Answer Key

Q1 (3)	**Q2** (4)	**Q3** (3)	**Q4** (4)
Q5 (3)	**Q6** (3)	**Q7** (2)	**Q8** (4)
Q9 (4)	**Q10** (1)	**Q11** (4)	**Q12** (4)
Q13 (4)	**Q14** (2)	**Q15** (4)	**Q16** (4)
Q17 (3)	**Q18** (3)	**Q19** (1)	**Q20** (4)
Q21 (2)	**Q22** (2)	**Q23** (40)	**Q24** (8)
Q25 (74)	**Q26** (7)	**Q27** (2)	**Q28** (11)
Q29 (100)	**Q30** (195)	**Q31** (1)	**Q32** (4)
Q33 (4)	**Q34** (4)	**Q35** (3)	**Q36** (1)
Q37 (1)	**Q38** (2)	**Q39** (1)	**Q40** (4)
Q41 (3)	**Q42** (4)	**Q43** (2)	**Q44** (3)
Q45 (1)	**Q46** (1)	**Q47** (2)	**Q48** (3)
Q49 (3)	**Q50** (4)	**Q51** (4)	**Q52** (25)
Q53 (57)	**Q54** (27)	**Q55** (6)	**Q56** (3)
Q57 (4)	**Q58** (7)	**Q59** (24)	**Q60** (8)
Q61 (2)	**Q62** (2)	**Q63** (3)	**Q64** (2)
Q65 (3)	**Q66** (4)	**Q67** (1)	**Q68** (4)

Q69 (2)	**Q70** (2)	**Q71** (3)	**Q72** (2)
Q73 (3)	**Q74** (2)	**Q75** (2)	**Q76** (2)
Q77 (1)	**Q78** (3)	**Q79** (2)	**Q80** (4)
Q81 (8)	**Q82** (1)	**Q83** (7)	**Q84** (9)
Q85 (15)	**Q86** (7)	**Q87** (0)	**Q88** (227)
Q89 (36)	**Q90** (4)		

Q1

Efficiency (η) of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$, where T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 500$ K.

$\therefore \quad 0.5 = 1 - \frac{500}{T_1} \Rightarrow T_1 = 1000 \ \text{K}$

Now, $\quad \eta' = 0.6 = 1 - \frac{T_2'}{1000} \left(T_2' \text{ is the new sink temperature}\right)$

$T_2' = 400$ K

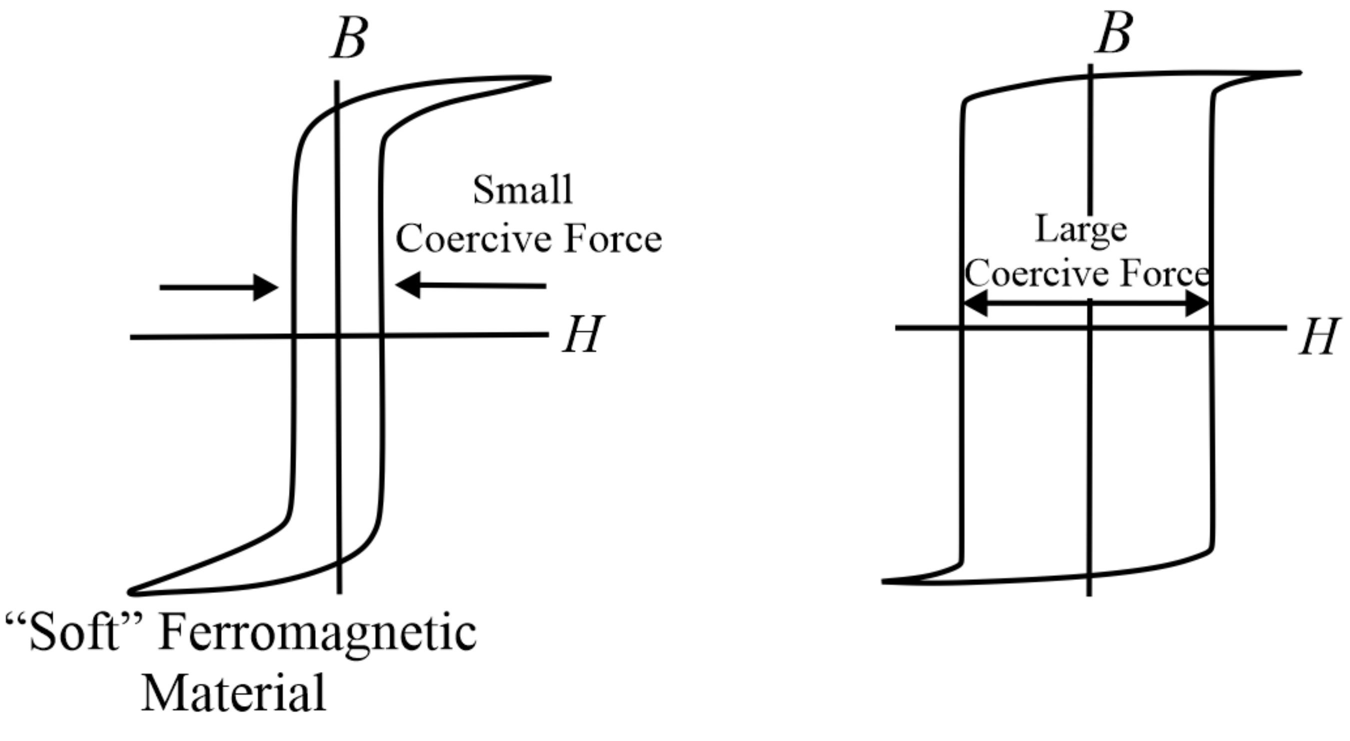
Q2

$\text{Band width} = 2\text{f}_\text{m}$

$\omega_\text{m} = 1.57 \times 10^8 = 2\pi\text{f}_\text{m}$

$\text{BW} = 2\text{f}_\text{m} = \frac{10^8}{2}\ \text{Hz} = 50\,\text{MHz}$

Q3



From the hysteresis curve of soft iron we know Soft iron has high retentivity and low coercive force therefore the loop (i) is for soft iron and the loop (ii) is for steel.

Q4

The relation between Y, η and B is $\frac{9}{Y}=\frac{1}{B}+\frac{3}{\eta}$

Where Y is Young's modulus, B is Bulk modulus and η is modulus of rigidity.

Q5

From conservation of angular momentum.

$\mathrm{mv_0R_0}=\mathrm{mv}^{'}\left(\frac{\mathrm{R_0}}{2}\right)$

$\Rightarrow \mathrm{v}^{'}=2\mathrm{v_0}$

Hence, final KE $=\frac{1}{2}\mathrm{mv}^{'2}=\frac{1}{2}\mathrm{m}(2\mathrm{v_0})^2$

$=2\mathrm{mv_0^2}$

Q6

From Work Energy Theorem,

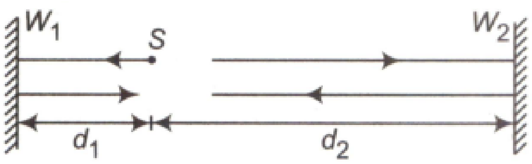
Change in Kinetic energy (KE)= Work done

$KE_f-KE_i=$ area under the graph of F vs x

$KE_f-0=5$

$KE_f=5$ J

Q7



$2d_1=340\times t_1$

$\therefore\ d_1=340\ \mathrm{m}\quad (t=2\ \mathrm{s})$

$2d_2=340\times t_2$

$\therefore\ d_2 = 680\ \mathrm{m}(t_2 = t_1 + 2 = 4\ \mathrm{s})$

$\therefore$ Distance between walls $= d_1 + d_2 = 1020$ m.

Next echo will be heard at 6 s not at 8 s. Because sound wave reflected from W_2 will be reflected by W_1 in next 2 s.

Q8

Here, distance between parallel plates $d = 4$ mm $= 0.004$ m, $K = 3$, thickness $t = 3$ mm $= 0.003$ m and d_1 =?

