Chapter 17

Permutations and Combinations

The Multiplication Principle.

If one operation, A, can be performed in m different ways, and a second operation, B, can be performed in n different ways, then

$$\boxed{\text{the number of ways of performing the joint operation “}A\ \textbf{and}\ B\text{” is } m \times n.}$$

The Addition Principle.

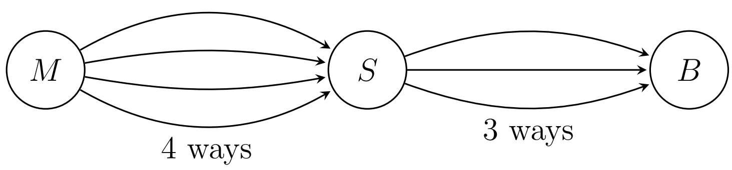
If one operation, A, can be performed in m different ways, and a second operation, B, can be performed in n different ways, then

$$\boxed{\text{the number of ways of performing the joint operation “}A\ \textbf{or}\ B\text{” is } m + n.}$$

Example 1. If there are 4 ways to travel from Melbourne to Sydney, and 3 ways to travel from Sydney to Brisbane, then how many ways are there to travel from Melbourne to Brisbane via Sydney?

Solution:

We need to travel from Melbourne to Sydney, **and** then from Sydney to Brisbane.



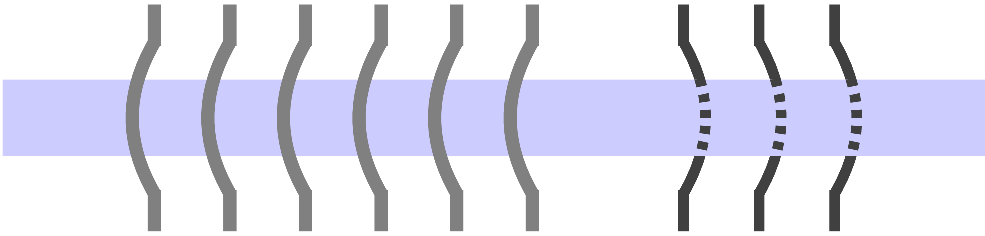
So the number of ways to travel from Melbourne to Brisbane via Sydney is

$$4 \times 3 = 12. \qquad \square$$

Example 2. Suppose that there are 6 bridges over a particular river and 3 tunnels under the river. How many ways are there to cross the river?

Solution:

We can choose one of the 6 bridges, **or** one of the 3 tunnels.



So the number of ways to cross the river is

$$6 + 3 = 9.$$

□

The words "**permutation**" and "**arrangement**" refer to examples in which objects are *arranged.* In particular, the **order** in which the objects are arranged is important.

- For example, the number 29 is different from the number 92.

We will study permutations in Section 17.1.

The word "**combination**" refers to an example in which objects are *selected* (or chosen). The **order** in which the objects are chosen is **not** important.

- For example, if I say 'My favourite numbers are 2 and 9', the order is **not** important.

We will study combinations in Section 17.4.

17.1 Arrangements (i.e. Permutations)

Note: In the following examples, the **order is important**.

Example 3. How many ways are there of arranging the letters A, C and T?

Solution:

The arrangements are:

$$\begin{array}{cc} ACT & ATC \\ CAT & CTA \\ TAC & TCA \end{array}$$

We can see (simply by counting these listed possibilities) that there are

6 arrangements.

Alternatively, we can find out how many arrangements there are without having to list them all. This is a useful skill because the approach of writing all the possibilities, and then counting them, is very time-consuming and tedious for the 'larger' examples. We first demonstrate with a 'small' example, by solving Example 3 again:

We draw a box for each position, then fill in the number of choices available for each.

$$\boxed{3}\boxed{2}\boxed{1}$$

3 choices for first position (A, C or T)

After filling 1st position, there are only **2** choices (of a letter) for the 2nd position

After filling both of the other positions, there is only **1** choice (of a letter) for the final position

We need to put a letter into the first position, **and** put a letter into the second position, **and** put a letter into the final position. So the number of arrangements is given by

$$3 \times 2 \times 1 \;=\; 6$$

(which is the same answer as obtained previously).

□

Example 4. How many arrangements are there of 4 people in a row?

