Mathematics 1

Exercise Sheet 19: Statistics

For this sheet, where an answer is not an integer and also not an integer number of tenths (e.g. -4.1), ***write the answer to two decimal places****.* *For example, approximate the number* $14.21593\ldots$ *as* 14.22 (2 d.p.) *for this sheet.*

1. In the group stage of the 2010 FIFA World Cup, the number of goals (**G**) scored in each match had the following frequency distribution:

G	0	1	2	3	4	5	6	7	Total
Frequency	6	13	12	9	5	2	0	1	T

   (a) Find the value of T.

   (b) Find the mean of these numbers of goals scored.

   (c) Find the mode of these numbers of goals scored.

   (d) Find the median of these numbers of goals scored.

2. Gaballa and Abraham (2008) investigated the distance covered when various food products were transported from where they were produced to Melbourne supermarkets. Some of the food products and the distances transported (measured in kilometres) were:

   apples 112, oranges 567, bananas 2746, tomatoes 1618, potatoes 155, lettuce 54, carrots 311, and onions 782.

   (a) Without using the statistical functions on your calculator,

   i. find the mean of these distances.

   ii. find the variance of these distances.

   iii. find the standard deviation of these distances.

   (b) Use the statistical functions on your calculator to confirm the values of the mean and the standard deviation found in (a).

   Note: The mean, the standard deviation and the variance of given values can be found by using statistical functions on your calculator. Touch here to see a demonstration video

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Reference: Gaballa, S. & Abraham, A.B. (2008) *Food Miles in Australia: A Preliminary study of Melbourne, Victoria.* East Brunswick: CERES Community Environment Park

3. Suppose that the heights (measured in centimetres) of 12 randomly selected children are as follows:

133	135	129	134
118	218	127	130
127	137	133	132

(a) Find the mean of these heights.

(b) Find the standard deviation of these heights.

(c) Find the variance of these heights.

Hint: variance = $(\text{standard deviation})^2$

(d) Find the median of these heights.

4. The golf scores of 12 people are

82	87	100	105
90	79	88	91
79	89	96	86

(a) Find the mean of these scores.

(b) Find the standard deviation of these scores.

(c) Find the variance of these scores.

(d) Find the range of these scores.

(e) Find the median of these scores.

(f) Find the lower quartile and upper quartile of these scores.

5. A health–conscious commerce student takes her resting pulse each night after eating. The past fortnight's measurements (in beats per minute) were

53	69	53	62
59	61	57	69
60	58	56	64
	61	62	

(a) Find the mean of these measurements.

(b) Find the standard deviation of these measurements.

(c) Find the variance of these measurements.

6. The length (in metres) of 85 snakes is summarised in the following frequency table.

Length (m)	0.6 –	0.8 –	1 –	1.2 –	1.4 –	1.6 – 1.8
Frequency	3	10	35	23	12	2

(a) Construct a cumulative frequency distribution table for the lengths.

(b) Sketch a cumulative frequency curve for the lengths.

(c) Use this curve to estimate the median length of the snakes.

7. The rental rate per week of 2-bedroom dwellings in Carlton was investigated in 40 randomly selected cases from the website *www.realestate.com.au* in July 2010. The rates (in dollars) obtained were:

495	350	460	445	440	495	465	440
520	465	440	445	425	465	438	450
550	485	425	380	440	380	445	438
422	410	425	400	422	445	505	438
325	425	485	445	425	435	425	400

In this exercise, assume that rental rate is a continuous variable.

(a) Construct a frequency distribution table for the rental rates.

(b) Sketch a cumulative frequency curve for the rental rates.

(c) Use this curve to estimate the proportion of 2-bedroom dwellings for which the rental was

(i) less than \$450 per week

(ii) more than \$430 per week.

Revision Exercises on Derivatives

Reference: Chapter 9.

8. Differentiate the following functions:

(a) $f(x) = 3x^2 + \dfrac{2}{5x} - 11$ (b) $f(x) = 3x + 2e^x$

(c) $f(x) = e^3 + \dfrac{1}{x^2}$ (d) $f(x) = x^2 \tan x$

(e) $f(x) = e^x \ln x$ (f) $f(x) = x^2 e^{3x}$

(g) $f(x) = x\sqrt{1+3x}$ (h) $f(x) = \dfrac{x}{\ln x}$

(i) $f(x) = \dfrac{\cos x}{e^x}$ (j) $f(x) = \dfrac{\sin x}{x+1}$

(k) $f(x) = \dfrac{e^{x-1}}{x^2+1}$ (l) $f(x) = \dfrac{x^2+1}{2x+1}$

(m) $f(x) = (1-2x^2)^7$ (n) $f(x) = \sqrt{8x+x^4}$

(o) $f(x) = \dfrac{x^3}{\sqrt{8x+1}}$ (p) $f(x) = e^{3x+2e^x}$

(q) $f(x) = xe^{-x^2+1}$ (r) $f(x) = \ln(12x^2+11x)$

(s) $f(x) = \ln\sqrt{x^2-1}$ (t) $f(x) = e^{\tan x}$

(u) $f(x) = \ln(\sin x)$ (v) $f(x) = x\cos x - \ln(x^2)$

(w) $f(x) = \sin(e^x+1)$ (x) $f(x) = \sin^{-1}(3x)$

(y) $f(x) = x^2\cos^{-1} x$ (z) $f(x) = \tan^{-1}(x^2)$.