$\therefore C = \frac{\varepsilon_0 A}{d}$ and $C_1 = \frac{\varepsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)}$

since $C_1 = \frac{2}{3}C$ (given)

$$\therefore \quad \frac{\varepsilon_0 A}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3}\frac{\varepsilon_0 A}{d}$$

$$\frac{1}{d_1 - t\left(1 - \frac{1}{K}\right)} = \frac{2}{3d}$$

$$\frac{1}{d_1 - 0.003\left(1 - \frac{1}{3}\right)} = \frac{2}{3 \times 0.004}$$

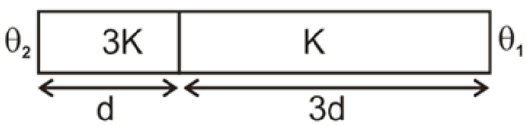
$$\frac{1}{d_1 - 0.003 \times \frac{2}{3}} = \frac{1}{0.006}$$

$$\frac{1}{d_1 - 0.002} = \frac{1}{0.006}$$

$$d_1 - 0.002 = 0.006$$

$$d_1 = 0.006 + 0.002 = 0.008\ \mathrm{m} = 8\ \mathrm{mm}.$$

Q9



Let the temperature of the junction $\mathrm{T}\ ^\circ\mathrm{C}$.

Rate of heat flow in Rod 1 = rate of heat flow in Rod 2

$\frac{3\,\mathrm{kA}}{\mathrm{d}}(\theta_2 - \mathrm{T}) = \frac{\mathrm{kA}}{3\mathrm{d}}(\mathrm{T} - \theta_1)$

$\Rightarrow 9(\theta_2 - \mathrm{T}) = (\mathrm{T} - \theta_1)$

$\Rightarrow 10\mathrm{T} = 9\theta_2 + \theta_1$

$\Rightarrow \mathrm{T} = \frac{9\theta_2+\theta_1}{10} = \frac{\theta_1}{10} + \frac{9\theta_2}{10}$

Q10

Acceleration of body$= g\sin\theta - \mu g\cos\theta$

$= 9.8[\sin 45^\circ - 0.5\cos 45^\circ] = \frac{4.9}{\sqrt{2}}\ \mathrm{m\ sec}^{-2}$

Q11

Since, the particle starts from rest, this means, initial velocity, $u = 0$

Also, it moves with uniform acceleration along positive X -axis. This means, its acceleration (a) is constant.

$\therefore$ Given, $a - t$ graph in (A) is correct. As we know, for velocity-time graph, slope $=$ acceleration.

Since, the given $v - t$ graph in (B) represents that its slope is constant and non-zero.

Also, the displacement of such a particle w.r.t. time is given by

$x = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$

$\Rightarrow\ x \propto t^2$

So, x versus t graph would be a parabola with starting from origin.

This is correctly represented in displacement time graph given in (D).

Q12

${}_4Li^7 + {}_1H^1 \rightarrow {}_2\left({}_2He^4\right)$

BE of products$= ((5.6\ \mathrm{MeV})\times 7) + 0$

$= 39.2\ \mathrm{MeV}$

$E_i = -39.2\ \mathrm{MeV}$

BE of reactant$=(7.06)\times 4\times 2$

$=56.48$ MeV

$E_f=-56.48$ MeV

As nuclear energy decreases, some energy will be released.

$Q_{\text{release}}=E_i-E_f=(-39.2)-(-56.48)=17.28$ MeV

Q13

Here, the dimension of $\frac{a}{V^2}$ will be equal to pressure so $\frac{a}{(L^3)^2}=ML^{-1}T^{-2}$ [Principle of homogeneity]

$\therefore\quad [a]=\left[ML^5T^{-2}\right]$

Aliter:

According to gas equation, for one mol of a real gas.

$\left[P+\frac{a}{V^2}\right](V-b)=RT$

$PV+\frac{a}{V}-Pb+\frac{ab}{V^2}=RT$

As this equation is dimensionally correct, each term on either side will have same dimensions, i.e.,

$\left[\frac{a}{V}\right]=[PV]$

or $\quad [a]=\left[ML^{-1}T^{-2}\right]\left[L^3\right]\left[L^3\right]=\left[ML^5T^{-2}\right]$

and $[P\times b]=(PV)$

or $\quad [b]=[V]=[L]^3$

Note: Actually vander Waals equations for μ mol is

$\left[P+\frac{\mu^2a}{V^2}\right][V-\mu b]=\mu RT$

So that $[\mu b]=[V]\quad i.e.,\quad [b]=\left[L^3\mu^{-1}\right]$ with units $m^3/$ mol

and $\left[\mu^2a\right]=\left[PV^2\right]\quad i.e.,\quad [a]=\left[ML^5T^{-2}\mu^{-2}\right]$ with units $J\ m^3/\text{mol}^2$

Q14

Given that,

$B_0=10^{-4}$ T,

the speed of the electromagnetic wave is

$c=\frac{E_0}{B_0}$

$(c=3\times 10^8\ \text{m s}^{-1})$

$E_0 = cB_0$
$E_0 = 3 \times 10^8 \times 10^{-4}$
$E_0 = 3 \times 10^4 \text{ V m}^{-1}$

Q15

Instantaneous current in AC circuit, at instant t

(Assuming $I = 0$ at $t = 0$).

$I = I_0 \sin(wt) = I_0 \sin(2\pi ft)$

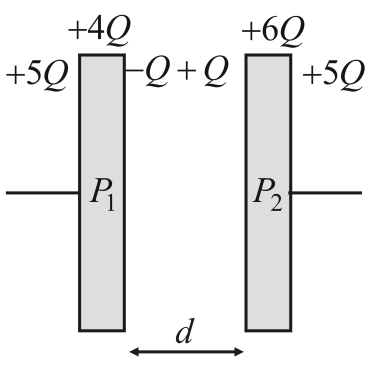
$I_0, \ w \ and \ f$ are peak current, angular frequency and frequency, respectively.

We know, rms current $I_{rms} = \frac{I_0}{\sqrt{2}}$. if rms current is equal to the instantaneous current at time t, then

$I = \frac{I_0}{\sqrt{2}} = I_0 \sin(wt), \ \sin(wt) = \frac{1}{\sqrt{2}}$
$\Rightarrow wt = 2\pi ft = \frac{\pi}{4}$
$\Rightarrow t = \frac{\pi}{4(2\pi f)} = \frac{\pi}{4(2\pi \times 50)} = \frac{1}{400}$
$\Rightarrow t = 2.5 \text{ ms}$

Q16

From the above questions charges on different plates will be,



When we connected both the plates with the wires, the charges will start flowing from one plate to the other and the final charges are as shown in the figure. Energy stored in the capacitor will be zero finally, i.e., $U_f = 0$, but initially, the energy will be stored in the electric field between the charges which can be given by,

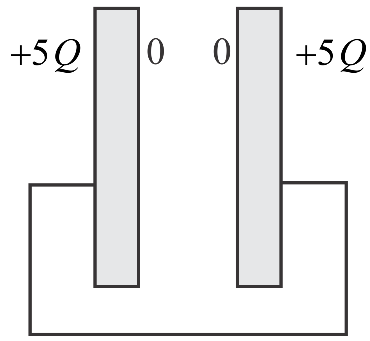
$E=\frac{Q}{2A\varepsilon_0}+\frac{Q}{2A\varepsilon_0}=\frac{Q}{A\varepsilon_0}$

So, the initial energy in the plates will be,

$U_{\mathrm{i}}=\frac{1}{2}\varepsilon_0 E^2\times Ad=\frac{1}{2}\times\varepsilon_0\times\left(\frac{Q}{A\varepsilon_0}\right)^2\times Ad$

The energy lost in heat will be,

$\Delta H=U_{\mathrm{i}}-U_{\mathrm{f}}=\frac{Q^2}{2A\varepsilon_0}d$



Q17

The magnetic field at a point along the axis at distance R from the centre of a circular coil of radius R carrying i is,
$B=\frac{\mu_0 i R^2}{2\left(R^2+R^2\right)^{3/2}}$, by using formula magnetic field at axial point at a distance equal to the radius of coil.
$=\frac{\mu_0 i}{2\sqrt{8}R}=\frac{B}{\sqrt{8}}\left[B_{\text{centre}}=B=\frac{\mu_0 i}{2R}\right]$

Q18

The electrical appliances with metallic body like heater, press, etc, have three pin connections. Two pins are for supply line and the third pin is for earth connection for safety purposes.

Q19

We know that, radius of Bohr orbit is $r_{\mathrm{n}}=\left(\frac{n^2h^2}{4\pi^2 mkZe^2}\right)$

where, m is the reduced mass of the electron.

