Answer Key

Q1 (1)	**Q2** (3)	**Q3** (3)	**Q4** (4)
Q5 (4)	**Q6** (2)	**Q7** (1)	**Q8** (3)
Q9 (1)	**Q10** (1)	**Q11** (4)	**Q12** (1)
Q13 (3)	**Q14** (3)	**Q15** (4)	**Q16** (3)
Q17 (4)	**Q18** (1)	**Q19** (1)	**Q20** (2)
Q21 (27)	**Q22** (8)	**Q23** (41)	**Q24** (15)
Q25 (9)	**Q26** (11)	**Q27** (1)	**Q28** (8)
Q29 (336)	**Q30** (8)	**Q31** (1)	**Q32** (3)
Q33 (3)	**Q34** (3)	**Q35** (3)	**Q36** (2)
Q37 (3)	**Q38** (2)	**Q39** (2)	**Q40** (1)
Q41 (4)	**Q42** (4)	**Q43** (2)	**Q44** (1)
Q45 (2)	**Q46** (3)	**Q47** (2)	**Q48** (3)
Q49 (2)	**Q50** (4)	**Q51** (313)	**Q52** (8)
Q53 (11)	**Q54** (4)	**Q55** (2)	**Q56** (5)
Q57 (3)	**Q58** (76)	**Q59** (4)	**Q60** (280)
Q61 (2)	**Q62** (3)	**Q63** (1)	**Q64** (1)
Q65 (4)	**Q66** (1)	**Q67** (2)	**Q68** (3)

Q69 (1)	**Q70** (1)	**Q71** (1)	**Q72** (1)
Q73 (2)	**Q74** (2)	**Q75** (4)	**Q76** (1)
Q77 (3)	**Q78** (1)	**Q79** (1)	**Q80** (3)
Q81 (5)	**Q82** (23)	**Q83** (288)	**Q84** (2550)
Q85 (4)	**Q86** (18)	**Q87** (2002)	**Q88** (190)
Q89 (6)	**Q90** (7)		

Q1

$\mathrm{U(x)}=\left(\mathrm{x}^2-3\mathrm{x}\right)\mathrm{J}$

For a conservative field, Force, $\mathrm{F}=-\frac{\mathrm{dU}}{\mathrm{dx}}$

$\therefore \quad \mathrm{F}=-\frac{\mathrm{d}}{\mathrm{dx}}\left(\mathrm{x}^2-3\mathrm{x}\right)=-(2\mathrm{x}-3)=-2\mathrm{x}+3$

At equilibrium position, $\mathrm{F}=0$

$-2\mathrm{x}+3=0 \quad \Rightarrow \quad \mathrm{x}=\frac{3}{2}\mathrm{m}=1\cdot 5\ \mathrm{m}$

Q2

$\mathrm{T} \propto \mathrm{R}^{\frac{3}{2}}$

$\frac{(\mathrm{T}+\Delta\mathrm{T})}{\mathrm{T}}=\left(\frac{\mathrm{R}+\Delta\mathrm{R}}{\mathrm{R}}\right)^{3/2}$

$1+\frac{\Delta\mathrm{T}}{\mathrm{T}}=\left(1+\frac{\Delta\mathrm{R}}{\mathrm{R}}\right)^{3/2}$

$\therefore \Delta\mathrm{R} << \mathrm{R}$

$1+\frac{\Delta\mathrm{T}}{\mathrm{T}}=1+\frac{3}{2}\frac{\Delta\mathrm{R}}{\mathrm{R}}$

$\frac{\Delta\mathrm{T}}{\mathrm{T}}=\frac{3}{2}\frac{\Delta\mathrm{R}}{\mathrm{R}}$

$\Delta\mathrm{T}=\frac{3}{2}\mathrm{T}\cdot\frac{\Delta\mathrm{R}}{\mathrm{R}}$

Q3

The bridge will be balanced when the shunted resistance of the value of $3\ \Omega$

$\therefore 3=\frac{4\times\mathrm{R}}{4+\mathrm{R}}$

$\Rightarrow 12+3\mathrm{R}=4\mathrm{R}$

$\Rightarrow R=12\ \Omega$

Q4

Each half part has a focal length f. The rays passing through first part will converge downwards while passing through the second part, the rays will equally bend upwards. So the power of the combination will be zero.

Q5

Total MI of the system,

$\mathrm{I}=\mathrm{I}_1+\mathrm{I}_2+\mathrm{I}_3$

$I_2 = I_3 = \frac{2}{3}mr^2 + mr^2 = \frac{5mr^2}{3}$

$I_1 = \frac{2}{3}mr^2$

$\therefore I = 2 \times 5\frac{mr^2}{3} + \frac{2}{3}mr^2$

$= \frac{12mr^2}{3} = 4\ mr^2$

Q6

Path difference on circles around 'P' is same. So, The fringes obtained on the screen in the given condition will be concentric circles.

Q7

After first half hour with the process A ,

$N = \frac{N_0}{2}$

for next one hour with the process B , i.e. $t = \frac{1}{2}$ hr to $t = 1\frac{1}{2}$ hr

$N = \frac{N_0}{2}\left(\frac{1}{2}\right)^4 = N_0\left(\frac{1}{2}\right)^5$

Half life for both A and B ,

$\frac{1}{t_{1/2}} = \frac{1}{1/2} = \frac{1}{1/4}$

$\frac{1}{t_{1/2}} = 2 + 4$

$t_{1/2} = \frac{1}{6}hr$

for further half an hour with both A and B , i.e. $t = 1\frac{1}{2}$ hr to $t = 2$ hr

$N = \left[N_0\left(\frac{1}{2}\right)^5\right]\ \left[\frac{1}{2}\right]^3 = N_0\left[\frac{1}{2}\right]^8$

Q8

Let the rod be depressed by a small amount x (in the figure). Both the springs are compressed by x. When the rod is released, the restoring torque is given by

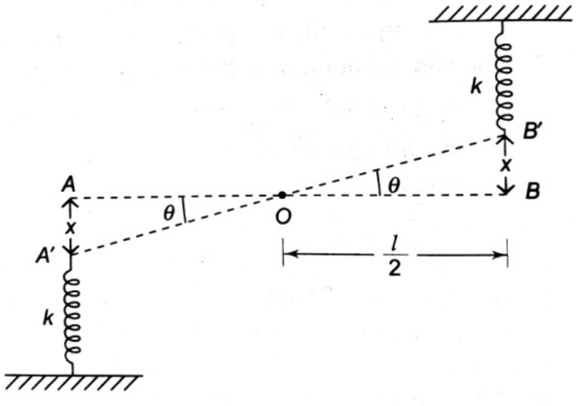
$\tau = (kx)\times\frac{l}{2} + (kx)\times\frac{l}{2} = (kx)l$

Now $\tan\theta = \frac{x}{l/2} = \frac{2x}{l}$.

Since θ is small, $\tan\theta \simeq \theta$, where θ is expressed in radian.

Thus $\theta = \frac{2x}{l}$ or $x = \frac{\theta l}{2}$

$\tau = \mathrm{k}\left(\frac{\theta \mathrm{l}}{2}\right) \times \mathrm{l} = \frac{\mathrm{k}\theta \mathrm{l}^2}{2}$



If I is the moment of inertia of the rod about O, then

$\mathrm{I}\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2} = -\left(\frac{\mathrm{kl}^2}{2}\right)\theta$

Or $\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2} = -\left(\frac{\mathrm{kl}^2}{2\mathrm{I}}\right)\theta$

Since $\frac{\mathrm{d}^2\theta}{\mathrm{dt}^2} \propto (-\theta)$, the motion is simple harmonic whose angular frequency is given by

$\omega = \sqrt{\frac{\mathrm{kl}^2}{2\mathrm{I}}}$

Now, $\omega = \frac{2\pi}{\mathrm{T}}$ and $\mathrm{I} = \frac{\mathrm{ml}^2}{12}$. Therefore, we have

$\frac{2\pi}{\mathrm{T}} = \sqrt{\frac{\mathrm{kl}^2}{2} \times \frac{12}{\mathrm{ml}^2}} = \sqrt{\frac{6\mathrm{k}}{\mathrm{m}}}$