9. Using implicit differentiation, find $\dfrac{dy}{dx}$ for

(a) $y + xe^y = x + 1$ (b) $xy^3 - y^2 = x^2$.

Answers:

1. (a) $T = 48$
   (b) The mean is 2.10 (2 d.p.).
   (c) The mode is 1.
   (d) The median is 2.

2. (a) (i) The mean is 793.13 km (2 d.p.).
   (ii) The variance is 885 040.13 km^2 (2 d.p.).
   (iii) The standard deviation is 940.77 km (2 d.p.).
   (b) Omitted.

3. (a) The mean is 137.75 cm.
   (b) The standard deviation is 25.76 cm (2 d.p.).
   (c) The variance is 663.48 cm^2 (2 d.p.).
   (d) The median is 132.5 cm.

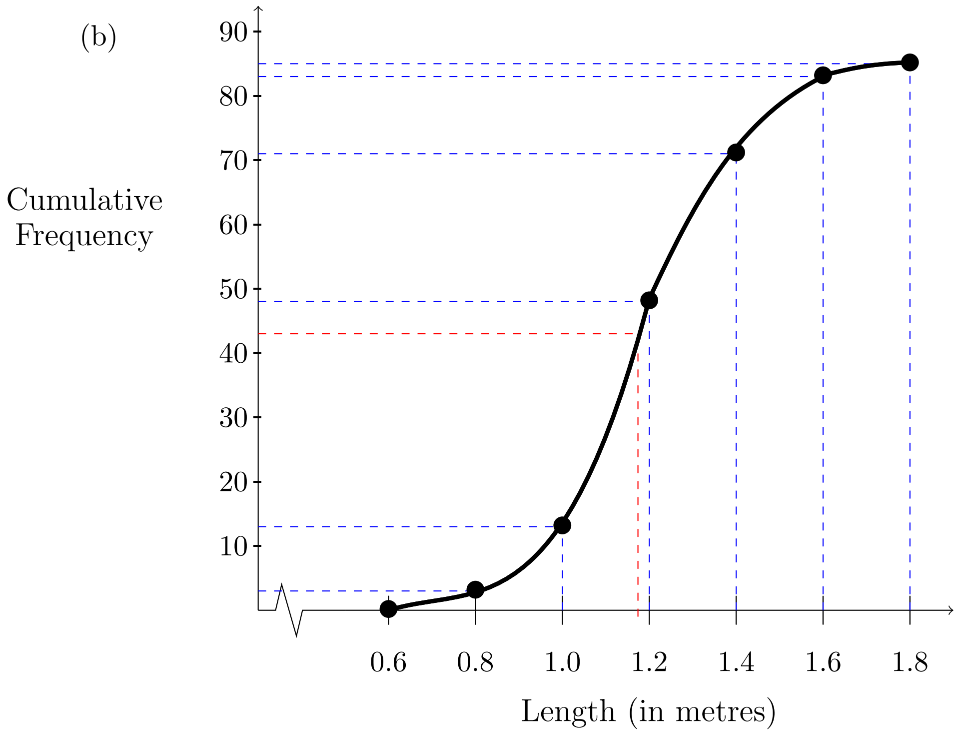
4. (a) The mean is 89.33 (2 d.p.).
   (b) The standard deviation is 7.94 (2 d.p.).
   (c) The variance is 62.97 (2 d.p.)
   (d) The range is 26.
   (e) The median is 88.5.
   (f) The lower quartile is 84, and the upper quartile is 93.5.

5. (a) The mean is 60.29 beats per minute (2 d.p.).
   (b) The standard deviation is 4.92 beats per minute (2 d.p.).
   (c) The variance is 24.22 (beats per minute)^2, to 2 d.p.

6. (a)

Length (in metres)	Cumulative frequency
< 0.6	0
< 0.8	3
< 1.0	13
< 1.2	48
< 1.4	71
< 1.6	83
≤ 1.8	85

(b)



(c) The median length is ***approximately*** 1.17 metres (obtained by looking at the red dashed line).

7. (a) The answer depends on the intervals chosen for the distribution. For example, *two of the many* possibilities are shown here:

