Answer Key

Q1 (4)	**Q2** (2)	**Q3** (3)	**Q4** (1)
Q5 (1)	**Q6** (4)	**Q7** (3)	**Q8** (1)
Q9 (4)	**Q10** (4)	**Q11** (3)	**Q12** (2)
Q13 (1)	**Q14** (2)	**Q15** (4)	**Q16** (1)
Q17 (3)	**Q18** (4)	**Q19** (4)	**Q20** (4)
Q21 (21)	**Q22** (5)	**Q23** (11)	**Q24** (40)
Q25 (9)	**Q26** (154)	**Q27** (350)	**Q28** (2)
Q29 (10)	**Q30** (0)	**Q31** (1)	**Q32** (2)
Q33 (2)	**Q34** (3)	**Q35** (4)	**Q36** (3)
Q37 (3)	**Q38** (2)	**Q39** (1)	**Q40** (3)
Q41 (4)	**Q42** (2)	**Q43** (3)	**Q44** (1)
Q45 (2)	**Q46** (1)	**Q47** (3)	**Q48** (3)
Q49 (1)	**Q50** (2)	**Q51** (2)	**Q52** (4)
Q53 (525)	**Q54** (1)	**Q55** (2)	**Q56** (24)
Q57 (4)	**Q58** (9)	**Q59** (5)	**Q60** (0)
Q61 (1)	**Q62** (2)	**Q63** (3)	**Q64** (4)
Q65 (2)	**Q66** (2)	**Q67** (1)	**Q68** (2)
Q69 (1)	**Q70** (4)	**Q71** (2)	**Q72** (2)

Q69 (1) **Q70** (4) **Q71** (2) **Q72** (2)

Q73 (3) **Q74** (1) **Q75** (2) **Q76** (2)

Q77 (3) **Q78** (4) **Q79** (4) **Q80** (2)

Q81 (2) **Q82** (4) **Q83** (4) **Q84** (4)

Q85 (59) **Q86** (36) **Q87** (9) **Q88** (8)

Q89 (51) **Q90** (3)

Q1

By perpendicular axes theorem,

$$I_{\mathrm{EF}} = M\frac{a^2+b^2}{12} = \frac{M\left(a^2+a^2\right)}{12} = M\frac{2a^2}{12}$$

$$I_{\mathrm{z}} = \frac{M\left(2a^2\right)}{12} + \frac{M\left(2a^2\right)}{12} = \frac{Ma^2}{3}.$$

By perpendicular axes theorem,

$$I_{\mathrm{AC}} + I_{\mathrm{BD}} = I_{\mathrm{z}} \Rightarrow I_{\mathrm{AC}} = \frac{I_{\mathrm{z}}}{2} = \frac{Ma^2}{6}.$$

By the same theorem,

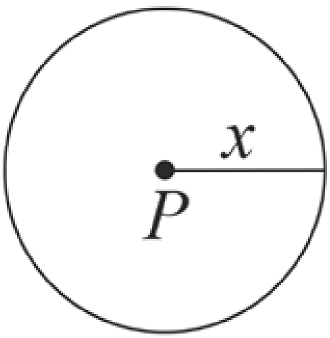
$$I_{\mathrm{EF}} = \frac{I_{\mathrm{z}}}{2} = \frac{Ma^2}{6}$$

$$\therefore\ I_{\mathrm{AC}} = I_{\mathrm{EF}}.$$

Q2

In telescope, the aperture diameter of objective lens is taken more so that more light can enter, so that image becomes sharper. Also due to more aperture diameter, the resolution power of telescope increases.

Q3



For a point source of power $= P$, then intensity at a point at a separation x from the source is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi x^2}$$

$\because$ Average intensity of e.m wave is given by,

$I=\frac{1}{2} C \in_0 E_o^2$

$\Rightarrow \frac{1}{2} C \in_0 E_o^2=\frac{P}{4 \pi x^2}$

$\Rightarrow E_o^2=\frac{2 P}{4 \pi \in_0 C\, x^2}$

$\Rightarrow E_0=\sqrt{\frac{2 P}{4 \pi \in_0 C\, x^2}}$

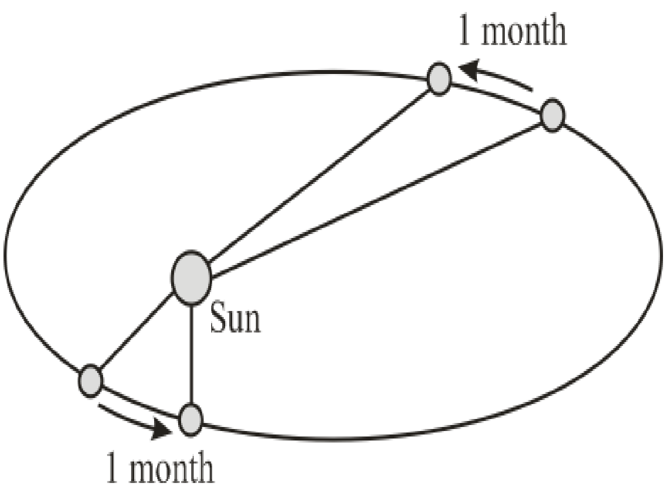
$\because \frac{1}{4 \pi \in_0 \in}=9 \times 10^9,\ P=0.1\, W,\ x=1\, m$

$C=\text{Speed of light}=3 \times 10^8 \mathrm{~m} \mathrm{~s}^{-1}$

$\Rightarrow E_0=\sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{3 \times 10^8 \times 1^2}}=\sqrt{6}=2.45 \mathrm{~v} \mathrm{~m}^{-1}$

Q4

The kinetic energy is maximum when the particle is near to the sun because we know that from Kepler's law of areal speed, that the particle swept the equal area in equal time so at point A to cover equal-area it should move faster than other points.



Q5

Let,

E =energy of electron

m =mass of electron

$r =$ The radius of the orbit

then

$E \propto m$

$\Rightarrow \frac{E}{-13.6} = \frac{2m}{m}$

$\Rightarrow E = -27.2\ \text{eV}$

Also,

$r \propto \frac{1}{m}$

$\frac{r_0}{a_0} = \frac{m}{2m}$

$r_0 = \frac{a_0}{2}$

Q6

Here, the dimension of $\frac{a}{V^2}$ will be equal to pressure so $\frac{a}{(L^3)^2} = ML^{-1}T^{-2}$ [Principle of homogeneity]

$\therefore \quad [a] = \left[ML^5T^{-2}\right]$

Aliter:

According to gas equation, for one mol of a real gas.

$\left[P + \frac{a}{V^2}\right](V-b) = RT$

$PV + \frac{a}{V} - Pb + \frac{ab}{V^2} = RT$

As this equation is dimensionally correct, each term on either side will have same dimensions, i.e.,

$\left[\frac{a}{V}\right] = [PV]$

or $\quad [a] = \left[ML^{-1}T^{-2}\right]\left[L^3\right]\left[L^3\right] = \left[ML^5T^{-2}\right]$

and $[P \times b] = (PV)$

or $\quad [b] = [V] = [L]^3$

Note: Actually vander Waals equations for μ mol is

$\left[P + \frac{\mu^2 a}{V^2}\right][V - \mu b] = \mu RT$

So that $[\mu b] = [V] \quad i.e., \quad [b] = \left[L^3\mu^{-1}\right]$ with units $m^3/$ mol

and $\left[\mu^2 a\right] = \left[PV^2\right] \quad i.e., \quad [a] = \left[ML^5T^{-2}\right]$ with units $J\ m^3/\text{mol}^2$

Q7

The expression of path difference is given as $\Delta x = d\sin\theta$, here θ is the angle relative to the incident direction.

For bright or maxima fringe, path difference is stated as$\Delta x = m\lambda$, here $m = 1, 2, 3, ..$

Giver here $D = 1$ m, d $= 1$ mm, $\lambda = 5000$ nm and $m = 100$

For 100^{th} maximum, $d\sin\theta = 100\lambda$

$\sin\theta = \frac{100\times5000\times10^{-9}}{1\times10^{-3}} = \frac{5\times10^{-4}}{10^{-3}} \times 0.5 = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

We know that separation of 100^{th} maxima from central maxima is given as $y = D\tan\theta$

Thus, distance $y = 1 \times \tan 30 = \frac{1}{\sqrt{3}}$ m.

Q8

Volume remains constant.

So, the volume of big drop $=$ volume of 1000 small drops

$\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3$

$R = 10r$

Recall the formula of surface energy,

$E = T \times A$

$\therefore \frac{\text{Surface energy of big drop}}{\text{Surface energy of 1000 small drops}}$

$= \frac{4\pi R^2 T}{1000 \times 4\pi r^2 T}$

$= \frac{10 \times 10r}{1000r} \ [\because R = 10r]$

$= \frac{1}{10}$

Q9

The acceleration of the system $= \left(\frac{\mathrm{m_1 - m_2}}{\mathrm{m_1 + m_2}}\right)\mathrm{g}$ and hence the tension T in the string is $\left(\frac{\mathrm{2m_1m_2}}{\mathrm{m_1+m_2}}\right)\mathrm{g}$

The reading of the spring balance is T (in units of force) and T/g(in units of mass).