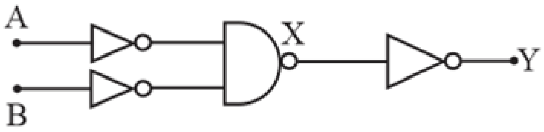
$m=\frac{m_e M}{m_e+M}$

where, m_e is the mass of the electron and M is the mass of the nucleus.

The mass of deuterium is more than that of hydrogen. So, the reduced mass of electron is more for deuterium than that for hydrogen.

Hence, the radius of first Bohr orbit of deuterium is less than that of hydrogen.

Q20



$\mathrm{Y} = \overline{\overline{\overline{\mathrm{A}} \cdot \overline{\mathrm{B}}}} = \overline{\mathrm{A}} \cdot \overline{\mathrm{B}} = \overline{\mathrm{A} + \mathrm{B}}$

Q21

Pressure of a gas is given by $P = \frac{1}{3} \frac{mN}{V} (v_{\text{rms}})^2$.

Where, m =mass of the gas,

N = Number of gas molecules,

V =Volume of the vessel,

v_{rms} = RMS speed of gas molecules.

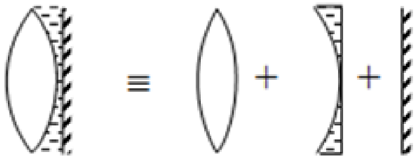
So, $P_0 = \frac{1}{3} \frac{mN}{V} (v_{\text{rms}})^2$.

If the mass of all the molecules are halved and their speed is doubled,

$$P = \frac{1}{3} \frac{(m/2)N}{V} (2v_{\text{rms}})^2$$
$$\Rightarrow \ P = 2 \left[\frac{1}{3} \frac{mN}{V} (v_{\text{rms}})^2\right]$$
$$\Rightarrow \ P = 2P_0$$

Therefore, $n = 2$.

Q22



focal length of convex mirror $(f_1) = Rm$

focal length of concave lens (f_2)=$-3Rm$

effective lens (f_{eq})=$\frac{f_1 f_2}{f_1+f_2} = \frac{-3R}{-2} = \frac{3R}{2} = 1\text{m}$

$\frac{1}{f} = \frac{1}{f_M} - \frac{2}{f_1} = \frac{1}{\infty} - \frac{2}{1}$

$f = -\frac{1}{2}\text{m}$

P =$-2D$

Q23

$i = \frac{(12-8)}{(200+200)}\text{A} = \frac{4}{400} = 10^{-2}\text{ A}$

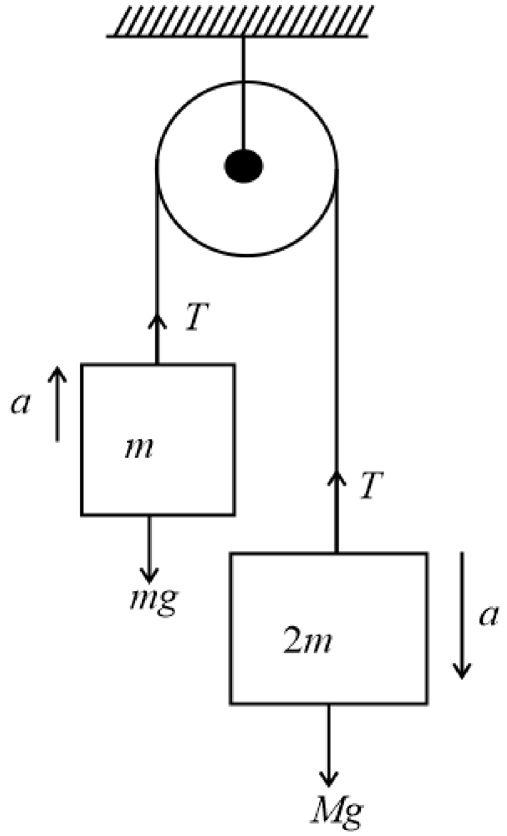
Power loss in each diode=$(4)\left(10^{-2}\right)\text{ W} = 40\text{ mW}$

Q24

Given, $m = 0.36\text{ kg}$ and $M = 2m = 0.72\text{ kg}$

Let a be the acceleration when the system is released.

Forces on m and M are shown in figure.



From the figure, we have

$T - mg = ma \quad \ldots(1)$

and $Mg - T = Ma \quad \ldots(2)$

Adding above two equation,

$g(M - m) = (M + m)a$
$\Rightarrow a = \frac{g(M-m)}{(M+m)}$

$a = \frac{g(0.72-0.36)}{(0.72+0.36)} = \frac{g \times 0.36}{1.08} = \frac{g}{3}$

Putting these values in (1),

$T = \left(m \times \frac{g}{3}\right) + (m \times g)$

$T = \frac{4mg}{3}$

Now, displacement of block is $s = ut + \frac{1}{2}at^2$

Here, initial velocity $u = 0$, then $s = \frac{1}{2}at^2$.

Work done by the string on the block is $W = T \times s = T \times \frac{1}{2}at^2$

$= \frac{4mg}{3} \times \frac{1}{2} \times \frac{g}{3} \times t^2$

$= \frac{4 \times 0.36 \times 10}{3} \times \frac{1}{2} \times \frac{10}{3} \times 1$

$W = 8 \text{ J}$

Q25

$I_{\max} = \frac{V}{R} = \frac{20 \text{ V}}{10 \text{ K}\Omega} = 2 \text{ mA}$

For LR-decay circuit,

$I = I_{\max} e^{-Rt/L}$

$I = 2 \text{ mA } \mathrm{e}^{\frac{-10 \times 10^3 \times 1 \times 10^{-6}}{10 \times 10^{-3}}}$

$I = 2 \text{ mA } \mathrm{e}^{-1}$

$I = 2 \times 0.37 \text{ mA}$

$I = \frac{74}{100} \text{ mA}$

$x = 74$

Q26

$n_1 \overline{X}_1 = n_2 \overline{X}_2$

$n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d}$

$n_1 \lambda_1 = n_1 \lambda_2$

$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{4000}{5600} = \frac{40}{58} = \frac{5}{7}$

$\therefore \quad \frac{n_2}{n_1} = \frac{5}{7} \Rightarrow \frac{X_1}{X_2} = \frac{7}{5}$

$\therefore \quad y = 7\overline{X}_1 = \frac{7D\lambda_1}{d}$

Q27

Given,

Energy of two photons are $E_1 = 4\ \text{eV}$ and $E_2 = 2.5\ \text{eV}$

The ratio of maximum speeds of the photoelectrons emitted in the two cases is $\frac{v_1}{v_2} = 2$

Using Einstein equation of photoelectric effect,

$KE_{max} = \frac{1}{2}mv^2 = E - \phi\ \ldots\left(1\right)$

Where, ϕ is the work function of metal and E is the energy of photon

Now using equation for both the cases we get,

$\frac{1}{2}mv_1^2 = 4 - \phi\ \ \ldots\left(2\right)$

$\frac{1}{2}mv_2^2 = 2.5 - \phi\ \ldots\left(3\right)$

Dividing equation (2) and (3) and substitute given values, we get,

$\frac{{v_1}^2}{{v_2}^2} = \frac{4-\phi}{2.5-\phi} = (2)^2$

$\Rightarrow 3\phi = 6$

$\Rightarrow \phi = 2\ \text{eV}$

Q28

The ball, B, follows horizontal and angular projectile and the ball A follows only horizontal projectile,

the height of the tower is, $h = 490\ \text{m}$, and both the particle follows the same range,

now for particle A,

$R = u\sqrt{\frac{2h}{g}} = 10 \times \sqrt{\frac{2\times490}{9.8}} = 100\ \text{m}$

and for oblique projectile,

$R = 100\ \text{m} = u\cos\theta \times t + u\cos\theta \times \left(490 + \frac{u\sin^2\theta}{2g}\right)$

it means,

$$R = u\cos\theta \times \frac{u\sin\theta}{g} + u\cos\theta \times \sqrt{\frac{\left(490 + \frac{u^2\sin^2\theta}{2g}\right)}{g}}$$

$$\Rightarrow R = 100 = u\cos 30^\circ \times \frac{u\sin 30^\circ}{9.8} + u\cos 30^\circ \times \sqrt{\frac{\left(490 + \frac{u^2\sin^2 30^\circ}{2\times 9.8}\right)}{9.8}}$$

$$\Rightarrow u = 10.9 \text{ m s}^{-1}$$

Q29

The given Wheat stone's bridge is in a balanced condition

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{100}{l_1} = \frac{\left(\frac{100\,x}{100+x}\right)}{l_2}$$

$$\because \quad \frac{l_1}{l_2} = 2 \text{ So, } \frac{100}{\left(\frac{100x}{100+x}\right)} = 2$$