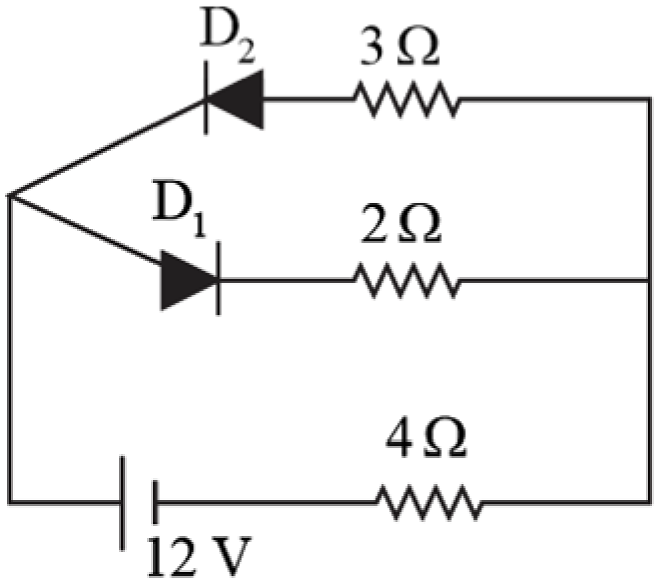
Or $\mathrm{T} = \pi\sqrt{\frac{2\mathrm{m}}{3\mathrm{k}}}$, which is choice $\pi\sqrt{\frac{2\mathrm{m}}{3\mathrm{k}}}$

Q9

According to Kepler's law, all the planets move in elliptical orbits with the sun as one of the foci. The comets are frozen leftovers from the formation of the solar system composed of dust, rock and ices. Comets do not revolve in an elliptical orbit around the sun, so they do not obey Kepler's law.

Q10

The given circuit can be redrawn as,



This means, D_2 is reversed based. Hence, this arm of the circuit will act as open arm i.e., the close circuit will have two resistors 2 Ω and 4 Ω which will be in series connection.

$\Rightarrow \ R_{eq} = 2\ \Omega + 4\ \Omega$

$= 6\ \Omega$

So, current through 4 Ω resistor will be,

$I = \frac{V}{R}$

$= \frac{12}{6}$

$= 2\ \text{A}$

Q11

If two identical balls undergo elastic head-on collision then their velocities are exchanged.

Therefore in the collision of A and B, A comes to rest and B acquires velocity v.

Subsequently, in the collision of B and C, B comes to rest and C acquires the velocity v.

Q12

Energy stored in the wire

$U = \frac{1}{2} Y \times \left(\text{strain}\right)^2 \times \text{volume}$

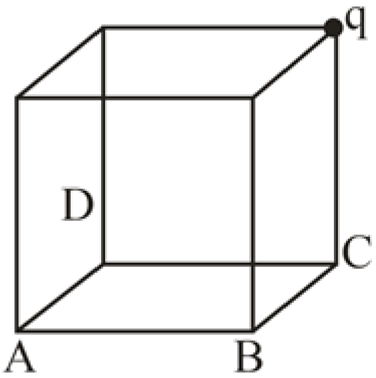
or $U = \frac{1}{2} Y \times \left(\frac{x}{l}\right)^2 \times Al$

or $U=\frac{1}{2} \frac{Y x^2}{l} \times A$

or $U=\frac{1}{2} \frac{Y A}{l} x^2$

Q13

Complete the square by symmetric faces



So charge will appear at corner of the cube

So $\phi_{\mathrm{ABCD}}=\frac{\mathrm{q}}{24 \varepsilon_0}$

Q14

Least count $=\frac{0.5}{50}=0.01 \quad \mathrm{mm}$

Diameter of ball $D=2.5$ mm $+(20)(0.01)$

$D=2.7 \mathrm{~mm}$

$\rho=\frac{M}{vol}=\frac{M}{\frac{4}{3} \pi\left(\frac{D}{2}\right)^3}$

$\left(\frac{\Delta \rho}{\rho}\right)_{max}=\frac{\Delta M}{M}+3 \frac{\Delta D}{D} ;\left(\frac{\Delta \rho}{\rho}\right)_{max}=2 \%+3\left(\frac{0.01}{2.7}\right) \times 100 \%$

$\frac{\Delta \rho}{\rho}=3.1 \%$

Q15

The saturation photocurrent (i) depends on intensity (I) of light ie,

$i \propto I$.

So, when intensity changes, the saturation current also changes. Hence the statement I false.

$hv = \phi + KE_{\max}$

when v is doubled KE will change but it will be not be doubled.

Q16

Rate of loss of energy by unit area of the planet $= \sigma \mathrm{T}^4$, where σ is the Stefan's constant. Let Q be the total energy emitted by the sun every second. If d is the distance of the planet from sun, then Q falls uniformly over the inner surface of the sphere of radius d. Rate of gain of heat by unit area of planet,

$$= \frac{\mathrm{Q}}{4\pi \mathrm{d}^2}$$

For steady temperature of planet,

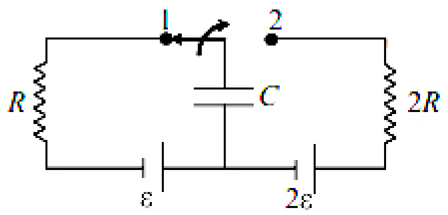
$$\sigma \mathrm{T}^4 = \frac{\mathrm{Q}}{4\pi \mathrm{d}^2}$$

$$\mathrm{T}^4 = \frac{\mathrm{Q}}{4\pi\sigma \mathrm{d}^2} \text{ or } \mathrm{T} = \left(\frac{\mathrm{Q}}{4\pi\sigma \mathrm{d}^2}\right)^{1/4}$$

$$\mathrm{T} \propto \frac{1}{\sqrt{\mathrm{d}}}$$

Q17

In the given circuit when the switch is in position 1.



Then the charge variation with time is

$$q = c\varepsilon\left(e^{-\frac{t}{Rc}} - 1\right)$$

thus, at $t = \infty,\ q = -c\varepsilon$

Now when the switch is changed to position 2, then the charge variation with time will be given by

$$q = c\varepsilon\left(2 - 3e^{\frac{-t}{2Rc}}\right)$$

$\therefore$At $t = 0,\ q = -c\varepsilon$

and at $t=\infty,\ q=2c\varepsilon$

Q18

Let $F \propto P^x V^y T^z$

by substituting the following dimensions :

$\left[P\right]=\left[\mathrm{ML}^{-1}\,\mathrm{T}^{-2}\right]\ ,\ \left[V\right]=\left[\mathrm{LT}^{-1}\right],\ \left[T\right]=\left[\mathrm{T}\right]$

and comparing the dimension of both sides

$x=1,\ y=2,\ z=2$

$F=PV^2T^2$

Q19

On heating water

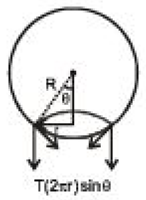
$s\mathrm{in}\theta=\frac{\mathrm{r}}{\mathrm{R}}$

The bubble will detach if Buoyant force 2 Surface tension force

$(\rho_{\mathrm{w}})\left(\frac{4}{3}\pi\mathrm{R}^3\right)\mathrm{g}\geq\left(\mathrm{T}\right)\left(2\pi\mathrm{r}\right)\sin\theta$

Solving $\mathrm{r}=\sqrt{\frac{2\mathrm{p_w R^4 g}}{3\mathrm{T}}}$

$\sin\theta=\frac{\mathrm{r}}{\mathrm{R}}$



Q20

On increasing temperature, KE of free electron increases. They collide more rapidly hence drift velocity decreases. Also with increase in temperature resistance increases, so conductivity decreases.

$\therefore$ Both statements are true, but reason is not correct explanation of assertion.

