TRINITY COLLEGE
THE UNIVERSITY OF MELBOURNE
FOUNDATION STUDIES

Family name: Chuah
Given name(s): Xin Yu
Student ID number: 961919

February Main 2020 Mathematics 1

COVER PAGE

Assignment 2 (8%)

Due Date and Time: **Monday 24th August, 5:30pm**

Assignment Overview

This assignment contains three questions from the topic of **Linear Approximations**. You are required to answer each question in the assignment. You will be marked on the correctness of your responses, your working and the quality of your mathematical communication (as described in the Mathematics 1 Assignment Guide for Students available on the TCOLE page).

Examiners Only	
1.	
2.	
3.	
Q.	
Total	
	/33
	%

Assignment Instructions

Format: Your assignment must be submitted as a single portable document format (.pdf) file of no more than 10 pages in length and 10MB in size. Pictures, graphs, etc. must be included in the same file as your assignment responses. Your responses may be typed or handwritten in either pen or pencil. Check that your file opens properly on your iPad (e.g. with GoodReader). If you cannot open your .pdf file on your iPad your marker will also be unable to open it.

Submission: Include this cover page in your submission (it does not count towards the page limit). You must submit your assignment via TCOLE. Please write your name and student number on each page of your submission. You must only submit responses to the assignment version you have been allocated on TCOLE. Submitting the wrong version will incur a marks penalty of 10%.

Late Submission: Late submissions will receive a marks penalty of 10% per day past the due date. Assignments will not be accepted more than four days past the due date.

Academic integrity: This assignment must be completed in accordance with the TCFS Academic Integrity Policy.

Question 1a

Let $L(x)$ be the linear approximation of $f(x)$ at $x = \frac{\pi}{2}$

$$f(x) = 40 + 4x\cos(x)$$

$$\begin{aligned} f'(x) &= 0 + udv + vdu \text{ , where } u = 4x \text{ and } v = \cos(x) \\ &= 0 + (4x)(-\sin(x)) + (\cos(x))(4) \\ &= -4x\sin(x) + 4\cos(x) \end{aligned}$$

$$\begin{aligned} f(a) &= f(\frac{\pi}{2}) \\ &= 40 + 4(\frac{\pi}{2})\cos(\frac{\pi}{2}) \\ &= 40 + 2\pi(0) \\ &= 40 \end{aligned}$$

$$\begin{aligned} f'(a) &= f'(\frac{\pi}{2}) \\ &= -4(\frac{\pi}{2})\sin(\frac{\pi}{2}) + 4\cos(\frac{\pi}{2}) \\ &= -2\pi(1) + 4(0) \\ &= -2\pi \end{aligned}$$

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= 40 - 2\pi(x - \frac{\pi}{2}) \\ &= -2\pi x + \pi^2 + 40 \end{aligned}$$

$$\boxed{\therefore L(x) = -2\pi x + \pi^2 + 40}$$

Question 1b

Let $Q(x)$ be the quadratic approximation of $f(x)$ at $x = \frac{\pi}{2}$

$$f(x) = 40 + 4x\cos(x)$$

From Question 1(a):

$$\begin{aligned} f'(x) &= -4x\sin(x) + 4\cos(x) \\ f(a) &= 40 \\ f'(a) &= -4 \\ L(x) &= -2\pi x + \pi^2 + 40 \end{aligned}$$

$$\begin{aligned} f''(x) &= f'(-4x\sin(x) + 4\cos(x)) \\ &= udv + vdu - 4\sin(x), \text{ where } u = -4x \text{ and } v = \sin(x) \\ &= (-4x)(\cos(x)) + (\sin(x))(-4) - 4\sin(x) \\ &= -4x\cos(x) - 4\sin(x) - 4\sin(x) \\ &= -4x\cos(x) - 8\sin(x) \end{aligned}$$

$$\begin{aligned} f''(a) &= f''(\frac{\pi}{2}) \\ &= -4x\cos(\frac{\pi}{2}) - 8\sin(\frac{\pi}{2}) \\ &= -2\pi(0) - 8(1) \\ &= -8 \end{aligned}$$

$$\begin{aligned} Q(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2} \\ &= L(x) + \frac{-8(x-\frac{\pi}{2})^2}{2} \\ &= -2\pi x + \pi^2 + 40 - 4(x - \frac{\pi}{2})(x - \frac{\pi}{2}) \\ &= -2\pi x + \pi^2 + 40 - 4x^2 + 4\pi x - \pi^2 \\ &= -4x^2 + 2\pi x + 40 \end{aligned}$$