$\Rightarrow$ The unknown resistance is $\mathrm{x} = 100\ \Omega$

Q30

$$\overrightarrow{r} = (4-1)\hat{i} + (3-2)\hat{j} + (-1-1)\hat{k}$$

$$= 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\tau = \overrightarrow{r} \times \overrightarrow{F} = \begin{vmatrix} \hat{i} & j & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

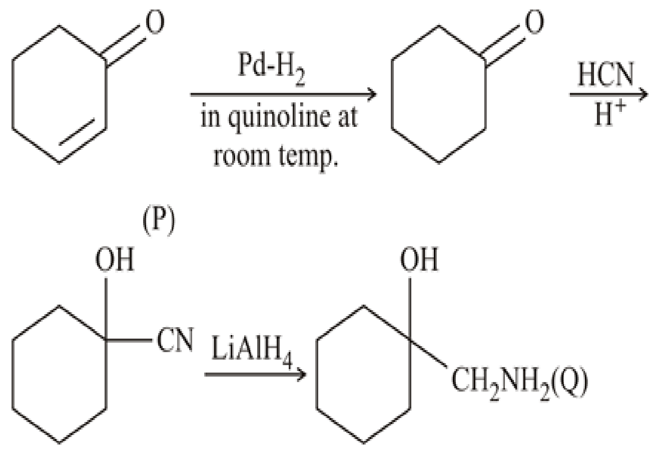
$$= \hat{i}(7) - \hat{j}(11) + \hat{k}(5) = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$= \sqrt{49 + 121 + 25} = \sqrt{195}$$

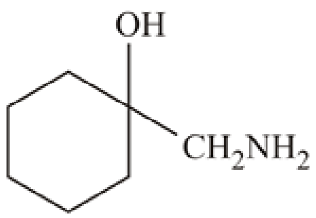
Q31

In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst. Ni catalysts enables the hydrogenation of fats. $CuCl_2$ is used as catalyst in Deacon's process. $\mathrm{ZSM} - 5$ used as catalyst in cracking of hydrocarbons.

Q32



$\therefore$ P is $\mathrm{Pd-H_2}$, in quinoline at room temp.,



Q is

Q33

He $\Rightarrow$ Inert gas

Cl $\Rightarrow$ Electron gain enthalpy is highest

Ca $\Rightarrow$ Most electropositive metal

Li $\Rightarrow$ Strong reducing agent

He has the electronic configuration of $1s^2$. It is completely filled electronic configuration, due to this, it has the highest ionisation enthalpy.

$\mathrm{Cl} \longrightarrow \mathrm{Cl^-}$ gives the highest $\Delta_{eg}H$ as $\mathrm{Cl^-}$ after getting one electron, it will achieve the stable electronic configuration $18[\mathrm{Ar}]$. Due to this, large amount of energy is released during this process.

${}_{20}\mathrm{Ca} \xrightarrow[-2e^-]{} \mathrm{Ca^{2+}}$

Alkaline earth metal and present in the fourth period, the outermost electron is far away from the nucleus. As a result, less energy is required to remove the electron and easily donate the electron, hence, shows maximum metallic nature.

E^{o}_{red} of Li is -3.07 V, a strong reducing agent.

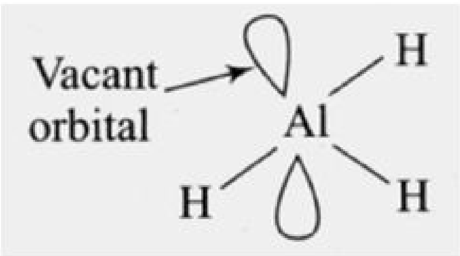
It has very high hydration energy due to smaller in size. The hydration energy is directly proportional to the charge density of the ion.

Q34

On being preferrentially wetted by oil, the ore particles rise to the surface in the form of froth and from there we can separate them.

Q35

AlH_3 is an electron deficient hydride.



It is generally formed by the group 13 element, and have lesser number of electrons than that required for writing its Lewis structure. Being electron deficient, this hydride generally behaves as a Lewis acid, which act as electron acceptor. This is a polynuclear hydride. According to the octet rule, each element tends to completely fill it's outermost shell with $8e^-$ in it. The electronic configuration of aluminium is $2, 8, 3$, and it still needs 5 more electrons to complete its octet. Al has 3 valence electrons to complete its octet, while each hydrogen has one valence electron.

Q36

$\mathrm{pH} = \mathrm{pk_a} + \log\left[\frac{\text{Salt}}{\text{Acid}}\right]$ $(\because [\text{Salt}]=[\text{Anion}])$

$\Rightarrow 6 = 5 + \log\ \frac{\text{Salt}}{\text{Acid}}$

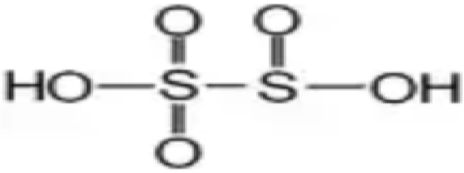
$\Rightarrow 1 = \log\ \frac{\text{Salt}}{\text{Acid}}$

$\Rightarrow \log 10 = \log \frac{\text{Salt}}{\text{Acid}}$

$\frac{\text{Salt}}{\text{acid}} = \frac{10}{1}$

Q37

Disulphurous acid $(H_2\ S_2O_5)$ contains $S - S$ in its structure.



Q38

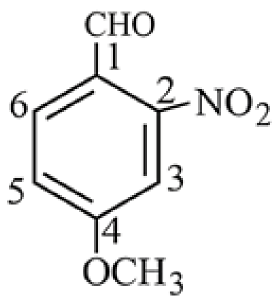
$\left[Fe(H_2O)_6\right]^{3+}$

$Fe^{+2} = 3d^5\left(t_{2g}^{1,1,1}e_g^{1,1}\right)$

So C.F.S.E is $= [-0.4 \times 3 + 0.6 \times 2]\Delta_0 = 0$

Q39

The -CHO functional group is the highest priority functional group. The carbons of the benzene ring are numbered accordingly.



4, methoxy-2 nitrobenzaldehyde

Q40

Non – stoichiometric Schottky defect is a type of point defect. This defect is forms when oppositely charged ion leave their lattice site creating vacancies, thus lowers the density of crystal.

Q41

Mercury poisoning often produces a crippling and fatal disease called Minamata disease.

Q42

$\frac{1}{\lambda}=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ (Rydberg equation)

For H-atom, Z = 1

For visible radiation, $n_1=2$

$\therefore \frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)=\frac{R(n^2-4)}{4n^2}$

Or $\lambda=\frac{4n^2}{R(n^2-4)}=\frac{kn^2}{n^2-4}$

$\therefore k=\frac{4}{R}$

Q43

Statement-I is incorrect

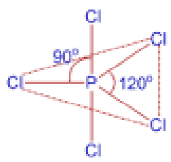
$Be(OH)_2$ dissolve in alkali due to it's amphoteric nature.

Statement-II is correct

Solubility of alkaline earth metal hydroxide in water increases down the group due to rapid decreases in lattice energy as compared to hydration energy.

Q44

On orbital overlap between phosphorus and chlorine, five sp^3 d-p sigma covalent bonds are formed. This statement is true, as PCl_5 carries five sigma bonds and all these sigma bonds are used for hybridisation according to the valence shell electron pair repulsion theory. Structure of PCl_5 is shown below:



And we can see here, that axial bonds and equatorial bonds, both are not of the same length.

Q45

A substance which is used for the purpose of diagnosis, prevention, cure or relief of a disease is called drug. Drugs are the chemicals of low molecular masses, these interact with macromolecular targets and produce a response. When the biological response is therapeutic and useful, these chemicals are called medicines and are used in diagnosis, prevention and treatment of diseases.

Analgesics are the drugs used to reduce or abolish pain without causing impairment of consciousness, mental confusion or paralysis or some other nervous system disturbances.

Q46

We can say that if the sequence of bases in one strand of DNA is I, then, the sequence in the second strand should be II. The base pairs on one of the strands of DNA bind with the base pairs of the other strand very specifically. Adenine always pairs with thymine with two hydrogen bonds and guanine always pairs with cytosine with three hydrogen bonds.