Solution:

There are 4 possible people for the first position. After filling that position there will only be 3 possible people remaining to choose from for the second position. Then there will only be 2 people left to choose from for the third position, which only leaves one choice for the fourth position:

4	3	2	1

Because we need to put someone into the first position
and put someone into the second position
and put someone into the third position
and put someone into the fourth position,
we should **multiply** these numbers together.

Thus the number of arrangements is given by

$$4 \times 3 \times 2 \times 1 \;=\; 24. \qquad \square$$

Note: We use the notation 4! to mean $4 \times 3 \times 2 \times 1$.

We introduce the following notation, which is pronounced as "n factorial":

$$\boxed{n! \;=\; n(n-1)(n-2)(n-3) \times \ldots \times 3 \times 2 \times 1}$$

Example 5.

(a) $6! \;=\; 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(b) $5! \;=\; 5 \times 4 \times 3 \times 2 \times 1 = 120$

(c) $4! \;=\; 4 \times 3 \times 2 \times 1 = 24$

(d) $3! \;=\; 3 \times 2 \times 1 = 6$

(e) $2! \;=\; 2 \times 1 = 2$

(f) $1! \;=\; 1$

(g) By convention, we take $0! \;=\; 1$.

This definition is made because it allows us to write certain formulae in a very nice way. For example, see the formula for ${}^{n}\mathrm{P}_{r}$ on page 9 and the formula for ${}^{n}\mathrm{C}_{r}$ on page 17.

$\square$

Note: Most calculators have a button for calculating factorials.

Example 6. Suppose that 4 girls and 2 boys are to be arranged in a row.

(a) How many arrangements are there if there is no restriction?

(b) In how many of the arrangements are the 2 boys together?

(c) In how many arrangements are the 2 boys **not** together?

(d) In how many arrangements are there at least 2 girls **between** the boys?

Solution:

(a) We just need to arrange the 6 people:

6	5	4	3	2	1

There are $6! = 720$ arrangements of the 6 people.

(b) Since the two boys must remain together, we consider them as a single "unit":

$$G_1 \quad G_2 \quad G_3 \quad G_4 \quad B_1 \quad B_2$$

So we have 5 "units", namely the 4 girls plus the "unit" of boys.

Note that we need to

arrange the 5 "units" **and** we need to arrange the 2 boys *within* their unit.

Thus there are $5! \times 2! = 240$ arrangements with the 2 boys together.

(c) The number of arrangements such that the boys are separated is equal to

the total number of arrangements $-$

the number of arrangements with the boys together.

Thus (using the results for (a) and (b)), there are

$$\begin{aligned} 6! - 5! \times 2! &= 720 - 240 \\ &= 480 \end{aligned}$$

arrangements such that the boys are not together.

(d) Let us call the boys B_1 and B_2. We first need to count how many ways the boys can be arranged. For each position B_1 can occupy, we count the possible positions for B_2.

We either have B_1 in the **first** position:

$$\boxed{\boldsymbol{B_1}}\boxed{*}\boxed{*}\overbrace{\boxed{B_2}\boxed{B_2}\boxed{B_2}}^{\text{3 possible positions for } B_2}$$

or we have B_1 in the **second** position:

$$\boxed{*}\boxed{\boldsymbol{B_1}}\boxed{*}\boxed{*}\overbrace{\boxed{B_2}\boxed{B_2}}^{\text{2 possible positions for } B_2}$$

or we have B_1 in the **third** position:

$$\boxed{*}\boxed{*}\boxed{\boldsymbol{B_1}}\boxed{*}\boxed{*}\boxed{B_2}$$

or we have B_1 in the **fourth** position:

$$\boxed{B_2}\boxed{*}\boxed{*}\boxed{\boldsymbol{B_1}}\boxed{*}\boxed{*}$$

or we have B_1 in the **fifth** position:

$$\overbrace{\boxed{B_2}\boxed{B_2}}^{\text{2 possible positions for } B_2}\boxed{*}\boxed{*}\boxed{\boldsymbol{B_1}}\boxed{*}$$

or we have B_1 in the **sixth** position:

$$\overbrace{\boxed{B_2}\boxed{B_2}\boxed{B_2}}^{\text{3 possible positions for } B_2}\boxed{*}\boxed{*}\boxed{\boldsymbol{B_1}}$$

Thus we see that there are

$$3 + 2 + 1 + 1 + 2 + 3 \;=\; 12$$

arrangements of the boys.

For each of these arrangements, the 4 girls can fill the remaining positions in 4! ways.