$$\boxed{\therefore Q(x) = -4x^2 + 2\pi x + 40}$$

Question 2a

Values

x	f(x)	L(x)	Q(x)	|f(x)-L(x)|	|f(x)-Q(x)|
0.5	41.7551651	46.7280117	42.1415927	4.9728466	0.3864275
0.6	41.9808055	46.0996932	42.3299112	4.1188877	0.3491057
0.7	42.1415581	45.4713747	42.4382297	3.3298166	0.2966716
0.8	42.2294615	44.8430562	42.4665482	2.6135947	0.2370868
0.9	42.2377959	44.2147376	42.4148668	1.9769417	0.1770709
1	42.1612092	43.5864191	42.2831853	1.4252099	0.1219761
1.1	41.9958229	42.9581006	42.0715038	0.9622776	0.0756809
1.2	41.7393172	42.3297820	41.7798224	0.5904648	0.0405051
1.3	41.3909939	41.7014635	41.4081409	0.3104696	0.0171470
1.4	40.9518160	41.0731450	40.9564594	0.1213290	0.0046434
1.5	40.4244232	40.4448264	40.4247780	0.0204032	0.0003548
1.6	39.8131231	39.8165079	39.8130965	0.0033849	0.0000266
1.7	39.1238574	39.1881894	39.1214150	0.0643319	0.0024424
1.8	38.3641449	38.5598708	38.3497336	0.1957259	0.0144114
1.9	37.5429993	37.9315523	37.4980521	0.3885530	0.0449472
2	36.6708253	37.3032338	36.5663706	0.6324085	0.1044547
2.1	35.7592927	36.6749153	35.5546891	0.9156225	0.2046036
2.2	34.8211902	36.0465967	34.4630077	1.2254066	0.3581825
2.3	33.8702606	35.4182782	33.2913262	1.5480176	0.5789344
2.4	32.9210203	34.7899597	32.0396447	1.8689393	0.8813756
2.5	31.9885638	34.1616411	30.7079633	2.1730773	1.2806006

Name: Chuah Xin Yu
Student ID: 961919
Question: 2(a)

Question 2b

Formulas

x	f(x)	L(x)	Q(x)	|f(x)-L(x)|	|f(x)-Q(x)|
0.5	=40+4*A2*COS(A2)	=-2*PI()*A2+(PI()^2)+40	=-4*(A2^2)+2*PI()*(A2)+40	=ABS(B2-C2)	=ABS(B2-D2)
0.6	=40+4*A3*COS(A3)	=-2*PI()*A3+(PI()^2)+40	=-4*(A3^2)+2*PI()*(A3)+40	=ABS(B3-C3)	=ABS(B3-D3)
0.7	=40+4*A4*COS(A4)	=-2*PI()*A4+(PI()^2)+40	=-4*(A4^2)+2*PI()*(A4)+40	=ABS(B4-C4)	=ABS(B4-D4)
0.8	=40+4*A5*COS(A5)	=-2*PI()*A5+(PI()^2)+40	=-4*(A5^2)+2*PI()*(A5)+40	=ABS(B5-C5)	=ABS(B5-D5)
0.9	=40+4*A6*COS(A6)	=-2*PI()*A6+(PI()^2)+40	=-4*(A6^2)+2*PI()*(A6)+40	=ABS(B6-C6)	=ABS(B6-D6)
1	=40+4*A7*COS(A7)	=-2*PI()*A7+(PI()^2)+40	=-4*(A7^2)+2*PI()*(A7)+40	=ABS(B7-C7)	=ABS(B7-D7)
1.1	=40+4*A8*COS(A8)	=-2*PI()*A8+(PI()^2)+40	=-4*(A8^2)+2*PI()*(A8)+40	=ABS(B8-C8)	=ABS(B8-D8)
1.2	=40+4*A9*COS(A9)	=-2*PI()*A9+(PI()^2)+40	=-4*(A9^2)+2*PI()*(A9)+40	=ABS(B9-C9)	=ABS(B9-D9)
1.3	=40+4*A10*COS(A10)	=-2*PI()*A10+(PI()^2)+40	=-4*(A10^2)+2*PI()*(A10)+40	=ABS(B10-C10)	=ABS(B10-D10)
1.4	=40+4*A11*COS(A11)	=-2*PI()*A11+(PI()^2)+40	=-4*(A11^2)+2*PI()*(A11)+40	=ABS(B11-C11)	=ABS(B11-D11)
1.5	=40+4*A12*COS(A12)	=-2*PI()*A12+(PI()^2)+40	=-4*(A12^2)+2*PI()*(A12)+40	=ABS(B12-C12)	=ABS(B12-D12)
1.6	=40+4*A13*COS(A13)	=-2*PI()*A13+(PI()^2)+40	=-4*(A13^2)+2*PI()*(A13)+40	=ABS(B13-C13)	=ABS(B13-D13)
1.7	=40+4*A14*COS(A14)	=-2*PI()*A14+(PI()^2)+40	=-4*(A14^2)+2*PI()*(A14)+40	=ABS(B14-C14)	=ABS(B14-D14)
1.8	=40+4*A15*COS(A15)	=-2*PI()*A15+(PI()^2)+40	=-4*(A15^2)+2*PI()*(A15)+40	=ABS(B15-C15)	=ABS(B15-D15)
1.9	=40+4*A16*COS(A16)	=-2*PI()*A16+(PI()^2)+40	=-4*(A16^2)+2*PI()*(A16)+40	=ABS(B16-C16)	=ABS(B16-D16)
2	=40+4*A17*COS(A17)	=-2*PI()*A17+(PI()^2)+40	=-4*(A17^2)+2*PI()*(A17)+40	=ABS(B17-C17)	=ABS(B17-D17)
2.1	=40+4*A18*COS(A18)	=-2*PI()*A18+(PI()^2)+40	=-4*(A18^2)+2*PI()*(A18)+40	=ABS(B18-C18)	=ABS(B18-D18)
2.2	=40+4*A19*COS(A19)	=-2*PI()*A19+(PI()^2)+40	=-4*(A19^2)+2*PI()*(A19)+40	=ABS(B19-C19)	=ABS(B19-D19)
2.3	=40+4*A20*COS(A20)	=-2*PI()*A20+(PI()^2)+40	=-4*(A20^2)+2*PI()*(A20)+40	=ABS(B20-C20)	=ABS(B20-D20)
2.4	=40+4*A21*COS(A21)	=-2*PI()*A21+(PI()^2)+40	=-4*(A21^2)+2*PI()*(A21)+40	=ABS(B21-C21)	=ABS(B21-D21)
2.5	=40+4*A22*COS(A22)	=-2*PI()*A22+(PI()^2)+40	=-4*(A22^2)+2*PI()*(A22)+40	=ABS(B22-C22)	=ABS(B22-D22)