$\mathrm{A : T : G : C : T : T : G : A} \rightarrow \mathrm{I}$

$\mathrm{T : A : C : G : A : A : C : T} \rightarrow \mathrm{II}$

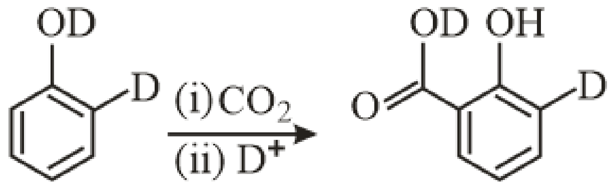
Q47

Explanation :- aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is delocalised to more electronegative element oxygen.

In Aniline lone pair of nitrogen delocalised over benzene ring.

Q48

In the first step of the reaction, $NaOH$ is given which is a strong base, abstracts the hydrogen and phenoxide ion is formed, which is more reactive than phenol. It further undergoes electrophilic substitution reaction with CO_2 to form salicylic acid. But further reaction with D^+ abstracts the hydrogen of acid to form deuterated acid. It follows the mechanism of Kolbe's reaction.

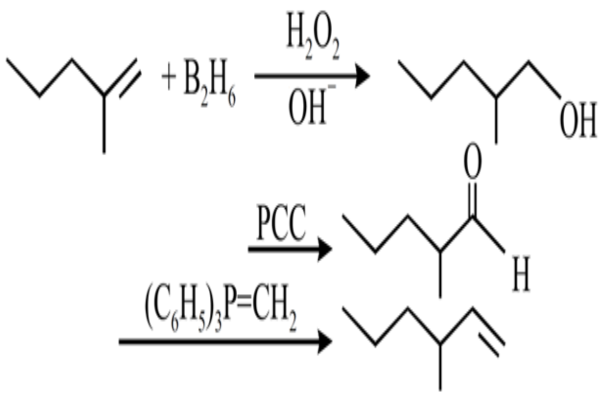


Q49

High density polythene: It is formed when addition polymerisation of ethene takes place in a hydrocarbon solvent in the presence of a catalyst such as triethylaluminium and titanium tetrachloride (Ziegler-Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6-7 atmospheres. High density polythene (HDP) thus produced, consists of linear molecules and has a high density due to close packing. It is also chemically inert and more tougher and harder. It is used for manufacturing buckets, dustbins, bottles, pipes, etc.

Q50

Here the final product is 3-methyl-1-hexene and it is formed as follows



Q51

$\frac{R_{H_2}}{R_{H.C}} = \sqrt{\frac{M_{H.C}}{2}} = 3\sqrt{3}$

$M_{H.C} = 54 \text{ g/mol}$

$M_{H.C} = 12n + (2n-2) = 54$

$\mathrm{n} = 4$

Q52

$A + B$		$\rightleftharpoons 2C$
1	1	1
-x	-x	2x
1-x	1-x	1 + 2x

$\mathrm{K} = \frac{[\mathrm{C}]_{\mathrm{eq}}^{2}}{[\mathrm{A}]_{\mathrm{eq}}[\mathrm{B}]_{\mathrm{eq}}} = \frac{(1+2\mathrm{x})^{2}}{(1-\mathrm{x})(1-\mathrm{x})}$

$100 = \left(\frac{1+2\mathrm{x}}{1-\mathrm{x}}\right)^{2}$

$\left(\frac{1+2\mathrm{x}}{1-\mathrm{x}}\right) = 10$

$\mathrm{x} = \frac{3}{4}$

$[\mathrm{C}]\mathrm{e}_{\mathrm{q.}} = 1 + 2\mathrm{x}$

$= 1 + 2\left(\frac{3}{4}\right)$

$= 2.5\ \mathrm{M}$

$= 25 \times 10^{-1}\ \mathrm{M}$

Q53

$\kappa = \frac{1}{R} \cdot G^{*}$

For same conductivity cell, G^{*} is constant and hence $\kappa \cdot R.$ = constant.

$\therefore 0.14 \times 4.19 = \kappa \times 1.03$

or, $= \frac{0.14 \times 4.19}{1.03}$

$= 0.5695\ \mathrm{Sm}^{-1}$

$= 56.95 \times 10^{-2}\,\mathrm{Sm}^{-1} \approx 57 \times 10^{-2}\,\mathrm{Sm}^{-1}$

Q54

Reaction : $2\,\mathrm{A} + \mathrm{B}_2 \longrightarrow 2\,\mathrm{AB}$

As the reaction is elementary, the rate of reaction is

$\mathrm{r} = \mathrm{K}.\left[\mathrm{A}\right]^2[\,\mathrm{B}_2]$

on reducing the volume by a factor of 3, the concentrations of A and B_2 will become 3 times and hence, the rate becomes $3^2 \times 3 = 27$ times of initial rate.

Q55

$8\mathrm{MnO}_4^- + 3\mathrm{S}_2\mathrm{O}_3^{2-} + \mathrm{H}_2\mathrm{O} \rightarrow 8\mathrm{MnO}_2 + 6\mathrm{SO}_4^{2-} + 2\mathrm{OH}^{\ominus}$

Q56

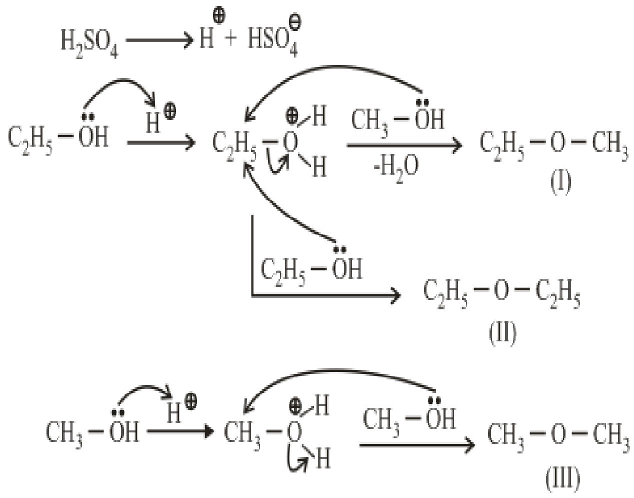
When alcohol is treated with concentrated $\mathrm{H}_2\,\mathrm{SO}_4$, ethers are formed by dehydration of alcohols.

Same alcohol form symmetric ether.

Different alcohol form unsymmetric ether.

The reaction follows SN^2 mechanism.

Mechanism:



Hence, three different alcohol are formed.

Q57

Complexes (i), (iii),(iv) and (v) are optically inactive due to the presence of plane of symmetry.

Q58

Considering 100gof solid, mass of anhydrous slat $= 56.25\ g$ and mass of water $= 100 - 56.25$

$43.75\ g$

Moles of H_2O in 288 g of solid $= \frac{43.75}{100} \times 288$

$= 126\ g$

$\therefore$ Moles of $H_2O = \frac{126}{18} = 7\ mol$

$\therefore n = 7$

Q59

$\mathrm{n_{eq}\,Fe^{2+} = n_{eq}\,Cr_2\,O_7^{2-}}$

or $\left(\frac{15\times \mathrm{M_{Fe^{2+}}}}{1000}\right)\times 1 = \left(\frac{20\times 0.03}{1000}\right)\times 6$

$\therefore \mathrm{M}_{\mathrm{Fe}^{2+}} = 0.24\mathrm{M} = 24 \times 10^{-2}\mathrm{M}$

Q60

$\mathrm{PdV} + \mathrm{VdP} = \mathrm{nRdT}$ ...(i)

along $\mathrm{AB} \rightarrow \mathrm{PT} = \text{constant}$

$\mathrm{PdT} + \mathrm{TdP} = 0$

$\mathrm{dP} = -\frac{\mathrm{P}}{\mathrm{T}}\mathrm{dT}$

Substitute in (i)

$\mathrm{PdV} + \mathrm{V}\left(-\frac{\mathrm{P}}{\mathrm{T}}\mathrm{dT}\right) = \mathrm{nRdT}$

$\mathrm{PdV} = \mathrm{nRdT} + \frac{\mathrm{PV}}{\mathrm{T}}\mathrm{dT} = 2\mathrm{nRdT}$

$\mathrm{W}_{\mathrm{AB}} = -\int_{\mathrm{V}_1}^{\mathrm{V}_2} \mathrm{PdV} = -\mathrm{R}\int_{2\mathrm{T}_1}^{\mathrm{T}_1} 2\mathrm{ndT} = -2\mathrm{nR}[\mathrm{T}_1 - 2\mathrm{T}_1]$

$= -2 \times 2 \times \mathrm{R}[300 - 600]$

$= 1200\ \mathrm{R} = (150\ \mathrm{R})\mathrm{x}$

$\therefore\ \mathrm{x} = 8$

Q61

For non trival solution, $\Delta = 0$

$$\begin{vmatrix} {}^nC_3 & {}^nC_4 & 35 \\ {}^nC_4 & 35 & {}^nC_3 \\ 35 & {}^nC_3 & {}^nC_4 \end{vmatrix} = 0$$

$\Rightarrow {}^nC_3 + {}^nC_4 + 35 = 0$ (not possible)

or ${}^nC_3 = {}^nC_4 = 35 \Rightarrow n = 7$

Q62

Given, $\mathrm{CV}_1 = 60, \mathrm{CV}_2 = 75, \sigma_1 = 18$ and $\sigma_2 = 15$

Let $\overline{x}_1$ and $\overline{x}_2$ be the means of 1^{st} and 2^{nd} distribution respectively.

