SEM1 REVISION

T2REV

Qatar Academy Doha

Mathematics 1

Semester 1 Revision Exercises

1. Solve the following inequations for x:
   (a) $|3x+1| \leq 4$
   (b) $x^2 + 2x - 24 > 0$.

2. Solve $\tan\left(2x - \frac{\pi}{4}\right) + 1 = 0$ for $x \in [0, 2\pi]$.

3. Solve $1 + \sqrt{3}\tan 2\left(x - \frac{\pi}{4}\right) = 0$ for $0 \leq x \leq \pi$.

4. The depth, $D(t)$ metres, of water at the entrance to a harbour at t hours after midnight on a particular day is given by
$$D(t) = 10 + 3\sin(0.5t) \quad \text{for } 0 \leq t \leq 24.$$
   (a) Find, to one decimal place, the depth of the water at 5:00 am.
   (b) Write down the maximum depth of the water.
   (c) Find, to the nearest minute, the time in the afternoon at which the maximum depth occurs.

1

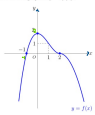
5. Sketch a graph for each of the following equations.
   (a) $y = x^3 - 12x^2 + 45x - 34$
   (b) $y = -(2^x) + 1$
   (c) $y = \log_4(x+2)$
   (d) $y = -\log_3(-x)$
   (e) $y = \sqrt{4 - 2^x}$
   (f) $y = \frac{x}{x-1} + 2$

6. Sketch the graph of $y = x^3 - 8x^2 + 19x - 12$.

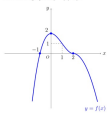
You will need to do careful calculations to find the y–coordinates of the stationary points.

2

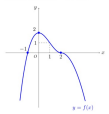
7. (a) The graph of $y = f(x)$ is shown below. Sketch the graph of $y = 2f(x)$.



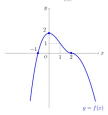
(b) The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f(-x)$.



(c) The graph of $y = f(x)$ is shown below. Sketch the graph of $y = f(x+1)$.



(d) The graph of $y = f(x)$ is shown below. Sketch the graph of $y = \frac{1}{f(x)}$.



3

8. Solve $2^x + 3 \times 2^{-x} = 7$ for x.
Write your answer(s) to 2 decimal places.

9. Consider the functions
$$f : [-5, 5] \longrightarrow \mathbf{R} \quad \text{where } f(x) = 2x + 1$$
and
$$g : (1, \infty) \longrightarrow \mathbf{R} \quad \text{where } g(x) = \frac{1}{x-1}.$$
   (a) For which values of x does $g(f(x))$ exist?
   (b) Find the rule for $g(f(x))$.
   (c) Find the range of $g \circ f$.

10. Consider the functions
$$f : \mathbf{R}^+ \longrightarrow \mathbf{R} \quad \text{where } f(x) = \frac{1}{x}$$
and
$$g : [4, \infty) \longrightarrow \mathbf{R} \quad \text{where } g(x) = 1 + x^2.$$
   (a) Find the rule for $g(f(x))$.
   (b) For which values of x is $g \circ f$ defined?

11. Consider the function
$$f : [0, a] \longrightarrow \mathbf{R} \quad \text{where } f(x) = (x-2)^2.$$
   (a) Find the largest value of a so that f has an inverse function.
   (b) Using this value of a, fully determine f^{-1}.
   That is, find the rule for $f^{-1}(x)$, as well as the domain and range of f^{-1}.

12. Consider the function $f(x) = 2\sin x$.
   (a) Find the domain and range of f, and sketch the graph of $y = f(x)$.
   (b) Does f have an inverse function?
   If f has an inverse function, then
   i. state the domain and range of f^{-1},
   ii. give the rule for f^{-1}, and
   iii. sketch the graph of $y = f^{-1}(x)$.

13. Consider the function $f(x) = 7^x$.
   (a) Find the domain and range of f, and sketch the graph of $y = f(x)$.
   (b) Does f have an inverse function, then
   If f has an inverse function, then
   i. state the domain and range of f^{-1},
   ii. give the rule for f^{-1}, and
   iii. sketch the graph of $y = f^{-1}(x)$.

4

14. Consider the function given by
$$f(x) = \begin{cases} x+3 & \text{if } x \leq 0 \\ 2x-1 & \text{if } x > 0. \end{cases}$$
Is f continuous at $x = 0$?
Give a reason for your answer.

15. Consider the function given by
$$f(x) = \begin{cases} x^2+1 & \text{if } x \leq 1 \\ 2x & \text{if } x > 1. \end{cases}$$
Is f continuous at $x = 1$?
Give a reason for your answer.

16. Find the derivative of
$$f(x) = x^3$$
by using first principles.

17. Differentiate the following functions:
   (a) $f(x) = x^{5.8}$ (b) $f(x) =$
   (c) $f(x) = \sqrt{4-4x^2}$ (d) $f(x) =$
   (e) $f(x) = \cos^2 x$ (f) $f(x) =$
   (g) $f(x) = \sin(3x+2)$ (h) $f(x) =$