Mathematics 1

Exercise Sheet 17: Matrices

1. Suppose $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & -3 & 1 \\ 5 & 3 & -1 & 2 \end{bmatrix}$.

   Find the following:

   (a) a_{32} (b) a_{23} (c) the order of A.

2. Evaluate $2\begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 0 \end{bmatrix} - 3\begin{bmatrix} 0 & 2 & 1 \\ 1 & 4 & -3 \end{bmatrix}$.

3. Evaluate $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 6 & 4 \\ 3 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 2 & 3 \end{bmatrix}$.

4. Evaluate

   (a) $\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}\left(\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}\right)$.

5. Suppose $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & -3 & 1 \\ 5 & 3 & -1 & 2 \end{bmatrix}$. Find A^T.

6. Find the following determinants:

   (a) $\begin{vmatrix} 4 & 3 \\ -2 & 5 \end{vmatrix}$ (b) $\begin{vmatrix} a & -b \\ -b & a \end{vmatrix}$.

7. Find the inverses of the following matrices:

   (a) $\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & 3 \\ 2 & -2 \end{bmatrix}$.

8. Find the matrix A such that

   (a) $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} A = \begin{bmatrix} 4 & 2 \\ 6 & 1 \end{bmatrix}$

   (b) $A\begin{bmatrix} -4 & 5 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

   (c) $A\begin{bmatrix} -11 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$.

9. Use matrices to solve the following systems of linear equations:

   (a) $\begin{aligned} 2x - y &= 7 \\ -5x + 3y &= -19 \end{aligned}$

   (b) $\begin{aligned} x + 3y &= -7 \\ 2x + 7y &= -16 \end{aligned}$

   (c) $\begin{aligned} 5x + 4y &= 2 \\ 3x + 2y &= 0 \end{aligned}$

10. (a) What does it mean when we say that a square matrix A is *singular*?

    (b) For what values of t is the matrix $\begin{bmatrix} t & 2 \\ 1 & t-1 \end{bmatrix}$ singular?

11. Does the system

$$\begin{aligned} 2x &+ 3y &= 8 \\ 4x &+ 6y &= 2 \end{aligned}$$

have a *unique* solution? Justify your answer.

12. For what value(s) of p does the system

$$\begin{aligned} x &+ 2y &= 1 \\ 3x &+ py &= p \end{aligned}$$

have a unique solution?

Answers

1. (a) $a_{32} = 3$ (b) $a_{23} = -3$ (c) 3×4

2. $\begin{bmatrix} 2 & -2 & 7 \\ 3 & -4 & 9 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2 \\ 7 & 5 \\ 7 & 8 \end{bmatrix}$

4. (a) $\begin{bmatrix} 6 & 0 \\ -4 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 0 \\ -4 & 0 \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 3 \\ 2 & -3 & -1 \\ 3 & 1 & 2 \end{bmatrix}$

6. (a) 26 (b) $a^2 - b^2$

7. (a) $\frac{1}{6}\begin{bmatrix} 4 & -2 \\ -7 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1.5 \\ -1 & -2 \end{bmatrix}$

8. (a) $\begin{bmatrix} -30 & -1 \\ 22 & 1 \end{bmatrix}$ (b) $-\frac{1}{5}\begin{bmatrix} 7 & 6 \\ 16 & 13 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 6 \\ -8 & -22 \end{bmatrix}$

9. (a) $(x, y) = (2, -3)$ (b) $(x, y) = (-1, -2)$ (c) $(x, y) = (-2, 3)$

10. (a) A is *singular* if $\det(A) = 0$ (i.e. if A does **not** have an inverse).

    (b) $t = 2$ or $t = -1$.

    Full solution

    We need $\begin{vmatrix} t & 2 \\ 1 & t-1 \end{vmatrix} = 0$.

    That is, $t(t-1) - 2 \times 1 = 0$.

    That is, $t^2 - t - 2 = 0$.

    That is, $(t-2)(t+1) = 0$.

    That is, $t = 2$ or $t = -1$.

11. The system does *not* have a *unique* solution because $\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 0$.

12. $p \neq 6$.

    Full solution

    There is a unique solution if and only if $\begin{vmatrix} 1 & 2 \\ 3 & p \end{vmatrix} \neq 0$.

    That is, $p - 6 \neq 0$.

    That is, $p \neq 6$.