Because we need to arrange the boys **and** we need to arrange the 4 girls, the total number of arrangements is given by

$$\begin{aligned} 12 \times 4! &= 12 \times 24 \\ &= 288\,. \end{aligned}$$

$\square$

Example 7. How many numbers greater than 6000 can be formed with the digits

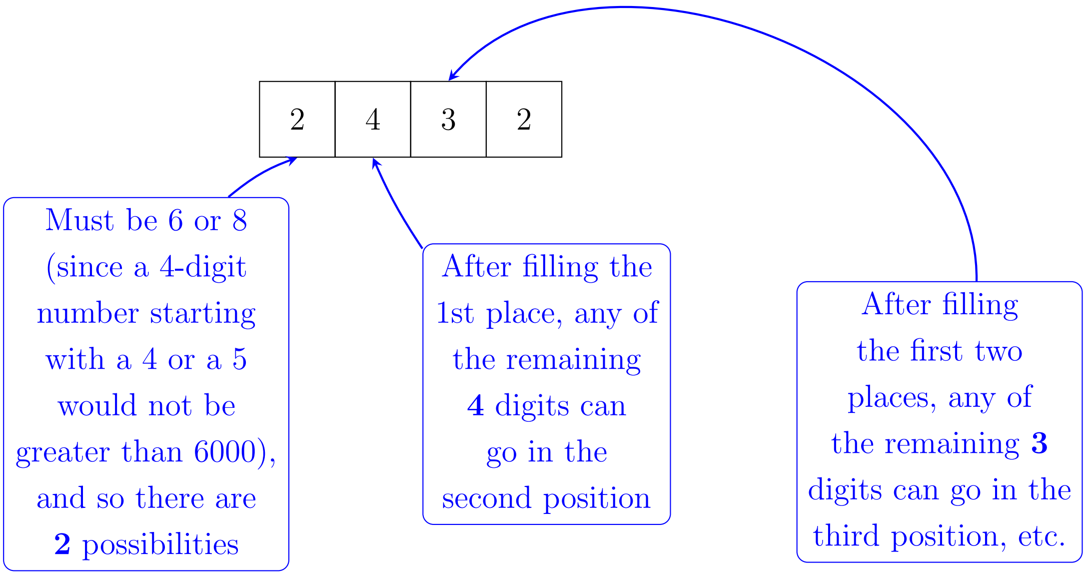
$$4,\ 5,\ 6,\ 8 \text{ and } 0$$

(without repetitions)?

Solution:

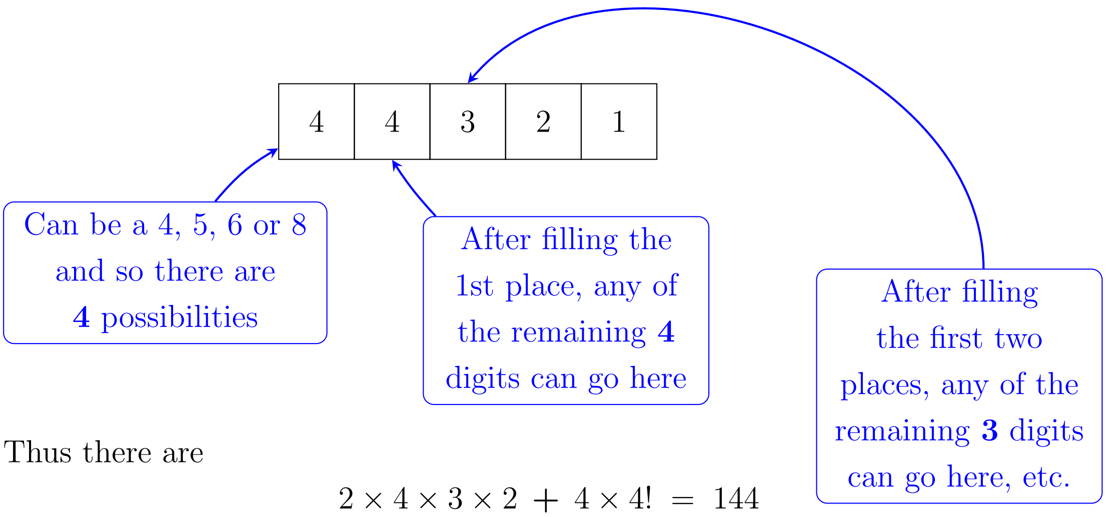
A number with just 1 digit, or 2 digits, or 3 digits **cannot** be greater than 6000. Therefore, we must consider numbers with **at least** 4 digits. We can either

- use 4 of the given digits (in which case we consider 4 boxes)



or we can

- use all 5 of the given digits (in which case we consider 5 boxes).