Q10

The relation between Y, η and B is $\frac{9}{Y} = \frac{1}{B} + \frac{3}{\eta}$
Where Y is Young's modulus, B is Bulk modulus and η is modulus of rigidity.

Q11

Focal length of combination

$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2} \quad \ldots(\mathrm{i})$

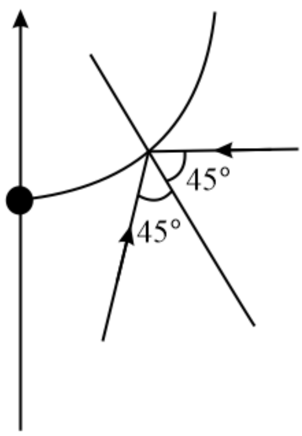
Using lens maker's formula

$\frac{1}{f_2}=\left(\mu_2-1\right)\left(\frac{1}{-R}-\frac{1}{\infty}\right)=-\frac{\left(\mu_2-1\right)}{R}$

$\frac{1}{f_1}=\left(\mu_1-1\right)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{\mu_1-1}{R}$

or $\frac{1}{f_1}=\frac{\left(\mu_1-1\right)}{R}$

$\frac{1}{f_2}=-\frac{\left(\mu_2-1\right)}{R}$



Putting these values in Equation (i), we get

$\Rightarrow \frac{1}{f}=\frac{\left(\mu_1-1\right)}{R}-\frac{\left(\mu_2-1\right)}{R}$

$\Rightarrow \frac{1}{f}=\frac{\left(\mu_1-1-\mu_2+1\right)}{R}$

$\Rightarrow \frac{1}{f}=\frac{\mu_1-\mu_2}{R}$

$f=\frac{R}{\left(\mu_1-\mu_2\right)}$

Q12

When the ball of mass m falls from a height$h$, it reaches the surface of earth in time

$\sqrt{\frac{2h}{g}}$ and Its velocity is$v=\sqrt{2gh}$. It then moves in to the tunnel and reaches on the other side of earth and goes again upto a height h from that side of earth. The ball again returns back and thus executes periodic motion.

Outside the earth ball crosses distance h four times.

When the ball is in the tunnel at distance x from the centre of the earth, then gravitational force acting on ball is

$F=\frac{Gm}{x^2}\times\left(\frac{4}{3}\pi x^2\rho\right)=G\times\left(\frac{4}{3}\pi\rho\right)mx$

Mass of the earth, $M=\frac{4}{3}\pi R^2\rho$

or $\frac{4}{3}\pi\rho=\frac{\mathrm{M}}{\mathrm{R}^3}$

$\therefore \quad F=\frac{GMmx}{R^3}\ \ ie,\ F\propto x$

As this F is directed towards the centre of the earth i.e., the mean position so the ball will execute periodic motion about the centre of the earth

Here inertia factor=mass of ball$=m$

Spring factor$=\frac{GMm}{R^3}=\frac{gm}{R}$

$\therefore$ time period of oscillation of ball in the tunnel is

$T^{'}=2\pi\sqrt{\frac{\text{inertia factor}}{\text{spring factor}}}$

$=2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{gm/R}}}=2\pi\sqrt{\frac{\mathrm{R}}{\mathrm{g}}}$

Time spent by ball outside the tunnel on both the sides will be

$4\sqrt{\frac{2h}{g}}$

Therefore, total time period of oscillation of ball is

$= 2\pi\sqrt{\frac{R}{g}} + 4\sqrt{\frac{2h}{g}}$

Q13

The rate of flow of charge gives electric current, while charge per unit change in potential gives capacitance. So,

$i = \frac{\mathrm{d}q}{\mathrm{d}t}$ & $C = \frac{\mathrm{d}q}{\mathrm{d}V}$

Thus, we get,

$$\begin{aligned} i &= \frac{q}{t} = \frac{CV}{t} \\ \Rightarrow t &= \frac{CV}{i} \\ &= \frac{20\times10^{-6}\times300}{10\times10^{-3}} \\ &= 0.6 \text{ s} \end{aligned}$$

Q14

Potential gradient of the potentiometer is given as 0.2 V m^{-1}.

So, $x = 0.2 \text{ V m}^{-1}$.

Here, the potential difference across the coil is, $V = IR = 0.1 \times 2 = 0.2 \text{ V}$.

If the balancing length is l, then,

$V = xl$

$\Rightarrow 0.2 = 0.2 \times l$

$\Rightarrow l = 1 \text{ m}$.

Q15

Rate of heat transfer in a slab is given as $Q = KA\frac{dt}{dx}$, where A is the area of the surface and $\frac{dt}{dx}$ is the temperature gradient.

Equating heat current in both slabs

$\frac{K(\theta-\theta_1)}{3d}=\frac{3K(\theta_2-\theta)}{d}$

$\theta-\theta_1=9\theta_2-9\theta$

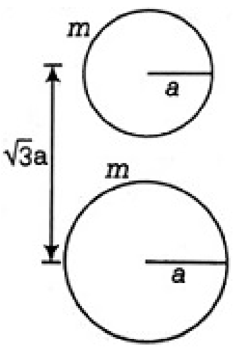
$10\theta=9\theta_2+\theta_1$

$\theta=\frac{9\theta_2+\theta_1}{10}$

Q16

Now, the gravitational field due to the ring at a distance $d=\sqrt{3}a$ on its axis is

$E=\frac{Gmd}{\left(a^2+d^2\right)^{\frac{3}{2}}}=\frac{\sqrt{3}Gm}{8a^2}$



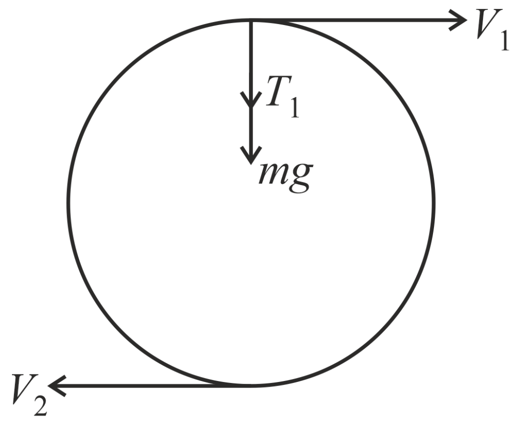
The force in a particle of mass M placed here is

$F=ME=\frac{\sqrt{3}GMm}{8a^2}$

This is also the force due to the sphere in the ring.

Q17

To just complete the loop, speed v_1 at the highest point must be minimum.



Applying Newton's 2nd law at the highest point.

$mg + T_1 = \frac{mv_1^2}{R}$.

For minimum v_1, tension must be minimum which can be zero, i.e. $T_1 = 0$.

$\therefore\ v_1 = \sqrt{Rg} \quad \ldots.(1)$.

Now from conservation of mechanical energy,

Loss in gravitational PE =Gain in KE

$\Rightarrow mg(2R) = \frac{1}{2}\mathrm{mv}_2^2 - \frac{1}{2}\mathrm{mv}_1^2$

$\Rightarrow v_2^2 = v_1^2 + 4gR$

$\Rightarrow v_2 = \sqrt{5gR} \quad$ [Using (1)].

Hence, assertion is true, reason is false.

Q18

By the law of conservation of momentum $m\,u = (M+m)\nu$

$0.50 \times 2.00 = \left(1 + 0\,.\,50\right)\nu, \frac{1\,.\,00}{1.50} = \nu$

Initial K.E.= $\left(1/2\right) \times 0.50 \times \left(2\,.\,00\right)^2 = 1\,.\,00\mathrm{J}$

Final K.E. $= \frac{1}{2} \times 1.50 \times \frac{1.00^2}{\left(1.50\right)^2} = \frac{1.00}{3.00} = 0.33$

$\therefore$ Loss of energy $= 1.00 - 0.33 = 0.67\ \mathrm{J}$.

