EXERCISE SHEET 22 (SELECTED QUESTIONS)

SEP 14

Conditional Formula

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

① a) $\Pr(\text{Jogger}|\text{Complaint}) = 0.2$

$\Pr(\text{Jogger} \cap \text{Complaint}) = 0.2 \times 0.25$

$= 0.05$

$\Pr(\text{Non Jogger}|\text{Complaint}) = 0.05$

$\Pr(\text{Non Jogger} \cap \text{Complaint}) = 0.05 \times 0.75$

$= 0.0375$

$\Pr(\text{Complaint}) = 0.05 + 0.0375$

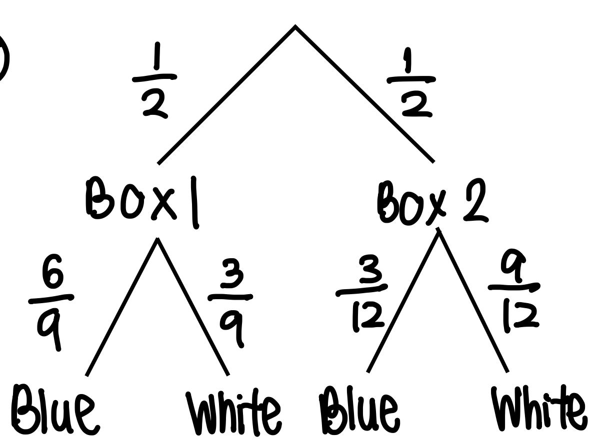
$= 0.0875$ //

b) $\Pr(\text{Complaint}|\text{Non Jogger}) = \dfrac{\Pr(\text{Non Jogger} \cap \text{Complaint})}{\Pr(\text{Complaint})}$

$= \dfrac{0.0375}{0.0875}$

$= \dfrac{3}{7}$ //

③



a) $\Pr(\text{Blue}) = \frac{1}{2} \times \frac{6}{9} + \frac{1}{2} \times \frac{3}{12}$

$= \dfrac{11}{24}$ //

b) $\Pr(\text{Blue}|\text{Box2}) = \dfrac{\Pr(\text{Blue} \cap \text{Box 2})}{\Pr(\text{Box 2})}$

$= \left(\frac{1}{2} \times \frac{3}{12}\right) \div \frac{11}{24}$

$$= \frac{3}{11} //$$

4) $$Pr(\text{Forecast Fine} \mid \text{Fine}) = \frac{Pr(\text{Forecast Fine} \cap \text{Fine})}{Pr(\text{Fine})}$$

$$= \frac{0.7 \times 0.9}{0.3 \times 0.05 \times 0.7 \times 0.9}$$

$$= \frac{42}{43} //$$

8) a) $$Pr(A \cap B) = 0.6 + 0.5 - 0.8$$

$$= 0.3 //$$

b) Since $Pr(A \cap B) \neq 0$, the events A and B are not mutually exclusive

c) $$Pr(A)Pr(B) = 0.6 \times 0.5$$

$$= 0.3$$

Since $Pr(A)Pr(B) = Pr(A \cap B)$, the events A and B are independent

d) i) $$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)}$$

$$= \frac{0.3}{0.5}$$

$$= 0.6 //$$

ii) $$Pr(A' \mid B) = \frac{Pr(A' \cap B)}{Pr(B)}$$

$$= \frac{0.5 - 0.3}{0.5}$$

$$= 0.4 //$$

iii) $$Pr(B \mid A') = \frac{Pr(B \cap A')}{Pr(A')}$$

$$= \frac{0.5 - 0.3}{1 - 0.6}$$

$= 0.5$ //

EXERCISE SHEET 22

SEP 14

① <Selected question>

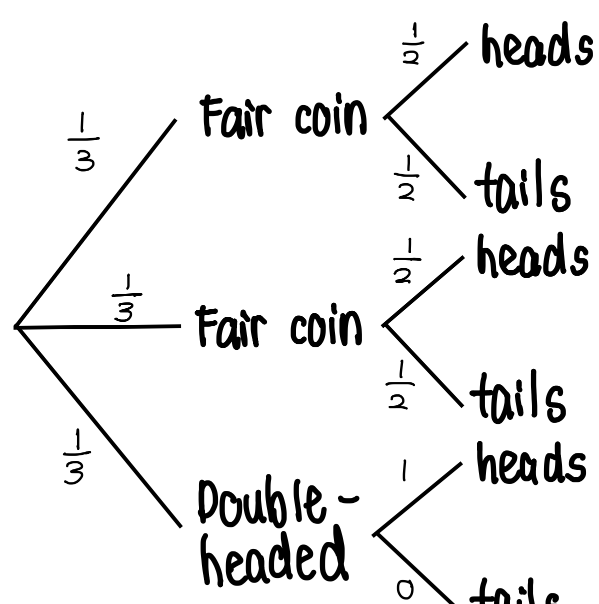
② $$\Pr(\text{tall} \mid \text{man}) = \frac{0.4 \times 0.04}{0.4 \times 0.04 + 0.6 \times 0.01}$$

$$= \frac{8}{11}$$

③ <selected question>

④ <selected question>

⑤



a) $$\Pr(\text{head}) = \Pr(\text{fair} \cap \text{head}) + \Pr(\text{fair} \cap \text{head}) + \Pr(\text{double-headed} \cap \text{head})$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times 1$$