Then, $\mathrm{CV}_1 = \frac{\sigma_1}{\overline{\mathrm{x}}_1} \times 100 \Rightarrow \overline{x}_1 = \frac{18 \times 100}{60} = 30$

$\mathrm{CV}_2 = \frac{\sigma_2}{\overline{x}_2} \times 100 \Rightarrow \overline{x}_2 = \frac{15 \times 100}{75} = 20$

Hence, $\overline{x}_1 = 30$ and $\overline{x}_2 = 20$

Q63

$$I=\int \frac{3(\tan x-1)\sec^2 x}{(\tan x+1)\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx=3\int \frac{(t-1)}{(t+1)\sqrt{t^3+t^2+t}}dt$$

$$=3\int \frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}+2\right)\sqrt{t+\frac{1}{t}+1}}dt \quad \text{Let } t+\frac{1}{t}+1=z^2 \Rightarrow \left(1-\frac{1}{t^2}\right)dt=2zdz$$

$$=6\int \frac{dz}{(z^2+1)}=6\tan^{-1}\sqrt{1+\frac{1}{t}+t}+C$$

Q64

$\overrightarrow{\mathbf{a}}=\overrightarrow{\mathbf{b}}\times\overrightarrow{\mathbf{c}}+2\overrightarrow{\mathbf{b}}$

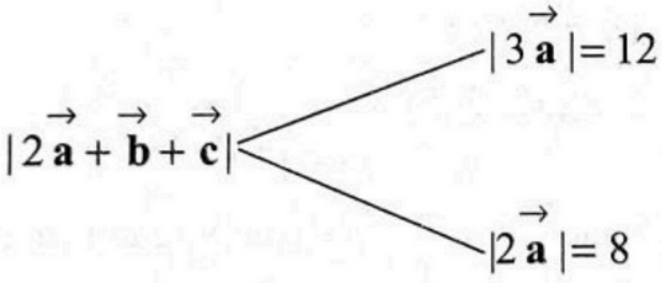
Taking dot product with $\overrightarrow{\mathbf{b}}$

$\vec{a}\cdot\overrightarrow{\mathbf{b}}=2|\overrightarrow{\mathbf{b}}|^2 \Rightarrow |\overrightarrow{\mathbf{a}}||\overrightarrow{\mathbf{b}}|\cos\theta=2|\overrightarrow{\mathbf{b}}|^2$

$\Rightarrow \quad \cos\theta=\dfrac{4}{|\overrightarrow{\mathbf{a}}|} \Rightarrow |\overrightarrow{\mathbf{a}}|=4 \Rightarrow \theta=0^\circ$

$\Rightarrow \quad \overrightarrow{\mathbf{a}}=2\overrightarrow{\mathbf{b}}$

Now, $\quad \overrightarrow{\mathbf{b}}\times\overrightarrow{\mathbf{c}}=0 \Rightarrow \overrightarrow{\mathbf{b}}=\overrightarrow{\mathbf{c}} \quad$ or $\quad \overrightarrow{\mathbf{b}}=-\overrightarrow{\mathbf{c}}$



$\therefore \quad$ Required sum $=12+8=20$

Q65

Given $g(x)=\cos^{-1}[x+1]+\sin^{-1}[x]$

For domain

$-1\leq[x+1]\leq 1$ and $-1\leq[x]\leq 1$

$\Rightarrow \quad -1\leq x+1<2$ and $-1\leq x<2$

$\Rightarrow -2 \leq x < 1$ and $-1 \leq x < 2$

$\Rightarrow x \in[-1,1)$

Now $g(x)=\begin{cases} \frac{\pi}{2}-\frac{\pi}{2} & ; \quad -1 \leq x < 0 \\ 0+0 & ; \quad 0 \leq x < 1 \end{cases}$

$g(x)= 0;\ x \in[-1,1)$

Now $f(x)+g(x)= 4$

$\Rightarrow \sin^{-1}(\sin x)+0 = 4,$

$\Rightarrow \sin^{-1}(\sin x)= 4$, which is not possible. Hence no solution.

Q66

$\det.(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}(\mathrm{adj}A))))= |A|^{(3-1)^4}$

$= |A|^{16} = 4^8 . 5^{16}$

$\Rightarrow |A|= \pm 10$

$|A|=\begin{vmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}= x+y+z = \pm 10$

$\because x, y, z \in N \Rightarrow x+y+z=-10$ (not possible)

Hence, $x+y+z=10$

The number of such matrices $= {}^9C_2$

$= 36$

Q67

$$\lim_{n\to\infty}\sum_{k=0}^{n}\frac{{}^nC_k}{n^k}\int_0^1 x^{k+2}dx = \int_0^1\left[\lim_{n\to\infty}\sum_{k=0}^{n}{}^nC_k\left(\frac{x}{n}\right)^k x^2\right]dx = \int_0^1\left[\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n x^2\right]dx$$
$$= \int_0^1 e^x x^2 dx = \int_0^1 e^x\left(x^2+2x\right)dx - 2\int_0^1 e^x(x+1)dx + 2\int_0^1 e^x dx$$
$$= e^x\left(x^2-2x+2\right)\Big]_0^1 = e-2$$

Q68

1) $(p \vee q) \wedge (\sim p \wedge q)$

$\equiv [(p \vee q) \wedge \sim p] \wedge q$ (Using associative property)

$\equiv [q \wedge \sim p] \wedge q$

$\equiv \sim p \wedge q$

2) $= (p \wedge q) \wedge (\sim p \wedge q)$

$\equiv (p \wedge \sim p) \wedge q$

$\equiv F \wedge q \equiv F$

3) $(p \vee q) \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim p) \vee q$

$\Rightarrow F \vee q \equiv q$

4) $(p \wedge q) \wedge (\sim p \vee q)$

$\equiv (q \wedge p) \wedge (\sim p \vee q)$

$\equiv q \wedge [p \wedge (\sim p \vee q)]$ (using associative property)

$\equiv q \wedge (p \wedge q)$

$\equiv p \wedge q$

Q69

Case-I : Both equations have both the roots in common.

i.e., $\frac{1}{1}=\frac{2k-6}{2k-2}=\frac{7-3k}{3k-5} \Rightarrow$ no value of k

Case-II : Equation $x^2+(2k-6)x+7-3k=0$ has equal roots and equation $x^2+(2k-2)x+(3k-5)=0$ has equal roots

$\therefore \quad (2k-6)^2-4(7-3k)=0 \Rightarrow 4k^2-12k+8=0 \Rightarrow k^2-3k+2=0 \Rightarrow k=1,2$

$(2k-2)^2-4(3k-5)=0 \Rightarrow 4k^2-20k+24=0 \Rightarrow (k-2)(k-3)=0 \Rightarrow k=2$

$\therefore k=2$

Q70

Consider the numbers $a/2, a/2, b/3, b/3, b/3, c/4, c/4, c/4, c/4$ using A.M. $\geq$ G.M. we get