Q19

The given equation of alternating voltage is,

$e = 200\sqrt{2}\sin 100t\ \ldots\left(\mathrm{i}\right)$

The standard equation of alternating voltage is,

$e = e_0 \sin \omega t\ \ldots(\mathrm{ii})$

Comparing equations (i) and (ii), we get,

$e_0 = 200\sqrt{2}\ \mathrm{V}, \omega = 100\ \mathrm{rad\ s}^{-1}$

The capacitive reactance is,

$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 1 \times 10^{-6}}\ \Omega$

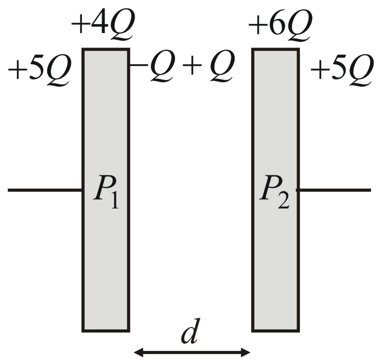
The rms value of current in the circuit is,

$i_{rms} = \frac{v_{\mathrm{rms}}}{X_C} = \frac{e_0/\sqrt{2}}{1/\omega C} = \frac{\left(200\sqrt{2}/\sqrt{2}\right)}{1/100 \times 10^{-6}}$

$= 200 \times 100 \times 10^{-6}\ \mathrm{A} = 2 \times 10^{-2}\ \mathrm{A} = 20\ \mathrm{mA}$

Q20

From the above questions charges on different plates will be,



When we connected both the plates with the wires, the charges will start flowing from one plate to the other and the final charges are as shown in the figure. Energy stored in the capacitor will be zero finally, i.e., $U_f = 0$, but initially, the energy will be stored in the electric field between the charges which can be given by,

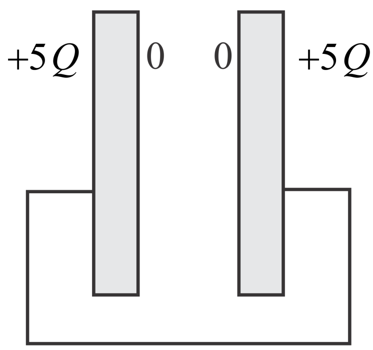
$E = \frac{Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

So, the initial energy in the plates will be,

$U_{\mathrm{i}} = \frac{1}{2}\varepsilon_0 E^2 \times Ad = \frac{1}{2} \times \varepsilon_0 \times \left(\frac{Q}{A\varepsilon_0}\right)^2 \times Ad$

The energy lost in heat will be,

$\Delta H = U_{\mathrm{i}} - U_{\mathrm{f}} = \frac{Q^2}{2A\varepsilon_0}d$



Q21

Change in velocity = area under $a - t$ graph.

$v_{\mathrm{f}}-v_{\mathrm{i}}=\frac{1}{2}(4)\,(8)=16\ \mathrm{m\,s^{-1}}.$

$\therefore v_{\mathrm{f}}=v_{\mathrm{i}}+16=(5+16)\ \mathrm{m\,s^{-1}}=21\ \mathrm{m\,s^{-1}}.$

Q22

In series LCR, current is maximum at resonance.

$\therefore$ Resonant frequency, $\omega=\frac{1}{\sqrt{LC}}$

$\therefore\ \omega^2=\frac{1}{LC}$ or, $L=\frac{1}{\omega^2 C}$

Given, $\omega=1000\ \mathrm{s}^{-1}$ and $C=10\ \mu\mathrm{F}$

$\therefore\ L=\frac{1}{1000\times1000\times10\times10^{-6}}=0.1\ \mathrm{H}=100\ \mathrm{mH}\Rightarrow n=5$

Q23

The density can be computed by

$\rho=\frac{\mathrm{m}}{\mathrm{v}}=\frac{\mathrm{m}}{\frac{\pi \mathrm{d}^3}{6}}$

or $\rho\ \propto\ \frac{\mathrm{m}}{\mathrm{d}^3}$

$\therefore \frac{\Delta\rho}{\rho}\times100\%=\frac{\Delta\mathrm{m}}{\mathrm{m}}\times100\%\ +\ 3\frac{\Delta\mathrm{d}}{\mathrm{d}}\times100\%$

or $\frac{\Delta\rho}{\rho}\times100=2+(3\times3)=11$

Hence, the percentage error in measurement of density is 11%

Q24

$i=\frac{(12-8)}{(200+200)}\mathrm{A}=\frac{4}{400}=10^{-2}\ \mathrm{A}$

Power loss in each diode$=(4)\left(10^{-2}\right)\ \mathrm{W}=40\ \mathrm{mW}$

Q25

Given, initial temperature T1 = 80oC

Final temperature $T_2 = 50^{o}C$

Temperature of the surroundings $T_0 = 20^{o}C$

$t_1 = 5$ min

According to Newton's law of cooling.

Rate of cooling, $\frac{dT}{dt} = k\left[\frac{T_1+T_2}{2} - T_0\right]$

$$\frac{(80-50)}{5} = k\left[\frac{80+50}{2} - 0\right]$$

$$\frac{30}{5} = k(65-20)$$

$$6 = k \times 45$$

or $k = \frac{6}{45} = \frac{2}{15}$

In second condition,

Initial temperature $T_1^{'} = 60^{o}C$

Final temperature $T_2^{'} = 30^{o}C$

$$t^{'} = ?$$

Now, $\frac{(60-30)}{t'} = \frac{2}{15}\left[\frac{60+30}{2} - 20\right]$

$\frac{30}{t'} = \frac{2}{15}(45 - 20)$

or $t' = \frac{30 \times 15}{2 \times 25}$

= 9 min.

Q26

By Mosely law $\frac{1}{\lambda} \propto (Z - 1)^2$
$\frac{\lambda_2}{193} = \frac{(26-1)^2}{(29-1)^2}$
$\lambda_2 = \frac{193 \times 25^2}{28^2}$
λ_2 = 153.8 pm $\simeq$ 154 pm

Q27

Given,

the fundamental frequency of an open pipe, $L = 50$ cm is f,

the speed, $v = 350 \text{ m s}^{-1}$

Now the fundamental frequency,

$f = \frac{v}{\lambda} = \frac{v}{2l} = \frac{350}{\left(2 \times \frac{50}{100}\right)}$

$= 350$ Hz

Q28

Pressure of a gas is given by $P = \frac{1}{3} \frac{mN}{V} (v_{\text{rms}})^2$.

Where, m =mass of the gas,

$N=$ Number of gas molecules,

$V=$Volume of the vessel,

$v_{\text{rms}}=$ RMS speed of gas molecules.

So, $P_0=\frac{1}{3}\frac{mN}{V}(v_{\text{rms}})^2$.

If the mass of all the molecules are halved and their speed is doubled,

$$P=\frac{1}{3}\frac{(m/2)N}{V}(2v_{\text{rms}})^2$$
$$\Rightarrow\ P=2\left[\frac{1}{3}\frac{mN}{V}(v_{\text{rms}})^2\right]$$
$$\Rightarrow\ P=2P_0$$

Therefore, $n=2$.

Q29

Coverage distance d is

$d=\sqrt{2Rh}$

coverage area can be calculated as

$$\mathrm{A}=\pi\mathrm{d}^2=2\pi\mathrm{Rh}$$
$$\mathrm{A}=\pi\mathrm{d}^2=2\times\pi\times6400\times1000\times400$$
$$\mathrm{A}=\pi\mathrm{d}^2=1.6\times10^{10}$$

Q30

Given values,

the electric field, $\overrightarrow{E}=\left(y\hat{\mathrm{i}}+x\hat{\mathrm{j}}\right)$ N C^{-1}, charge, $q=1$ C and displacement,

$\overrightarrow{r}_{\mathrm{A}}=\left(2\hat{\mathrm{i}}+2\hat{\mathrm{j}}\right)$ m & $\overrightarrow{r}_{\mathrm{B}}=\left(4\hat{\mathrm{i}}+\hat{\mathrm{j}}\right)$ m

$d\overrightarrow{r}=\left(dx\hat{\mathrm{i}}+dy\hat{\mathrm{j}}\right)$

Use the relation between force and work done,

$$dW=\overrightarrow{F}\cdot d\overrightarrow{r}=q\left(y\hat{\mathrm{i}}+x\hat{\mathrm{j}}\right)\cdot\left(dx\hat{\mathrm{i}}+dy\hat{\mathrm{j}}\right)$$

$$=q(dx\cdot y+dy\cdot x)=qd(xy)$$

$$\Rightarrow W=q\int_{2,\,2}^{4,\,1}d(xy)=1\times[xy]_{2,\,2}^{4,\,1}=4\times1-2\times2=0$$

Q31

Due to inert pair effect Pb has four electrons in its valence shell but it shows $+2$ oxidation state. In other words due to inert pair effect $+2$ oxidation state is more stable than $+4$ of Pb.

So, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Q32

Enzymes are a group of biomolecules which act as bio-catalyst, speeding up the reaction. These are made up of proteins. When such proteins encounter high temperature, it causes certain bonds to break in the structure, resulting in conformational change (denaturation). This denaturation makes the enzyme inactive and useless as catalyst. For an enzyme to be active, it may need support of non-protein constituents called cofactors, which bind to it, thus, making it catalytically active. These can be organic compounds, vitamins or mineral ions. The working of an enzyme binding to a single substrate was explained using a lock and key analogy given by Emil Fischer. In this analogy, the lock is the enzyme and the key is the substrate; on binding of substrate, the enzyme unlocks and changes the substrate to product. Also, this model explains the specificity of an enzyme as there is only one type of key to open the lock.

