SAMPLE 1

Sunday, 14 June 2020 1:15 pm

sample semester 1

TRINITY COLLEGE
THE UNIVERSITY OF MELBOURNE

30 YEARS FOUNDATION STUDIES

FEBRUARY MAIN PROGRAM

Mathematics 2

Semester 1 Sample Examination #1

Reading Time: 10 minutes
Writing Time: 3 hours
Weighting: 30%

Instructions to Candidates:

On the following 3 pages there are 11 questions.
Write your answers to these questions in the booklets provided.
You are required to show working for each of your solutions.
The total marks available is 66.
A formula sheet is provided.

Materials Allowed:

Calculators and dictionaries are not allowed.

Examiner's Use Only:

THE EXAMINATION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM.

1. Let b represent the statement "I ride a bike", t represent the statement "I take a tram", and w represent the statement "I walk". Analyse the logical forms of the following statements:

(a) If I take a tram then I walk.
(b) Either I take a tram and I walk, or I ride a bike.
(c) If I do not ride a bike then I take a tram, but I take a tram if and only if I walk.

With b, t and w having the same meaning as above, what English sentences are represented by the following expressions?

(d) $\neg b \vee \neg t$
(e) $\neg b \vee (t \vee w)$

[1+1+1+1+1=5 marks]

2. (a) Construct a truth table for $(\neg(p \vee q) \rightarrow r) \leftrightarrow ((\neg p \rightarrow q) \vee r)$.
(b) Is this formula a tautology? Explain your answer.

[5+2=7 marks]

3. Given the complex number $z = -1 + i$

(a) work in Cartesian form to simplify $\frac{1}{z}$
(b) convert z to polar form
(c) hence use De Moivre's theorem to find z^6 (give your answer in Cartesian form)

[2+3+3=8 marks]

4. Sketch the region of the Argand plane represented by all $z \in C$ such that

$|z| < 4$ and p.v. $\arg(z) \geq \frac{\pi}{2}$

[5 marks]

5. Use a suitable inverse matrix to solve the following matrix equation for A:

$$A\begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} - 2\begin{bmatrix} -2 & 4 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 9 & -2 \end{bmatrix}$$

[5 marks]

6. Consider the transformation given by the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

(a) Find the image of the point $(-2, 3)$ under this transformation.
(b) If the image of a point under this transformation is $(1, -4)$, find the object.
(c) Find the image of the line $y = 3x + 7$ under this transformation.

[1+2+3=6 marks]

7. Find an equation for the curve with centre $(0, 0)$, a focus at $(4, 0)$, and a vertex at $(-1, 0)$

[6 marks]

8. (a) If the coordinates of a triangle are $A(1, 2, 3)$, $B(-1, 0, 2)$ and $C(2, -1, -1)$ find the cosine of the angle between the median at A and an edge at A.
(b) If the length of the median is 3 units what is the distance from the centroid to the vertex?

[6+2=8 marks]

9. Given the vectors $q = -\underline{i} + 2\underline{j} + 5\underline{k}$, $b = -\underline{i} + 2\underline{j} + 2\underline{k}$, $c = 4\underline{j} - 3\underline{j} + x\underline{k}$ where $x \in R$, find

(a) the vector projection of $\underline{b}$ onto $\underline{q}$
(b) the component of $\underline{b}$ perpendicular to $\underline{q}$
(c) the value of x so that $\underline{q}$ and $\underline{c}$ are orthogonal

[3+2+3=8 marks]

10. Determine the output of the following Octave code.
Remember that your answer should be presented exactly as it would be in Octave.

```
A=89
B=floor(A/10)
if A>=0 & A<=99 & B>10
  C = A - 10*B
else
  D = 1
end
```

[3 marks]

11. Determine the output of the Octave code

```
-1-2*3&&4-5|6<=7*-1
```

by copying and completing the following table into your answer booklet.
Each row of the table should eliminate one operator only, according to Octave's order of precedence. For example, in the first row the – operator has been eliminated.

-1-2*3&&4-5|6<=7*-1
ans =

[5 marks]

END OF EXAM.