Name: Chuah Xin Yu
Student ID: 961919
Question: 2(b)

Question 2c

Referring to the sheet of Values created in response to Question 2(a), the x-value that corresponds to the smallest error when $L(x)$ is used to approximate $f(x)$ is 1.6 with an error of 0.0033849 and the x-value with the second smallest error $L(x)$ is used to approximate $f(x)$ is 1.5 with an error of 0.0204032. This is because $y = L(x)$ is the equation of the tangent of $y = f(x)$ at $x = \frac{\pi}{2} = 1.570796327$ and the closer the value of x to the point where the tangent meets the function (at $x = \frac{\pi}{2} = 1.570796327$) the smaller the error.

Question 2d

Referring to the sheet of Values created in response to Question 2(a), Q has less associated error when approximating f on the interval $[0.5, 2.5]$ because the value of errors when $Q(x)$ is used to approximate $f(x)$ is less than the value of errors when $L(x)$ is used to approximate $f(x)$. For example, the smallest error when $Q(x)$ is used to approximate $f(x)$ (at $x = 1.6$) is 0.0003548 and this is approximately 0.2036772 less than the smallest error when $L(x)$ is used to approximate $f(x)$ (at $x = 1.6$) which is 0.0204032.

Question 3a

Let $P(x)$ be the linear approximation of h(x) at $x = 0.05$

$$h(x) = 100 + 10x\sin(10\pi x)$$

$$\begin{aligned} h'(x) &= 0 + udv + vdu, \text{ where } u = 10x \text{ and } v = \sin(10\pi x) \\ &= 0 + (10x)(10\pi)(\cos(10\pi x)) + (sin(10\pi x))(10) \\ &= 100\pi x\cos(10\pi x) + 10\sin(10\pi x) \\ &= 10(10\pi x\cos(10\pi x) + \sin(10\pi x)) \end{aligned}$$

$$\begin{aligned} h(a) &= h(0.05) \\ &= 100 + 10(0.05)\sin(10\pi(0.05)) \\ &= 100 + 0.5\sin(\frac{\pi}{2}) \\ &= 100 + 0.5(1) \\ &= 100.5 \end{aligned}$$

$$\begin{aligned} h'(a) &= h'(0.05) \\ &= 10(10\pi(0.05)\cos(10\pi(0.05)) + \sin(10\pi(0.05))) \\ &= 10(\frac{\pi}{2}\cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})) \\ &= 10(\frac{\pi}{2}(0) + (1) \\ &= 10 \end{aligned}$$

$$\begin{aligned} P(x) &= h(a) + h'(a)(x - a) \\ &= 100.5 + 10(x - 0.05) \\ &= 100.5 + 10x - 0.5 \\ &= 100 + 10x \\ &= 10(x + 10) \end{aligned}$$

$$\boxed{\therefore P(x) = 10(x + 10)}$$

Question 3b

Values

x	h(x)	P(x)	|h(x)-P(x)|
-0.74	92.9621818	92.6000000	0.3621818
0.055	100.5432286	100.5500000	0.0067714
0.85	108.5000000	108.5000000	0