Q21

In Carnot cycle, temperature varies during adiabatic change. For adiabatic change,

$\mathrm{T}_1 \mathrm{V}_1^{\gamma-1}=\mathrm{T}_2 \mathrm{V}_2^{\gamma-1} \ldots .(\mathrm{i})$

Given that, $\mathrm{V}_1: \mathrm{V}_2=1: 8$,

$\gamma=\frac{5}{3}$(for monoatomic gas)

$T_1=927+273$

$=1200 \mathrm{~K}$

$\mathrm{T}_2=\mathrm{T}_1\left(\frac{\mathrm{V}_1}{\mathrm{~V}_2}\right)^{\gamma-1} \quad \ldots . .[\text{From } (i)]$

$=1200 \times\left(\frac{1}{8}\right)^{\frac{5}{3}-1}$

$=1200 \times\left(\frac{1}{8}\right)^{\frac{2}{3}}$

$=1200 \times \frac{1}{4}$

$=300 \mathrm{K}$

$\therefore \quad \mathrm{T}_2=300-273=27^{\circ} \mathrm{C}$

Q22

The magnetic field produced by long-straight wire carrying current of 20A rests on a table on small wire,

$\mathrm{B}=\frac{\mu_0 \mathrm{I}}{2 \pi \mathrm{h}}$ .... (i)

The magnetic force on smal conductor is

$\mathrm{F}_{\mathrm{B}}=\mathrm{I} / \mathrm{B} \sin \theta=\mathrm{I} / \mathrm{B}=\mathrm{I} l\left(\frac{\mu_0 \mathrm{I}}{2 \pi \mathrm{h}}\right)$

$= \frac{\mu_0 I^2 l}{2\pi h} \quad \ldots . \left(\text{ ii }\right)$

At equilibrium,

$\Rightarrow \mathrm{F_B} = \mathrm{F_G}$

$\Rightarrow \frac{\mu_0 I^2 l}{2\pi h} = mg$

$\Rightarrow \mathrm{h} = \frac{\mu_0 \mathrm{I}^2 l}{2\pi \, \mathrm{mg}} = \frac{4\pi \times 10^{-7} \times 20 \times 20 \times 2}{2\pi \times 2 \times 10^{-3} \times 10}$

$= 8 \times 10^{-3} \mathrm{m}$

$\therefore \quad \mathrm{h} = 0.80 \, \mathrm{cm}$

Q23

$\because$ the cut-off wavelength for continuous x-ray is given as : $\lambda_{\min} = \frac{hc}{eV_0}$

$\therefore$

$\frac{hc}{eV_0} = \lambda \ldots \; Eq\left(1.\right)$

$\frac{hc}{3eV_0} = \lambda - \Delta\lambda \ldots \; Eq\left(2.\right)$

$(\mathrm{V}_0 = \text{accelerating voltage})$

$\Rightarrow \quad V_0 = \frac{2hc}{3e\Delta\lambda} = \frac{2 \times 6.6 \times 10^{-34} \times 3 \times 10^8}{3 \times 1.6 \times 10^{-19} \times 20 \times 10^{-12}} \approx 41 \; kV$

Q24

$\mathrm{As}, \mathrm{m_a} = \frac{\mathrm{A_m}}{\mathrm{A_c}}$

$\therefore \quad A_m = m_a \times A_c = 0.75 \times 20 = 15\mathrm{V}$

Q25

Given, PV^5 =constant

This is polytropic process with $n=5$.

$\therefore$Specific heat capcity,

$C=C_V+\frac{R}{1-n}$

$=\frac{R}{\gamma-1}+\frac{R}{1-n}$

$=\frac{\mathrm{R}}{\frac{7}{5}-1}+\frac{\mathrm{R}}{1-5} \quad \ldots \ldots\left(\because \gamma_{\mathrm{di}}=\frac{7}{5}\right)$

$=\frac{5R}{2}-\frac{R}{4}=\frac{9R}{4}$

$\therefore \quad C=(2.25)R$

Q26

From lens maker's equation,

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

For objective,

$\frac{1}{f_o}=\frac{1}{v_o}-\frac{1}{u_o}$

$\therefore \quad v_0=\frac{u_0 f_0}{(u_0+f_0)} \qquad \ldots(i)$

For eye-piece,

$\frac{1}{\mathrm{f_e}}=\frac{1}{\mathrm{V_e}}-\frac{1}{\mathrm{u_e}}$

Here,$v_e=D=25\,\mathrm{cm}$

$\therefore \quad \frac{1}{\mathrm{u_e}}=\frac{1}{\mathrm{D}}-\frac{1}{\mathrm{f_c}}$

$\therefore \quad \mathrm{u_e} = \frac{\mathrm{f_e D}}{\mathrm{f_e - D}} \quad \ldots (ii)$

From equations (i) and (ii),

$\mathrm{v_0} = \frac{(-2)(1.5)}{(-2)+(1.5)} = 6$

$\mathrm{u_e} = \frac{(6.25)(-25)}{(6.25)-(-25)} = -5$

As negative sign denotes direction neglecting it,

$\therefore \quad L = v_0 + u_\mathrm{e} = 6 + 5 = 11\,\mathrm{cm}$

Q27

Given,

Magnetic flux linked with the larger coil of radius R is $\phi = 0.5 \times 10^{-3}\ \mathrm{Wb}$

Strength of current through small neighbouring coil $I = 0.5\ \mathrm{A}$

The total amount of magnetic flux linked with all the turns of one coil is directly proportional to the strength of current in neighbouring coil.

$\phi = MI$

$\Rightarrow M = \frac{\phi}{I}$

Substituting the values, we get

$M = \frac{0.5 \times 10^{-3}}{0.5} = 1\ \mathrm{mH}$

So, the coefficient of mutual inductance for the given pair of coils is $1\ \mathrm{mH}$.

Q28

In y direction

$u_y = 20 m/s$

$a_y = -10m/s^2$

$s_y = u_y t + \frac{1}{2} a_y t^2$

$t = \frac{2u_y}{a_y} = \frac{2\times20}{10} = 4sec$

In - α direction

$s_x = 0 = u_x t + \frac{1}{2} a \times d^2$

$0 = u_x(4) - \frac{8}{2}(4)^2$

$16 = 4x$

$\tan\theta = \frac{4x}{4y} = \frac{16}{20}$

$10\tan\theta = 10 \times \frac{16}{20} = \ 8$

Q29

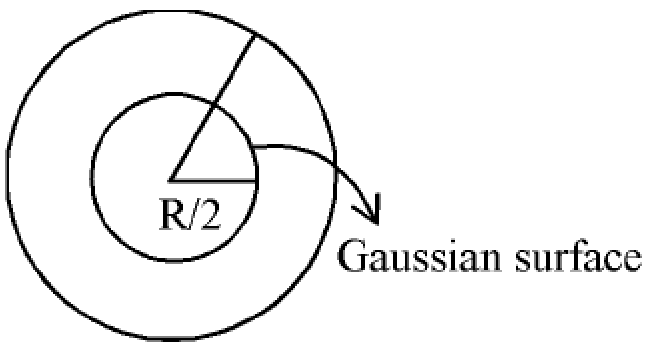
$\frac{v}{4[L+0.6r]} = 480$ (frequency)

$\Rightarrow\ v\ =\ 480\ \times\ 4\ \times\ [L\ +\ 0.6r]$

$\Rightarrow\ v\ =\ 480\ \times\ 4\ \times\ [0.16\ +\ 0.6 \times 0.025]$

$\Rightarrow v = 336\ \mathrm{m\ s^{-1}}$

Q30



$\oint \overrightarrow{\mathrm{E}} \cdot \mathrm{d}\overrightarrow{\mathrm{s}} = \frac{\mathrm{q_{in}}}{\varepsilon_0}$

$\mathrm{E}4\pi\left(\frac{\mathrm{R}}{2}\right)^2 = \frac{\int_0^{\mathrm{R}/2} \rho 4\pi \mathrm{r}^2\, \mathrm{dr}}{\varepsilon_0}$

$$\Rightarrow \quad E\pi R^2 = \frac{\int_0^{R/2} Kr\,4\pi r^2\,dr}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{4K\int_0^{R/2} r^3\,dr}{R^2\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{4K}{R^2\varepsilon_0}\left[\frac{r^4}{4}\right]_0^{R/2}$$

$$\Rightarrow \quad E = \frac{K}{R^2\varepsilon_0}\left[\frac{R^4}{16}\right]$$

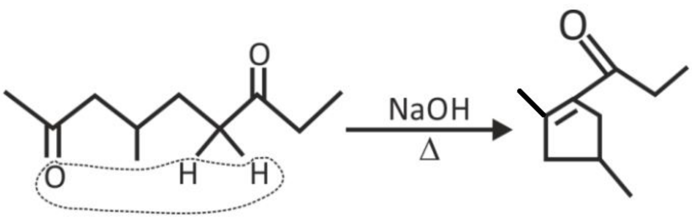
$$\Rightarrow \quad E = \frac{KR^2}{2\times 8\varepsilon_0}$$

Q31

Coagulation power is directly proportional to charge on the ions.

Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1.

Q32



Q33

Element ‘S’ forms electron deficient compounds.