Q33

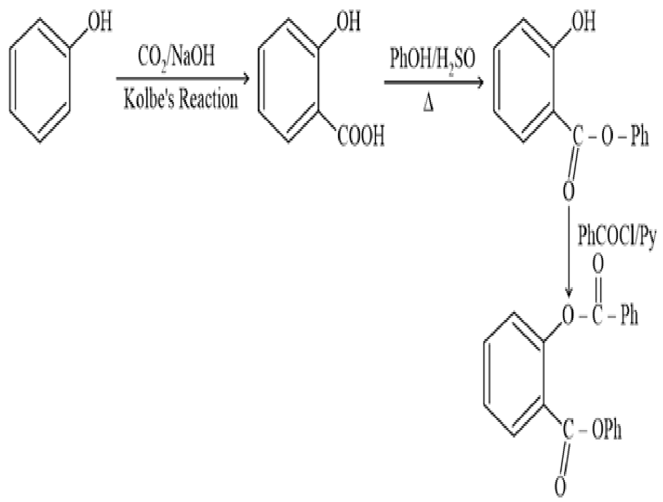
Statement-I is incorrect

Be $(\mathrm{OH})_2$ dissolve in alkali due to it's amphoteric nature.

Statement-II is correct

Solubility of alkaline earth metal hydroxide in water increases down the group due to rapid decreases in lattice energy as compared to hydration energy.

Q34



In this reaction, first Kolbe's reaction product is formed and then phenol is attached to carboxylic acid in the presence of sulphuric acid to form an ester. This reaction is called esterification and again, an ester is formed when acyl chloride reacts with another alcohol.

Q35

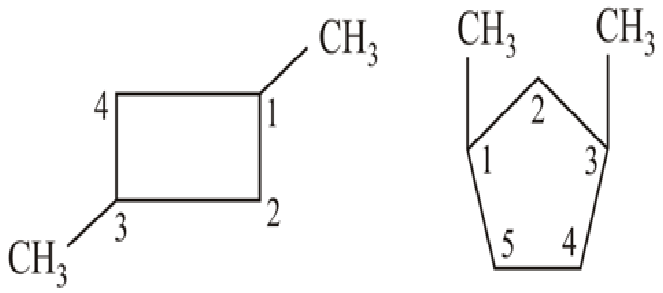
Water will pass through the membrane from the lower concentration region to the higher concentration region. Therefore, it will pass into the egg immersed in water, and out of the egg immersed in syrup.

Q36

Geometrical isomers (also known as cis-trans isomers or E-Z isomers), are chemical species with the same type and quantity of atoms as another species, but having a different geometrical structure. Atoms or groups exhibit different spatial arrangements on either side of a chemical bond or a ring structure.

In case of a closed chain, the ring must be substituted at least at two different positions.

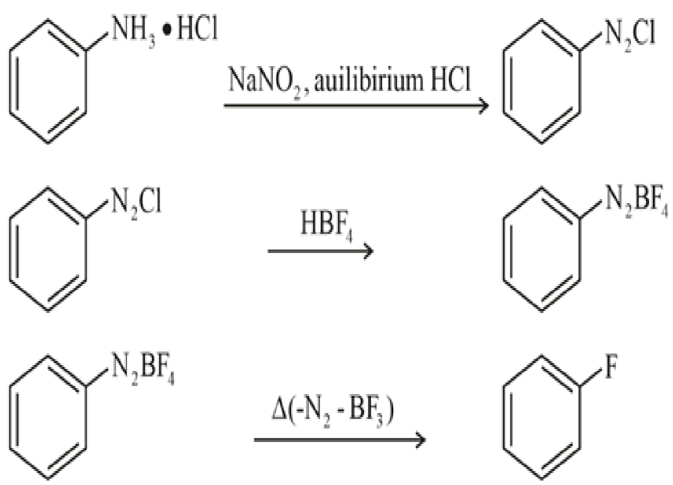
In options (a) and (b), there are no two ring carbons, containing different substituents, but in options (c) and (d), C-1 and C-3 have different substituents.



Q37

Aniline reacts with sodium nitrite and hydrochloric acid which forms benzene diazonium ion chloride which further reacts with hydrofluoric acid which gives fluoro benzene.

A is diazonium salt and B is flourobenzene.



Q38

In this particular case Zn is undergoing oxidation and Ag is undergoing reduction.

According to that first we will write the equation

For the cell

$\mathrm{Zn}\left|\mathrm{Zn}^{2+} \| \mathrm{Ag}^{+}\right| \mathrm{Ag}$

$\mathrm{E}^{\circ}{}_{\text{cell}} = \mathrm{E}^{\circ}{}_{\text{cathode}} + \mathrm{E}^{\circ}{}_{\text{anode}}$

$1.56 = 0.8 + \mathrm{E}^{\circ}{}_{\mathrm{Zn} / \mathrm{Zn}^{2+}}$

$\mathrm{E}^{\circ}{}_{\mathrm{Zn} / \mathrm{Zn}^{2+}} = 0.76\ V$

Q39

Antacids: Antacid is a substance, which neutralizes stomach acidity and is used to relieve heartburn, indigestion or upset stomach.

Until 1970, the only treatment for acidity was the administration of antacids such as sodium hydrogencarbonate or a mixture of aluminium and magnesium hydroxide. But, excessive hydrogencarbonate can make the stomach alkaline and triggers the production of even more acid. Metal hydroxides are better alternatives because of being insoluble. These do not increase the pH above neutrality. These treatments control only symptoms, not the cause.

Antipyretics: These are the substances that reduce fever. Antipyretics cause the hypothalamus to override a prostaglandin-induced increase in temperature. The body then works to lower the temperature, which results in a reduction in fever.

Analgesics: An analgesic or painkiller is any member of the group of drugs used to achieve analgesia and relief from pain.

Antibiotics: Antibiotics also called antibacterials that are type of antimicrobial drugs used in the treatment and prevention of bacterial infections. They may either kill or inhibit the growth of bacteria.

Q40

The Wacker process is originally referred to the oxidation of ethylene to acetaldehyde by oxygen in water, where the catalyst is tetrachloropalladate(II).

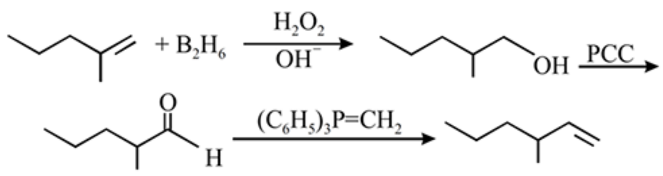
In the contact process, Platinum used to be the catalyst for this reaction, however, as it is susceptible to react with arsenic impurities in the sulphur feedstock, the preferred catalyst today is vanadium (V) oxide (V_2O_5).

In the Deacon's process, the reaction takes place at about 400 to $450^{\circ}C$ in the presence of a variety of catalysts, which include copper chloride $(CuCl_2)$.

In Ziegler-Natta polymerisation, homogenous catalysts, usually based on complexes of Ti, Zr or Hf, are used. They are usually used in combination with different catalysts.

Q41

Here the final product is 3-methyl-1-hexene and it is formed as follows



Q42

In A, Iodine is oxidized from -1 to zero whereas in B, chlorine gets reduced from $+1$ to -1. So hydrogen peroxide acts as an oxidising agent in (A) and reducing agent in (B).

Q43

Mercury poisoning often produces a crippling and fatal disease called Minamata disease.

Q44

Statement 1: $\text{B. P. of chloroform} = 334\text{ K}$

$\text{B. P. of aniline} = 457\text{ K}$

thus can be seprated of simple distillation.

Statement 2 : Mixture of aniline and water seprated by simple distillation.

Q45

For first reaction:

$$\mathrm{E_1=\frac{2.303RT_1\,T_2}{(T_1-T_2)}\log\frac{k_1'}{k_1}\quad .....(i)}$$

For second reaction:

$$\mathrm{E_2=\frac{2.303RT_1\,T_2}{(T_1-T_2)}\log\frac{k_2'}{k_2}\quad .....(ii)}$$

Given: $\mathrm{E_1>E_2}$

$$\Rightarrow \mathrm{\frac{2.303RT_1\,T_2}{(T_1-T_2)}\log\frac{k_1'}{k_1}>\frac{2.303RT_1\,T_2}{(T_1-T_2)}\log\frac{k_2'}{k_2}}$$

$$\therefore \mathrm{\frac{k_1'}{k_1}>\frac{k_2'}{k_2}}$$

Q46

Smelting : It is a form of extractive metallurgy; it's main use is to produce a metal from it's ore. Smelting uses heat and a chemical reducing agent like C, CO, etc. to decompose the ore driving off other elements as gases or slag and leaving just the metal behind.