Weekly Rent (in dollars)	Frequency
300–	1
350–	3
400–	24
450–	9
500 – 550	3
Total	40

or

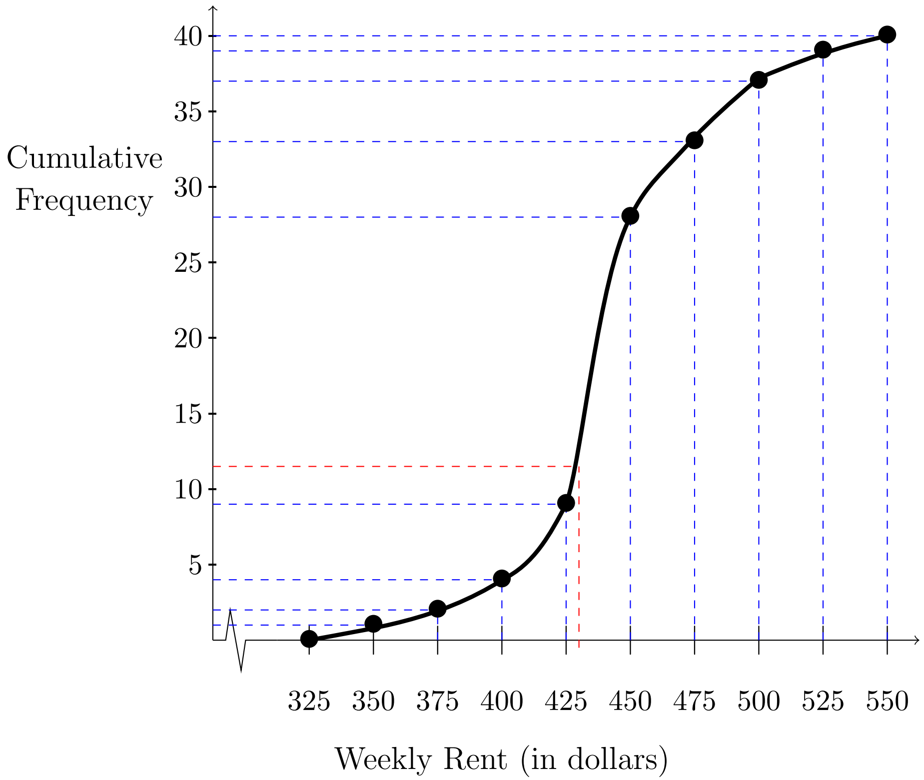
Weekly Rent (in dollars)	Frequency
325–	1
350–	1
375–	2
400–	5
425–	19
450–	5
475–	4
500–	2
525 – 550	1
Total	40

Other tables are also possible!

(b) The answer depends on the intervals chosen for the distribution. For example, *one of the many* possible cumulative frequency distribution tables (based on the second of the two frequency distribution tables shown above) is as follows:

Weekly Rent (in dollars)	Cumulative frequency
< 325	0
< 350	1
< 375	2
< 400	4
< 425	9
< 450	28
< 475	33
< 500	37
< 525	39
≤ 550	40

This particular cumulative frequency distribution table leads to the following cumulative frequency curve (although other curves are also possible).



(c) Answers will be ***approximately***

(i) 70 % (since 28 of the 40 rents were less than \$450).

(ii) 70 % (since the red dashed line shows that approximately 12 of the 40 rents were less than or equal to \$430, which means that the other (approximately) 28 of the 40 rents were more than \$430.

These answers are ***approximate*** since they depend on the particular curve drawn.

8. (a) $f'(x) = 6x - \dfrac{2}{5x^2}$ (b) $f'(x) = 3 + 2e^x$

(c) $f'(x) = -\dfrac{2}{x^3}$ (d) $f'(x) = 2x\tan x + x^2\sec^2 x$

(e) $f'(x) = e^x\left(\ln x + \dfrac{1}{x}\right)$ (f) $f'(x) = xe^{3x}(2+3x)$

(g) $f'(x) = \sqrt{1+3x} + \dfrac{3x}{2\sqrt{1+3x}}$ (h) $f'(x) = \dfrac{(\ln x) - 1}{(\ln x)^2}$

(i) $f'(x) = -\dfrac{\sin x + \cos x}{e^x}$ (j) $f'(x) = \dfrac{x\cos x + \cos x - \sin x}{(x+1)^2}$

(k) $f'(x) = \dfrac{e^{x-1}(x-1)^2}{(x^2+1)^2}$ (l) $f'(x) = \dfrac{2(x^2+x-1)}{(2x+1)^2}$

(m) $f'(x) = -28x(1-2x^2)^6$ (n) $f'(x) = \dfrac{4+2x^3}{\sqrt{8x+x^4}}$

(o) $f'(x) = \dfrac{20x^3+3x^2}{(8x+1)^{\frac{3}{2}}}$ (p) $f'(x) = (3+2e^x)e^{3x+2e^x}$

(q) $f'(x) = e^{-x^2+1}(1-2x^2)$ (r) $f'(x) = \dfrac{24x+11}{12x^2+11x}$

(s) $f'(x) = \dfrac{x}{x^2-1}$ (t) $f'(x) = \sec^2 x\, e^{\tan x}$

(u) $f'(x) = \cot x$ (v) $f'(x) = -x\sin x + \cos x - \dfrac{2}{x}$

(w) $f'(x) = e^x\cos(e^x+1)$ (x) $f'(x) = \dfrac{3}{\sqrt{1-9x^2}}$

(y) $f'(x) = 2x\cos^{-1} x - \dfrac{x^2}{\sqrt{1-x^2}}$ (z) $f'(x) = \dfrac{2x}{1+x^4}$

9. (a) $\dfrac{dy}{dx} = \dfrac{1-e^y}{1+xe^y}$ (b) $\dfrac{dy}{dx} = \dfrac{2x-y^3}{3y^2x-2y}$