Q34

Van Arkel's method is used to purify crude titanium metal. It is heated with iodine to about 500K to form volatile compound. TiI_4 leaving behind the impurities . TiI_4 is further heated to 1700K when it decomposes to give pure titanium.

Q35

The reason for temporary hardness is owing to the presence of bicarbonates of Ca and Mg.

To remove this temporary hardness, we have to use agents that can precipitate out the bicarbonates in an easy-to-filter-out form:

$$Ca(HCO_3)_2 + Na_2CO_3 \rightarrow CaCO_3 \downarrow + NaHCO_3$$
$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 \downarrow + 2H_2O$$

The hypochlorite anion is oxidizing and gets hydrolyzed to ^{-}OH.

$$OCl^{-}(aq) + H_2O(I) \rightleftharpoons HOCl(aq) + {}^{-}OH(aq)$$

Calcium phosphate is just insoluble in water. It is useful in this context.

Q36

$$K_{sp} \text{ for } ZnS = \left[Zn^{2+}\right]\left[S^{2-}\right]$$
$$\left[S^{2-}\right] = \frac{10^{-21}}{0.01} = 10^{-19}$$

$$H_2S \rightleftharpoons H^+ + HS^-$$
$$K_{a_1} = \frac{\left[H^+\right]\left[S^{2-}\right]}{\left[HS^-\right]}$$
$$HS^- \rightleftharpoons H^+ + S^{2-}$$
$$K_{a_2} = \frac{\left[H^+\right]\left[S^{2-}\right]}{\left[HS^-\right]}$$

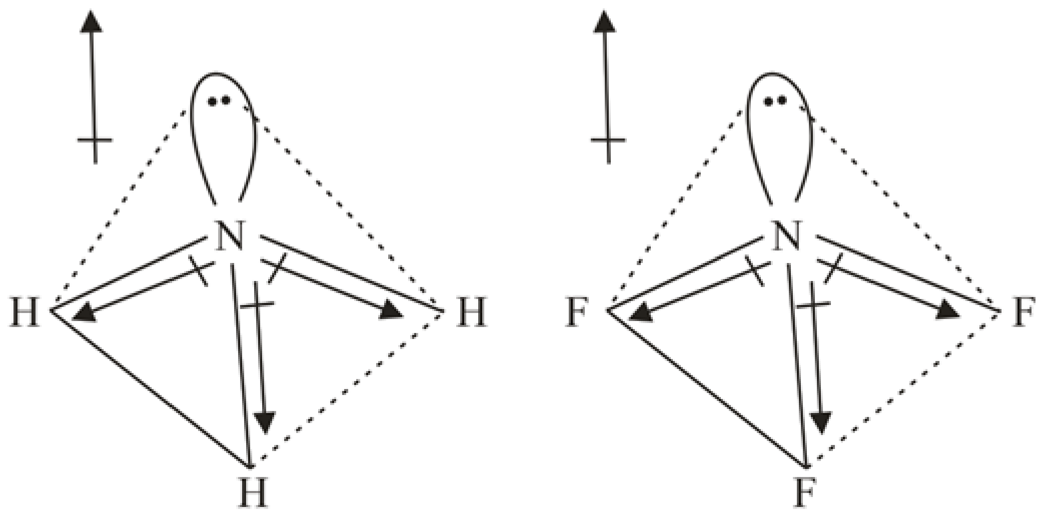
$$\text{for } K_{a_1}.\ K_{a_2} = \frac{[H^+]^2[S^{2-}]}{[H_2S]}$$

$$10^{-20} = \frac{\left[H^+\right]^2 \times 10^{-19}}{0.1} \Rightarrow \left[H^+\right] = 0\ .1$$

or $pH = 1$

Q37

Dipole moment of NH_3 (0.49×10^{-29} cm) is more than that of $NF_3 \left(0.07 \times 10^{-29} \mathrm{cm}\right)$. This is due to the fact that the direction of the dipole moments of the bonding $N - H$ and non-bonding electron pairs coincide in NH_3 and therefore, vector addition of all dipole moments yield a large resultant dipole moment. In NF_3 , on the contrary, the dipole moments of $N-F$ bonds and of the non-bonding electron pair are in opposite directions, so that they are partially compensated when summated.



A. From the above figre, it is clear that NF_3 has a trigonal pyramidal structure.

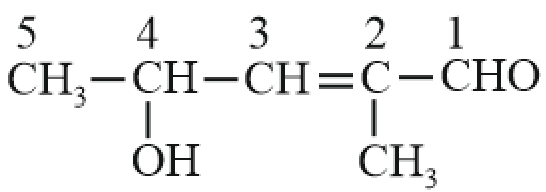
B. $2NF_3 + 3H_2O(\text{vapour}) \rightarrow N_2O_3 + 6HF$

D. $N_2O_3(l) + H_2O(l) \rightarrow \underset{(\text{Acid})}{2HNO_2(l)}$

Q38

In $\left[Fe(CN)_6\right]^{4-}$ Fe is in $+2$ state with elctronic configuration $4s^o 3d^6$ and CN^- is strong ligand causes inward pairing, thus shows d^2sp^3 , Octahedral. $Ni(CO)_4$ is sp^3 tetrahedral and $\left[Ni(CN)_4\right]^{2-}$ is dsp^2 , square planar.

Q39



4-Hydroxy-2-methylpent-2-en-1-al

Q40

An FCC contains 4 atoms per unit cell whereas BCC and SC contain 2 and 1 atoms respectively.

The Packing fraction for FCC= 0.74

The Packing fraction for BCC= 0.68

The Packing fraction for SC= 0.52

Q41

Industries present nearby Taj Mahal produce a lot of NO_2 and SO_2 gases which react with water, oxygen and other chemicals to form sulphuric acid and nitric acid. These then mix with water and make the rain acidic which than react with marble to decolourise it as follows

$CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + CO_2 + H_2O$

Q42

For H-atom E belongs to visible region when $n_1 = 2$and $n_2 = 3$. B, C and D lines belongs to Infrared region.

Q43

$BeCl_2$ and $AlCl_3$ both have bridged structure in solid phase.

Beryllium does not exhibit coordination number more than four as in valence shells there are only four orbitals. The remaining members of the group can have a coordination number of six by making use of d-orbitals.

Q44

M.O. configuration of N_2 is

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \pi^2 {p_x}^2 = \pi 2{p_y}^2 < \sigma 2{p_z}^2$

B.O. of $N_2 = \frac{10-4}{2} = 3$

M.O. configuration of N_2^+ is

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \pi 2{p_x}^2 = \pi 2{p_y}^2 < \sigma 2{p_z}^1$

B.O. of $N_2^+ = \frac{9-4}{2} = 2.5$

M.O. of configuration of O_2 is

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \sigma 2p_z{}^2 < \pi 2p_x{}^2 < \pi 2p_y{}^2 < \pi^* 2p_x{}^1 < \pi^* 2p_y{}^1$

$\therefore$ B.O. of $O_2 = \frac{10-6}{2} = 2$

As it is paramagnetic so option (a) is incorrect.

M.O. configuration of O_2^+ is

$\sigma 1s^2 < \sigma^* 1s^2 < \sigma 2s^2 < \sigma^* 2s^2 < \sigma 2p_x{}^2 < \pi 2p_y{}^2 \cong \pi 2p_z{}^2 < \pi^* 2p_y{}^1 \cong \pi^* 2p_z{}^0$

B.O. of $O_2^+ = \frac{10-5}{2} = 2.5$

B.O. $\propto$ Bond energy $\propto \frac{1}{\text{Bond length}}$

Q45

(1) Norethindrone — Antifertility drug

(2) Ofloxacin — Anti – Biotic

(3) Equanil — Hypertension (tranquailizer)

Q46

Statement

1. Correct

2. Incorrect- Including fructose all monosaccharides give the fehling's test

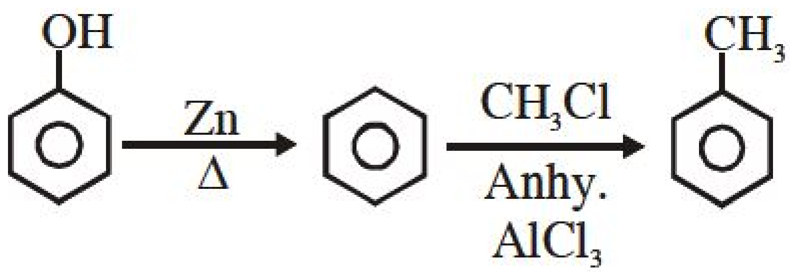
3. Correct

4. Incorrect- Glucose & mannose are epimers not anomers.

Q47

Mixture of $1^\circ, 2^\circ$ and 3° amines can be distinguished by Hinsberg's method. In Hinsberg's method benzene sulphonyl chlotide $C_6H_5SO_2Cl$ (Hinsberg's reagent) is used. 1° and 2° amines reacts to give sulphonamide derivatives with loss of HCl, whereas 3° amines do not give any isolable products other than the starting amine. In the latter case a quanternary ammonium salt may be as an intermediate, but this rapidly breaks down in water to liberate the original 3° amine.

