CHAPTER 6: ELLIPSES AND HYPERBOLAS

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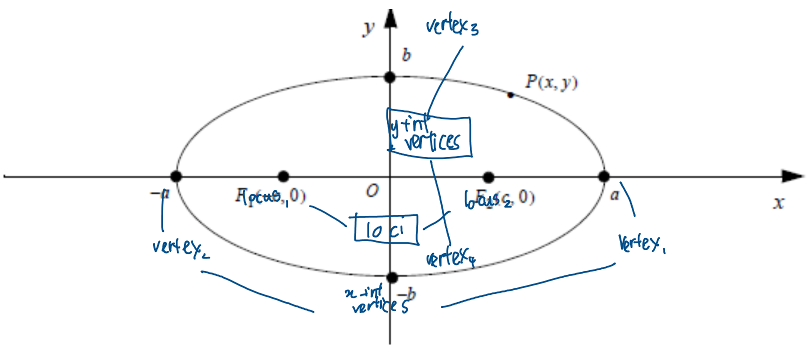
chapter 6

ELLIPSE

Cartesian equation of an ellipse with centre (h, k): $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

If $a > b$ the ellipse has vertices $(h \pm a, k)$, foci $(h \pm c, k)$ and eccentricity $e = \frac{c}{a}$, where $c = \sqrt{a^2 - b^2}$.

If $b > a$ the ellipse has vertices $(h, k \pm b)$, foci $(h, k \pm c)$ and eccentricity $e = \frac{c}{b}$, where $c = \sqrt{b^2 - a^2}$.



HYPERBOLA

Cartesian equation of a hyperbola with centre (h, k) is either

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ with vertices $(h \pm a, k)$, foci $(h \pm c, k)$,

asymptotes $(y-k) = \pm\left(\frac{b}{a}\right)(x-h)$,

or

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ with vertices $(h, k \pm a)$, foci $(h, k \pm c)$,

asymptotes $(y-k) = \pm\left(\frac{a}{b}\right)(x-h)$.

In both cases $c = \sqrt{a^2 + b^2}$.

	Horizontal Orientation	Vertical Orientation	Notes
Ellipse	$a > b$	$b > a$	a^2 is in the denominator of the x^2 term.
Hyperbola	x^2 term has a positive coefficient	y^2 term has a positive coefficient	a^2 is in the denominator of the term with the positive coefficient.

Note: "Horizontal Orientation" means that important points (focus, centre, vertices) lie on a horizontal line; "Vertical Orientation" means that important points lie on a vertical line.

Note: c measures the distance between centre and focus in **all** cases. This is particularly useful for questions 6 and 7 in the tutorial.

Important: When you sketch a hyperbola take care to ensure that the branches of the hyperbola *approach* the asymptotes.