

Answer Key

| | | | |
|------------------|------------------|-----------------|------------------|
| Q1 (3) | Q2 (3) | Q3 (4) | Q4 (4) |
| Q5 (2) | Q6 (1) | Q7 (3) | Q8 (1) |
| Q9 (3) | Q10 (4) | Q11 (4) | Q12 (4) |
| Q13 (2) | Q14 (1) | Q15 (4) | Q16 (4) |
| Q17 (3) | Q18 (4) | Q19 (3) | Q20 (4) |
| Q21 (2) | Q22 (2) | Q23 (40) | Q24 (74) |
| Q25 (7) | Q26 (2) | Q27 (11) | Q28 (8) |
| Q29 (100) | Q30 (195) | Q31 (4) | Q32 (1) |
| Q33 (1) | Q34 (1) | Q35 (1) | Q36 (3) |
| Q37 (1) | Q38 (2) | Q39 (3) | Q40 (1) |
| Q41 (3) | Q42 (1) | Q43 (1) | Q44 (1) |
| Q45 (1) | Q46 (2) | Q47 (2) | Q48 (4) |
| Q49 (4) | Q50 (2) | Q51 (12) | Q52 (8) |
| Q53 (2) | Q54 (230) | Q55 (4) | Q56 (900) |
| Q57 (2) | Q58 (0) | Q59 (1) | Q60 (4) |

Q65 (3)

Q66 (2)

Q67 (4)

Q68 (3)

Q69 (4)

Q70 (2)

Q71 (1)

Q72 (3)

Q73 (2)

Q74 (2)

Q75 (3)

Q76 (2)

Q77 (2)

Q78 (4)

Q79 (4)

Q80 (3)

Q81 (7)

Q82 (768)

Q83 (1)

Q84 (10)

Q85 (0)

Q86 (6)

Q87 (36)

Q88 (2)

Q89 (2)

Q90 (1)

Q1

As, $\beta = 2.1 \times 10^9 \text{ N/m}^2$

Also given that $\frac{\Delta V}{V} = 0.1\% = 10^{-4}$

We know that, $\beta = \frac{pV}{\Delta V}$

$$\Rightarrow p = \beta \left(\frac{\Delta V}{V} \right)$$

$$= 2.1 \times 10^9 \times 10^{-4}$$

$$= 2.1 \times 10^5 \text{ N/m}^2$$

Q2

Obviously, the magnetic moment of an atom is due to both orbital motion and spin motion. Thus the assertion statement is a correct statement.

While all the charges do not produce a magnetic field, only moving charges produce a magnetic field. Thus the reason statement is incorrect.

Q3

Here, E stands for emf of the cells and r is the internal resistance of the cell.

Case I:

$$E + E = (r + r + 5) \times 1.0$$

$$\Rightarrow 2E = 2r + 5 \quad \dots(\text{i})$$

Case II:

$$E = \left(\frac{r \times r}{r+r} + 5 \right) \times 0.8$$

$$\Rightarrow E = \left(\frac{r}{2} + 5 \right) 0.8$$

$$\Rightarrow E = 0.4r + 4.0 \quad \dots(\text{ii})$$

Multiplying equation (ii) by two and equating with equation (i), we get,

$$2r + 5 = 0.8r + 8$$

$$\Rightarrow 1.2r = 3$$

$$\Rightarrow r = \frac{3}{1.2} = 2.5 \Omega$$

Q4

(1) Pressure, Young's modulus and stress

Pressure and stress are defined as the force acting per unit area, $[P]=[σ]=\frac{MLT^{-2}}{L^2}=ML^{-1}T^{-2}$.

Young's modulus is defined as $Y = \frac{\text{Linear stress}}{\text{Linear strain}}$. Since linear strain is dimensionless, thus Young's modulus has the same dimension of stress.

(2) E.M.F., potential difference and electric potential are energy difference.

