

# Mathematics 1

## Exercise Sheet 22: Conditional Probability

1. At a particular school, 25% of the staff jog. It has been found that 20% of the joggers and 5% of the non-joggers develop foot complaints.

Find the probability that a randomly selected member of staff

- (a) will develop a foot complaint;
- (b) will be a non-jogger, given that he/she has a foot complaint.

2. In a certain college,

- 4 per cent of the men and 1 per cent of the women are taller than 1.8 metres;
- 60 per cent of the students are women, and the other 40 per cent of the students are men.

If a student is selected at random and is taller than 1.8 metres, what is the probability that the selected student is a man?

3. A box contains 6 blue cubes and 3 white cubes. A second box contains 3 blue cubes and 9 white cubes. A box is chosen at random, and a cube is chosen at random from that box.

**Hint for drawing the tree diagram:**

- When we choose a box there are two possibilities (namely “first box” or “second box”).
- Then when we choose a cube there are two possibilities (namely “blue” or “white”).

- (a) What is the probability that a blue cube is chosen?
- (b) Suppose that a blue cube was chosen. What is the probability that it was chosen from the second box?

4. In a certain district it rains on 30% of the days, and is fine on 70% of the days. A weather forecaster is not always correct in his forecasts.

- On the rainy days, 5% of the time the forecaster had (mistakenly) predicted that the weather was going to be fine, and
- on the fine days, 10% of the time the forecaster had (mistakenly) predicted that the weather was going to be rainy.

A particular day has been forecast to be fine. What is the probability that the weather will be fine?

5. A bag contains two fair coins and one double-headed coin.

**Note:** A “fair coin” is an ordinary coin, with “heads” on one side and “tails” on the other side.

A “double-headed coin” is an unusual coin, which has “heads” on both sides (and no “tails”).

Suppose that a coin is taken from the bag and tossed.

- (a) What is the probability of getting a head?
- (b) Suppose that the coin falls with a head facing upwards. What is the probability that it was the double-headed coin?

6. **A man is on time for work on Monday.** It has been found that

- when the man is **on time** for work, then on 90% of occasions he is on time again the next day, and
- when the man is **late** for work, then on 20% of occasions he is late again the next day.

- (a) What is the probability that he is on time on Wednesday?
- (b) What is the probability that he was on time on Tuesday, if it is known that he is on time on Wednesday? Write your answer accurate to two decimal places.

7. For two events,  $X$  and  $Y$ , we have

$$\Pr(X) = 0.35, \quad \Pr(Y) = 0.4 \quad \text{and} \quad \Pr(X | Y) = 0.5.$$

Find

- (a)  $\Pr(X \cap Y)$
- (b)  $\Pr(X \cup Y)$
- (c)  $\Pr(Y | X)$
- (d)  $\Pr(Y' | X)$ .

8. For two events,  $A$  and  $B$ , we have

$$\Pr(A) = 0.6, \quad \Pr(B) = 0.5 \quad \text{and} \quad \Pr(A \cup B) = 0.8.$$

- (a) Find  $\Pr(A \cap B)$ .
- (b) State whether the events  $A$  and  $B$  are mutually exclusive.
- (c) State whether the events  $A$  and  $B$  are independent.
- (d) Find

$$(i) \Pr(A | B) \quad (ii) \Pr(A' | B) \quad (iii) \Pr(B | A').$$

9. Consider two events  $A$  and  $B$  such that

$$\Pr(A) = \frac{1}{3}, \quad \Pr(B | A) = \frac{3}{4} \quad \text{and} \quad \Pr(A | B) = \frac{1}{2}.$$

Find

- (a)  $\Pr(A \cap B)$
- (b)  $\Pr(A' | B')$
- (c)  $\Pr(A | B')$ .

10. In the general population, 1 in 1000 people have a particular disease. A test is available for detecting this disease. Unfortunately, only 95% of the people who have the disease give positive results for the test, and only 99% of the people who do **not** have the disease give negative results.

Suppose that I had a test which gave a positive result. What, to three decimal places, is the probability that I have the disease?

11. Five cards are chosen at random from a pack.
- (a) What is the probability that exactly three hearts were chosen? Write the answer to four decimal places.
  - (b) What is the probability that exactly three hearts were chosen, if it is known that at least three hearts were chosen? Write the answer to four decimal places.
12. Modern wireless communications systems consist of a transmitting unit (for example, a satellite) and a receiving unit (for example, a mobile phone). The transmitting unit sends the information to the receiving unit as a series of 0's and 1's. A particular system has the following characteristics:
- The probability that a 0 is sent is 0.03.
  - The probability that a 0 is received when a 0 is sent is 0.95.