Thus there are

$$2 \times 4 \times 3 \times 2 \;+\; 4 \times 4! \;=\; 144$$

suitable numbers. $\square$

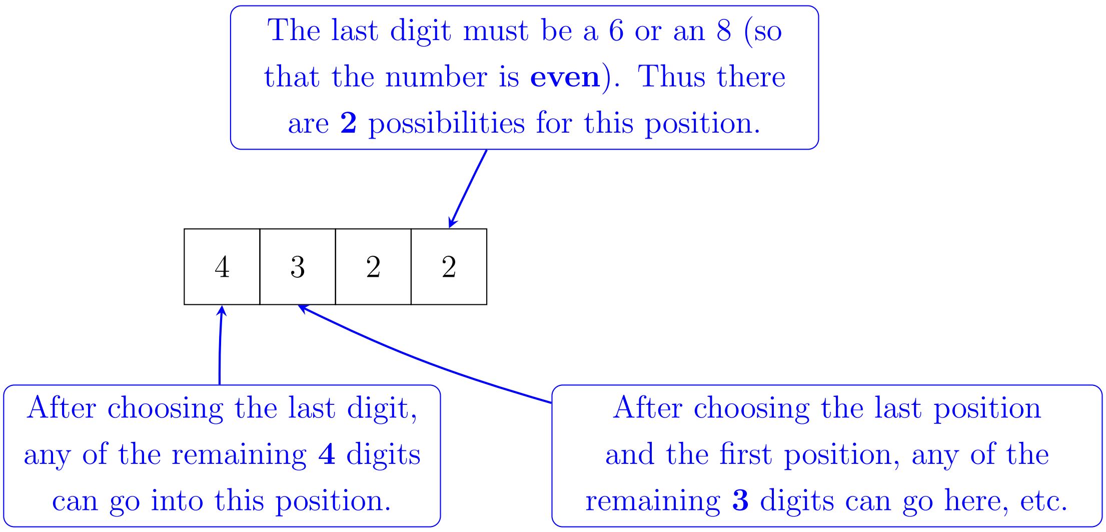
Example 8. How many **even** four–digit numbers can be made from the digits

$$5,\ 6,\ 7,\ 8 \text{ and } 9$$

(without repetitions)?

Solution:

We want to count four–digit numbers, so we need 4 boxes.
We must fill the final box first, to ensure that the number is even.
Then we can fill the remaining boxes.



So there are $4 \times 3 \times 2 \times 2 \ = \ 48$ suitable numbers.

$\square$

Example 9. How many arrangements are there of 2 people taken from a set of 4 people?

Solution:

We want to arrange 2 people, so we need 2 boxes.

4	3

There are $4 \times 3 \ = \ 12$ arrangements.

This is often represented by the notation ${}^{4}\mathrm{P}_{2}$.

$\square$

$^{n}\mathrm{P}_{r}$ is the number of arrangements
of r objects taken from a set of n different objects:

n	$n-1$	$n-2$	......	$n-r+1$

$$\begin{aligned}\text{Thus } {}^{n}\mathrm{P}_{r} &= n(n-1)(n-2)\times\ldots\times(n-r+1)\\ &= \frac{n(n-1)(n-2)\ldots(n-r+1)\,(n-r)(n-r-1)\times\ldots\times3\times2\times1}{(n-r)(n-r-1)\times\ldots\times3\times2\times1}\\ &= \frac{n!}{(n-r)!}\end{aligned}$$

Example 10. How many arrangements are there of 8 people taken from a group of 20 people?

Solution:

Because we want to arrange 8 people, we draw 8 boxes.

There are 20 possible people for the first box. After someone is chosen for the first box, there remain 19 possible people for the second box. Then there will remain 18 possible people for the third box, and so on.

20	19	18	17	16	15	14	13

So the number of arrangements is given by

$$\begin{aligned}20\times19\times18\times\cdots\times13 &= 20\times19\times18\times\ldots\times13\ \times\ \frac{12!}{12!}\\ &= \frac{20!}{12!}\\ &= 5\,079\,110\,400.\end{aligned}$$

$$\begin{aligned}\text{That is, } {}^{20}\mathrm{P}_{8} &= \frac{20!}{12!}\\ &= 5\,079\,110\,400.\end{aligned}$$

□

Many calculators have a button for calculating $^{n}\mathrm{P}_{r}$ directly.