Name: Chuah Xin Yu
Student ID: 961919
Question: 3(b)

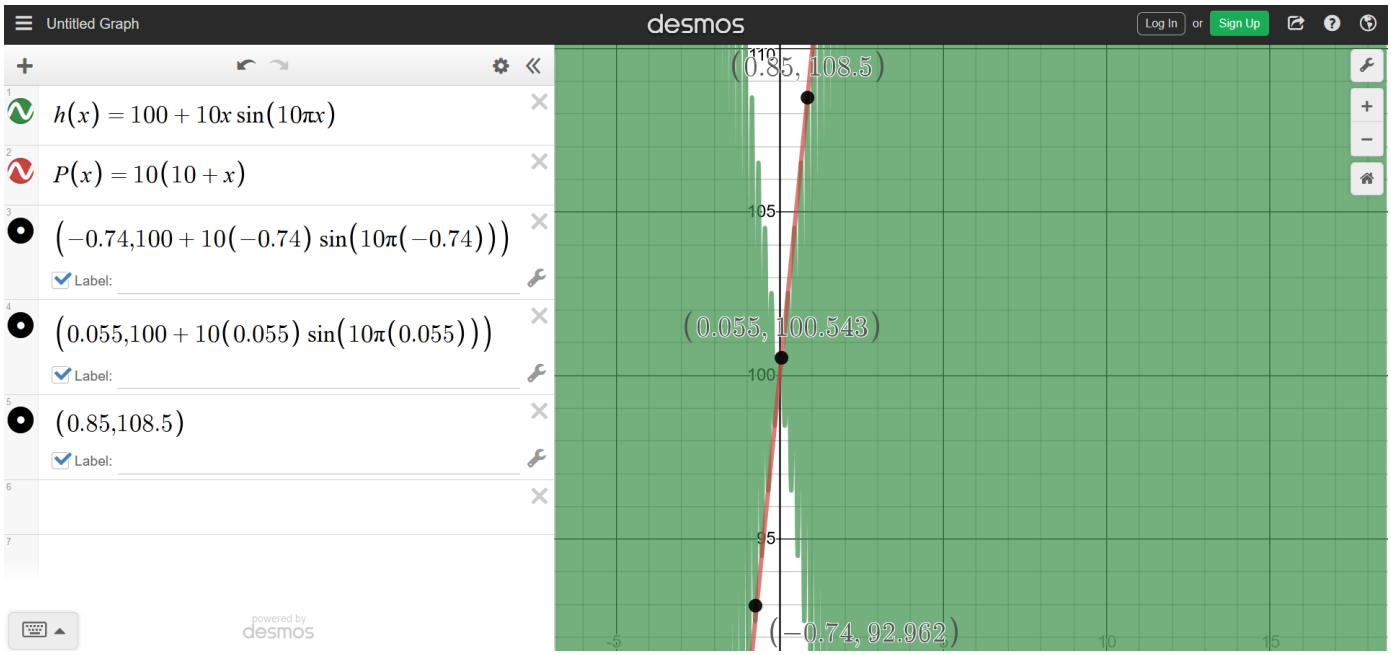
Question 3c

Formula

x	h(x)	P(x)	|h(x)-P(x)|
-0.74	=100+10*A2*SIN(10*PI()*A2)	=10*(A2+10)	=ABS(B2-C2)
0.055	=100+10*A3*SIN(10*PI()*A3)	=10*(A3+10)	=ABS(B3-C3)
0.85	=100+10*A4*SIN(10*PI()*A4)	=10*(A4+10)	=ABS(B4-C4)

Name: Chuah Xin Yu
Student ID: 961919
Question: 3(c)

Question 3d



Question 3e

The errors in the table in Question 3(b) do meet Edgar's current project requirements as they are all less than 1. Referring to the sheet of Values created in response to Question 3(b), the x-value that creates the smallest error when $P(x)$ is used to approximate $h(x)$ is 0.85 with an error of 0. The error of 0 indicates that the point created at $x = 0.85$ is a point of intersection between the line of $y = h(x)$ and $y = P(x)$.

Link to Microsoft Excel document for Questions 2(a), 2(b), 3(b), 3(c) and 3(d).