- The probability that a 0 is received when a 1 is sent is 0.10.

- (a) What is the probability that a 0 is received?
- (b) What is the probability a 0 was sent when a 0 is received? Approximate your answer to four decimal places.
- (c) What do you conclude about the reliability of the communications system?

13. An advertising agency notes that

- 1 in 40 potential buyers have seen an internet pop-up advertisement for Product X,
- 1 in 4 have seen the corresponding television advertisement, and
- 1 in 100 have seen both.

Suppose that 1 in 3 people buy Product X after seeing the advertisement, and 1 in 8 without seeing it.

- (a) What is the probability that a potential buyer has seen an advertisement?
- (b) What, to four decimal places, is the probability that a randomly selected potential buyer will actually buy Product X?
- (c) What, to two decimal places, is the probability that a person who actually buys Product X has seen the advertisement?

14. Each year a company either makes an annual profit or an annual loss.

- When the company makes an annual profit, the probability that it makes another annual profit during the following year is 0.73.
- When the company makes an annual *loss*, the probability that it makes another annual *loss* during the following year is 0.37.

Suppose that the company has just made an annual profit.

- (a) Find the probability that the company makes another annual profit in one year's time.
- (b) Find the probability that the company makes an annual profit in two years' time.
- (c) Find the probability that the company makes an annual profit in three years' time.

15. Each year, the cows in a breeding program give birth to either single calves or else to *twin* calves.

- If a cow gives birth to a single calf, the probability that she gives birth to a single calf again the next year is  $\frac{9}{10}$ .
- If a cow gives birth to *twin* calves, the probability that she gives birth to twin calves again the next year is  $\frac{3}{10}$ .

Suppose that a cow has just given birth to *twin* calves.

- (a) Find the probability that she gives birth to twin calves again in one year's time.
- (b) Find the probability that she gives birth to twin calves again in two years' time.
- (c) Find the probability that she gives birth to twin calves again in three years' time.

## Answers:

1. (a) The probability that the staff member has a foot complaint is 0.0875.  
(b) The probability that the staff member is a non-jogger, given that he/she has a foot complaint, is  $\frac{3}{7}$ .
2. The probability that the tall student is a man is  $\frac{8}{11}$ .
3. (a) The probability of a blue cube is  $\frac{11}{24}$ .  
(b) The probability that the cube was from the second box, given that it was blue, is  $\frac{3}{11}$ .
4. The probability that it is fine, given that it was forecast to be fine, is  $\frac{42}{43}$ .
5. (a) The probability of getting a head is  $\frac{2}{3}$ .  
(b) The required probability is  $\frac{1}{2}$ .
6. (a) The probability that he will be on time on Wednesday is 0.89.  
(b) The required probability is 0.91 (2 d.p.).
7. (a) 0.2                      (b) 0.55                      (c)  $\frac{4}{7}$                       (d)  $\frac{3}{7}$ .
8. (a)  $\Pr(A \cap B) = 0.3$ .  
(b) Since  $\Pr(A \cap B) \neq 0$  then  $A$  and  $B$  are **not** mutually exclusive.  
(c) Since  $\Pr(A) \Pr(B) = \Pr(A \cap B)$  then  $A$  and  $B$  **are** independent.  
(d) (i) 0.6                      (ii) 0.4                      (iii) 0.5.
9. (a)  $\frac{1}{4}$                       (b)  $\frac{5}{6}$                       (c)  $\frac{1}{6}$ .
10. The probability that I have the disease is 0.087 (3 d.p.).
11. (a) The required probability is 0.0815 (4 d.p.).  
(b) The required probability is 0.8790 (4 d.p.).
12. (a) The probability that a 0 is received is 0.1255.  
(b) The probability that a zero was sent given that a zero is received is 0.2271 (4 d.p.).  
(c) The probability that a zero is sent given that a zero is received is quite low so this communication system is not very reliable.
13. (a) The probability that a potential buyer has seen the advertisement is 0.265.  
(b) The probability that a randomly selected potential buyer will actually buy the Product X is 0.1802 (4 d.p.).  
(c) The probability that a person who actually buys the Product X has seen the advertisement is 0.49 (2 d.p.).
14. (a) The probability that the company makes another annual profit in one year's time is 0.73.  
(b) The probability that the company makes another annual profit in two years' time is 0.703.  
(c) The probability that the company makes another annual profit in three years' time is 0.7003.
15. (a) The probability that the cow gives birth to twin calves again in one year's time is 0.3.  
(b) The probability that the cow gives birth to twin calves again in two years' time is 0.16.  
(c) The probability that the cow gives birth to twin calves again in three years' time is 0.132.