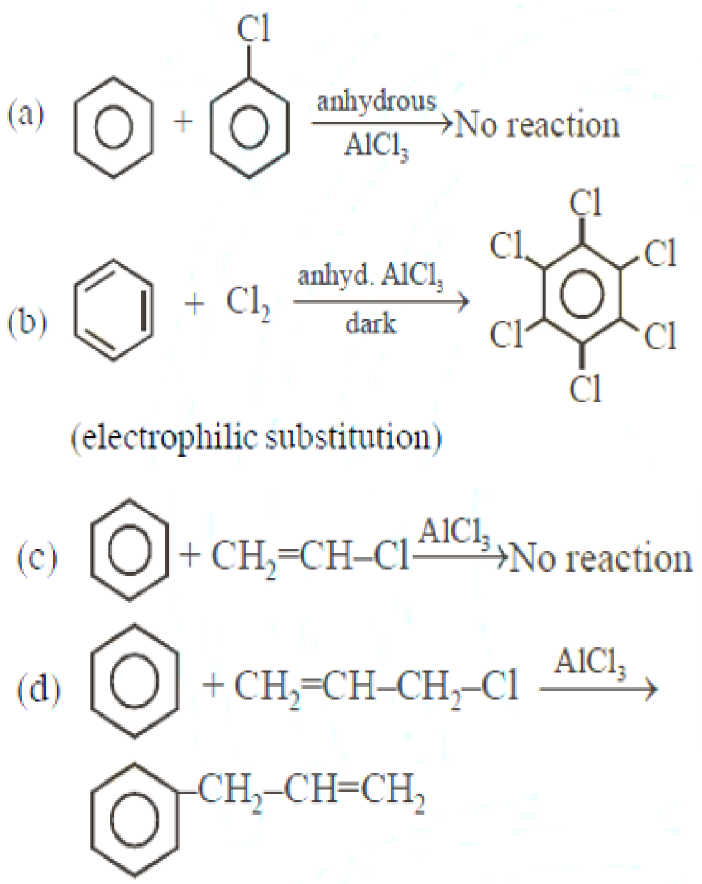
Q48



Q49

As neoprene is a polymer of 2- chloro buta 1, 3- di-ene $-(CH_2-\underset{\displaystyle Cl}{\underset{|}{C}}=CH-CH_2-)_n$

Q50



Q51

Average kinetic energy $\overline{\mathrm{KE}}$ (per molecule)$= \frac{3}{2}\mathrm{k_b T}$

$\frac{\overline{\mathrm{KE}}(40^\circ \mathrm{C})}{\overline{\mathrm{KE}}(20^\circ \mathrm{C})} = \frac{\frac{3}{2}\mathrm{k_b}}{\frac{3}{2}\mathrm{k_b}}\frac{(313)}{(293)} = \frac{313}{293}$

(Here, $\mathrm{k_b}$ =Boltzmann's constant=$\frac{\mathrm{R}}{\mathrm{N_A}}$)

Q52

The number of Faraday's passed $= \frac{2\times10^{-3}\times16\times60}{96500}$

$= 1.99 \times 10^{-5}$

$\Rightarrow$ number of gram equivalent of $\mathrm{Cu^{2+}}$ deposited as Cu(s)

$= 1.99 \times 10^{-5}$

$\Rightarrow$ number of moles of $\mathrm{Cu^{2+}}$ deposited

$= \frac{1.99}{2} \times 10^{-5} \approx 10^{-5}$

Absorbance is directly proportional to $\left[\mathrm{Cu^{2+}}\right]$.

No. of moles of $\mathrm{Cu^{2+}}$ initially present = 2 x 10^{-5} moles

$\left[\mathrm{Cu^{2+}}\right]_{\mathrm{Initial}} = 2 \times 10^{-5} \times \frac{1000}{250}$

= 8 x10^{-5}M

Q53

Since the partial pressure reduces to half after

6.93 minutes, the half-life,$t_{1/2} = 6.93\mathrm{min}$

$\mathrm{k} = \frac{0.693}{\mathrm{t_{1/2}}} = \frac{0.693}{6.93} = 0.10\,\mathrm{min}^{-1}$

For a gas phase first order reaction,

$\mathrm{k} = \frac{2.303}{\mathrm{t}}\log_{10}\left(\frac{\mathrm{p_i}}{\mathrm{p_A}}\right)$

$\mathrm{p_i} = 6.0 \ \mathrm{atm}\ ;\ \mathrm{p_A} = 6.0 - 4.0 = 2.0 \ \mathrm{atm}$

$\therefore \ \ 0.10 = \frac{2.303}{\mathrm{t}}\log_{10}\left(\frac{6.0}{2.0}\right)$

$\therefore\ \mathrm{t}=23.03\ \log\ (3.0)=11.05\ \mathrm{min}$

Q54

Ziegler-Natta catalyst is $\mathrm{TiCl_4}+\mathrm{Al(C_2H_5)_3}$

Q55

Tertiary alcohols like 2-methylpropan-2-ol, $2,4$-dimethylpentan-2-ol react immediately with Lucas reagent $(conc. HCl\ +\ anhydrous\ \mathrm{ZnCl_2}\ )$ at room temperature.

Q56

Total count of geometrical isomers of square planar complex $[\mathrm{Mabcd}]$ is 3 and $[\mathrm{Ma_4\,b_2}]$ is 2.

$\mathrm{Total}=5$

Q57

Mixtures $(\mathrm{i}),\ (\mathrm{ii})$ and (vi) behave as ideal solutions.

Q58

Chemical reaction is :

$5\,\mathrm{Fe}^{2+}+\mathrm{MnO_4}+8\mathrm{H}^{+}$

$\rightarrow \mathrm{Mn}^{2+}+4\mathrm{H_2O}+5\,\mathrm{Fe}^{3+}$

$\therefore\ \ 5\,\mathrm{molFe}^{2x}\equiv 1\,\mathrm{molMnO_4}$Now, $1000\,\mathrm{mL}$ of

$100\,\mathrm{mL}$ of $0.1\,\mathrm{MKM}_n\,\mathrm{O_4}=0.01\,\mathrm{molMnO_4}$

$0.01\,\mathrm{molMnO_4}=0.05\,\mathrm{molFe}^{2+}$

$=\ 0.05\ \mathrm{mol}\ \ \mathrm{FeSO_4}$

$= 0.05 \times 152$

$= 7.6 \text{ g } FeSO_4$

Q59

	$3A \rightleftharpoons$	C	$+$	$2D$
Initially :	C			
at eq :	$C - x$	$\frac{x}{3}$		$\frac{2x}{3}$

Now, $\frac{2x}{3} = 2 \times (C - x)$

$x = 3C - 3x$

$C = \frac{4}{3}x$

$K_c = \frac{[C]\,[D]^2}{[A]^3}$

$K_c = \frac{\frac{x}{3} \times \left(\frac{2x}{3}\right)^2}{\left(\frac{4}{3}x - x\right)^3} = \frac{\left(\frac{4}{27}\right)}{\left(\frac{1}{27}\right)} = 4$

Q60

$q = q_{AB} + q_{BC} + q_{CD} + q_{DA}$

$= nRT_B \cdot \ln \frac{2V_0}{V_0} + n \cdot C_{v,m} \cdot (T_C - T_B) + nRT_c \ln \frac{V_0}{2V_0} + n \cdot C_{v,m}(T_A - T_D)$

$= nR \ln 2 (T_B - T_C) = 1 \times 2 \times 0.7 \times 200 = 280 \text{ cal / mol}$

Q61

$$\text{Given integral } = \int_0^1 0 dx + \int_1^2 1\{x\} dx + \int_2^3 2^3\{x\} dx + \ldots\ldots + \int_9^{10} 9^3\{x\} dx$$
$$= \left(1^3 + 2^3 + \ldots\ldots 9^3\right) \int_0^1 \{x\} dx = \left(\frac{9 \times 10}{2}\right)^2 \times \int_0^1 x dx = \frac{2025}{2}$$

Q62

Any tangent to $y = x^2$ can be written as $y = mx - \frac{1}{4}m^2$ if it touches 2^{nd} , then

$$mx - \frac{m^2}{4} = x^2 - 2x + 2$$

$$\Rightarrow \quad x^2 - (2+m)x + \left(2 + \frac{m^2}{4}\right) = 0$$

$$D = 0 \Rightarrow (m+2)^2 - 4\left(2 + \frac{m^2}{4}\right)$$

$$4m - 4 = 0 \Rightarrow m = 1 \Rightarrow 4x - 4y - 1 = 0$$

Q63

Given, $f(f(1)) = 0 \Rightarrow f(1 + \alpha + \beta) = 0$

$f(f(2)) = 0 \Rightarrow f(4 + 2\alpha + \beta) = 0$

Hence, roots of $f(x)$ are $\alpha + \beta + 1$ and $2\alpha + \beta + 4$

Now, sum of roots $= 3\alpha + 2\beta + 5 = -\alpha$

$\Rightarrow \quad 4\alpha + 2\beta = -5$

Now, product of roots $= (\alpha + \beta + 1)(2\alpha + \beta + 4) = \beta$

$(\alpha + \beta + 1)\frac{3}{2} = \beta$

From Equs. (1) and (2), $b = f(0) = \frac{-3}{2}$

Hence, $2|f(0)| = 3$

Q64

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$\Rightarrow$ a(b − 1)(c − 1) $- (1-a)(b-1) - (c-1)(1-a) = 0$

Dividing by $(1-a)(1-b)(1-c)$

$$\Rightarrow \frac{a}{1-a} + \frac{1}{1-c} + \frac{1}{1-b} = 0$$

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

Q65

Given,

$$\text{Slope of normal } = \frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$$

$x^2y^2dx + dx - xydx = x^2dy$
$x^2y^2dx + dx = x^2dy + xydx$
$x^2y^2dx + dx = x(xdy + ydx)$
$x^2y^2dx + dx = xd(xy)$

$$\frac{dx}{x} = \frac{dxy}{1 + x^2y^2}$$

$\ln kx = \tan^{-1}(xy)$ ... (i)

Curve passes though $(1, 1)$

So, In $\mathrm{k} = \frac{\pi}{4} \Rightarrow \mathrm{k} = \mathrm{e}^{\frac{\pi}{4}}$

Now from equation (i)

We get, $\frac{\pi}{4} + \ln x = \tan^{-1}(xy)$

$xy = \tan\left(\frac{\pi}{4} + \ell nx\right)$

$xy = \left(\frac{1-\tan(\ln x)}{1-\tan(\ln x)}\right)$...(ii)

Put $x = e$ in (ii)

We get, e. $y(e) = \frac{1+\tan 1}{1-\tan 1}$

Q66

$$\begin{aligned}(125)\ & A + B = 3I \Rightarrow AA^T + BA^T = 3A^T \Rightarrow AA^T = 3A^T - BA^T \Rightarrow 4I = 3A^T - BA^T \\ & \Rightarrow 12A^{-1} - BA^T + I = 5I \\ & \therefore \det\left(12A^{-1} - BA^T + I\right) = 125\end{aligned}$$

Q67

$S_n = {}^nC_0{}^nC_1 + {}^nC_1{}^nC_2 + \ldots + {}^nC_{n-1} \cdot {}^nC_n = {}^{2n}C_{n-1}$
$S_{n+1} = {}^{2n+2}C_n$

$$\frac{S_{n+1}}{S_n} = \frac{{}^{2n+2}C_n}{{}^{2n}C_{n-1}} = \frac{15}{4}$$

$$\Rightarrow \quad \frac{(2n+2)(2n+1)}{n(n+2)} = \frac{15}{4}$$

$$\Rightarrow \quad n^2 - 6n + 8 = 0$$

sum of roots $= 6$

Q68

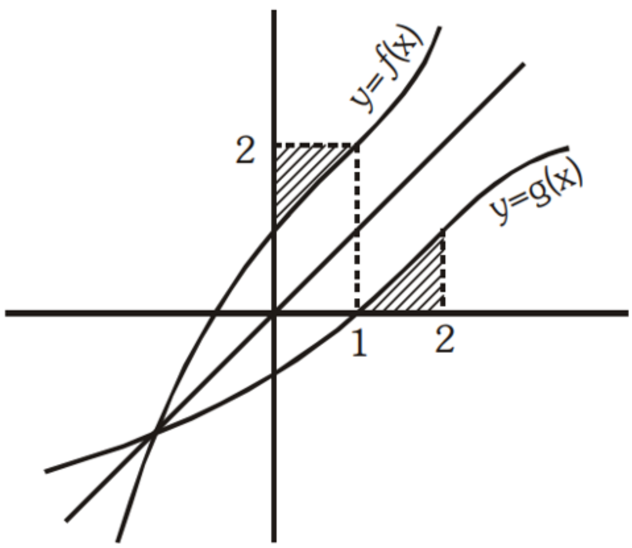
$$\frac{1}{d}\left(\frac{a_2-a_1}{a_2a_1}+\frac{a_3-a_2}{a_3a_2}+\ldots.+\frac{a_{4001}-a_{4000}}{a_{4001}a_{4000}}\right)=10$$
$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\ldots+\frac{1}{a_{4000}}-\frac{1}{a_{4001}}\right)=10$$
$$=\frac{1}{d}\left(\frac{1}{a_1}-\frac{1}{a_{4001}}\right)=10$$
$$=\frac{1}{d}\left(\frac{a_{4001}-a_1}{a_1\cdot a_{4001}}\right)=10$$
$$=\frac{4000}{a_1\cdot a_{4001}}=10 \quad (\text{ as } a_{4001}=a_1+4000d)$$
$$\Rightarrow \quad a_1a_{4001}=400; a_2+a_{4000}=50 \Rightarrow (a_1+d)+(a_1+3999d)=50$$
$$(a_1-a_{4001})^2=(a_1+a_{4001})^2-4a_1a_{4001}$$
$$|a_1-a_{4001}|=30$$

Q69

$$f(x)=x^3-3x^2+3x+1$$
$$f'(x)=3x^2-6x+3=3\left(x^2-2x+1\right)$$
$$f'(x)=0 \quad \Rightarrow \quad x=1$$

sign of $f'(x)$ does not change about $x=1$, then $x=1$ is point of inflection.

Area bounded by $y=g(x)$ and x-axis from $x=1$ to $\mathrm{x}=2$ is same as area bounded by $\mathrm{y}=f(\mathrm{x})$ and y-axis from $\mathrm{y}=1$ to $\mathrm{y}=2$.



Area of shaded region = area of rectangle with 1 and 2 as sides - area bound by $f(x)$ from $x=0$ to $1=1/4$

Q70

$p \Rightarrow (p \vee {\sim}q) \equiv {\sim}p \vee (p \vee {\sim}q) \equiv ({\sim}p \vee p) \vee {\sim}q$