$\frac{a+b+c}{9} \geq \left(\frac{a^2b^3c^4}{2^{10}3^3}\right)^{1/9}$

$\Rightarrow$ maximum value of $a^2b^3c^4$ is $2^{10}\times 3^3$

Hence $x=10$ and $y=3$

$\therefore \quad \log_{10}(x^y)=\log_{10}\left(10^3\right)=3$

Q71

Equation of tangent at (2,4) on the parabola $y^2=8x$ is

$y(4)=8\left(\frac{x+2}{2}\right) \Rightarrow y=x+2$

Let the equation of the circle touching line $y=x+2$ at (2,4) is

$(x-2)^2+(y-4)^2+\lambda(x-y+2)=0$ which passes through $(0,4)$

$\Rightarrow 4+0+\lambda(0-4+2) \Rightarrow \lambda=2$

$\Rightarrow$ Required circle is $x^2+y^2-2x-10y+24=0$

$\Rightarrow (x-1)^2+(y-5)^2=2$

If x and y are integers, then

$(x-1)^2=1=(y-5)^2$

$\Rightarrow x=0,2$ and $y=4,6$

$\Rightarrow$ 4 integral points lie on the circle

Q72

Given,

$S_n = \left(1^2 - 1 + 1\right)\left(1!\right) + \left(2^2 - 2 + 1\right)\left(2!\right) + \cdots + \left(n^2 - n + 1\right)\left(n!\right)$

Its general term t_r is given as,

$\left(r^2 - r + 1\right) r!$

$t_r = \left[\left(r^2 - 1\right) - \left(r - 2\right)\right]\left(r!\right)$

$t_r = (r-1)(r+1)! - (r-2)(r)!$

$S_n = \sum_{r=1}^{n} t_r$

$S_n = (0 - (-1)) + (3! - 0) + (2(4)! - 3!) + \ldots + (n-1)(n+1)! - (n-2)n!$

$S_{50} = 1 + 49(51)!$.

Q73

We have,

$\arg zw = \pi$

$\Rightarrow \arg z + \arg w = \pi \quad \ldots(1)$

Now,

$\bar{z} + i\overline{w} = 0 \Rightarrow \bar{z} = -i\overline{w}$

$\therefore z = iw$

$\Rightarrow \arg z = \arg i + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \arg w$

$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z$ [from (1)]

$\therefore \arg z = \frac{3\pi}{4}$

Q74

$\because \quad 3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$ ...(1)

Replace x by $\frac{x+59}{x-1}$, we get

$3f\left(\frac{x+59}{x-1}\right) + 2f(x) = 10\left(\frac{x+59}{x-1}\right) + 30$ ...(2)

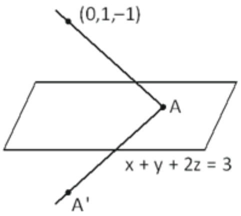
From eqn. (1) $\times 3$ - eqn. (2)$\times 2$, we get

$$5f(x) = 30x - 20\left(\frac{x+59}{x-1}\right) + 30$$

$$5f(7) = 210 - 20 \times 11 + 30 = 20 \quad \Rightarrow \quad f(7) = 4$$

Q75

Any general point on the line is $(2\lambda, 5\lambda + 1, 3\lambda - 1)$



On satisfying this point on the plane, we get,

$2\lambda + 5\lambda + 1 + 6\lambda - 2 = 3$

$13\lambda = 4 \Rightarrow \lambda = \frac{4}{13}$

So, coordinates of the point are $\left(\frac{8}{13}, \frac{33}{13}, \frac{-1}{13}\right)$

This point also lies on the image of the line

Image of point $(0,1,-1)$ also lies on the image of the line

$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z+1}{2} = -2\frac{(-4)}{6}$

$x = \frac{4}{3}, y = \frac{7}{3}, z = \frac{5}{3}$

Point is $\left(\frac{4}{3}, \frac{7}{3}, \frac{5}{3}\right)$

Equation of image of the line is $\frac{x-\frac{4}{3}}{28} = \frac{y-\frac{7}{3}}{-8} = \frac{z-\frac{5}{3}}{68}$

For $xz-$plane, putting $y = 0$, we get,

$\frac{x-\frac{4}{3}}{28} = \frac{7}{24} = \frac{z-\frac{5}{3}}{68}$

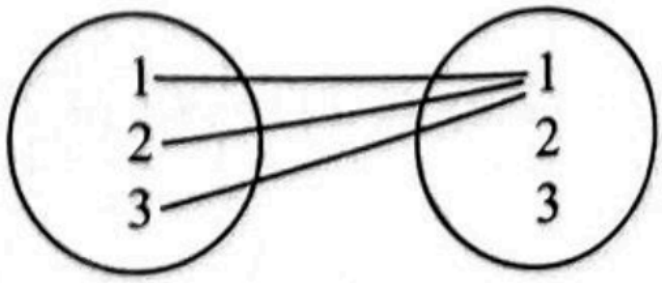
$\Rightarrow z = \frac{129}{6}$

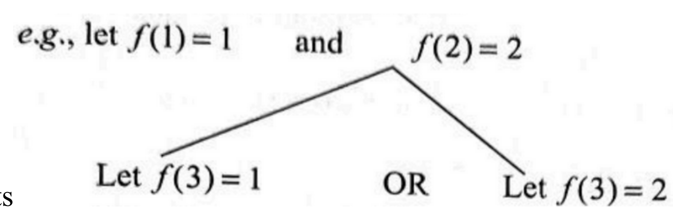
Q76

$f[f(x)] = f(x) \forall x \in S = \{1, 2, 3\}$

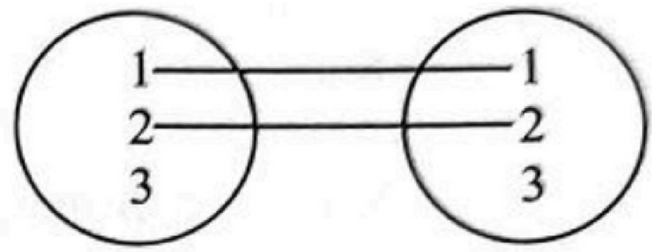
I. When range contains 1 element

${}^3C_1 \times 1 = 3$





II. When range contains 2 elements



(i) If $x = 1$, LHS $= f[f(x)] = f(1)$

RHS $= 1$

In this case also LHS $=$ RHS $\forall x \in S$

(ii) If $x = 2$, LHS $=$ RHS

(iii) If $x = 3$, LHS $=$ RHS

$\therefore \quad {}^3C_2 \times 2 = 6$

Remaining element can be mapped 2 ways.

III. When range contains 3 elements

$f(1) = 1 \quad f(2) = 2 \quad f(3) = 3$

Q77

$f(x)= \frac{ax^3}{3}+\frac{bx^2}{2}+cx$

$f(0)= 0$

$f(1)= \frac{a}{3}+\frac{b}{2}+c$

$= \frac{2a+3b+6c}{6}=0$

From Rolle's theorem,

$\exists$ at least one point $x=\alpha$ in $(0,1)$ such that $f'(\alpha)= 0$

Where $f'(x)= ax^2+bx+c$

$\Rightarrow ax^2+bx+c$ has at least one root in $(0,1)$.

Q78

Here $\frac{z_1+z_2+z_3}{3}=z_0 \Rightarrow z_3=4+3z_0$

Therefore, center coincides with the circumcentre

$\Rightarrow$ Triangle is equilateral $\Rightarrow |z_1-z_2|= 4$

Clearly, z_3 either lie in the second or third quadrant

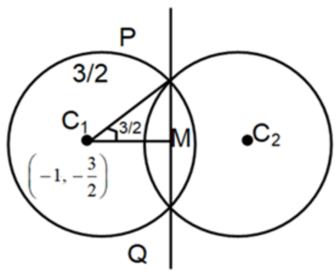
So the centre z_0 also lies in the second or third quadrant.

