[ES8] LIMITS, CONTINUITY AND DIFFERENTIABILITY

Monday, April 27, 2020 6:55 AM

ATHS 1] SHEET 8: LIMITS, CONTINUITY AND DIFFERENTIABLITY

Evaluate the following limits:

a) $\lim_{x\to 4}(x^2-1)$

$\lim_{x\to 4}(x^2-1) = (4)^2 - 1$
$= 15$ ✓

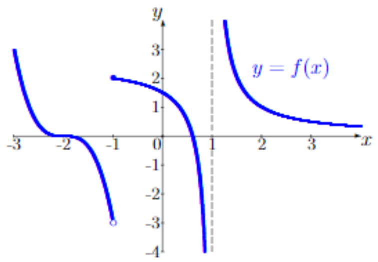
) $\lim_{x\to 1}\frac{x-1}{x^2-1}$

$\lim_{x\to 1}\frac{x-1}{x^2-1} = x\to 1 \ \frac{\cancel{x-1}}{(x+1)\cancel{(x-1)}}$
$= \frac{1}{1+1}$
$= \frac{1}{2}$ ✓

) $\lim_{x\to 3^-}\sqrt{3-x}$

$\lim_{x\to 3^-}\sqrt{3-x} = \sqrt{3-3}$
$= 0$ ✓

2. Consider the graph of $y = f(x)$ given below:



Find the following limits:

) $\lim_{x\to -1^-} f(x)$

$\lim_{x\to -1^-} f(x) = -3$ ✓

) $\lim_{x\to -1^+} f(x)$

$\lim_{x\to -1^+} f(x) = 2$ ✓

) $\lim_{x\to -1} f(x)$

Since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$
$\therefore \lim_{x\to -1} f(x)$ does not exist ✓

$\lim_{x\to 1^-} f(x)$

$\lim_{x\to 1^-} f(x) = -\infty$
$\therefore \lim_{x\to 1^-} f(x)$ does not exist ✓

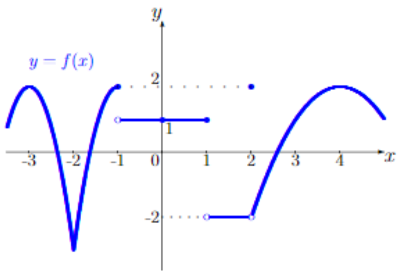
$\lim_{x\to 1^+} f(x)$

$\lim_{x\to 1^+} f(x) = \infty$
$\therefore \lim_{x\to 1^+} f(x)$ does not exist ✓

$\lim_{x\to 1} f(x)$

$\lim_{x\to 1} f(x)$ does not exist ✓

Consider the graph of $y = f(x)$ shown below:



At which x-values in the interval $[-3, 4]$ is f discontinuous?
Justify your answers.

Let $f(x) = \dfrac{x^2 - 2x + 1}{x - 1}$.

(a) Find $\lim_{x \to 1} f(x)$.

) Is $f(x)$ continuous at $x = 1$? Justify your answer.

Sketch the graph of $y = f(x)$.

. Let $f(x) = \dfrac{x - 1}{x^2 - 2x + 1}$.

) Find $\lim_{x \to 1} f(x)$.

Is $f(x)$ continuous at $x = 1$? Justify your answer.

Sketch the graph of $y = f(x)$.

. Let $f(x) = \begin{cases} x & \text{if } x \leq -1 \\ x^2 + 2x & \text{if } -1 < x \leq 0 \\ \frac{1}{x} & \text{if } x > 0 \end{cases}$

) Sketch the graph of $y = f(x)$.

) Find $\lim_{x \to -1^+} f(x)$.

) Find $\lim_{x \to -1^-} f(x)$.

Find $\lim_{x \to -1} f(x)$.

Is $f(x)$ continuous at $x = -1$? Justify your answer.

Find $\lim_{x \to 0^-} f(x)$.

) Find $\lim_{x \to 0^+} f(x)$.

) Find $\lim_{x \to 0} f(x)$.

Is $f(x)$ continuous at $x = 0$? Justify your answer.

State whether the following functions are continuous. If a point of discontinuity occurs, explain why it is a point of discontinuity.

(a) $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 2 \\ 2x - 1 & \text{for } x \geq 2 \end{cases}$

) $f(x) = \begin{cases} \dfrac{1}{x+1} & \text{for } x \neq -1 \\ 2 & \text{for } x = -1 \end{cases}$

Find the derivative of $f(x) = 4x + 7$ by using first principles.

. Find the derivative of $f(x) = 5x^2 - 2$ by using first principles.

Find the derivative of $f(x) = 3x^2 - 4x + 1$ by using first principles.

. **[Revision from Chapter 2]**

(a) Solve the equation

$$\sin 2x = \sin x \quad \text{for } x \in [0, 2\pi].$$

Hint: Rewrite $\sin 2x$ by using a double-angle formula.

) Hence solve the inequality

$$\sin 2x > \sin x \quad \text{for } x \in [0, 2\pi].$$

Hint: Look at the graphs of $y = \sin 2x$ and $y = \sin x$.

[Revision from Chapter 1]

Solve the inequality $|2x - 5| \leq |x + 4|$.

[Revision from Chapter 4]

On the same set of axes sketch the graphs of

$$y = \log_2(3x - 6) \quad \text{and} \quad y = \frac{1}{\log_2(3x - 6)}.$$