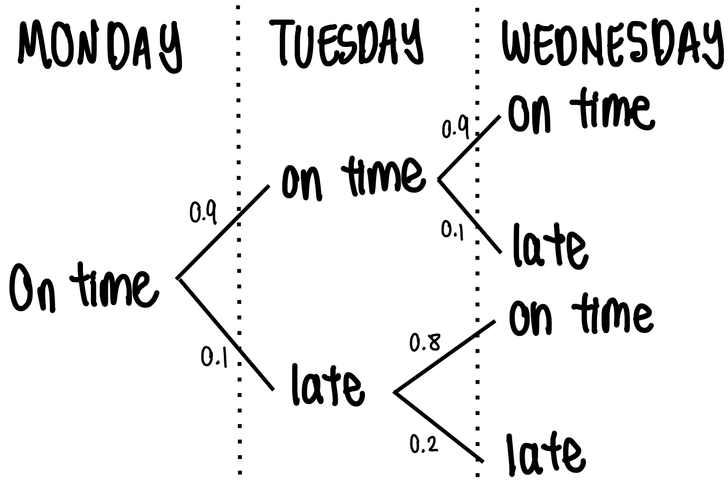
$$= \frac{2}{3} \ //$$

b) $$\Pr(\text{head} \mid \text{double-headed}) = \frac{\Pr(\text{head} \cap \text{double-headed})}{\Pr(\text{head})}$$

$$= \left(\frac{1}{3}\right) \div \left(\frac{2}{3}\right)$$

$$= \frac{1}{2} \ //$$

6



a) Pr(on time | late/on time | on time) $= 0.9 \times 0.9 + 0.1 \times 0.8$

$= 0.89$ //

b) $\text{Pr(Wed on time} \mid \text{Tues on time)} = \dfrac{0.9 \times 0.9}{0.89}$

$= 0.9101123595506$

≈ 0.91 <2dp> //

7 a) $\Pr(X \cap Y) = \Pr(Y) \times \Pr(X \mid Y)$

$= 0.4 \times 0.5$

$= 0.2$ //

b) $\Pr(X \cup Y) = 0.35 - 0.2 + 0.4$

$= 0.55$ //

c) $\Pr(Y \mid X) = \dfrac{\Pr(Y \cap X)}{\Pr(X)}$

$= \dfrac{0.2}{0.35}$

$= 0.5714285714286$

≈ 0.57 <2dp> //

d) $\Pr(Y' \mid X) = \dfrac{\Pr(Y' \cap X)}{\Pr(X)}$

$= \dfrac{0.35 - 0.2}{0.35}$

$= 0.4285714285714$

≈ 0.43 <2dp> //

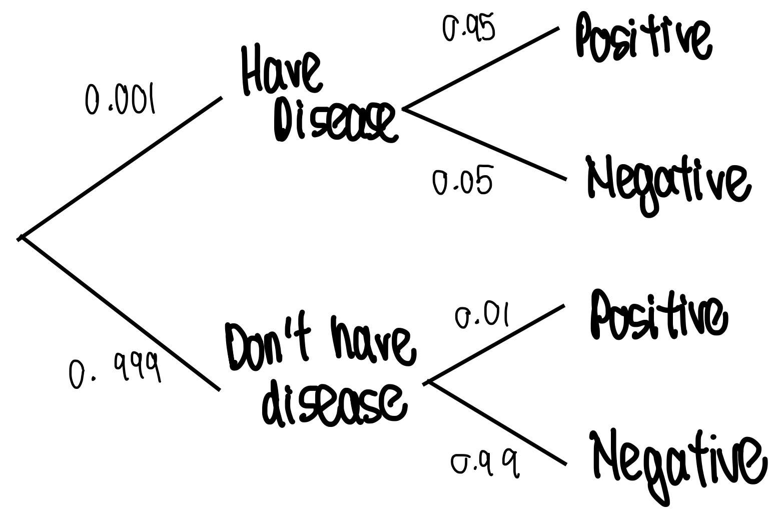
8 <selected question>

9 <selected question>

10 $Pr(\text{Positive} \mid \text{Have disease}) = \dfrac{0.001 \times 0.95}{0.001 \times 0.95 + 0.999 \times 0.01}$

$= 0.0868372943327$

≈ 0.087 <2dp>



11 a) $Pr(\text{3 Hearts chosen}) = \dfrac{{}^{13}C_3 \times {}^{52-13}C_1}{{}^{52}C_5}$