Example 11.

(a) ${}^{4}\mathrm{P}_{2} \;=\; \dfrac{4!}{2!} \;=\; \dfrac{4\,\times\,3\,\times\,2!}{2!} \;=\; 4\times 3 \;=\; 12.$

Alternatively, we can obtain this answer by

- simply using the ${}^{n}\mathrm{P}_{r}$ button on our calculators, or else by
- considering boxes: $\boxed{4}\boxed{3}$

(b) ${}^{6}\mathrm{P}_{2} \;=\; \dfrac{6!}{4!} \;=\; \dfrac{6\,\times\,5\,\times\,4!}{4!} \;=\; 6\times 5 \;=\; 30.$

Again, we can obtain this answer by

- simply using the ${}^{n}\mathrm{P}_{r}$ button on our calculators, or else by
- considering boxes: $\boxed{6}\boxed{5}$

(c) ${}^{4}\mathrm{P}_{4} \;=\; \dfrac{4!}{0!} \;=\; \dfrac{4\times 3\times 2\times 1}{1} \;=\; 24.$

Again, we can obtain this answer by

- simply using the ${}^{n}\mathrm{P}_{r}$ button on our calculators, or else by
- considering boxes: $\boxed{4}\boxed{3}\boxed{2}\boxed{1}$

□

Example 12. How many arrangements are there of 3 people taken from a set of 7 people?

Solution:

The number of arrangements is given by ${}^{7}\mathrm{P}_{3}$.

We can find ${}^{7}\mathrm{P}_{3}$ by using the ${}^{n}\mathrm{P}_{r}$ button on our calculators, or else by using the formula:

$${}^{7}\mathrm{P}_{3} \;=\; \frac{7!}{4!} \;=\; \frac{7\,\times\,6\,\times\,5\,\times\,4!}{4!} \;=\; 7\times 6\times 5 \;=\; 210.$$

Either way, we conclude that the number of arrangements is 210.

Alternatively, this answer is easily seen by using boxes:

$$\boxed{7}\boxed{6}\boxed{5}$$

Again we see that there are $7\times 6\times 5 \;=\; 210$ arrangements.

□

Exercises for Section 17.1

1. (a) How many three digit numbers can be formed using the digits 1, 2, 3, 4, and 5 (without repetitions)?

   (b) How many **even** three digit numbers can be formed using the digits 1, 2, 3, 4, and 5 (without repetitions)?

   (c) How many **odd** three digit numbers can be formed using the digits 1, 2, 3, 4, and 5 (without repetitions)?

2. Seven students and three teachers are to be seated in a row. In how many different ways can this be done if:

   (a) there are no restrictions.

   (b) the three teachers must be seated together.

   (c) there must be a teacher at each end of the row.
   (That is, there must be a teacher at the *left end* of the row, and also a teacher at the *right end* of the row.)

   (d) two particular students may not sit together.

   (e) there must be at least six people between the two particular students.

17.2 Arrangements with Repetitions

Example 13.

(a) How many three–digit numbers can be formed using the digits

$$1,\ 3,\ 6,\ 8 \text{ and } 9$$

if repetitions **are** allowed?

(b) How many **even** three–digit numbers can be formed using the digits

$$1,\ 3,\ 6,\ 8 \text{ and } 9$$

if repetitions **are** allowed?

Solution:

(a) There are 5 digits available for the first place, **and** 5 digits available for the second place (since we can reuse the digit from the first place if we want to), **and** 5 digits available for the third place:

5	5	5

So there are $5 \times 5 \times 5 \ = \ 125$ possible three–digit numbers that can be formed.

(b) Our numbers will have to end with the digit "6" or the digit "8" (to make sure that our number is **even**). Thus there are only 2 digits available for the last place. However, any of the 5 digits is available for each of the first and second places:

5	5	2

So there are $5 \times 5 \times 2 \ = \ 50$ possible **even** three–digit numbers that can be formed.

$\square$

Example 14. How many different number–plates can be formed if each number–plate has 3 letters of the alphabet followed by 3 digits?
(Note that repetitions **are** allowed, and the digits **are** allowed to begin with a zero.)

Solution:

There are 26 letters in the alphabet, and there are 10 digits. Repetitions are allowed, so there are 26 choices for the first 3 positions and 10 for the last 3 positions.

