CHAPTER 2

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M2 Chap2 M2 CHAP3

CHAPTER 2 COMPLEX NUMBERS

IMAGINARY NUMBERS I

imaginary number, i

$i = \sqrt{-1}$

$i^2 = -1$

real number, c

$\sqrt{-c} = \sqrt{c \times -1} = \sqrt{c}\sqrt{-1} = \sqrt{c}\, i$

COMPLEX NUMBERS

complex number, z

$z = a + bi$ *CARTESIAN FORM

a: real part, $\text{Re } z = a$; b: imaginary part, $\text{Im } z = b$

set of complex numbers

$C = \{z : z = x + yi,\ x, y \in \mathbb{R}\}$

All real numbers are complex

$c = c + 0i$

Real numbers are a subset of complex numbers

Example 4.1

Factorise within C via CTS & solve $P(z) = 0$

$P(z) = z^2 - 4z + 13$

$0 = (z-2)^2 - 4 + 13$

$= (z-2)^2 + 9$

$= (z-2)^2 + 9$

$= (z-2)^2 - (-3)^2$

$= (z-2)^2 - (3i)^2$

$= (z-2-3i)(z-2+3i)$

$z-2-3i = 0$ $\quad z-2+3i = 0$

$z = 2+3i$ $\quad z = 2-3i$

$\therefore z = 2 \pm 3i$

via quadratic formula

$P(z) = z^2 - 4z + 13$, $a=1$, b, c

$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{4 \pm \sqrt{16-4(1)(13)}}{2}$

$= \frac{4 \pm \sqrt{-36}}{2}$

$= \frac{4 \pm 6i}{2}$

$= 2 \pm 3i$

BASIC OPERATIONS (CARTESIAN FORM)

EQUALITY

$a+bi = c+di$ $\quad a=c$, $b=d$

ADDITION

$(a+bi) \pm (c+di) = (a \pm c) + (b \pm d)i$

MULTIPLICATION

$(a+bi)(c+di) = (ac-bd) + (ad+bc)i$

FRACTIONS~rationalise

Example 4.2 II

$\frac{4+5i}{2-3i} = \frac{(4+5i)(2+3i)}{(2-3i)(2+3i)}$

$= \frac{(8-15)+(12+10)i}{4+9}$

$= -\frac{7}{13} + \frac{22}{13}i$

COMPLEX CONJUGATE

$z = x+yi$

COMPLEX CONJUGATE of z: