Answer Key

Q1 (4)	**Q2** (4)	**Q3** (2)	**Q4** (1)
Q5 (1)	**Q6** (1)	**Q7** (1)	**Q8** (3)
Q9 (2)	**Q10** (3)	**Q11** (1)	**Q12** (2)
Q13 (4)	**Q14** (1)	**Q15** (1)	**Q16** (1)
Q17 (3)	**Q18** (1)	**Q19** (3)	**Q20** (1)
Q21 (4)	**Q22** (3)	**Q23** (2)	**Q24** (2)
Q25 (2)	**Q26** (4)	**Q27** (26)	**Q28** (25)
Q29 (6)	**Q30** (5)	**Q31** (4)	**Q32** (4)
Q33 (3)	**Q34** (1)	**Q35** (1)	**Q36** (4)
Q37 (4)	**Q38** (3)	**Q39** (1)	**Q40** (4)
Q41 (2)	**Q42** (2)	**Q43** (1)	**Q44** (3)
Q45 (2)	**Q46** (4)	**Q47** (2)	**Q48** (3)
Q49 (4)	**Q50** (3)	**Q51** (4)	**Q52** (8)
Q53 (25)	**Q54** (57)	**Q55** (231)	**Q56** (6)
Q57 (4)	**Q58** (7)	**Q59** (3)	**Q60** (24)
Q61 (2)	**Q62** (3)	**Q63** (2)	**Q64** (4)

Q65 (1)	**Q66** (3)	**Q67** (4)	**Q68** (4)
Q69 (1)	**Q70** (2)	**Q71** (2)	**Q72** (2)
Q73 (2)	**Q74** (2)	**Q75** (1)	**Q76** (1)
Q77 (4)	**Q78** (4)	**Q79** (4)	**Q80** (3)
Q81 (12)	**Q82** (3)	**Q83** (3)	**Q84** (16)
Q85 (6)	**Q86** (6)	**Q87** (3)	**Q88** (24)
Q89 (4)	**Q90** (10)		

Q1

$P=a^{\frac{1}{2}}\ b^2c^3d^{-4}$

$\left(\frac{\Delta p}{p}\right)_{max}=\frac{1}{2}\ \frac{\Delta a}{a}+2\frac{\Delta b}{b}+3\frac{\Delta c}{c}+4\frac{\Delta d}{d}$

$100\times\frac{\Delta P}{P}=\left(\frac{1}{2}\ \frac{\Delta a}{a}+2\frac{\Delta b}{b}+3\frac{\Delta c}{c}+4\frac{\Delta d}{d}\right)\times 100$

$\%\ error\ in\ P=\left(\frac{1}{2}\times 2+2\times 1+3\ \times 3+4\ \times 5\right)$

$=32\%$

Q2

Centripetal acceleration

$\frac{v^2}{r}\propto r^{-2}$

Or $\frac{v^2}{r}=\frac{k}{r^2}$

Or $v^2=\frac{k}{r}$

$\therefore\ \ KE=\frac{1}{2}mv^2$

$=\frac{1}{2}\ \frac{mk}{r}$

Or $KE\propto r^{-1}$

Q3

We have, $x=a\sin\omega t$

$\frac{x}{a}=\sin\omega t\ .\ .\ .\ .\ .(i)$

and

$y = a \cos \omega t$

$\Rightarrow \frac{y}{a} = \cos \omega t \ldots . . (ii)$

Squaring and adding, we get

$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \qquad (\because \ \cos^2 \omega t + \sin^2 \omega t = 1)$

or $x^2 + y^2 = a^2$

This is the equation of a circle.

Hence, particle follows a circular path.

Q4

The moment of inertia of complete disc about a perpendicular axis passing through centre O is,

$I_1 = \frac{1}{2}(9M) R^2$

$= \frac{9}{2} M R^2$

The mass of cut out disc of radius $\frac{R}{3}$ is,

$m = \frac{9M}{\pi R^2} \pi \left(\frac{R}{3}\right)^2$

$= M$

Now, using the theorem of the parallel axis, the moment of inertia of cut out disc about the perpendicular axis passing through centre O is,

$I_2 = \frac{1}{2} M \left(\frac{R}{3}\right)^2 + M \left(\frac{2R}{3}\right)^2$

$= \frac{1}{2} M R^2$

The moment of inertia of residue disc is,

$I = I_1 - I_2$

$= \frac{9}{2} M R^2 - \frac{1}{2} M R^2$

Q5

The period of a satellite is the time it takes it to make one full orbit around an object. The period of a satellite are only dependent upon the radius of orbit and the mass of the central body that the satellite is orbiting.

As per Kepler's third law,

$T^2 \propto r^3 \Rightarrow \frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3}$

Given,

$T_A = 8T_B$

$\therefore \frac{r_A}{r_B} = \left(\frac{8T_B}{T_B}\right)^{2/3} = 4 = n$

Q6

Increase in surface energy

$= 4\pi R^2 T\left(n^{1/3} - 1\right) = 4\pi\left(2 \times 10^{-3}\right)^2\left(0.465\right)\ \left(8^{1/3} - 1\right) = 23.4 \times 10^{-6}\,J = 23.4\,\mu\ J$

Q7

When a system undergoes a change under the condition that there is no exchange of heat between the system and surroundings, then the process is called an adiabatic process. The leaking air of balloon undergoes adiabatic expansion. Air cools down due to adiabatic expansion as air has to do work against the external pressure at the cost of its internal energy. Due to work done against external pressure, the internal energy of air reduces. Thus, it becomes cooler.

Q8

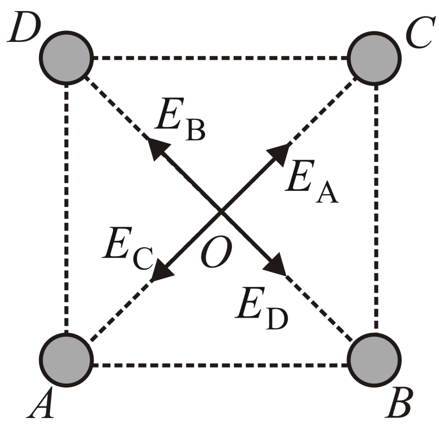
When an object is under a conservative force moving from one position to another, then the value of the net work done depends only on the initial and final position of the object, i.e., the work done by a conservative force is path independent.

In the case of a closed-loop, the initial and final position of the object is the same. The net displacement of the object is zero. Thus, the net work done by a conservative force in a closed loop is zero.

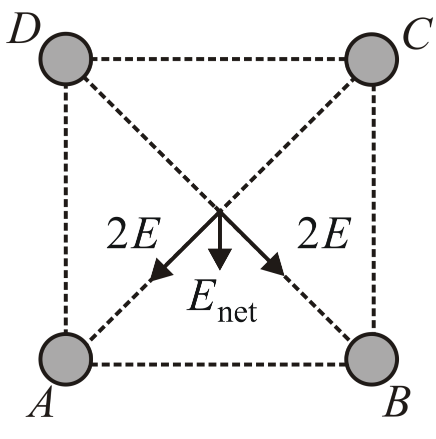
In a constant electrostatic field, the magnitude of the electric field $(E)$is constant at every point inside the field. When a charged particle is moving from one point to another in the presence of an electrostatic field, the force (F) acting on the particle will be constant since $F = qE$. Thus, this is a conservative force, and work done will be independent of the path connecting P to Q.

Thus, both statements are true, but statement 2 is not the correct explanation of statement 1.

Q9



Let the electric fields generated at the center of the square due to the charge present at the vertices of the square be $\overrightarrow{E_A}$, $\overrightarrow{E_B}$, $\overrightarrow{E_C}$ and $\overrightarrow{E_D}$. Since the magnitude of the electric field is proportional to the magnitude of the charge producing it, let $\left|\overrightarrow{E_A}\right| = E$, $\left|\overrightarrow{E_B}\right| = 2E$, $\left|\overrightarrow{E_C}\right| = 3E$ and $\left|\overrightarrow{E_D}\right| = 4E$.



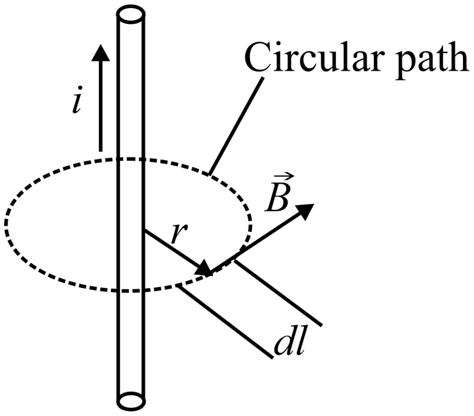
We have,

$\left|\overrightarrow{E_c} + \overrightarrow{E_A}\right| = 2E$ in the direction CA.

$\left|\overrightarrow{E_D}+\overrightarrow{E_B}\right|=2E$ in the direction DB.

These two vectors of equal magnitude at an angle of 90° add up to give the net electric field at the center due to all four charges. The resultant electric field is clearly in the downward direction, or in the direction along CB.

Q10



Consider a long current-carrying wire i, and B is magnetic field at any distance r from the wire.

From Ampere's circuital law,

$\oint \overrightarrow{B}.d\overrightarrow{l}=\mu_0 i$

$\Rightarrow B(2\pi r)=\mu_0 i$

$\Rightarrow B=\frac{\mu_0 i}{2\pi r}$

$\Rightarrow B\propto\frac{1}{r}$.

So, the graph between magnetic field B and distance r will be a rectangular hyperbola.

Q11

Given that the time delay between generation of a wave and the reception of its echo $(T)=77.0$ s.

The generated wave moves through water and strikes the objects under water. After striking with objects, these waves are reflected back.

$\therefore$ Time delay between generation of a wave and the reception of its echo $=\ 2x$ time taken by the wave to reach the enemy submarine

$\therefore$ Time taken by the wave to reach the enemy submarine,

$t=\frac{T}{2}=\frac{77.0}{2}$ s

$=38.5$ s

Speed of sound in water $(v)=1450$ m s^{-1}

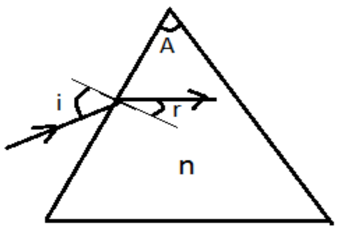
$\therefore$ Distance of enemy submarine $(D)=v\times t$

$=1450\times 38.5$

$=55825$ m

$=55.825$ km

Q12



at angle of minimum deviation

$\angle i=\angle e\ and\ \angle r=\frac{A}{2}$
$\therefore \partial=i+e-A$
$at\ \ minimum\ deviation$
$\partial_{min}=2i-A$
$i=\frac{\partial_{min}+A}{2}$

by snell`s law

$n = \frac{\sin\ i}{\sin\ r} \qquad given\ that\ n = cot\left(\frac{A}{2}\right)$

$\Rightarrow cot\left(\frac{A}{2}\right) = \frac{\sin\ \left(\frac{\partial m + A}{2}\right)}{\sin\ \frac{A}{2}}$

$\Rightarrow \frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\ \left(\frac{\partial m + A}{2}\right)}{\sin\ \frac{A}{2}}$

$\Rightarrow \cos\left(\frac{A}{2}\right) = \sin\ \left(\frac{\partial_m + A}{2}\right)$

$or\ \ \sin\left(90^\circ - \frac{A}{2}\right) = \sin\left(\frac{A + \delta m}{2}\right)$

$or\ \ 90^\circ - \frac{A}{2} = \left(\frac{A + \delta m}{2}\right)$

$or\ \ 180^\circ - A = A + \ \delta m$

$\delta m = 180^\circ - 2A = \pi - 2A$

Q13

The separation between the objective and the eye piece $=$ Length of the telescope, tube $L = f_0 + f_e$

Here, $f_0 = 144\ \text{cm} = 1.44\ \text{m},\ f_e = 6.0\text{cm} = 0.06\ \text{m}$

$\therefore\ L = 1.44 + 0.06 = 1.5\ \text{m}$

Q14

The de-Broglie wavelength λ associated with same potential V is

$\lambda = \frac{h}{\sqrt{2meV}}$

$\therefore \lambda\ \alpha \frac{1}{\sqrt{m}}$ [For same potential]

Since alpha particle is ${}_{2}^{4}\text{He}$ and proton is ${}_{1}^{1}\text{H}$

$\therefore$ mass of alpha particle = 4 times mass of proton

$\therefore \lambda_{\text{proton}} \alpha \frac{1}{\sqrt{m}}$ and $\lambda_{\text{alpha}} \alpha \frac{1}{\sqrt{4m}}$

$\therefore \frac{\lambda_{\text{proton}}}{\lambda_{\text{alpha}}} = \frac{1}{\sqrt{m}} \times \frac{\sqrt{4m}}{1} = \frac{2}{1}$

Therefore $\lambda_{\text{proton}} : \lambda_{\text{alpha}} = 2 : 1$

Q15

Kinetic energy of a thermal neutron can be neglected; even for a temperature of $10^6 K$, the thermal energy is only 130 eV. The Q value is given by the equation

$$Q = \left\{M\left({}^{235}U\right) + m(n) - \left[M\left({}^{99}Zr\right) + M\left({}^{134}Te\right) + 3m(n)\right]\right\}c^2$$

$$= [235.0439 - 98.9165 - 133.9115 - 2(1.0087)]u$$

$$= 0.1985 \times 931.5 = 185\ MeV .$$

Q16

The input resistance r_{i} of a transistor is in the range of $1 - 10$ kΩ.

The output resistance r_0 of a transistor is in the order of 100 kΩ.

So, the ratio of output resistance and input resistance,

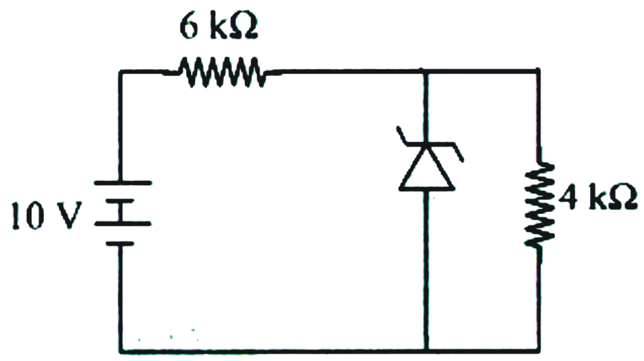
$$R = \frac{r_0}{r_i} = \frac{100\ \text{k}\Omega}{(1-10)\ \text{k}\Omega}$$

$$\Rightarrow\ R \sim 10^2 - 10^3$$

Q17

When $p - n$ junction diode is forward biased, both the depletion region and barrier height are reduced.

Q18



Given, break down voltage is 6V

Then, potential difference across load $4 \mathrm{k} \Omega$ will be $6 \mathrm{V}$.

Then potential difference across resistance $6 \mathrm{k} \Omega=10-6=4 \mathrm{V}$

So, current in given resistance is given by Ohm's Law

From $I=\frac{V}{R}$

$=\frac{4}{6000}=\frac{2}{3} \mathrm{mA}$

Q19

The modulated signal (the message is superimposed with high frequency carrier wave) or sender signal is demodulated or reconstruction by a receiver in which a high-frequency carrier wave is knocked out from our information. The reconstruction is always required to get our original message or data.

Q20

Given, wavelength $\lambda_1=1 \mathrm{~m}$ and $\lambda_2=1.01 \mathrm{~m}$ and beats per second is $\Delta f=\frac{10}{3}$

Beats is the difference of frequency, then

$\Delta f=f_1-f_2 \ldots(1)$,

where frequency of a wave is $f=\frac{v}{\lambda}$, here v is speed of sound wave.

From equation (1),

$\Delta f=\frac{v}{\lambda_1}-\frac{v}{\lambda_2}$

$\therefore \Delta f=v\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$

Substituting the values in above equation,

$\frac{10}{3}=v\left(\frac{1}{1}-\frac{1}{1.01}\right)$
$\frac{10}{3}=v\left(\frac{1.01-1}{1\times1.01}\right)$
$\frac{10}{3}=v\left(\frac{0.01}{1.01}\right)$
$v=336.66\ \mathrm{ms}^{-1}\simeq 336.67\ \mathrm{ms}^{-1}$

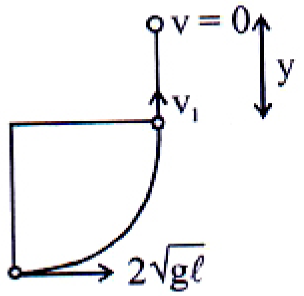
Q21

By energy conservation
$\frac{1}{2}m\left(2\sqrt{gl}\right)^2=mgh$

$h=2l$

$y=l$

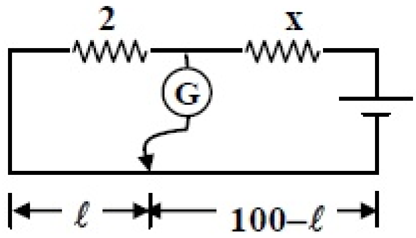
distance $=\frac{\pi\ell}{2}+\ell=\frac{14}{9}\left[\frac{11}{7}+1\right]=4\mathrm{m}$



Q22

Unknown resistor $R>2\Omega$

$\therefore$ its balencing length is 100 - $l > l$



It form Wheatstone network, then balencing condition for Wheatstone bridge is

Applying $\frac{P}{Q} = \frac{R}{S}$

We have $\frac{2}{X} = \frac{l}{100-l}$ .... (i)

On intercharging balance point must shift to right as R$>2\Omega$

$\frac{X}{2} = \frac{l+20}{100-l-20}$ .... (ii)

From (i) and (ii)

$l^2 + 20l = (100 - l)(80 - l)$

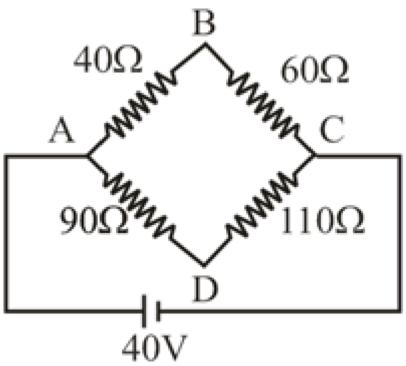
$200l = 8000$

$l{=}40\ cm;\ R{=}2\Omega$

$For\ 60\ cm,\ resis\tan ce\ will\ be\ 3\Omega$

$\therefore$ X $= 3\,\Omega$

Q23



From KVL

$\mathrm{V_B} - 60\left(\frac{40}{100}\right) + 110\left(\frac{40}{200}\right) = \mathrm{V_D}$

$\mathrm{V_B} - \mathrm{V_D} = 2$

Q24

$R = 50\ \Omega$

$X_L = \omega L = 50\ \Omega$

$X_L = \frac{1}{\omega C} = 100\ \Omega$

$\Rightarrow Z = \sqrt{R^2 + (X_L - X_C)^2}$

$Z = 50\sqrt{2}\,\Omega$

$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{Z} = \frac{200/\sqrt{2}}{50\sqrt{2}} = 2\ \mathrm{A}$

Q25

$I\omega_0 = 2I\omega$ ($M.I.$ of one disc)
$\therefore K = \frac{1}{2}2I\omega^2$
$= \frac{1}{2}\left(\frac{1}{2}I\omega_0^2\right)$
The ratio of kinetic energy before and after collision is 2

Q26

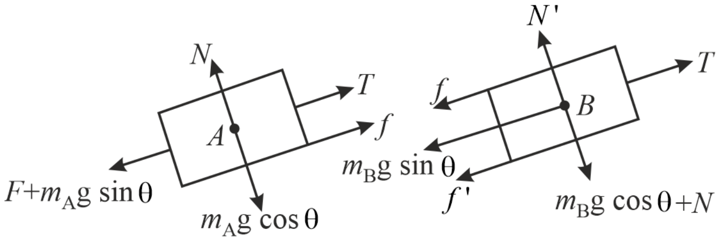
From momentum conservation, Velocity of B just after impact $= \frac{v}{2}$

Angular velocity, $\omega = \frac{v}{2l}$

So, $n = 2$

Q27

FBD of Block A and B.



By Newton's II law of motion,

For Block A,

$m_A g \sin\theta + F - T - f = m_A a \ \ldots$(i)

Kinetic frictional force on A, $\quad f = \mu_k m_A g \cos\theta \ \ldots$(ii)

For Block B,

$T - m_B g \sin\theta - f - f' = m_B a \ \ldots$(iii)

Kinetic frictional force on B, $\quad f' = \mu_k . N' \ldots$(iv)

$= \mu_k (m_A g \cos\theta + m_B g \cos\theta)$

Solving above equations, then

$a_A = 5.2 \ \mathrm{ms}^{-2}$

Q28

For ideal gas :

$\Delta \mathrm{U} = \mathrm{nC_V}[\mathrm{T_2} - \mathrm{T_1}]$

$\Rightarrow \quad 5000 = 4 \times \mathrm{C_V}[500 - 300]$

$\Rightarrow \mathrm{C_V} = \frac{5000}{800} = 6.25 \text{ J mole}^{-1}\mathrm{K}^{-1}$

Q29

$\sin\theta = \frac{L}{r} = \frac{1}{\frac{mv}{Bq}}$

$\therefore \quad v \propto \frac{1}{\sin\theta} \;\; or \;\; \frac{v}{v_0} = \frac{\sin 60^\circ}{\sin 30^\circ}$

$\therefore \quad v = \sqrt{3}v_0$

Q30

In series LCR, current is maximum at resonance.

$\therefore$ Resonant frequency, $\omega = \frac{1}{\sqrt{LC}}$

$\therefore \;\omega^2 = \frac{1}{LC}$ or, $L = \frac{1}{\omega^2 C}$

Given, $\omega = 1000\ \text{s}^{-1}$ and $C = 10\ \mu\text{F}$

$\therefore \;L = \frac{1}{1000\times1000\times10\times10^{-6}} = 0.1\ \text{H} = 100\ \text{mH} \Rightarrow n = 5$

Q31

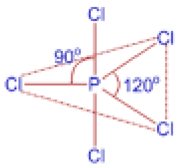
Non – stoichiometric Schottky defect is a type of point defect. This defect is forms when oppositely charged ion leave their lattice site creating vacancies, thus lowers the density of crystal.

Q32

For H-atom E belongs to visible region when $\mathrm{n}_1 = 2$and $\mathrm{n}_2 = 3$. B, C and D lines belongs to Infrared region.

Q33

On orbital overlap between phosphorus and chlorine, five sp^3 d-p sigma covalent bonds are formed. This statement is true, as PCl_5 carries five sigma bonds and all these sigma bonds are used for hybridisation according to the valence shell electron pair repulsion theory. Structure of PCl_5 is shown below:



And we can see here, that axial bonds and equatorial bonds, both are not of the same length.

Q34

$\mathrm{pH}=\mathrm{pk_a}+\log\left[\frac{\text{Salt}}{\text{Acid}}\right]$ $(\because [\text{Salt}]=[\text{Anion}])$
$\Rightarrow 6=5+\log\ \frac{\text{Salt}}{\text{Acid}}$
$\Rightarrow 1=\log\ \frac{\text{Salt}}{\text{Acid}}$
$\Rightarrow \log 10=\log\ \frac{\text{Salt}}{\text{Acid}}$
$\frac{\text{Salt}}{\text{acid}}=\frac{10}{1}$

Q35

In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst. Ni catalysts enables the hydrogenation of fats. $CuCl_2$ is used as catalyst in Deacon's process. $\mathrm{ZSM}-5$ used as catalyst in cracking of hydrocarbons.

Q36

He $\Rightarrow$ Inert gas

Cl $\Rightarrow$ Electron gain enthalpy is highest

Ca $\Rightarrow$ Most electropositive metal

Li $\Rightarrow$ Strong reducing agent

He has the electronic configuration of $1s^2$. It is completely filled electronic configuration, due to this, it has the highest ionisation enthalpy.

$\mathrm{Cl}\longrightarrow \mathrm{Cl}^-$ gives the highest $\Delta_{eg}H$ as Cl^- after getting one electron, it will achieve the stable electronic configuration $18[\mathrm{Ar}]$. Due to this, large amount of energy is released during this process.
${}_{20}\mathrm{Ca}\xrightarrow[-2e^-]{}\mathrm{Ca}^{2+}$

Alkaline earth metal and present in the fourth period, the outermost electron is far away from the nucleus. As a result, less energy is required to remove the electron and easily donate the electron, hence, shows maximum metallic nature.

E^{o}_{red} of Li is -3.07 V, a strong reducing agent.

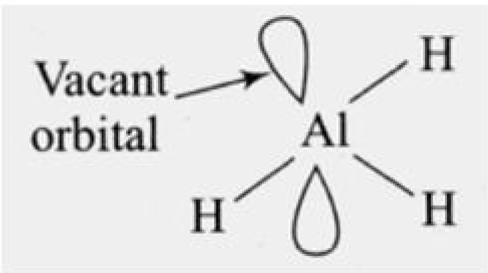
It has very high hydration energy due to smaller in size. The hydration energy is directly proportional to the charge density of the ion.

Q37

On being preferrentially wetted by oil, the ore particles rise to the surface in the form of froth and from there we can separate them.

Q38

AIH_3 is an electron deficient hydride.



It is generally formed by the group 13 element, and have lesser number of electrons than that required for writing its Lewis structure. Being electron deficient, this hydride generally behaves as a Lewis acid, which act as electron acceptor. This is a polynuclear hydride. According to the octet rule, each element tends to completely fill it's outermost shell with $8e^-$ in it. The electronic configuration of aluminium is $2, 8, 3$, and it still needs 5 more electrons to complete its octet. Al has 3 valence electrons to complete its octet, while each hydrogen has one valence electron.

Q39

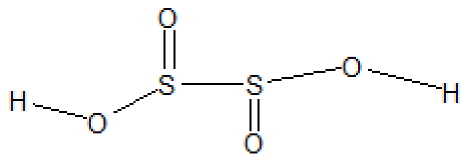
The stability of the oxide of alkali metals depends upon the comparability of size of cation and anion.

Therefore the main oxide of alkali metals formed on excess of air are as follows

Li	Li_2O
Na	Na_2O_2
K	KO_2
Rb	RbO_2
Cs	CsO_2

Q40

Hyposulphurous acid contains the $S-S$ bond linkage in the structure having the chemical formula $H_2S_2O_4$.



Q41

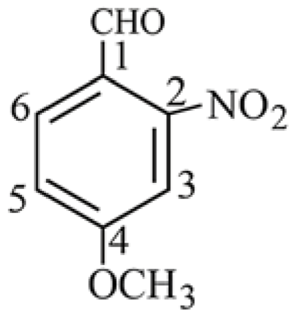
$\left[Fe(H_2O)_6\right]^{3+}$
$Fe^{+2}=3d^5\left(t_{2g}^{1,1,1}e_g^{1,1}\right)$
So C.F.S.E is $=[-0.4\times 3+0.6\times 2]\Delta_0=0$

Q42

In Calcination and roasting CO_2 and SO_2 are released which are responsible for Global warming and acid rain.

Q43

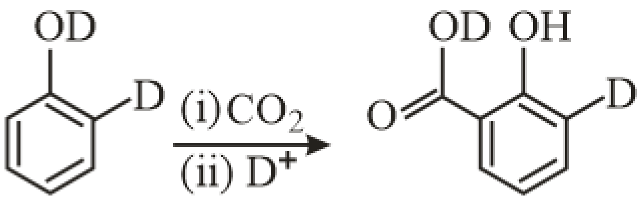
The -CHO functional group is the highest priority functional group. The carbons of the benzene ring are numbered accordingly.



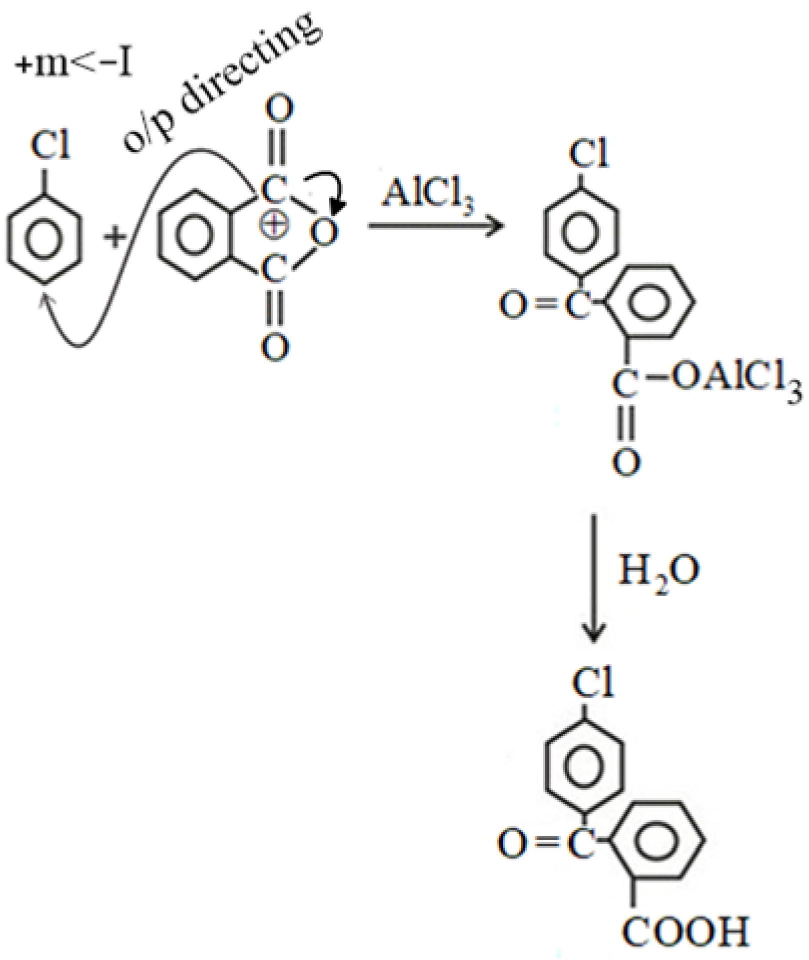
4-methoxy-2-nitrobenzaldehyde

Q44

In the first step of the reaction, NaOH is given which is a strong base, abstracts the hydrogen and phenoxide ion is formed, which is more reactive than phenol. It further undergoes electrophilic substitution reaction with CO_2 to form salicylic acid. But further reaction with D^+ abstracts the hydrogen of acid to form deuterated acid. It follows the mechanism of Kolbe's reaction.

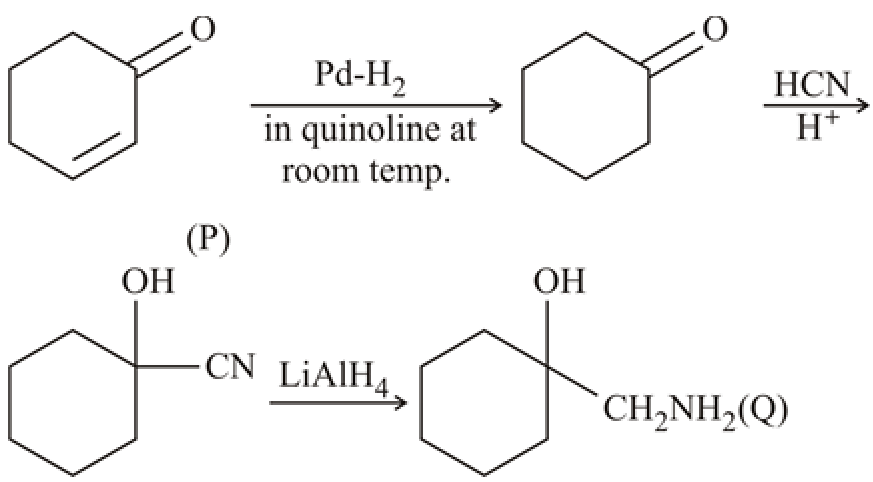


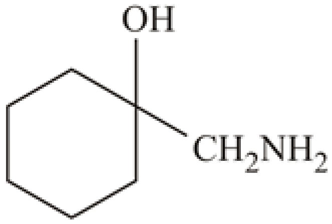
Q45



This reaction is Friedel-craft-acylation, $-\mathrm{Cl}$ group is an ortho and para directing.

Q46



$\therefore$ P is $\mathrm{Pd-H_2}$, in quinoline at room temp., Q is 

Q47

Explanation :- aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is delocalised to more electronegative element oxygen.

In Aniline lone pair of nitrogen delocalised over benzene ring.

Q48

High density polythene: It is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in the presence of a catalyst such as triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmospheres. High density polythene (HDP) thus produced, consists of linear molecules and has a high density due to close packing. It is also chemically inert and more tougher and harder. It is used for manufacturing buckets, dustbins, bottles, pipes, etc.

Q49

In double stranded DNA, the molar amount of adenine equals that of thymine, and the molar amount of guanine equals that of cytosine. Therefore, A + G = T + C, but A + T does not equal G + C.

Q50

Biodegradable detergents are defined as the type of detergent that has a straight hydrocarbon chain. These detergents are known as biodegradable as they are destroyed by the bacteria.

Branched alkyl groups are not dissociated easily and hence, such compounds are non-biodegradable.

Q51

$\frac{R_{H_2}}{R_{H.C}} = \sqrt{\frac{M_{H.C}}{2}} = 3\sqrt{3}$

$M_{H.C} = 54 \text{ g/mol}$

$M_{H.C} = 12n + (2n - 2) = 54$

$n = 4$

Q52

$PdV + VdP = nRdT \ ...(i)$

along $AB \rightarrow PT = \text{constant}$

$PdT + TdP = 0$

$dP = -\frac{P}{T}dT$

Substitute in (i)

$PdV + V\left(-\frac{P}{T}dT\right) = nRdT$

$PdV = nRdT + \frac{PV}{T}dT = 2nRdT$

$W_{AB} = -\int_{V_1}^{V_2} PdV = -R\int_{2T_1}^{T_1} 2ndT = -2nR[T_1 - 2T_1]$

$\quad = -2 \times 2 \times R[300 - 600]$

$\quad = 1200 \text{ R} = (150 \text{ R})x$

$\therefore \ x = 8$

Q53

A	$+\ B$	$\rightleftharpoons 2C$
1	1	1
-x	-x	2x
1-x	1-x	1 + 2x

$\mathrm{K}=\frac{[\mathrm{C}]_{\mathrm{eq}}^{2}}{[\mathrm{A}]_{\mathrm{eq}}[\mathrm{B}]_{\mathrm{eq}}}=\frac{(1+2\mathrm{x})^{2}}{(1-\mathrm{x})(1-\mathrm{x})}$

$100=\left(\frac{1+2\mathrm{x}}{1-\mathrm{x}}\right)^{2}$

$\left(\frac{1+2\mathrm{x}}{1-\mathrm{x}}\right)=10$

$\mathrm{x}=\frac{3}{4}$

$[\mathrm{C}]\mathrm{e}_{\mathrm{q.}}=1+2\mathrm{x}$

$=1+2\left(\frac{3}{4}\right)$

$=2.5\ \mathrm{M}$

$=25\times10^{-1}\ \mathrm{M}$

Q54

$\kappa=\frac{1}{R}\cdot G^{*}$

For same conductivity cell, G* is constant and hence $\kappa\cdot R.=$ constant.

$\therefore 0.14\times4.19=\kappa\times1.03$

or, $=\frac{0.14\times4.19}{1.03}$

$=0.5695\,\mathrm{Sm}^{-1}$

$=56.95\times10^{-2}\,\mathrm{Sm}^{-1}\approx57\times10^{-2}\,\mathrm{Sm}^{-1}$

Q55

From question

$\frac{2k_1}{2k_1+3k_2}=0.5$ and $k_1+k_2=\frac{0.693}{1.386\times10^{2}}=5\times10^{-3}h^{-1}$

Solving $\frac{k_1}{k_2}=\frac{2}{3}$ and $k_1=2\times10^{-3}h^{-1}$ and $k_2 = 3 \,'\, 10^{-3} h^{-1}$

$t_{1/2(A\to B)}=\frac{0.693}{k_2}=\frac{0.693}{3\times10^{-3}} = 231 h$

Q56

$8MnO_4^- + 3S_2O_3^{2-} + H_2O \to 8MnO_2 + 6SO_4^{2-} + 2OH^{\ominus}$

Q57

Complexes (i), (iii),(iv) and (v) are optically inactive due to the presence of plane of symmetry.

Q58

Considering 100 g of solid, mass of anhydrous salt $=56.25\ g$ and mass of water $=100-56.25$

$43.75\ g$

Moles of H_2O in 288 g of solid $=\frac{43.75}{100}\times288$

$=126\ g$

$\therefore$ Moles of $H_2O=\frac{126}{18}=7\ mol$

$\therefore n=7$

Q59

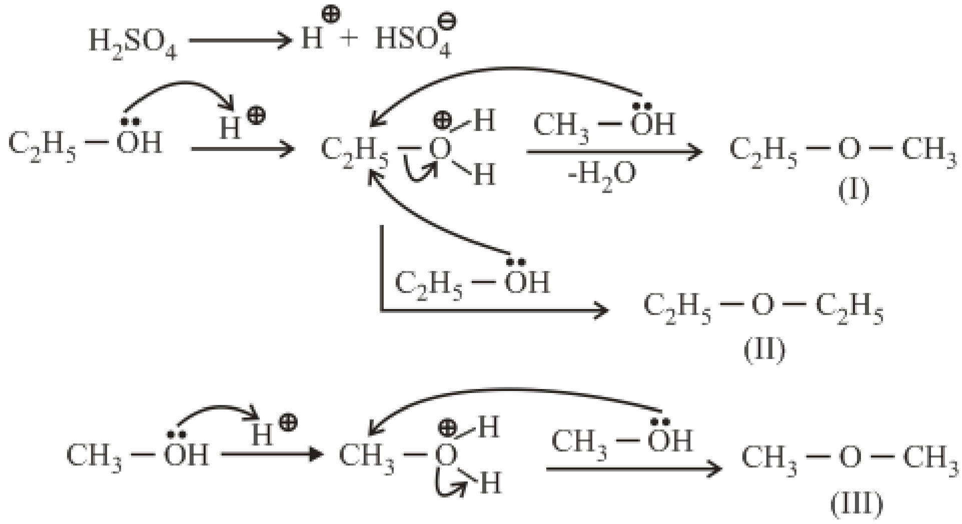
When alcohol is treated with concentrated H_2SO_4, ethers are formed by dehydration of alcohols.

Same alcohol form symmetric ether.

Different alcohol form unsymmetric ether.

The reaction follows SN^2 mechanism.

Mechanism:



Hence, three different ethers are formed.

Q60

$n_{eq}\,Fe^{2+} = n_{eq}\,Cr_2\,O_7^{2-}$

or $\left(\frac{15\times M_{Fe^{2+}}}{1000}\right)\times 1 = \left(\frac{20\times 0.03}{1000}\right)\times 6$

$\therefore M_{Fe^{2+}} = 0.24M = 24\times 10^{-2}M$

Q61

Since, $\alpha^2 - 2\alpha + 3 = 0$

$\Rightarrow \alpha^3 - 3\alpha^2 + 5\alpha - 2 = \alpha(\alpha^2 - 2\alpha + 3) - (\alpha^2 - 2\alpha + 3) + 1 = 1$

And $\beta^2 - 2\beta + 3 = 0$

$\Rightarrow \beta^3 - \beta^2 + \beta + 5 = \beta(\beta^2 - 2\beta + 3) + (\beta^2 - 2\beta + 3) + 2 = 2$

So, the required equation is

$x^2 - (2+1)x + 2.1 = 0$

$\Rightarrow x^2 - 3x + 2 = 0$

Sum of roots $= 3$

Q62

Here $\frac{z_1+z_2+z_3}{3}=z_0 \Rightarrow z_3=4+3z_0$

Therefore, centroid coincides with the circumcentre

$\Rightarrow$ Triangle is equilateral $\Rightarrow |z_1-z_2|=4$

Clearly, z_3 either lie in the second or third quadrant

So the centre z_0 also lies in the second or third quadrant.

$\therefore z_0$ can be $=-2+\frac{2}{\sqrt{3}}\mathrm{i},-2-\frac{2}{\sqrt{3}}\mathrm{i}$

$\Rightarrow \arg(z_0)=\frac{5\pi}{6},\frac{7\pi}{6}$

Q63

$\because P_n=\left[\frac{3i+j}{4^{2n}}\right]=\frac{1}{4^{2n}}[3i+j]$

$4^nP_n=\frac{1}{4^n}[3i+j]$

$T_r\left(4P_1+4^2P_2\ldots 4^nP_n\right)=T_r(4P_1)+T_r\left(4^2P_2\right)\ldots.+T_r(4^nP_n)$

$=\frac{24}{4}+\frac{24}{4^2}\ldots\ldots\ldots\frac{24}{4^n}$

$\lim\limits_{n\to\infty}\left(T_r\left(4P_1+4^2P_2\ldots.+4^nP_n\right)\right)$

$\lim\limits_{n\to\infty}\left(\frac{24}{4}+\frac{24}{4^2}\ldots\ldots.\frac{24}{4^n}\right)$

$=\frac{6}{1-\frac{1}{4}}=8$

Q64

$T_k=\frac{5+(k-1)4}{(3+(k-1)4)^2(7+(k-1)4)^2}$

$=\frac{4k+1}{(4k-1)^2(4k+3)^2}$

$=\frac{1}{8}\left\{\frac{1}{(4k-1)^2}-\frac{1}{(4k+3)^2}\right\}$

$S_n=T_1+T_2+T_3+\ldots.+T_n$

$=\frac{1}{8}\left\{\frac{1}{3^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{11^2}+\frac{1}{11^2}-\frac{1}{15^2}+\ldots.+\frac{1}{(4n-1)^2}-\frac{1}{(4n+3)^2}\right\}$

$= \frac{1}{8}\left\{\frac{1}{3^2} - \frac{1}{(4n+3)^2}\right\}$

$S_\infty = \lim_{n\to\infty} S_n = \frac{1}{8}\left\{\frac{1}{9} - 0\right\} = \frac{1}{72}$

Q65

$\lim_{x\to 0} \frac{x^{6000} - (\sin x)^{6000}}{x^2 \frac{(\sin x)^{6000}}{x^{6000}} . x^{6000}} \qquad \left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{x^{6000} - (\sin x)^{6000}}{x^{6002}}$

$= -\lim_{x\to 0} \frac{(\sin x)^{6000} - x^{6000}}{x^{6002}}$

$= -\lim_{x\to 0} \frac{\left(\frac{\sin x}{x}\right)^{6000} - 1}{x^2}$

$= -\lim_{x\to 0} \frac{6000\left(\frac{\sin x}{x}\right)^{5999}\left(\frac{x\cos x - \sin x}{x^2}\right)}{2x}$ (By L-Hospital Rule)

$= -\frac{6000}{2} \lim_{x\to 0} \frac{x\cos x - \sin x}{x^3}$

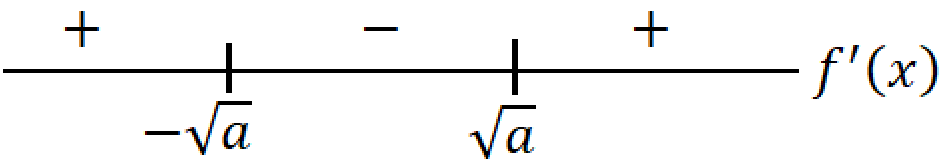
$= -3000 \lim_{x\to 0} \frac{\cos x - x\sin x - \cos x}{3x^2}$

$= 1000$

Q66

$f'(x) = 3x^2 - 3a$

$= 3(x - \sqrt{a})(x + \sqrt{a})$



$\therefore f(x)$ has a local minimum at $x = \sqrt{a}$

$\Rightarrow \sqrt{a} \geq 4$

$\Rightarrow a \geq 16$

$\therefore$ 'a' can be 16,17,18

Q67

$$\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)\in[1,\sqrt{2}]\forall x\in[0,1]$$

Hence, as $\quad x\in[0,1]$

$$10x^{\sqrt{2}}\leq f(x)\leq 10x$$

With equality holding if and only if $x=0$ or 1

As equality holds only for finitely many points, the inequalities become on strict integrating on all sides.

Hence, $\quad 4<10(\sqrt{2}-1)<\int_0^1 f(x)dx<5\Rightarrow\left[\int_0^1 f(x)dx\right]=4$

Q68

$f'(x)=\tan^2 x+K$ where $K=\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} f(x)dx$

$$\begin{aligned}&f(x)=\tan x-x+Kx+C\\&f\left(\frac{\pi}{4}\right)=1-\frac{\pi}{4}+\frac{K\pi}{4}+C=\frac{-\pi}{4}\\&C+1=\frac{-K\pi}{4}\\&f(x)=\tan x-x+Kx-\frac{K\pi}{4}-1\end{aligned}$$

Now,

$K=\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}(\underbrace{\tan x-x+Kx}_{\text{odd function}}-\frac{K\pi}{4}-1)dx=\frac{-\pi}{2}-\frac{K\pi^2}{8}$

Hence, $\quad K=\frac{-4\pi}{8+\pi^2}$

$$\therefore\frac{8+\pi^2}{\pi}\cdot\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} f(x)dx=-4\equiv m\Rightarrow m^2=16$$

Q69

Given equation is $x \cos y dy + \sin y dx = x dx$

Or $d(x \sin y) = x dx$

Integrating, we get $\int d(x \sin y) = \int x dx$

$\Rightarrow x \sin y = \frac{x^2}{2} + C$

Since, it passes through $(0, 0)$

$C = 0$

$\Rightarrow x \sin y = \frac{x^2}{2}$

Q70

Parameter corresponding to P is $t_1 = 3$

Hence, the parameter corresponding to Q is $t_2 = -t_1 - \frac{2}{t_1} = -3 - \frac{2}{3} = -\frac{11}{3}$

Let, $S(t_3)$ be the point where another tangent from R touches the parabola

Now, if the tangent at $Q(t_2)$ and $S(t_3)$ meet on the directrix at R then $t_2 t_3 = -1$

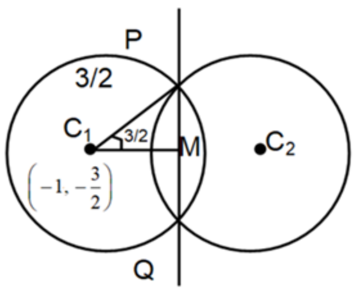
$\Rightarrow t_3 = \frac{3}{11}$

So the equation of the tangent at S is $\frac{3}{11} y = x + \frac{9}{121}$

Hence, the slope of the tangent at S is $\frac{11}{3}$

Q71

Equation of the common chord is $2x + 1 = 0$



$C_1 M = \left| \frac{-2+1}{2} \right| = \frac{1}{2}$

$PM=\sqrt{\frac{9}{4}-\frac{1}{4}}=\sqrt{2}$

Length of the common chord $=2\sqrt{2}$

Hence, the perimeter of $\Delta C_1PQ=\frac{3}{2}+\frac{3}{2}+2\sqrt{2}$

$=3+2\sqrt{2}$ units

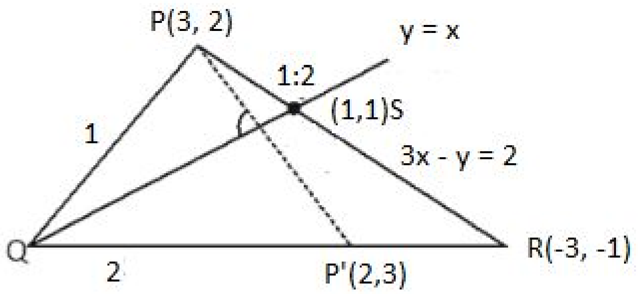
Q72

Plane P_1 is parallel to line AB

$\Rightarrow$ Shortest distance $=$ Perpendicular distance of any point on line to plane $P_1=0$

$=\left|\frac{0+9+4+1}{\sqrt{9+1}}\right|=\frac{14}{\sqrt{10}}=7\sqrt{\frac{2}{5}}$ units

Q73



Point of intersection of $y=x$ & $3x-y=2$ is $S(1{,}1)$

Now, $PS:SR=PR:RQ=1:2$

Hence, using section formula $R=(-3,-1)$

Let, reflection of P about the line $y=x$ is P'

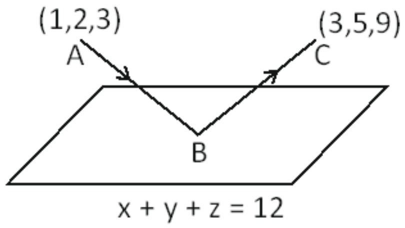
$\Rightarrow P'=(2{,}3)$ which lies on the side QR

$\Rightarrow$ equation of QR is $4x-5y+7=0$

Point of intersection of QR & $y=x$ is

$(7{,}7)=Q$

Q74



A plane which contains the incident ray and the reflected ray must contain the points A, B, C and should be perpendicular to the given plane.

So, equation of the plane is $\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & 1 \\ 2 & 3 & 6 \end{vmatrix} = 0$

$\Rightarrow (x-1)(3) - (y-2)(4) + (z-3)(3-2) = 0$

$\Rightarrow 3x - 3 - 4y + 8 + z - 3 = 0$

$\Rightarrow 3x - 4y + z + 2 = 0$

Hence, the distance of the plane from $(0,0,0)$ is $\left|\frac{0-0+0+2}{\sqrt{9+16+1}}\right| = \frac{2}{\sqrt{26}}$ units

Q75

Given, $\vec{r} \times \vec{a} = \vec{b}$

Taking right cross product with $\vec{c}$, we get,

$\left(\vec{r} \times \vec{a}\right) \times \vec{c} = \vec{b} \times \vec{c} \Rightarrow \left(\vec{r} \cdot \vec{c}\right)\vec{a} - \left(\vec{a} \cdot \vec{c}\right)\vec{r} = \vec{b} \times \vec{c}$

$\Rightarrow 3\vec{a} - \vec{r} = \vec{b} \times \vec{c} \Rightarrow \vec{r} = 3\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$\Rightarrow \vec{r} = 5\hat{i} + 7\hat{j} + 9\hat{k} \Rightarrow \left|\vec{r}\right| = \sqrt{155}$

Q76

For A,

$|x-3| = 1 \Rightarrow x = 2,4$ or

$x^2 - 5x + 6 = 0 \Rightarrow x = 2,3$ but $x \neq 3$

$\therefore A = \{2,4\}$

For B,

$B = \{4, 5, 6\}$

$n(A \times B) = 6$

$\therefore$ number of subsets $= 2^6$

Q77

$\bar{x}_{\text{old}} = 10 = \frac{\Sigma x_i}{10} \Rightarrow \Sigma x_{i\text{old}} = 100$

$\Sigma x_{i\text{new}} = 100 - 5 + 15 = 110$

$\overline{x}_{\text{new}} = \frac{110}{10} = 11$

$\text{Var}_{\text{old}} = 5 = \frac{\Sigma x_{i\text{old}}^2}{10} - (\bar{x}_{\text{old}})^2$

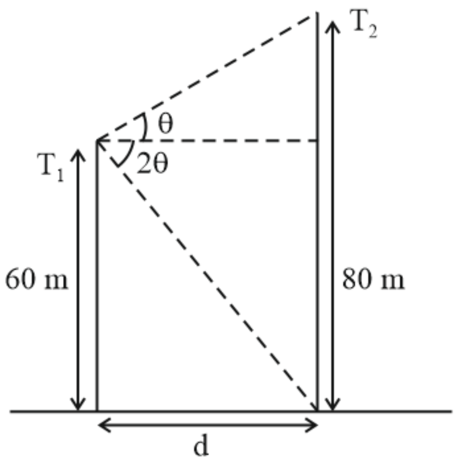
$\Sigma x_{i\text{old}}^2 = 1050$

$\Sigma x_{i\,\text{new}}^2 = 1050 - 25 + 225$

$= 1250$

$\text{Var}_{\text{new}} = \frac{1250}{10} - (11)^2 = 125 - 121 = 4$

Q78



Let, the width of the road the road is d.

If the angle of the elevation is θ, then

$\tan\theta = \frac{20}{d}$, here 20 is the difference between the heights of T_1 and T_2.

Given that, the angle of depression is twice of the angle of elevation.

$\tan 2\theta = \frac{60}{d}$

We know, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\therefore \frac{60}{d} = \frac{40/d}{1-(400/d^2)}$

$\Rightarrow \frac{400}{d^2} = \frac{1}{3}$

$d = 20\sqrt{3}$

Q79

$(p \wedge q \wedge r)$ is subset of r, so $(p \wedge q \wedge r)$ is a subset of $(\sim p \vee \sim q \vee r)$. Hence, $(p \wedge q \wedge r) \Rightarrow (\sim p \vee \sim q \vee r)$ is a tautology.

Similarly, $(p \wedge q \wedge r) \Rightarrow ((\sim p \wedge \sim q) \vee r)$ is a tautology because $(p \wedge q \wedge r)$ is a subset of $((\sim p \wedge \sim q) \vee r)$

Similarly, $(p \wedge \sim q \wedge r)$ is a subset of $(\sim p \vee q \vee r)$. Hence, $(p \wedge \sim q \wedge r) \Rightarrow (\sim p \vee q \vee r)$ is a tautology

$(p \wedge q \wedge \sim r) \Rightarrow r$ is false if p is true, q is true and r is false. Hence, $(p \wedge q \wedge \sim r) \Rightarrow r$ is not a tautology

Q80

$(1+x)^n = \sum_{r=0}^{n} C_r x^r$

$\Rightarrow x(1+x)^n = \sum_{r=0}^{n} C_r x^{r+1}$

Differentiating w.r.t. x we get

$xn(1+x)^{n-1} + (1+x)^n = \sum_{r=0}^{n} (r+1) C_r x^r$

Again multiplying both sides by x

$(1+x)^{n-1}(nx+1+x)x = \sum_{r=0}^{n} (r+1) C_r x^{r+1}$

Again differentiating w.r.t. x we get,

$\frac{d}{dx}\left((1+x)^{n-1}\left(nx^2+x^2+x\right)\right) = \sum_{r=0}^{n}\left((r+1)^2\right) C_r x^r$

$\sum_{r=0}^{n}\left((r+1)^2\right) C_r x^r = (1+x)^{n-1}(2nx+2x+1) + \left(nx^2+x^2+x\right)(n-1)(1+x)^{n-2}$

Putting $x=1$ on both sides, we get,

$\sum_{r=0}^{n}\left((r+1)^2\right) C_r = 2^{n-1}(2n+3) + (n+2)\cdot(n-1)2^{n-2} = 2^{n-2}\left(n^2+5n+4\right)$

Now, $f(x) = x^2+5x+4 = (x+1)(x+4) \Rightarrow \alpha = -1,\ \beta = -4$

Hence, $\alpha^2+\beta^2 = 17$

Q81

Let, $\overrightarrow{\alpha}=\hat{i}-\hat{j}+2\hat{k},\ \overrightarrow{\beta}=2\hat{i}-\hat{j}+\hat{k},\ \overrightarrow{\gamma}=\hat{i}-2\hat{j}+\hat{k}$

$\overrightarrow{a}=\frac{\overrightarrow{\gamma}\times\left(\overrightarrow{\alpha}\times\overrightarrow{\beta}\right)}{\left|\overrightarrow{\gamma}\times\left(\overrightarrow{\alpha}\times\overrightarrow{\beta}\right)\right|}$

$\overrightarrow{\gamma}\times\left(\overrightarrow{\alpha}\times\overrightarrow{\beta}\right)=\left(\overrightarrow{\gamma}\cdot\overrightarrow{\beta}\right)\overrightarrow{\alpha}-\left(\overrightarrow{\gamma}\cdot\overrightarrow{\alpha}\right)\overrightarrow{\beta}$

$=5\overrightarrow{\alpha}-5\overrightarrow{\beta}$

$=5\left(-\hat{i}+\hat{k}\right)$

$\overrightarrow{a}=\frac{-\hat{i}+\hat{k}}{\sqrt{2}}$

Projection of $\overrightarrow{a}$ on $\overrightarrow{\alpha}$ is $\overrightarrow{a}\cdot\widehat{\alpha}=\frac{\overrightarrow{a}\cdot\overrightarrow{\alpha}}{\left|\overrightarrow{\alpha}\right|}=\frac{-1+2}{\sqrt{2}\sqrt{6}}=\frac{1}{\sqrt{12}}$

Q82

$\frac{dy}{dx}=(1+x)+\left(\frac{y-3}{1+x}\right)$

$\frac{dy}{dx}-\frac{1}{(1+x)}y=(1+x)-\frac{3}{(1+x)}$

I.F$=e^{-\int\frac{1}{(1+x)}dx}=\frac{1}{(1+x)}$

$\therefore\frac{d}{dx}\left(\frac{y}{1+x}\right)=x+3(1+x)^{-1}+c$

$\Rightarrow\frac{y}{1+x}=x+3(1+x)^{-1}+c$

$\Rightarrow y=(1+x)\left[x+\frac{3}{(1+x)}+c\right]$, $\therefore$ at $x=2,\ y=0$

$\Rightarrow 0=3(2+1+c)\Rightarrow c=-3$

At $x=3,\ y=3$

Q83

$2\left(\frac{3x}{2}\right)+3\left(\frac{4y}{3}\right)=5$

$\frac{3x}{2}+\frac{3x}{2}+\frac{4y}{3}+\frac{4y}{3}+\frac{4y}{3}=5$

Using $A.M\geq G.M$ for $\frac{3x}{2},\frac{3x}{2},\frac{4y}{3},\frac{4y}{3},\frac{4y}{3}$

$\frac{\frac{3x}{2}+\frac{3x}{2}+\frac{4y}{3}+\frac{4y}{3}+\frac{4y}{3}}{5} \geq \left(\left(\frac{3x}{2}\right)^2\left(\frac{4y}{3}\right)^3\right)^{\frac{1}{5}}$

$\frac{3x+4y}{5} \geq \left(\frac{3^2 4^3}{2^2 3^3} x^2 y^3\right)^{\frac{1}{5}}$

$1 \geq \left(\frac{2^4}{3} x^2 y^3\right)^{\frac{1}{5}}$

$16x^2y^3 \leq 3$

Maximum value of $16x^2y^3 = 3$

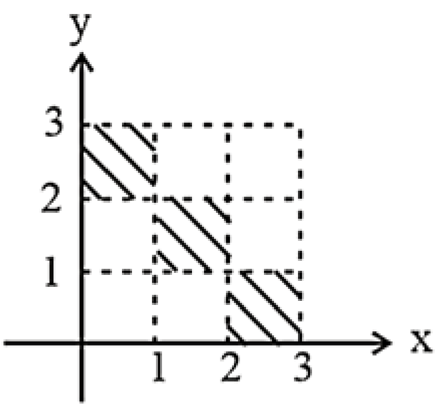
Q84

$\because \sin^{-1}\left(\frac{2\sqrt{15}}{|x|}\right)= \cos^{-1}\sqrt{1-\left(\frac{2\sqrt{15}}{|x|}\right)^2}$ for $0 < \frac{2\sqrt{15}}{|x|} \leq 1 \Rightarrow |x| \geq 2\sqrt{15}$

$\therefore \left(\frac{14}{|x|}\right)^2 = 1-\left(\frac{2\sqrt{15}}{|x|}\right)^2 \Rightarrow |x|= 16 \Rightarrow x = \pm 16$ which satisfies $|x| \geq 2\sqrt{15}$.

Hence, the maximum value of $x = 16$

Q85



$[x]+[y]= 2$ (Let $x, y \geq 0$)

$\Rightarrow [x], [y]= 2, 0$ or $1, 1$ or $0, 2$

If $[x]= 2$ and $[y]= 0$

$\Rightarrow x \in [2, 3)$ and $y \in [0, 1)$ and so on

Since, the curve is symmetric in the I[st] and III[rd] quadrant

Hence, the required area $= 6(1 \times 1)= 6$ sq. units

Q86

$$
\begin{aligned}
f(x) &= x^4-4x^3-8x^2+a \\
f'(x) &= 4\left(x^3-3x^2-4x\right) \\
&= 4x\left(x^2-3x-4\right) \\
&= 4x(x-4)(x+1)=0 \text{ at } x=-1,0,4 \\
f(-1) &= a-3 \leq 0, a \leq 3 \\
f(0) &\geq 0 \Rightarrow a \geq 0 \\
a &\in [0,3] \\
\text{Sum } &= 0+1+2+3=6
\end{aligned}
$$

Q87

Let, $Z=re^{i\theta}$

$\Rightarrow r^3e^{i\theta}+4re^{-i2\theta}=0$

$\Rightarrow r^2e^{i5\theta}=-4$

$\Rightarrow r^2=4$ and $e^{i5\theta}=-1$

$\Rightarrow r=2$ and $\theta=\frac{-3\pi}{5},\frac{-\pi}{5},\frac{\pi}{5},\frac{3\pi}{5},\pi$

$\Rightarrow$ least $\arg Z$ is $-\frac{3\pi}{5}$

$\Rightarrow k=0.6$

Q88

Let, $y=x+h$, where $h\geq 1$ and $z=x+h+k$, where $k\geq 1$.

Then, $x+y+z=20$

$\Rightarrow x+x+h+x+h+k=20$

$\Rightarrow 3x+2h+k=20$.

When $x=1\Rightarrow 2h+k=17$, then there are 8 ways.

When $x=2\Rightarrow 2h+k=14$, then there are 6 ways.

When $x=3\Rightarrow 2h+k=11$, then there are 5 ways.

When $x=4\Rightarrow 2h+k=8$, then there are 3 ways.

When $x=5\Rightarrow 2h+k=5$, then there are 2 ways.

When $x > 5 \Rightarrow$ not possible.

$\Rightarrow$ Total number of ways $= 24$

Q89

$\ell = n^2 - \frac{n(n-1)}{2} + 1$

$m = n^2 - n(n-1)+1$

$p = \frac{n(n-1)}{2}$

Given, $\ell + 5 = p + 2m$

$\frac{n^2}{2} + \frac{n}{2} + 1 + 5 = \frac{n^2}{2} - \frac{n}{2} + 2(n+1)$

$n + 6 = 2n + 2$

$n = 4$

Q90

$f[f(x)] = f(x) \forall x \in S = \{1, 2, 3\}$

I. When range contains 1 element

${}^3C_1 \times 1 = 3$

II. When range contains 2 elements e.g., let $f(1) = 1$ and $f(2) = 2$

For the case of $f(2) = 2$,

Let $f(3) = 1$ OR Let $f(3) = 2$

(i) If $x = 1$,

LHS $= f[f(x)] = f(1)$

RHS $= 1$

In this case also LHS $=$ RHS $\forall x \in S$

(ii) If $x = 2$, LHS $=$ RHS

(iii) If $x = 3$, LHS $=$ RHS

$\therefore {}^3C_2 \times 2 = 6$

Remaining element can be mapped 2 ways.

III. When range contains 3 elements

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$