\,dx = 0$

(b) $\displaystyle\int_a^b c\,dx = c(b-a)$

(c) $\displaystyle\int_a^b \Big[f(x)+g(x)\Big]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

(d) $\displaystyle\int_a^b \Big[f(x)-g(x)\Big]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

(e) $\displaystyle\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

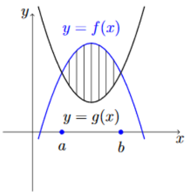
(f) $\displaystyle\int_a^a f(x)\,dx = 0$

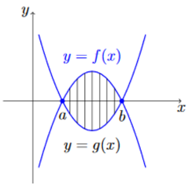
(g) $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

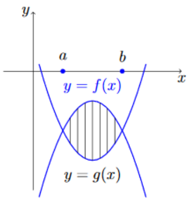
(h) $\displaystyle\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

AREA BETWEEN TWO CURVES

$$\boxed{\int_a^b \Big(f(x)-g(x)\Big)\,dx}$$







EXERCISES [p.14, 17]

1. Find the area under $f(x) = \dfrac{1}{x}$ from $x = \dfrac{1}{2}$ to $x = 1$.

2. Find the area under $f(x) = x + e^{-x}$ from $x = 0$ to $x = 2$.

3. Find the area of the region bounded by the x-axis, $y = \sqrt{x}$ and

$y = 6 - x$.

4. Find the area bounded by the $f(x) = (x-1)(x-2)(x-3)$ and the x –axis.

5. Find the area bounded by the $f(x) = x^4 - x^2$ and the x –axis.

6. Find the area bounded by $f(x) = x + 1$ and $g(x) = x^2 - x - 2$.

7. Find the area bounded by $f(x) = x + 3$ and $g(x) = 12 + x - x^2$.

8. Find the area bounded by $f(x) = 3x + 5$ and $g(x) = x^2 + 1$.

9. Find the area bounded by $f(x) = 3 - x^2$ and $g(x) = 2x^2$.

10. Find the area bounded by $f(x) = x^2$ and $g(x) = 3x$.

11. Using the same axes sketch $y = \sin x$ and $y = \sin 2x$ for $0 \le x \le \pi$. Calculate the smaller of the two areas bounded by the curves.

12. Find the area of the shaded region in the diagram given below.