$= 0.0815426170468$

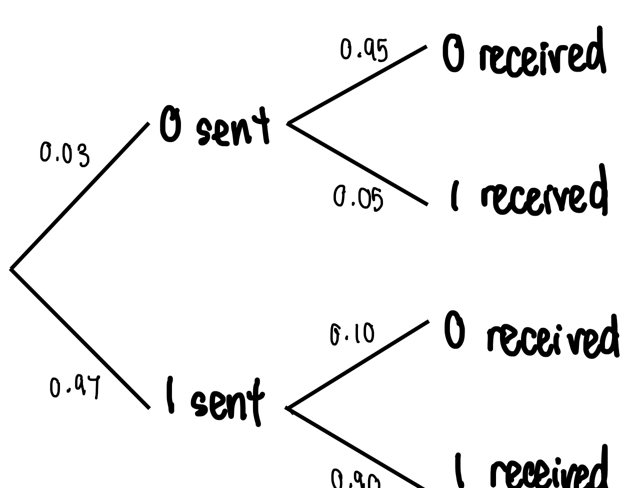
≈ 0.0815 <4dp>

b) $Pr(\text{at least 3 hearts} \mid \text{exactly 3 hearts}) = 0.0815426170468 \div \dfrac{{}^{13}C_3 \times {}^{52-13}C_2 + {}^{13}C_4 \times {}^{52-13}C_1 + {}^{13}C_5}{{}^{52}C_5}$

$= 0.8790035587189$

≈ 0.8790 <4dp>

12



a) $Pr(\text{0 received}) = 0.03 \times 0.95 + 0.97 \times 0.1$

$= 0.1255$

b) $Pr(\text{0 received} \mid \text{0 sent}) = \dfrac{0.03 \times 0.95}{0.1255}$

$= 0.2270916335$

≈ 0.2271 <4dp>

c) Probability that a 0 is sent when a 0 is received is low so the communication system is unreliable //

(13) $Pr(\text{Internet}) = 0.025$

$Pr(\text{TV}) = 0.25$

$Pr(\text{Internet} \cap \text{TV}) = 0.01$

$Pr(\text{Internet} / \text{TV} / \text{Internet} \cap \text{TV} \mid \text{Buy}) = \frac{1}{3}$

$Pr(\text{No Ad} \mid \text{Buy}) = 0.125$

a) $0.025 - 0.01 + 0.25 = 0.265$ //

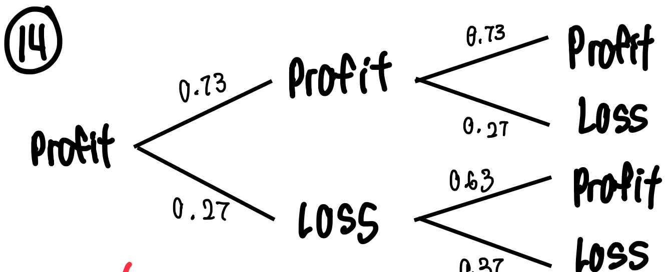
b) $0.265 \times \frac{1}{3} + (1-0.265) \times 0.125 = 0.1802083333$

$\approx 0.1802 \ <4dp>$ //

c) $\dfrac{0.265 \times \frac{1}{3}}{0.1802} = 0.4901734104$

$\approx 0.49 \ <2dp>$ //

(14)

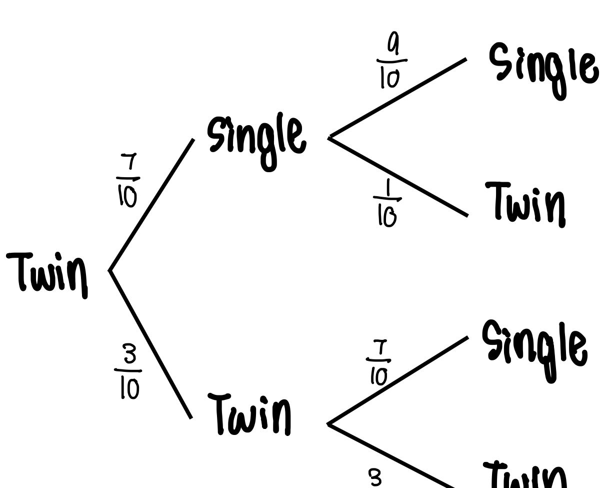


a) 0.73 //

b) $0.73 \times 0.73 + 0.27 \times 0.63 = 0.703$ //

c) $0.73 \times 0.73 \times 0.73 + 0.73 \times 0.27 \times 0.63 + 0.27 \times 0.63 \times 0.73 + 0.27 \times 0.37 \times 0.63 = 0.7003$ //

(15)



a) $\frac{3}{10}$ //

b) $\frac{3}{10} \times \frac{3}{10} + \frac{7}{10} \times \frac{1}{10} = \frac{4}{25}$ //

c) $\frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} + \frac{3}{10} \times \frac{7}{10} \times \frac{1}{10} + \frac{7}{10} \times \frac{1}{10} \times \frac{3}{10} + \frac{7}{10} \times \frac{9}{10} \times \frac{1}{10} = \frac{33}{250}$ //