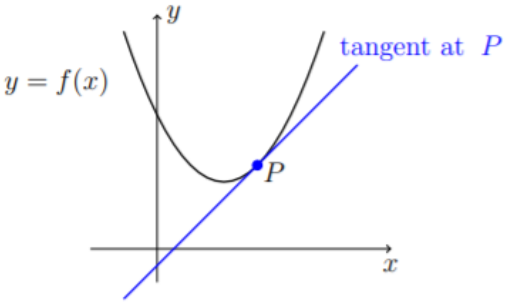
C8: DIFFERENTIABILITY

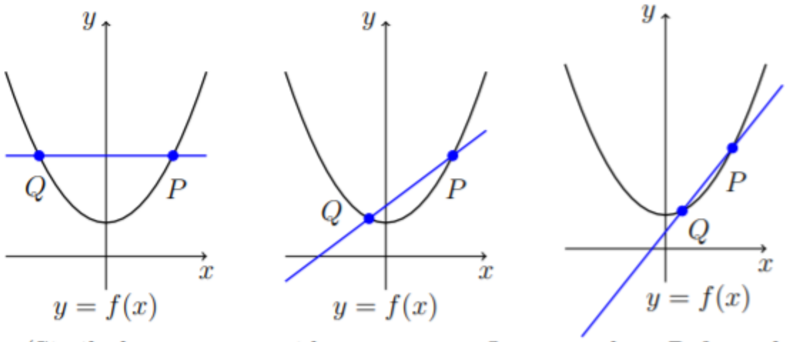
15 April 2020 23:35

8.1 TANGENTS



Suppose Q is any point on the curve, other than P. We can draw a straight line through P and Q; this straight line is known as the **secant** PQ. We can see in the diagrams below that when we consider

Q **very close** to P, the secant PQ looks very similar to the tangent.

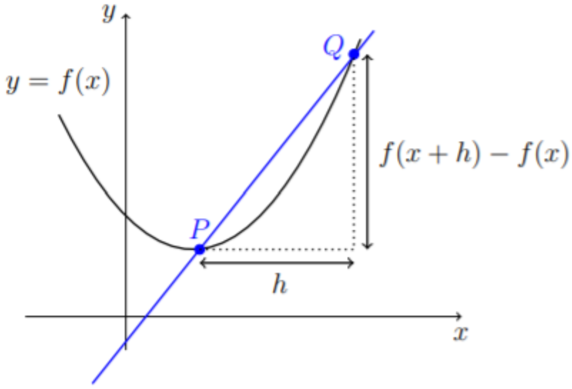


(Similarly, we can consider secants as Q approaches P from the right.)

If we let m denote the slope of the tangent at P, and m_{PQ} denote the slope of the secant line PQ, then we have

$$m = \lim_{Q \to P} m_{PQ}\,.$$

Let $(x, f(x))$ denote the coordinates of P, and let $(x+h, f(x+h))$ denote the coordinates of Q. Note that when Q is very close to P, then h will be very close to 0. That is, $Q \to P$ corresponds to $h \to 0$.



Recall that the slope of a straight line is given by $\frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line.

Thus we can write

$$m_{PQ} = \frac{f(x+h) - f(x)}{x+h - x} = \frac{f(x+h) - f(x)}{h}$$

and so

$$\boxed{m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

^ that's also first principle

Definition of tanget

The **tangent** of the curve $y = f(x)$ at point $P = (x, f(x))$ is defined to be the line through P with slope given by the above limit, **provided that this limit exists.**

If, at a particular point the limit $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ does not exist, then either

- the curve has a vertical tangent at that point, or
- the curve has no tangent at that point.

In general,

- tangents do **not** exist at sharp corners, kinks, or sudden jumps in a curve.

Furthermore,

- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \pm\infty$ when there is a vertical tangent,

8.2 DIFFERENTIABILITY

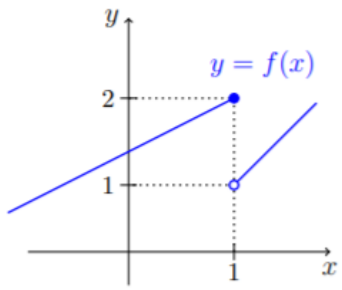
The derivative of f is usually denoted by $f'(x)$ or by $\frac{dy}{dx}$. So we have

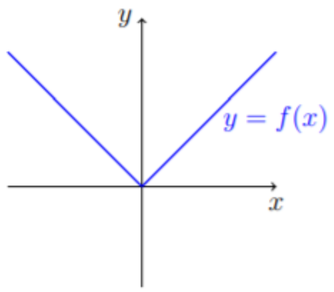
$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

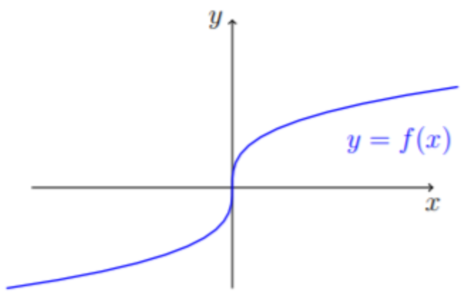
Then f is **differentiable** at $x = a$ if and only if

- f is continuous at $x = a$, and
- f does not have a sharp corner or kink at $x = a$, and

- the tangent of f at $x = a$ is not vertical.







$f'(x)$ is the gradient of the tangent (if it exists) to $y = f(x)$ at the point $(x, f(x))$.

$f'(x)$ is the gradient **of the curve** of $y = f(x)$ at the point $(x, f(x))$.

2. $f'(x)$ also represents the **rate of change** of y with respect to x , at the point $(x, f(x))$ on the curve $y = f(x)$.

8.2 DIFFERENTIABILITY

Establishing differentiability **from first principles** involves actually checking whether this limit exists.

Similarly, to find the derivative of $f(x)$ **using first principles**, we only use the definition of $f'(x)$. That is, we calculate $f'(x)$ by finding the following limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} .$$

EXERCESES (pg 12)

By differentiating from first principles, verify that

(a) when $f(x) = x$ then $f'(x) = 1$.

(b) when $f(x) = x^2$ then $f'(x) = 2x$.

(c) when $f(x) = x^3$ then $f'(x) = 3x^2$.

(d) when $f(x) = \sqrt{x}$ then $f'(x) = \dfrac{1}{2\sqrt{x}}$.