26	26	26	10	10	10

26 possible letters　　**10** possible digits

So there are $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17\,576\,000$ possible number-plates.

$\square$

17.3 Arrangements of Objects Which Are Not All Different

Suppose there are n objects of which p are of one kind, q of another kind, ...

$$\boxed{\text{Then the number of arrangements} = \frac{n!}{p!\; q!\; \ldots}}$$

Example 15. How many arrangements of the letters in the word **MAMMAL** are possible?

Solution:

List the letters: **AALMMM**.

We have 6 letters, consisting of 2 A's, 1 L and 3 M's.

$$\text{So the number of arrangements} = \frac{6!}{\underbrace{2!}_{2\text{ A's}}\;\underbrace{3!}_{3\text{ M's}}} = \frac{720}{12} = 60.$$

$\square$

Example 16.

(a) In how many ways can the letters of the word **PRECISION** be arranged?

(b) In how many of these arrangements do the vowels occupy the even places?

Solution:

(a) List the letters: **CEIINOPRS**.

We have 9 letters, including 2 I's.

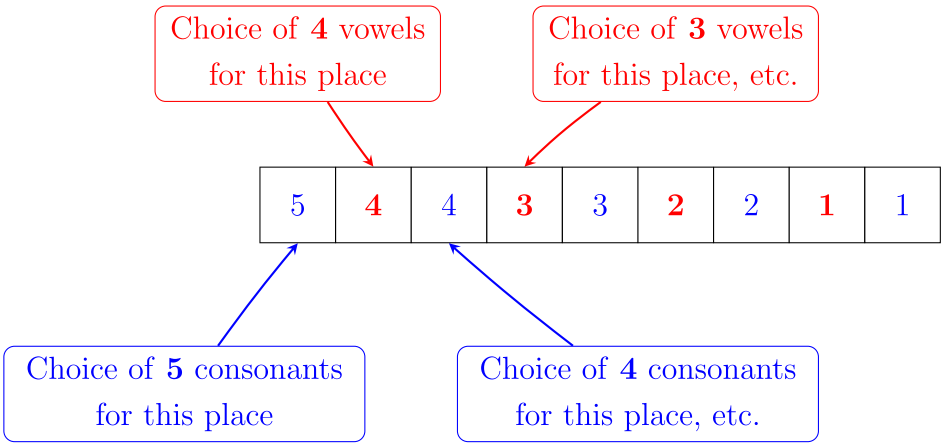
Therefore, the total number of arrangements $= \dfrac{9!}{2!} = 181\,440\,.$

(b) Our word contains **4 vowels: E, I, I, O**.

- These must go into the even positions (which are the 2^{nd}, 4^{th}, 6^{th} and 8^{th} positions).

The word also contains **5 consonants: C, N, P, R, S**.

- These must go into the remaining positions (namely the 1^{st}, 3^{rd}, 5^{th}, 7^{th} and 9^{th} positions).



We need to arrange the consonants **and** arrange the vowels. So the number of arrangements with the vowels in the even places

$$= 5! \times \frac{4!}{\underbrace{2!}_{\text{2 I's}}} = 1440.$$

$\square$

Example 17.

(a) In how many ways can the letters of the word **STATISTICS** be arranged?

(b) In how many ways can the letters of the word **STATISTICS** be arranged if the vowels are to remain together?

Solution:

(a) List the letters: **ACIISSSTTT**.

Note that we have 10 letters, consisting of

1 A, 1 C, 2 I's, 3 S's and 3 T's.

$$\text{So the number of arrangements} = \frac{10!}{2!\,3!\,3!} = \frac{3\,628\,800}{72} = 50\,400\,.$$

(b) We keep the vowels together, by considering them as a single unit. Then we have 8 units, consisting of 1 C, 3 S's, 3 T's and the unit of vowels:

C S S S T T T A I I

We need to

- arrange the 8 units,

and we need to

- arrange the 3 vowels *within* their unit.

$$\begin{aligned}\text{So the number of arrangements} &= \frac{\overset{\text{8 units}}{8!}}{\underset{\text{3 S's}}{3!}\,\underset{\text{3 T's}}{3!}} \times \frac{\overset{\text{Arrange the 3 vowels within their unit}}{3!}}{\underset{\text{2 I's}}{2!}}\\ &= \frac{40320}{6 \times 6} \times \frac{6}{2}\\ &= 1120 \times 3\\ &= 3360.\end{aligned}$$

□

Exercises for Section 17.3

1. (a) In how many ways can the letters of the word **PARALLEL** be arranged?