$\therefore z_0$ can be $=-2+\frac{2}{\sqrt{3}}\mathrm{i}, -2-\frac{2}{\sqrt{3}}\mathrm{i}$

$\Rightarrow \arg(z_0)= \frac{5\pi}{6}$

Q79

Equation of the common chord is $2x+1=0$



$C_1M = \left|\frac{-2+1}{2}\right|= \frac{1}{2}$

$PM=\sqrt{\frac{9}{4}-\frac{1}{4}}=\sqrt{2}$

Length of the common chord $=2\sqrt{2}$

Hence, the perimeter of $\Delta C_1PQ = \frac{3}{2} + \frac{3}{2} + 2\sqrt{2}$

$= 3 + 2\sqrt{2}$ units

Q80

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{\left(\frac{2j}{n} - \frac{1}{n} + 8\right)}{\left(\frac{2j}{n} - \frac{1}{n} + 4\right)}$$

$$\int_0^1 \frac{2x+8}{2x+4} dx = \int_0^1 dx + \int_0^1 \frac{4}{2x+4} dx$$

$$= 1 + 4\frac{1}{2}\left(\ln\left|2x+4\right|\right)\Big|_0^1$$

$$= 1 + 2\ell\text{n}\left(\frac{3}{2}\right)$$

Q81

$f(x) = -x^2$ for $x \le -1$,
$f(x) = 1$ for $-1 < x < 0$,

$f(x) = 2$ for $x = 0$,
$f(x) = 1$ for $0 < x < 1$ and
$f(x) = -x^2$ for $x \ge 1$

$\Rightarrow f(x)$ is even

$$\therefore I = 2\int_0^2 f(x)\,dx = 2\int_0^1 f(x)\,dx + 2\int_1^2 f(x)\,dx = 2\int_0^1 (1)\,dx + 2\int_1^2 -x^2 dx = \frac{-8}{3}$$

$\therefore\ |3I| = |-8| = 8$

Q82

The given equation is

$$e^{\frac{x}{y}}\left(dx - \frac{x}{y}dy\right) + e^{\frac{x}{y}}dy + dx = 0$$

or $e^{\frac{x}{y}} y d\left(\frac{x}{y}\right) + e^{\frac{x}{y}}dy + dx = 0$

$$\Rightarrow\ d\left(e^{\frac{x}{y}}y\right) + dx = 0$$

On integrating, we get,

$e^{\frac{x}{y}} y + x = C$

$\Rightarrow k = 1$

Q83

$\mathrm{P}\left(\frac{\mathrm{B}}{\mathrm{A} \cap \mathrm{C}}\right)=\frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C})}{\mathrm{P}(\mathrm{A} \cap \mathrm{C})}$
$\mathrm{A} \cap \mathrm{C}:\left(-\underline{\mathrm{T}}----\right)(---\underline{\mathrm{T}}--)-(-\underline{\mathrm{T}}-\underline{\mathrm{T}}--)$

$\mathrm{P}\left(\mathrm{A} \cap \mathrm{C}\right)={ }^5 \mathrm{C}_4 \cdot\left(\frac{1}{2}\right)^5 \cdot \frac{1}{2}+{ }^5 \mathrm{C}_4 \cdot\left(\frac{1}{2}\right)^5 \cdot \frac{1}{2}-\left(\frac{1}{2}\right)^6$
$=\frac{9}{2^6}$

$\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}:(\underline{\mathrm{H}}\ \ \underline{\mathrm{T}}\ \ \underline{\mathrm{HHHT}})$

or $(\underline{\mathrm{H}}, \underline{\mathrm{H}}\ \ \underline{\mathrm{HTHT}})$
$\mathrm{P}\left(\mathrm{A} \cap \mathrm{B} \cap \mathrm{C}\right)=\frac{2}{2^6}$

Required probability$=\frac{\frac{2}{2^6}}{\frac{9}{2^6}}=\frac{2}{9}=\frac{m}{n}$

$\therefore n-m=7$

Q84

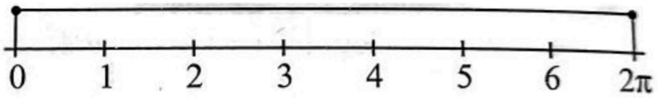
$y=f(x)$ in $[0,10]$

Use L.M.V.T.

$$\begin{aligned}
f'(x) &= \frac{f(10)-f(0)}{10}=\frac{19-f(0)}{10} \\
-4 &\leq \frac{19-f(0)}{10}-5 \leq 4 \\
1 &\leq \frac{19-f(0)}{10} \leq 9 \\
10 &\leq 19-f(0) \leq 90 \\
-71 &\leq f(0) \leq 9 \\
\therefore \quad & f(0)\big|_{\max.}=9
\end{aligned}$$

Q85

$x=I+\frac{1}{4}\left\{\frac{1}{4},\frac{5}{4},\frac{9}{4},\frac{13}{4},\frac{17}{4},\frac{21}{4},\frac{25}{4}\right\}$



$[x]\rightarrow\{1,2,3,4,5,6\}$

$(2x-1)^{1/3}\rightarrow\left\{\frac{1}{2}\right\}$

$\sin x\rightarrow\pi$

$\Rightarrow$ Total 15 points

Q86

$4x+7y+4z+81=0\quad\ldots.(\mathrm{i})$

$5x+3y+10z=25\quad\ldots.(\mathrm{ii})$

Equation of plane passing through line of intersection of planes (i) and (ii) is

$(4x+7y+4z+81)+\lambda(5x+3y+10z-25)=0\quad\ldots.(\mathrm{iii})$

or

$(4+5\lambda)x+(7+3\lambda)y+(4+10\lambda)z+81-25\lambda=0\quad\ldots(\mathrm{iv})$

Normal vector of plane$=4\hat{i}+7\hat{j}+4\hat{k}$

(iv) is perpendicular to plane (i). So,

$\left(4\hat{\mathrm{i}}+7\hat{\mathrm{j}}+4\widehat{\mathrm{k}}\right)\cdot\left((4+5\lambda)\hat{\mathrm{i}}+(7+3\lambda)\hat{\mathrm{j}}+(4+10\lambda)\widehat{\mathrm{k}}\right)=0$

$4(4+5\lambda)+7(7+3\lambda)+4(4+10\lambda)=0$

$\therefore\ \lambda=-1$

From (iii), equation of plane is

$-x+4y-6z+106=0\ \ldots(\mathrm{v})$

Distance of (v) from $(0,0,0)$, i.e.,

$k=\frac{0+0+0+106}{\sqrt{1^2+4^2+6^2}}=\frac{106}{\sqrt{1+16+36}}=\frac{106}{\sqrt{53}}$

Therefore,

$\left[\frac{k}{2}\right]=\left[\frac{2\sqrt{53}}{2}\right]=\left[\sqrt{53}\right]=7$

$\Rightarrow\left[\frac{k}{2}\right]=7$

Q87

$A_1^T=-A_1,\ A_2^T=-A_2,\ \ldots\ldots,\ A_{20}^T=-A_{20}$

$B=\sum_{r=1}^{20}2r(A_r)^{2r+1}$

$B^T=\left(\sum_{r=1}^{20}2r(A_r)^{2r+1}\right)^T$

$=\sum_{r=1}^{20}2r\left(A_r^T\right)^{2r+1}$

$=\sum_{r=1}^{20}2r(-A_r)^{2r+1}$

$=-\sum_{r=1}^{20}2r(A_r)^{2r+1}$

$=-B$

$\Rightarrow B$ is skew-symmetric

Hence, the sum of principal diagonal elements of $B=0$

Q88

We can have $17^{256}=(290-1)^{128}$

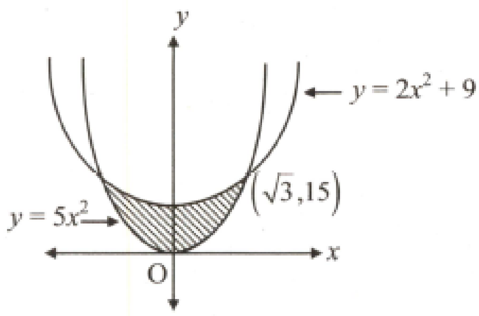
$=1000I+{}^{128}C_2(290)^2-{}^{128}C_1(290)+1$, where I is an integer

$=1000I+128(290)(18415-1)+1$

$=1000m+681$

Hence, $681/3=227$

Q89



Solving $y = 5x^2$ and $y = 2x^2 + 9$ we get $x = \pm\sqrt{3}$

Area, $A = \int_0^{\sqrt{3}} \left((2x^2+9) - 5x^2\right) dx$

$= 2\int_0^{\sqrt{3}} (9 - 3x^2) dx$

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left(9\sqrt{3} - 3\sqrt{3}\right) = 12\sqrt{3}$

$\Rightarrow \frac{A^2}{12} = \frac{\left(12\sqrt{3}\right)^2}{12} = 36$

Q90

$$\begin{aligned} f(x) &= x\left(2x^2 + ax + b\right) \\ D &> 0 \\ a^2 - 8b &> 0 \\ a^2 &> 8b \\ (a, b)|_{\text{min.}} &= (3, 1) \end{aligned}$$

Note that b can not be zero. (think!)

$\therefore \quad a + b|_{\text{min.}} = 4$