Q47

Oxidation state of P in $\mathrm{H_4P_2O_7,\ H_4P_2O_5}$ and $\mathrm{H_4P_2O_6}$ is $5,\ 3$ & 4 respectively

$\mathrm{H_4P_2O_7}$

$\mathrm{2x+4(+1)+7(-2)=0}$

$\mathrm{x=+5}$

$\mathrm{H_4\underline{P_2}O_5}$

$\mathrm{2x+4(+1)+5(-2)=0}$

$\mathrm{x=+3}$

$\mathrm{H_4\underline{P_2}O_6}$

$2x + 4(+1) + 6(-2) = 0$

$x = +4$

Q48

When O_2 changes to O_2 : $\left(\sigma 1s\right)^2 (\sigma^* 1s)^2 \left(\sigma 2s\right)^2 (\sigma^* 2s)^2 (\sigma 2p_z)^2 \left(\pi 2p_x^2 = \pi 2p_y^2\right)$

$\left(\pi^* 2p_x^1 = \pi^* 2p^1 y\right)$

Bond order : $\frac{1}{2}[N_b - N_a] = \frac{1}{2}\left[10 - 6\right] = 2$

Magnetic moment $\left(\mu\right)$: $\sqrt{n(n+2)} BM = \sqrt{2(2+2)}$

$= 2.828\,BM$

O_2^+ : $\left(\sigma 1s\right)^2 (\sigma^* 1s)^2 \left(\sigma 2s\right)^2 (\sigma^* 2s)^2 (\sigma 2p_z)^2 \left(\pi 2p_x^2 = \pi 2p^2 y\right)$

$\left(\pi^* 2p_x^1 = \pi^* 2p_y^0\right)$

Bond order : $\frac{1}{2}(N_b - N_a) = \frac{1}{2}\left(10 - 5\right) = 2.5$

Magnetic moment $\left(\mu\right)$: $\sqrt{n(n+2)} BM$

$= \sqrt{1(1+2)} = 1.732\,BM$

Q49

$H_2 + \frac{1}{2} O_2 \rightarrow H_2O$; $\Delta H = -188$ kJ/mol ...(i)

$H_2 + O_2 \rightarrow H_2O_2$; $\Delta H = -286$ kJ/mol ...(ii)

By Eq.(i) $-$ (ii)

$2H_2O_2 \rightarrow 2H_2O + O_2$;

$\Delta H = 2(-188) - 2(-286) = +196$ kJ

Q50

Nylon-66 is made from adipic acid.

Nylon 66 is synthesized by polycondensation of hexamethylenediamine and adipic acid. Equivalent amounts of hexamethylenediamine and adipic acid are combined with water in a reactor.

Q51

V = 1 L moles = Molar

$$\begin{array}{ccccc} 2\,H_2\,S(g) & \rightleftharpoons & 2H_2(g) & + & S_2(g) \\ 1-2x & & 2x & & x \end{array}$$

Total moles $= 1 - 2x + 2x + x = 1 + x$

$K_c = 4 \times 10^{-6} = \frac{(2x)^2(x)}{(1-2x)^2} = \frac{4x^3}{1}$

$(1 - 2x \approx 1)$

$x^3 = 10^{-6}$

$x = 10^{-2}$

$2x = 2 \times 10^{-2}$

% Dissociation = 2%

Q52

In Hoffmann's mustard oil reaction, the primary amine reacts with alcoholic carbon disulphide and then heated with excess of mercuric chloride and formed the isothiocyanate. Primary amines can only give Hoffmann's mustard oil reaction. Primary amine is the amine in which nitrogen atom is connected only with one carbon atom and two hydrogen atoms.

Q53

$3\ \mathrm{Pb}\left(\mathrm{NO}_3\right)_2+\mathrm{Cr}_2\left(\mathrm{SO}_4\right)_3 \rightarrow 3\,\mathrm{PbSO}_4+2\,\mathrm{Cr}\left(\mathrm{NO}_3\right)_3$

$35\ \mathrm{ml}$ $\quad 20\ \mathrm{ml}$

$0.15\ \mathrm{M}$ $\quad 0.12\ \mathrm{M}$

$=5.25\ \mathrm{m.mol}=2.4\ \mathrm{m.mol}\quad 5.25\ \mathrm{m.mol}$

$=5.25\times 10^{-3}\ \mathrm{mol}$

Therefore moles of PbSO_4 formed

$=5.25\times 10^{-3}=525\times 10^{-5}$

Q54

$\Delta \mathrm{T}_{\mathrm{f}}=\mathrm{iK}_{\mathrm{f}}\mathrm{m}$

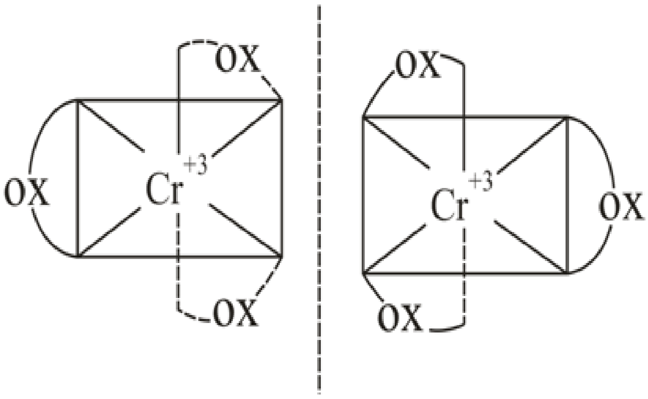
$0.0557=\mathrm{i}\times 1.86\times 0.01$

$\mathrm{i}=3$, means complex will give 3-ions

Hence, formula should be $\left[\mathrm{Co}\left(\mathrm{NH}_3\right)_5\mathrm{Br}\right]\mathrm{Br}_2$

Q55

The number of optical isomers for $[\mathrm{Cr}(\mathrm{C}_2\mathrm{O}_4)\mathrm{s}]^{3-}$ is two.



Q56

Solute in 200 g of 30% solution $= 60$ g

Solute in 300 g of 20% solution $= 60$ g

total grams of solute $= 120$ g

total grams of solution $= 200 + 300 = 500$ g

% of solute in the final solution $= \frac{\text{Wt. of solute}}{\text{Wt. of solution}} \times 100$

% of solute in the final solution $= \frac{120}{500} \times 100 = 24\%$

Q57

$\text{Energy}_{\text{photon}}\left(\text{eV}\right) = \frac{1240}{\text{wavelength(nm)}}$

$\text{Energy}_{\text{photon}} = \frac{1240}{300} = 4.1 \text{ eV}$

According to photoelectric effect, those metals will show this effect which have work function value less than the energy of photon.

So on comparing energy of photon and work function values for the given metal we can conculde that in $\text{Li, Na, K \& Mg}$ i.e. in 4 metals , photo emission will take place.

Q58

$\text{HA} + \text{OH}^- \rightleftharpoons \text{H}_2\text{O} + \text{A}_{\text{aq}}^{-1}$; $\text{K}_{\text{eq}} = 10^9$

Weak acid st. base

Then for hydrolysis reaction

$$A^{-}_{aq}+H_2O\rightleftharpoons HA+OH^{-}_{aq}\ ;\ K_h=10^{-9}$$

$$\left[OH^{-}\right]=\sqrt{K_h\cdot C}=\sqrt{10^{-9}10^{-1}}=10^{-5}$$

$$pOH=-\log 10^{-5}=5$$

$$\therefore\ pH=14-pOH=9$$

Q59

1 mole KBr $(=119\,gm)$ have $\frac{10^{-5}}{100}$ moles $SrBr_2$ and hence, 10^{-7} moles cation vacancy (as $1\,Sr^{2+}$ will result 1 cation vacancy)

$\therefore$ Required number of cation vacancies

$$=\frac{10^{-7}\times 6.023\times 10^{23}}{119}=5.06\times 10^{14}\simeq 5\times 10^{14}$$

Q60

Half-life is 500 minutes for all concentrations of sugar. Half-life is independent of the concentration of the sugar. So reaction is 1st order with respect to sugar. For H^{+} concentration

$$\frac{t_1}{t_2}=\left(\frac{a_2}{a_1}\right)^{n-1}$$

$$\frac{500}{50}=\left(\frac{10^{-6}}{10^{-5}}\right)^{n-1}$$

Therefore, $n=0$

Q61

Let,

$$t=f(x)=\log_a\left(x^2\right)=2\log_a|x|=2\log_a x \text{ as } x>0$$

$\Rightarrow t = 2\log_a x$

$\Rightarrow \frac{t}{2} = \log_a(x)$

$\Rightarrow x = a^{\frac{t}{2}}$

$f^{-1}(x) = a^{\frac{x}{2}}$

$f^{-1}(b+c) = a^{\frac{b+c}{2}} = a^{\frac{b}{2}} a^{\frac{c}{2}} = f^{-1}(b) \cdot f^{-1}(c)$