We know, ${\sim}p \vee p$ is always true

i.e. the given statement is always true, hence, a tautology

Q71

$$\underbrace{\int\left(\frac{1}{(\cos x)^{2019}}\right)}_{\mathrm{I}} \cdot \underbrace{\operatorname{cosec}^2 x}_{\mathrm{II}} dx - 2019\int\frac{dx}{(\cos x)^{2019}}$$

$$\frac{1}{(\cos x)^{2019}} \cdot (-\cot x) + 2019\int\frac{\sin x}{(\cos x)^{2020}} \cdot \cot x dx$$

$$\therefore \quad f(x) = \cot x; \quad g(x) = \cos x$$

$$\Rightarrow \quad \left|f\left(\frac{\pi}{4}\right) + g(0)\right| = 2$$

Q72

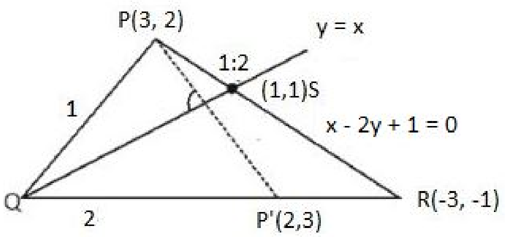
$$\begin{aligned}
&h(x) = g(x) + x \\
&\Rightarrow h'(x) = g'(x) + 1 \\
&\Rightarrow g'(x) = h'(x) - 1 \\
&\Rightarrow g''(x) = h''(x) \\
&\Rightarrow h''(x) - 3\left(h'(x) - 1\right) > 3 \\
&\Rightarrow h''(x) - 3h'(x) > 0 \\
&\Rightarrow \frac{d}{dx}\left(e^{-3x}h'(x)\right) > 0
\end{aligned}$$

$$\begin{aligned}
&\text{Let } P(x) = e^{-3x}h'(x) \\
&\Rightarrow P'(x) > 0 \\
&\Rightarrow P(x) \text{ is an increasing function.} \\
&P(0) = h'(0) = 0 \\
&\Rightarrow P(x) > 0 \forall x > 0 \\
&\Rightarrow h'(x) > 0 \forall x > 0 \\
&\Rightarrow h(x) \text{ is an increasing function } \forall x > 0
\end{aligned}$$

Q73

$|A|=4, |B|=6$, the number of one-one functions is ${}^{6}P_{4}=360$. The number of increasing functions is $\binom{6}{4}=15$. The desired number is $360-15=345$

Q74



Point of intersection of $y=x$ & $x-2y+1=0$ is $S(1,1)$

Now, $PS:SR=PR:RQ=1:2$

Hence, using section formula $R=(-3,-1)$

Let, reflection of P about the line $y=x$ is P'

$\Rightarrow P'=(2,3)$ which lies on the side QR

$\Rightarrow$ equation of QR is $4x-5y+7=0$

Point of intersection of QR & $y=x$ is

$(7,7)=Q$

Q75

Given,

$$\begin{aligned} 2x-y-z&=-a \\ x-2y+z&=b \\ x+y-2z&=c \end{aligned}$$

For consistent system of equations $\Delta \neq 0$ or $\Delta=\Delta_1=\Delta_2=\Delta_3=0$

$$\Delta=\begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{vmatrix}=0$$

$$\Delta_1=\begin{vmatrix} -a & -1 & -1 \\ b & -2 & 1 \\ c & 1 & -2 \end{vmatrix}=0$$

$\Rightarrow \mathrm{a}+\mathrm{b}+\mathrm{c}=0$

$$\Delta_2=\begin{vmatrix} 2 & -\mathrm{a} & -1 \\ 1 & \mathrm{b} & 1 \\ 1 & \mathrm{c} & -2 \end{vmatrix}=0$$

$\Rightarrow \mathrm{a}+\mathrm{b}+\mathrm{c}=0$

$$\Delta_3=\begin{vmatrix} 2 & -1 & -\mathrm{a} \\ 1 & -2 & \mathrm{b} \\ 1 & 1 & \mathrm{c} \end{vmatrix}=0$$

$\Rightarrow \mathrm{a}+\mathrm{b}+\mathrm{c}=0$

$$\left.\begin{array}{r} \mathrm{a}+\mathrm{b}+\mathrm{c}=0 \\ 16\mathrm{a}-4\mathrm{b}+\mathrm{c}=0 \end{array}\right\} \Rightarrow \frac{\mathrm{a}}{5}=\frac{\mathrm{b}}{15}=\frac{\mathrm{c}}{-20}$$

$\Rightarrow \mathrm{a}=\lambda$

$\mathrm{b}=3\lambda$

$\mathrm{c}=-4\lambda$

$\Rightarrow$ Sum of roots $=\frac{-\mathrm{b}}{\mathrm{a}}=-3$

Q76

Given, $f(x+y)=f(x)+f(y)-xy-1, \forall x, y \in R$

$\therefore f(x+1)=f(x)+f(1)-x-1[\text{ putting } y=1]$

$\Rightarrow f(x+1)=f(x)-x[\because f(1)=1] \text{ AARKS}$

$\therefore f(n+1)=f(n)-n<f(n)$

$\Rightarrow f(n+1)<f(n)$

So, $f(n)<f(n-1)<f(n-2)<\ldots<f(3)<f(2)<f(1)=1$

$\therefore f(n)=n$ holds only for $n=1$

Q77

$$T_n=\tan^{-1}\left(\frac{4}{4n^2+3}\right)=\tan^{-1}\left(\frac{1}{n^2+(3/4)}\right)=\tan^{-1}\left(\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$$

$$T_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(n-\frac{1}{2}\right)$$

$$S_n=\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right) \Rightarrow S_{\infty}=\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$$

Q78

Given, $R = \{(x, y) \in W \times W : \text{the word } x \text{ and } y \text{ have atleast one letter in common}\}$

Let, $W = \{\text{cat}, \text{ toy}, \text{ you}, \ldots\}$

Clearly, R is reflexive and symmetric but not transitive.

[Since, cat R toy, toy Ryou $\nRightarrow$ cat R you]

Q79

$$L = \lim_{x\to\infty} \sqrt{n} \int_0^1 \frac{dx}{\left(1+x^2\right)^n}$$
$$\left(1+x^2\right)^n = 1 + nx^2 + \ldots..$$
$$\therefore \quad \left(1+x^2\right)^n > 1 + nx^2$$
$$\frac{1}{\left(1+x^2\right)^n} < \frac{1}{1+nx^2}$$
$$\therefore \quad \int_0^1 \frac{dx}{\left(1+x^2\right)^n} < \int_0^1 \frac{dx}{\left(1+x^2\right)}$$
$$= \frac{1}{n} \int_0^1 \frac{dx}{\frac{1}{n} + x^2} = \frac{1}{n} \sqrt{n} \left(\tan^{-1} x\sqrt{n}\right)_0^1$$
$$= \frac{1}{\sqrt{n}} \tan^{-1} \sqrt{n}$$

$\therefore \quad \lim_{x\to\infty} \sqrt{n} \int_0^1 \frac{dx}{\left(1+x^2\right)^n} < \lim_{x\to\infty} \tan^{-1} \sqrt{n}$
$\therefore \quad L < \frac{\pi}{2}$
$\therefore \quad L < 2$

Q80

Here, $\lim_{x\to 0} \frac{x\sin(\sin x) - \sin^2 x}{x^6}$
$$= \lim_{t\to 0} \frac{\sin^{-1}(t) \cdot \sin t - t^2}{\left(\sin^{-1} t\right)^6}$$
$$= \lim_{t\to 0} \frac{\sin^{-1}(t) \cdot \sin t - t^2}{t^6} \cdot \frac{t^6}{\left(\sin^{-1} t\right)^6}$$

[put $\sin x = t$]

$= \lim_{t\to 0} \frac{\left\{t + \frac{t^3}{6} + \frac{9t^5}{120} + \frac{5t^7}{112} + \ldots\right\} \cdot \left\{t - \frac{t^3}{3!} + \frac{t^5}{5!} - \ldots\right\} - t^2}{t^6} \times 1$

$= \lim_{t \rightarrow 0} \frac{t^6\left(\frac{1}{5!}-\frac{1}{6\cdot(3!)}+\frac{9}{120}\right)+\text{ higher powers to } t}{t^6}$

$= \frac{1}{120}+\frac{9}{120}-\frac{1}{36}=\frac{1}{18}$

Q81

Point of intersection of the lines is $(0,1,1)$ which lies on the plane P.