   (b) In how many of these arrangements are the L's together?

2. In how many ways can the letters of the word **PROBABILITY** be arranged?

3. In how many ways can the letters of the word **PERMUTE** be arranged

   (a) without restriction?

   (b) if the first and last places are occupied by consonants?

   (c) if the vowels and consonants occupy alternate places?

17.4 Combinations

Recall that the word "**combination**" refers to an example in which objects are *selected* (or chosen). The **order** in which the objects are chosen is *not* important.

Example 18. How many ways are there of choosing a committee of 3 people from the 4 candidates **A**nh, **B**ai, **C**heng and **D**ewi?

Solution:

The combinations are:

$$\begin{aligned} &\mathbf{A}, \mathbf{B}, \mathbf{C} \quad (\text{omitting } \mathbf{D}),\\ &\mathbf{A}, \mathbf{B}, \mathbf{D} \quad (\text{omitting } \mathbf{C}),\\ &\mathbf{A}, \mathbf{C}, \mathbf{D} \quad (\text{omitting } \mathbf{B}),\\ &\mathbf{B}, \mathbf{C}, \mathbf{D} \quad (\text{omitting } \mathbf{A}).\end{aligned}$$

Thus (by simply counting the listed possibilities) we conclude that there are 4 ways to choose the committee.

□

We will want to find out how many combinations there are without having to list them all (since writing all the possibilities, and then counting them, is very time-consuming and tedious for the 'larger' examples).

The number of ways of selecting r objects from a set of n objects is written as

$$\boxed{{}^{n}\mathrm{C}_{r}\,,\text{ and is pronounced as “}n\text{ choose }r\text{”.}}$$

It can be shown that $$\boxed{{}^{n}\mathrm{C}_{r} = \frac{n!}{r!\,(n-r)!}\,.}$$

Notes:

- Sometimes the notation $\binom{n}{r}$ is used instead of ${}^{n}\mathrm{C}_{r}$.
- Most calculators have a button for calculating ${}^{n}\mathrm{C}_{r}$.

Let us consider Example 18 again:

Example 19. How many ways are there of choosing a committee of 3 people from the 4 candidates **A**nh, **B**ai, **C**heng and **D**ewi?

Solution:

We are choosing 3 people from 4 candidates, and so the number of committees is given by ${}^{4}\mathrm{C}_{3} = \dfrac{4!}{3!\,1!} = 4\,.$

Note that we can find this answer ${}^{4}C_{3} = 4$ ***immediately*** *by using the* ${}^{n}C_{r}$ *button on our calculators, without needing to use the formula.*

□

Example 20.

(a) In how many ways can a set of 4 girls be selected from a set of 10 girls?

(b) How many of these sets include the eldest girl?

Solution:

(a) The number of ways is given by ${}^{10}\mathrm{C}_{4} = \dfrac{10!}{4!\,6!} = 210\,.$

As before, we can find this answer ${}^{10}C_{4} = 210$ ***immediately*** *by using the* ${}^{n}C_{r}$ *button on our calculators, without needing to use the formula.*

(b) We want to choose the eldest girl, **and** also choose 3 of the remaining 9 girls. So the number of ways is given by

$$ {}^{1}\mathrm{C}_{1} \times {}^{9}\mathrm{C}_{3} = 1 \times 84 = 84\,. $$

□

Example 21. A group consists of 5 girls and 10 boys. A committee of 8 is to be selected.

(a) How many ways are there of choosing such a committee?

(b) How many of the committees contain *exactly* 2 girls?

(c) How many of the committees contain *at least* 3 girls?

Solution:

(a) We need to choose 8 people from 15 candidates, so the number of committees is given by ${}^{15}C_8 = 6435$.

(b) We need to choose

2 of the 5 girls, **and** 6 of the 10 boys.

So the number of committees $= {}^{5}C_2 \times {}^{10}C_6 = 10 \times 210 = 2100$.

(c) We need to choose

3 of the 5 girls, **and** 5 of the 10 boys,

or 4 of the 5 girls, **and** 4 of the 10 boys,

or 5 of the 5 girls, **and** 3 of the 10 boys.