Q62

$\overrightarrow{r} \times \overrightarrow{a} = \overrightarrow{b} \times \overrightarrow{r} \Rightarrow \overrightarrow{r} \times \left(\overrightarrow{a} + \overrightarrow{b}\right) = 0$

$\overrightarrow{r} = \overrightarrow{\lambda}\left(\overrightarrow{a} + \overrightarrow{b}\right) \Rightarrow \overrightarrow{r} = \overrightarrow{\lambda}\left(\hat{i} + 2\hat{j} - 3\hat{k} + 2\hat{i} - 3\hat{j} + 5\hat{k}\right)$

$\overrightarrow{\mathrm{r}} = \overrightarrow{\lambda}\left(3\hat{\mathrm{i}} - \hat{\mathrm{j}} + 2\widehat{\mathrm{k}}\right) \ \ldots(1)$

$\overrightarrow{r} \cdot \left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$

Put $\overrightarrow{r}$ from (1) $\alpha\lambda = 1 \ \ldots(2)$

$\overrightarrow{r} \cdot \left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$

Put $\overrightarrow{r}$ from (1) $2\lambda\alpha - \lambda = 1 \ \ldots(3)$

Solve (2) and (3)

$\alpha = 1, \lambda = 1$

$\Rightarrow \overrightarrow{\mathrm{r}} = 3\hat{\mathrm{i}} - \hat{\mathrm{j}} + 2\widehat{\mathrm{k}}$

$\left|\overrightarrow{r}\right|^2 = 14$ & $\alpha = 1$

$\alpha + \left|\overrightarrow{r}\right|^2 = 15$

Q63

Basically, we need to find the intersection of the ray and mirror.

Let us assume $A'(x_1,\ y_1)$ is the image of $A(2,\ 3)$ with respect to $x+y=0$.

$\Rightarrow A^{'}(-3,\ -2)$

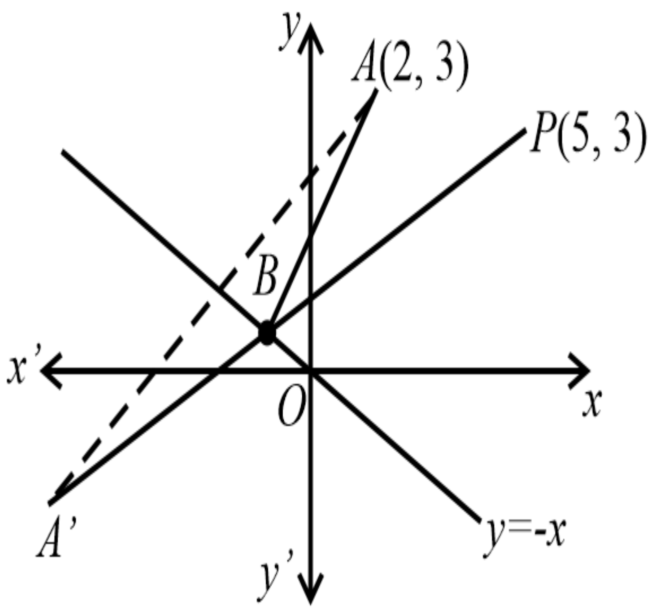
Now, we can say that B is the point of intersection of PA' with $x+y=0$.

The slope of PA' is $\frac{5}{8}$.

Hence, the equation of the line PA' is given as,

$y-3=\frac{5}{8}(x-5)$

$\Rightarrow 5x-8y-1=0$



Now, on solving the equations $y+x=0$ and $5x-8y-1=0$, we will get the point B as $\left(\frac{1}{13},\ -\frac{1}{13}\right)$

Q64

Given equation are $x+y+z=6,\ x+2y+3z=10$ and $x+2y+\lambda z=10$

Since, it is consistent.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow\ \ 1(2\lambda-6)-1(\lambda-3)+1(2-2)=0$$

$$\Rightarrow\ \ \lambda-3=0\ \Rightarrow\ \lambda=3$$

Q65

$$\cot^{-1}\frac{5}{\sqrt{3}}+\cot^{-1}\frac{9}{\sqrt{3}}+\cot^{-1}\frac{15}{\sqrt{3}}+\cot^{-1}\frac{23}{\sqrt{3}}+\ldots..\infty$$

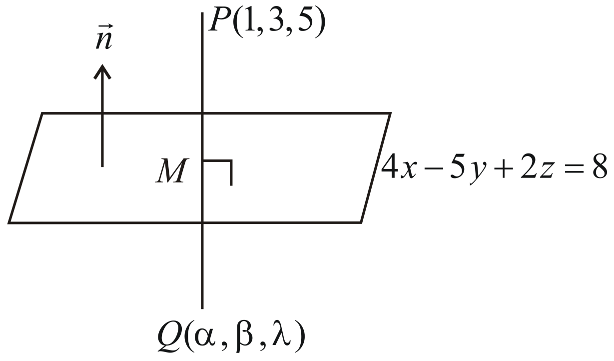
$$=\tan^{-1}\left(\frac{\frac{2}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{1+\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{3}}}\right)+\tan^{-1}\left(\frac{\frac{3}{\sqrt{3}}-\frac{2}{\sqrt{3}}}{1+\frac{3}{\sqrt{3}}\times\frac{2}{\sqrt{3}}}\right)+\ldots..+\ldots.+\tan^{-1}\left(\frac{\frac{n+1}{\sqrt{3}}-\frac{n}{\sqrt{3}}}{1+\frac{n+1}{\sqrt{3}}\times\frac{n}{\sqrt{3}}}\right)+\ldots..$$

$$=\left(\tan\frac{2}{\sqrt{3}}-\tan^{-1}\frac{1}{\sqrt{3}}\right)+\left(\tan^{-1}\frac{3}{\sqrt{3}}-\tan^{-1}\frac{2}{\sqrt{3}}\right)+\left(\tan^{-1}\frac{4}{\sqrt{3}}-\tan^{-1}\frac{3}{\sqrt{3}}\right)+\ldots..+\left(\tan^{-1}\frac{n+1}{\sqrt{3}}-\tan^{-1}\frac{n}{\sqrt{3}}\right)$$

$$\Rightarrow\ S_n=\tan^{-1}\frac{n+1}{\sqrt{3}}-\tan^{-1}\frac{1}{\sqrt{3}}$$

$$S_\infty=\frac{\pi}{2}-\frac{\pi}{6}=\frac{\pi}{3}$$

Q66



Point Q is image of point P w.r.to plane, M is mid point of P and Q, lies in plane

$M\left(\frac{1+\alpha}{2}, \frac{3+\beta}{2}, \frac{5+\gamma}{2}\right)$

$4x-5y+2z=8$

$4\left(\frac{1+\alpha}{2}\right)-5\left(\frac{3+\beta}{2}\right)+2\left(\frac{5+\gamma}{2}\right)=8\ \ldots(1)$

Also PQ perpendicualr to the plane

$\Rightarrow \overrightarrow{PQ} \| \vec{n}$

$\frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=k$ (let)

$\left.\begin{array}{l}\alpha=1+4k \\ \beta=3-5k \\ \gamma=5+2k\end{array}\right\} \ldots(2)$

use (2) in (1)

$2(1+4k)-5\left(\frac{6-5k}{2}\right)+(10+2k)=8$

$k=\frac{2}{5}$

from (2) $\alpha=\frac{13}{5}, \beta=1, \gamma=\frac{29}{5}$

$5(\alpha+\beta+\gamma)=13+5+29=47$

Q67

Let, $M=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$\therefore$ Sum of the diagonal elements, $\operatorname{Tr}\left(M^T M\right)=a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2+i^2=5$, where entries are $\{0,1,2\}$

Only two cases are possible.

(I) Five entries are 1 and other fourare 0

$\therefore \quad {}^9C_5 \times 1$

(II) One entry is 2, one entry is 1 and others are 0.

$\therefore \quad {}^9C_2 \times 2$

Total cases$=126+72=198$.

Q68

The given statement is $p \rightarrow(q \wedge r)$

Where,

p : The weather is fine.

q : My friends will come.

r : We will go for a picnic.

The contrapositive is $(\backsim(q \wedge r)) \rightarrow \backsim p$

Now, using De Morgan's Law, we have

$(\backsim q \vee \backsim r) \rightarrow \backsim p$

If my friends do not come or we do not go for picnic then weather will not be fine.

Q69

Let the probability of getting any number other than 5 is P, then the probability of getting 5 is 5P.