D.r. of L_1 is $(2,2,1)$& D.r. of L_2 is $(1,1,-1) \Rightarrow$ a vector parallel to P is

$\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & 1 & -1\end{array}\right|=-3 \hat{i}+3 \hat{j}$

$\therefore \quad \text{Normal to the plane } =\left|\begin{array}{ccc}\hat{i} & \hat{j} & \hat{k} \\ -3 & 3 & 0 \\ 1 & 1 & -1\end{array}\right|$

$=3 \hat{i}-3 \hat{j}-6 \hat{k}$

$\Rightarrow$ Equation of plane is

$1.(x-0)+(y-1)+2(z-1)=0$

$\Rightarrow \quad x+y+2 z=3$

$\because \quad(a,-2,0)$ lies on $P \Rightarrow \quad a=5$

Q82

Let $z=x+i y$

S_1 denotes the interior of circle of radius 4 units

S_3 denotes $x>0$

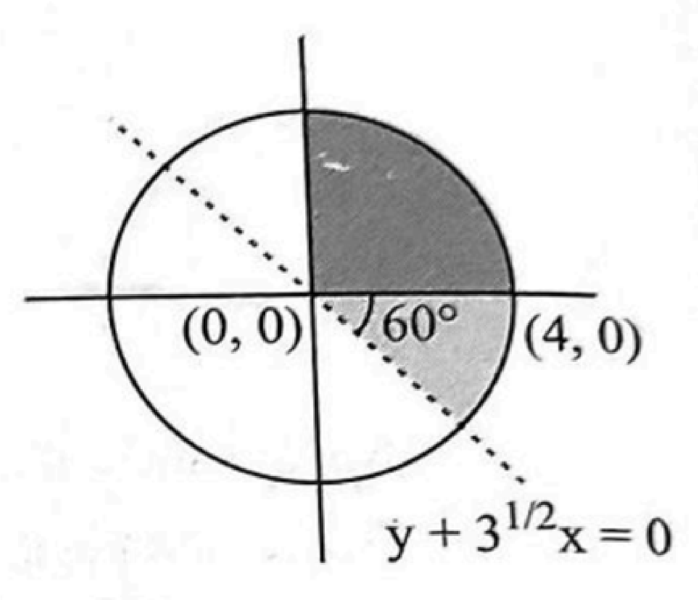
$$S_2=\operatorname{Im}\left(\frac{(x-1+i(y+\sqrt{3}))(1+i \sqrt{3})}{4}\right)>0$$

$S_2=i(y+\sqrt{3} x)>0$

Now the shaded region represents the required area

Required area = Area of quarter of circle + Area of sector

$= \frac{\pi r^2}{4} + \frac{\pi r^2 \theta}{360^\circ} = 4\pi + \frac{8}{3}\pi \Rightarrow \frac{20}{3}\pi$



Q83

$$V_1 = (\vec{a} + \vec{b}) \cdot ((\vec{b} + \overrightarrow{c}) \times (\overrightarrow{c} + \overrightarrow{a})) = 2[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}]$$
$$V_2 = \frac{1}{6}(\overrightarrow{a} \times \overrightarrow{b}) \cdot [(\overrightarrow{b} \times \overrightarrow{c}) \times (\overrightarrow{c} \times \overrightarrow{a})]$$
$$= \frac{1}{6}(\vec{a} \times \vec{b}) \cdot [((\vec{b} \times \vec{c}) \cdot \vec{a})\vec{c} - ((\vec{b} \times \vec{c}) \cdot \vec{c})\vec{a}]$$
$$= \frac{1}{6}[\vec{a}\vec{b}\vec{c}]^2$$
$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} -3 & 1 & 1 \\ 4 & 2 & 4 \\ 2 & 2 & 0 \end{vmatrix} = 36$$
$$\therefore V_1 = 72 \text{ and } V_2 = 216$$

Q84

$\bar{x} = \frac{\sum x_1}{50} = 50$

$\Rightarrow \sum x_i = 2500$

Correct $\sum x_i = 2500 - 99 = 2401$

Correct $\bar{x} = \frac{2401}{49} = 49$

$\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$

$\Rightarrow 100 = \frac{\sum x_i^2}{50} - 2500$

$\Rightarrow \sum x_i^2 = 130000$

Correct $\sum x_i^2 = 130000 - 99^2 = 120199$

Correct $\sigma^2 = \frac{\sum x_i^2}{n} - \overline{x}^2$

$= \frac{120199}{49} - (49)^2$

$= \frac{120199-117649}{49} = \frac{2550}{49} = \frac{\lambda}{49}$

Hence, $\lambda = 2550$

Q85

$h(x) = x^2$ for $x \geq \dfrac{1+\sqrt{5}}{2}$

$h(x) = 1 + x$ for $0 \leq x \leq \dfrac{1+\sqrt{5}}{2}$

$h(x) = 1 - x$ for $\dfrac{-81}{100} \leq x < 0$

$h(x) = 1 - x^2$ for $x < \dfrac{-81}{100}$

h is non-continuous at $\frac{-81}{100}$

h is non-differentiable at $\frac{1+\sqrt{5}}{2}$, 0 and $\frac{-81}{100}$ $m = 3$ and $n = 1$

Hence, $m + n = 4$

Q86

Let A be the event that the letter is from TATANAGAR and B be the event that letter is from CALCUTTA.

Also, let E be the event that on the letter, two consecutive letters TA are visible.

$P(A) = \frac{1}{2}; \quad P(B) = \frac{1}{2}$

And $P(E/A) = \frac{2}{8}$ and $P(E/B) = \frac{1}{7}$

[If the letter is TATANAGAR, we see that the events of two consecutive letters visible are TA, AT, TA, AN, NA, AG, GA, AR]

So, $\quad P(E/A) = \frac{2}{8}$ and same in case of CALCUTTA, so $P(E/B) = \frac{1}{7}$

Therefore, $P(A/E)=\frac{P(A)P(E/A)}{P(A)P(E/A)+P(B)P(E/B)}$

$$=\frac{(1/2)(2/8)}{[(1/2)(2/8)]+[(1/2)(1/7)]}=\frac{(1/8)}{(1/8)+(1/14)}=\frac{(1/8)}{(11/56)}=\frac{7}{11}$$

$p=7, q=11, p+q=11+7=18$

Q87

$x=-1$ is the root of the quadratic equation $ax^2+2bx+c=0$

$\Rightarrow a-2b+c=0 \Rightarrow 2b=a+c$

$\Rightarrow a$ and c both must be even or odd.

So, the total number of quadratic polynomials, $k={}^{1001}\mathrm{C}_2\times 2+{}^{1001}\mathrm{C}_2\times 2$

$={}^{1001}\mathrm{C}_2\times 2\times 2=\frac{1001\times 1000}{2}\times 2\times 2$

$\Rightarrow \frac{k}{10^3}=2002$

Q88

$B^2=I$

$$AB=\begin{bmatrix} a & x & p \\ y & q & b \\ r & c & z \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}=\begin{bmatrix} p & x & a \\ b & q & y \\ z & c & r \end{bmatrix}$$

$$AB=AB^3=\ldots\ldots=AB^{19}=\begin{bmatrix} p & x & a \\ b & q & y \\ z & c & r \end{bmatrix}$$

$\operatorname{tr.}\left(AB+AB^3+\ldots\ldots+AB^{19}\right)=210$

$\Rightarrow 10(p+q+r)=210 \Rightarrow p+q+r=21, p, q, r\in N$

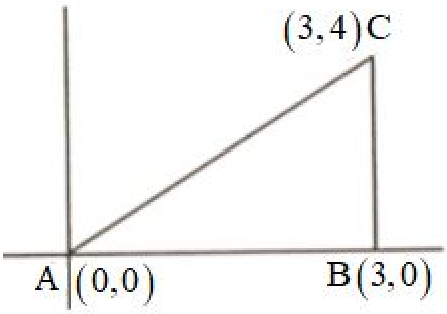
$p'+q'+r'=18, p', q', r'\in W$

$\therefore$ Number of ordered triplets $(p, q, r)={}^{20}C_2=\frac{20\times 19}{2}=190$

Q89

$$\begin{aligned}
f(x) &= x^4 - 4x^3 - 8x^2 + a \\
f'(x) &= 4\left(x^3 - 3x^2 - 4x\right) \\
&= 4x\left(x^2 - 3x - 4\right) \\
&= 4x(x-4)(x+1) = 0 \text{ at } x = -1, 0, 4 \\
f(-1) &= a - 3 \leq 0, a \leq 3 \\
f(0) &\geq 0 \Rightarrow a \geq 0 \\
a &\in [0,3] \\
\text{Sum } &= 0 + 1 + 2 + 3 = 6
\end{aligned}$$

Q90



$AC = 5 = 2ae$

$\Rightarrow 2ae = 5 \ldots$(i)

Also $AB + BC = 2a$

$\Rightarrow 2a = 7 \Rightarrow a = \frac{7}{2}$

$\therefore e = \frac{5}{7}$

$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{25}{49}$

$\Rightarrow b = \sqrt{6}$

Length of the latus rectum, $\frac{2b^2}{a} = \frac{2\times 6}{\frac{7}{2}}$ units

$\Rightarrow p = \frac{7}{2} = 3\ .5$