So the number of committees is given by

$$\begin{aligned} & {}^{5}C_3 \times {}^{10}C_5 \; + \; {}^{5}C_4 \times {}^{10}C_4 \; + \; {}^{5}C_5 \times {}^{10}C_3 \\ &= 10 \times 252 \; + \; 5 \times 210 \; + \; 1 \times 120 \\ &= 2520 \; + \; 1050 \; + \; 120 \\ &= 3690\,. \end{aligned}$$

□

Example 22. A committee of 4 people is to be selected from a group of 6 basketball players, 5 badminton players, and 4 soccer players. In how many ways can the committee be formed if each of the three sports is to be represented?

Solution:

Overall there are

6 basketball players, 5 badminton players and 4 soccer players.

Note that we either need to choose

- 2 basketball players **and** 1 badminton player **and** 1 soccer player, **or**
- 1 basketball player **and** 2 badminton players **and** 1 soccer player, **or**
- 1 basketball player **and** 1 badminton player **and** 2 soccer players.

So the number of committees is given by

$$
\begin{aligned}
& {}^{6}C_{2} \times {}^{5}C_{1} \times {}^{4}C_{1} \;+\; {}^{6}C_{1} \times {}^{5}C_{2} \times {}^{4}C_{1} \;+\; {}^{6}C_{1} \times {}^{5}C_{1} \times {}^{4}C_{2} \\
&= 15 \times 5 \times 4 \;+\; 6 \times 10 \times 4 \;+\; 6 \times 5 \times 6 \\
&= 300 \;+\; 240 \;+\; 180 \\
&= 720\,.
\end{aligned}
$$

□

Exercises for Section 17.4

1. From a group of 6 women and 4 men, a committee of 5 is to be selected.
   (a) How many ways are there of choosing such a committee?
   (b) How many of these committees contain 3 women and 2 men?
2. In how many ways can a jury of 12 be chosen from 10 women and 7 men so that there are at least 8 women on each jury?
3. In how many ways can three cards be selected from a pack of 52 cards if
   (a) at least one of them is an ace?
   (b) the number of aces chosen is less than or equal to one?

   Note: A pack of 52 cards contains 4 aces, (and 48 other cards).

17.5 Answers for the Chapter 17 Exercises

17.1 1. (a) There are $5 \times 4 \times 3 = 60$ possible numbers.

(b) There are $4 \times 3 \times 2 = 24$ possible even numbers.

(c) There are $60 - 24 = 36$ possible odd numbers.

2. (a) There are 10! (i.e. 3 628 800) arrangements if there are no restrictions.

(b) There are 8!3! (i.e. 241 920) arrangements with the teachers together.

(c) There are $8! \times 6$ (i.e. 241 920) arrangements with a teacher at each end of the row.

(d) There are $10! - 9!2!$ (i.e. 2 903 040) arrangements such that the two particular students are not together.

(e) There are $8! \times 12$ (i.e. 483 840) arrangements such that at least six people are between the two particular students.

17.3 1. (a) There are $\dfrac{8!}{2!\,3!} = 3360$ arrangements of the letters in the word PARALLEL.

(b) In $\dfrac{6!}{2!} = 360$ of these arrangements, the L's are together.

2. There are $\dfrac{11!}{2!\,2!} = 9979200$ arrangements of the letters in the word PROBABILITY.

3. (a) There are $\dfrac{7!}{2!} = 2520$ arrangements.

(b) There are $\dfrac{4 \times 3 \times 5!}{2!} = 720$ arrangements.

(c) There are $\dfrac{4! \times 3!}{2!} = 72$ arrangements.

17.4 1. (a) There are ${}^{10}C_5 = 252$ ways to choose a committee.

(b) There are ${}^{6}C_3 \times {}^{4}C_2 = 120$ ways to choose such a committee.

2. There are ${}^{10}C_8\,{}^{7}C_4 + {}^{10}C_9\,{}^{7}C_3 + {}^{10}C_{10}\,{}^{7}C_2 = 1946$ ways to choose such a jury.

3. (a) There are ${}^{4}C_1\,{}^{48}C_2 + {}^{4}C_2\,{}^{48}C_1 + {}^{4}C_3\,{}^{48}C_0 = 4804$ ways to choose at least one ace.

(Alternatively, you can get this answer by working out ${}^{52}C_3 - {}^{4}C_0\,{}^{48}C_3$.)

(b) There are ${}^{4}C_0\,{}^{48}C_3 + {}^{4}C_1\,{}^{48}C_2 = 21808$ ways to choose no aces or one ace.