$\because \mathrm{P}+\mathrm{P}+\mathrm{P}+\mathrm{P}+\mathrm{P}+5\mathrm{P}=1 \Rightarrow \mathrm{P}=\frac{1}{10}$

Expected income per throw $=\frac{5}{10}\times 5+\left(\frac{1}{10}\times 8\right)5$

$=\frac{65}{10}=6.5$

Q70

Given Expression is, $\left(x+{ }^{2n+1}C_0\right)\left(x+{ }^{2n+1}C_1\right)\left(x+{ }^{2n+1}C_2\right) \ldots . .\left(x+{ }^{2n+1}C_n\right)$

If P is coefficient of x^n then,

$P={ }^{2n+1}C_0+{ }^{2n+1}C_1+{ }^{2n+1}C_2+\ldots . .+{ }^{2n+1}C_n$ -------(1)

$\Rightarrow P={ }^{2n+1}C_{2n+1}+{ }^{2n+1}C_{2n}+{ }^{2n+1}C_{2n-1}+\ldots . .+{ }^{2n+1}C_{n+1}$ ------(2)$(\because { }^nC_r={ }^n C_{n-r})$

adding (1) and (2)

$2P=\left({ }^{2n+1}C_0+{ }^{2n+1} C_1+\ldots \ldots .+{ }^{2n+1} C_{2n+1}\right)$

$2P=2^{2n+1}$

$\therefore \quad P=2^{2n}$

Q71

(i) aRa, then GCD of a and a is a.

$\therefore$ R is not reflexive.

(ii) $aRb \Rightarrow bRa$

If GCD of a and b is 2, then GCD of b and a is 2.

$\therefore$ R is symmetric.

(iii) $aRa,\ bRc \Rightarrow cRa$

If GCD of a and b is 2 and GCD of b and c is 2, then it is need not to be GCD of c and a is 2.

$\therefore$ R is not transitive.

Q72

$1^2 + 2^2 - 3^2 + 4^2 + 5^2 - 6^2 + \ldots + 30$ terms

$= 1^2 + 2^2 + 3^2 + 4^2 + \ldots + 30^2 - 2\left[3^2 + 6^2 + 9^2 + \ldots + 30^2\right]$

$= 1^2 + 2^2 + 3^2 + 4^2 + \ldots + 30^2 - 2 \times 3^2\left[1^2 + 2^2 + 3^2 + \ldots + 10^2\right]$

$= \frac{30\times31\times61}{6} - \frac{18\times10\times11\times21}{6}\ \left(\because \sum_{r=1}^{n} r^2 = \frac{n\,(n+1)\,(2n+1)}{6}\right)$

$= 5 \times 31 \times 61 - 3 \times 10 \times 11 \times 21$

$= 9455 - 6930$

$= 2525$

Q73

Let $z = x + iy,\ \bar{z} = x - iy$

$(2iy)^2 = 4\left(x^2 + y^2\right) - 12$

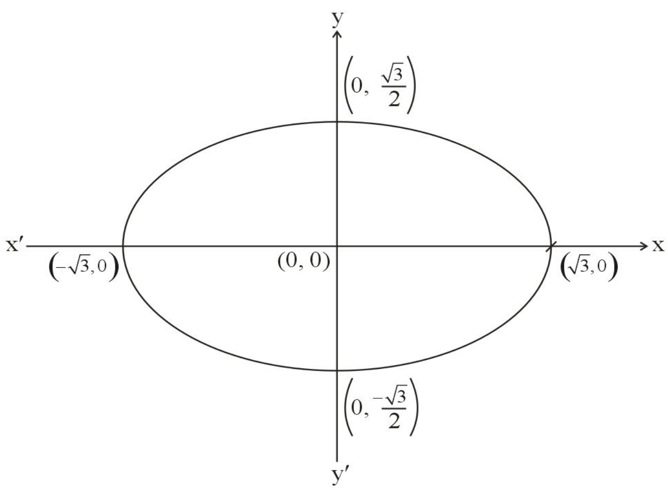
$-4y^2 = 4\left(x^2 + y^2\right) - 12$

$x^2 + 2y^2 = 3$

$\frac{x^2}{3} + \frac{y^2}{\frac{3}{2}} = 1$

It represents an ellipse

So,



$|z|_{max} = \sqrt{3}$

Q74

Equation of chord of contact AB is

$5x - 5y = 1$

$\Rightarrow x-y=1$

Therefore,

$x^2+(x-1)^2=5$

$\Rightarrow 2x^2-2x-4=0$

$\Rightarrow x=2,\ -1$

$\Rightarrow y=1,\ -2$

Hence, $A(2,1)$ and $B(-1,-2)$.

And

$P\equiv\left(\sqrt{5}\cos\theta,\sqrt{5}\sin\theta\right)$

Let the locus of orthocenter be (h,k). Now, we know that the centroid divides the line joining orthocenter and circumcenter in 2 : 1. Therefore,

$h=2-1+\sqrt{5}\cos\theta=1+\sqrt{5}\cos\theta$

$k=1+(-2)+\sqrt{5}\sin\theta=-1+\sqrt{5}\sin\theta$

$\Rightarrow (h-1)^2+(k+1)^2=5$

Hence, the required locus is

$\left(x-1\right)^2+\left(y+1\right)^2=5$

Q75

$n=2^\alpha\cdot 3^\beta$

Given number of divisors$=12$

$\Rightarrow(\alpha+1)(\beta+1)=12$

For $2n = 2^{\alpha+1} \cdot 3^{\beta}$

Given number of divisors$= 15$

$\Rightarrow(\alpha+2)(\beta+1)= 15$

Dividing both$= \frac{\alpha+1}{\alpha+2} = \frac{4}{5}$

$\Rightarrow 5\alpha + 5 = 4\alpha + 8$

$\Rightarrow \alpha = 3$

So, $\beta = 2$.

Now for $3n = 2^{\alpha} \cdot 3^{\beta+1}$

Number of divisors$=(\alpha+1)(\beta+2)$

$= 4 \times 4 = 16$

Q76

Let, $\mathrm{t} = 2^{11\mathrm{x}}$
$\Rightarrow \frac{\left(2^{11\mathrm{x}}\right)^3}{2^2} + 2^{11\mathrm{x}}.2^2 = \left(2^{11\mathrm{x}}\right)^2.2 + 1$
$\Rightarrow \frac{\mathrm{t}^3}{4} + 4\mathrm{t} = 2\mathrm{t}^2 + 1$
$\Rightarrow \mathrm{t}^3 - 8\mathrm{t}^2 + 16\mathrm{t} - 4 = 0$
Cubic in t has roots $\mathrm{t}_1, \mathrm{t}_2, \mathrm{t}_3$
i.e. $\mathrm{t}_1\mathrm{t}_2\mathrm{t}_3 = 4 \Rightarrow 2^{11\mathrm{x}_1}.2^{11\mathrm{x}_2}.2^{11\mathrm{x}_3} = 4$
$\Rightarrow 2^{11\left(\mathrm{x}_1+\mathrm{x}_2+\mathrm{x}_3\right)} = 2^2$
$\Rightarrow 11(\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3)= 2 \Rightarrow \mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 = \frac{2}{11}$

Q77

$f'(x)= 0$ must have two real and unequal roots

$f'(x)= 3x^2 - 6(2\lambda - 1)x + 6\lambda = 0$ must have

$D > 0 \Rightarrow 36(2\lambda - 1)^2 - 72\lambda > 0$

$\Rightarrow 4\lambda^2 + 1 - 4\lambda - 2\lambda > 0 \Rightarrow 4\lambda^2 - 6\lambda + 1 > 0,\ \lambda \in \left(-\infty,\ 3 - \sqrt{5}\right) \cup \left(3 + \sqrt{5},\ \infty\right)$

Q78

Given, $\sin\ 2x\left(\frac{dy}{dx} - \sqrt{\tan\ x}\right) - y = 0$

or, $\frac{dy}{dx} = \frac{y}{\sin\ 2x} + \sqrt{\tan\ x}$

or, $\frac{dy}{dx} - y\ \text{cosec}\ 2x = \sqrt{\tan\ x}$ ....(i)

Now, integrating factor (I.F) $(\text{I. F}) = e^{\int -\text{cosec}\ 2x}$

or, $\text{I. F} = e^{-\frac{1}{2}\log\ |\tan\ x|} = e^{\log\ \left(\sqrt{\tan\ x}\right)^{-1}}$

$= \frac{1}{\sqrt{\tan\ x}} = \sqrt{\cot\ x}$

Now, general solution of eq. (i) is written as

$y(\text{I. F.}) = \int Q(\text{I. F.})dx + c$

$\therefore\ y\sqrt{\cot\ x} = \int \sqrt{\tan\ x} \cdot \sqrt{\cot\ x} dx + c$

$\therefore\ y\sqrt{\cot\ x} = \int 1.\, dx + c$

$\therefore\ y\sqrt{\cot\ x} = x + c$

Q79

We know that

$\frac{1}{x} - 1 < \left[\frac{1}{x}\right] \leq \frac{1}{x}$

Hence,

$\frac{1+2+3...+15}{x} - 15 < \left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right] \leq \frac{1+2+...+15}{x}$

$\Rightarrow \lim_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + ... + \left[\frac{15}{x}\right]\right) = 15\left(\frac{16}{2}\right)$

$= 120$

Q80

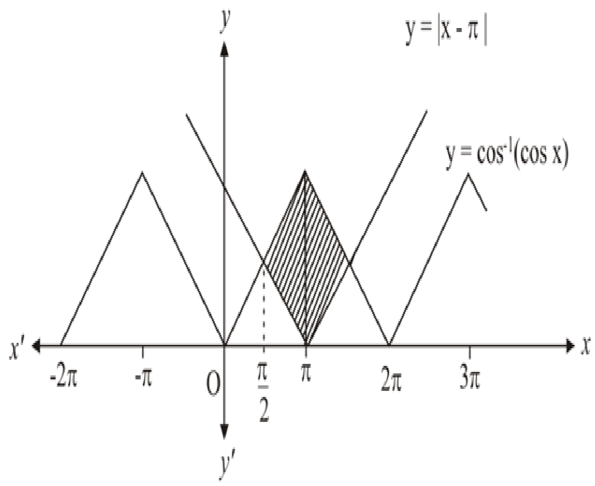
$T: y = mx \pm \sqrt{9\,m^2+1} \therefore 2 = \frac{\sqrt{9\,m^2+1}}{\sqrt{1+m^2}} \Rightarrow m = -\frac{\sqrt{3}}{\sqrt{5}}$
$e = \frac{2\sqrt{2}}{3} \therefore F\left(2\sqrt{2}, 0\right), RS: y = -\frac{\sqrt{3}}{\sqrt{5}}\left(x - 2\sqrt{2}\right) \Rightarrow \sqrt{5}y + \sqrt{3}x = 2\sqrt{6}$Let T be foot of perpendicular from $O(0,0)$ to RS Then,

$OT = \frac{2\sqrt{6}}{2\sqrt{2}} = \sqrt{3} \;\; \therefore \; R'S' = 2\sqrt{4-3}$

Q81

Given, $f(x) = \cos^{-1}(\cos x) = \begin{cases} x, & x \in [0,\ \pi] \\ 2\pi - x, & x \in [\pi,\ 2\pi] \end{cases}$

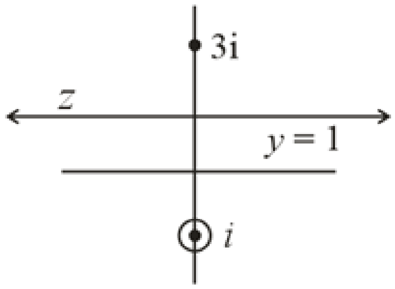
Graph of $y = f(x)$ has been shown in the figure.



$\therefore \; A = 2\int_{\frac{\pi}{2}}^{\pi} \left[x - (\pi - x)\right]dx = 2\left[x^2 - \pi x\right]_{\frac{\pi}{2}}^{\pi} = \frac{\pi^2}{2}$ sq. units

Hence, $n = 2$.

Q82



$\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow |z+i| = |z-3i|$

$\Rightarrow z = x + i,\ x \in R$

$\omega = (x+i)(x-i) - 2(x+i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

For min $(\text{Re}(\omega)), x = 1$

$\Rightarrow \omega = 2 - 2i = 2(1-i) = 2\sqrt{2}e^{-i\frac{\pi}{4}}$

$\omega^n = \left(2\sqrt{2}\right)^n e^{-i\frac{n\pi}{4}}$

For real and minimum value of n,

$n = 4$

Q83

$\frac{b}{c} = \frac{c}{b} = \text{(integer)}$

$b^2 = ac \ \Rightarrow \ \ c = \frac{b^2}{a}$

$\frac{a+b+c}{3}=b+2$

$a+b+c=3b+6 \Rightarrow a-2b+c=6$

$a-2b+\frac{b^2}{a}=6 \ \Rightarrow 1-\frac{2b}{a}+\frac{b^2}{a^2}=\frac{6}{a}$

$\left(\frac{b}{a}-1\right)^2=\frac{6}{a} \ \Rightarrow \ \ a=6 \text{ only}$

Q84

Let 2 distinct number are a, b

$\frac{a+b}{2}=60 \Rightarrow a+b=120$

if $a=0$ then $b=120$

$a=1$ then $b=119$

.

.

.

$a=59$ then $b=61$

favourable case $=60$

Total cases $={}^{181}C_2$

Required probability $=\frac{60}{{}^{181}C_2}=\frac{2}{543}=k$

$\Rightarrow 1086k=4$

Q85

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 0 & 1 & 2 \end{vmatrix} = \hat{i}(-8) - \hat{j}(6) + \hat{k}(3)$$

$$= -8\hat{i} - 6\hat{j} + 3\hat{k}$$

$$\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = \left(\vec{a} \times \vec{b}\right) \cdot \vec{c} \leq \left|\vec{a} \times \vec{b}\right|\left|\vec{c}\right|$$

$\Rightarrow$ maximum value of $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = \left|\vec{a} \times \vec{b}\right|\left|\vec{c}\right|$

$$\Rightarrow k = \sqrt{64 + 36 + 9}.1 = \sqrt{109}$$

$$\Rightarrow k^2 - 50 = 109 - 50 = 59$$

Q86

Let $M = \left(P^{-1}AP - I\right)^2$

$$= \left(P^{-1}AP\right)^2 - 2P^{-1}AP + I$$

$$= P^{-1}A^2P - 2P^{-1}AP + I$$

$$PM = A^2P - 2AP + P$$

$$= \left(A^2 - 2\, A.I + I^2\right)P$$

$$\Rightarrow \text{Det}(PM) = \text{Det}\left((A - I)^2 \times P\right)$$

$$\Rightarrow \text{Det}(PM) == \text{Det}\,(A - I)^2 \times \text{Det}(P)$$

$$\Rightarrow \text{Det}\ M == (\text{Det}(A - I))^2$$

Now $A-I=\begin{bmatrix}1 & 7 & w^2\\ -1 & -w-1 & 1\\ 0 & -w & -w\end{bmatrix}$

$\text{Det}(A-I)=\left(w^2+w+w\right)+7(-w)+w^3=-6w$

$\text{Det}\left((A-I)\right)^2=36w^2$

$\Rightarrow \alpha=36$

Q87

We have,

$n_1=20,\ \bar{x}_1=50,\ \sigma_1^2=1$

$n_1=40,\ \bar{x}_1=50,\ \sigma_2^2=4$

Then,

$\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

$\Rightarrow \bar{x}=\frac{20\times 50+40\times 50}{20+40}=50$

Then,

$d_1=\bar{x}-\bar{x}_1=0$

$d_1=\bar{x}-\bar{x}_2=0$

Then,

$\sigma^2=\frac{n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)}{n_1+n_2}$

$\Rightarrow \sigma^2=\frac{20\times 1+40\times 2^2}{60}=3$

Q88

Let $P(\alpha, \beta, \gamma)$, $Q(0,0, \gamma)$ and $R(\alpha, \beta, -\gamma)$

Now, $\overrightarrow{PQ} \parallel \left(\hat{i}+\hat{j}\right) \Rightarrow -\left(\alpha\hat{i}+\beta\hat{j}\right) \parallel \left(\hat{i}+\hat{j}\right)$

$\Rightarrow \alpha=\beta$

Also, mid point of PQ lies on the plane

$\Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}=3 \Rightarrow \alpha+\beta=6 \Rightarrow \alpha=3$

Now, distance of point P from x-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \beta^2+\gamma^2=25 \Rightarrow \gamma^2=16$

As $\beta=\alpha=3$

As $\gamma=4$

Hence, $PR=2\gamma=8$

Q89

$\sum_{r=0}^{25}(4r+1)^{25}C_r$

$=4\sum_{r=0}^{25} r.^{25}C_r+\sum_{r=0}^{25} {}^{25}C_r$

$=4\sum_{r=1}^{25} r \times \frac{25}{r} {}^{24}C_{r-1}+2^{25}$

$=100\sum_{r=1}^{25} {}^{25}C_{r-1}+2^{25}$

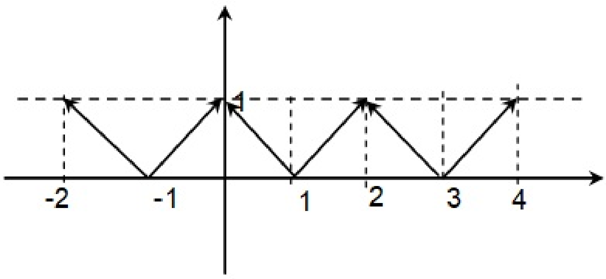
$=100.2^{24}+2^{25}$

$=2^{25}(50+1)$

$= 51\ .2^{25}$

So, $k = 51$

Q90



We know that,

$x - [x] = \{x\}$

$\Rightarrow x - [x+1] = \{x\} - 1$

Hence, $\int_{-2}^{4} f(x)\ dx = 6.\frac{1}{2}(1.1) = 3$