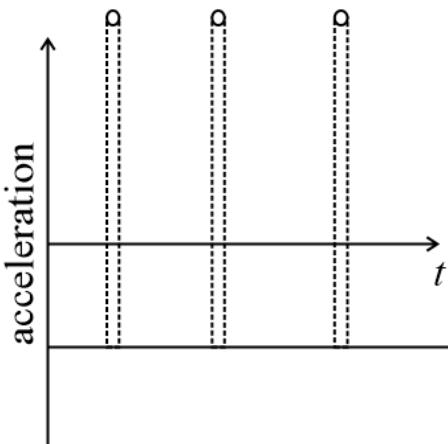
(3) Heat, work done and energy are all forms of energy thus, has the same dimensions ML^2T^{-2} .

(4) Dipole moment, electric flux and electric field have different units.

Q5

Gravitational force always operates on the body in the downward direction during the free fall of the ball, and thus the ball's acceleration lies in the downward vertical direction, which is constant over time. At the collision of the ball with the surface, the acceleration turns positive at a single span of time. The ground imposes an upward impulse and thus the upward acceleration on the ball that makes the ball move upward, and yet it accelerates afterward in the downward direction as fast as the ball loses contact with the ground.

Thus, the correct plot of ball is



Q6

Acceleration of body = $g \sin \theta - \mu g \cos \theta$

$$= 9.8[\sin 45^\circ - 0.5 \cos 45^\circ] = \frac{4.9}{\sqrt{2}} \text{ m sec}^{-2}$$

Q7

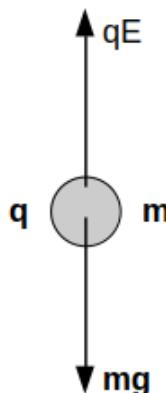
From conservation of angular momentum.

$$mv_0 R_0 = mv' \left(\frac{R_0}{2}\right)$$

$$\Rightarrow v' = 2v_0$$

$$\text{Hence, final KE} = \frac{1}{2}mv'^2 = \frac{1}{2}m(2v_0)^2$$

$$= 2mv_0^2$$

Q8

For equilibrium of charged oil drop,

$$qE = mg$$

$$\therefore q = \frac{mg}{E} = \frac{(9.9 \times 10^{-15}) \times 10}{(3 \times 10^4)} = 3.3 \times 10^{-18} \text{ C.}$$

Q9

From Work Energy Theorem,

Change in Kinetic energy (KE) = Work done

$KE_f - KE_i$ = area under the graph of F vs x

$$KE_f - 0 = 5$$

$$KE_f = 5 \text{ J}$$

Q10

Since there is no external torque acting on the planet revolving around sun, so when a planet revolves around the sun, its angular momentum ($L = I\omega = mr^2\omega$) remains conserved. Here, I is the moment of inertia, ω is the angular velocity, m is the mass of the planet and r is the orbit radius.

Angular momentum of planet when it is at minimum distance $L_1 = mr_{min}^2\omega$

Let the angular velocity of the planet at maximum distance be ω_2 . Then angular momentum at maximum distance $L_2 = mr_{max}^2\omega_2$.

Using $L_1 = L_2$,

$$\begin{aligned}\Rightarrow mr_{min}^2\omega &= mr_{max}^2\omega_2 \\ \Rightarrow \omega_2 &= \left(\frac{r_{min}}{r_{max}}\right)^2 \omega\end{aligned}$$

Q11

Instantaneous current in AC circuit, at instant t

(Assuming $I = 0$ at $t = 0$).

$$I = I_0 \sin(wt) = I_0 \sin(2\pi ft)$$

I_0 , w and f are peak current, angular frequency and frequency, respectively.

We know, rms current $I_{rms} = \frac{I_0}{\sqrt{2}}$. if rms current is equal to the instantaneous current at time t , then

$$I = \frac{I_0}{\sqrt{2}} = I_0 \sin(wt), \quad \sin(wt) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow wt = 2\pi ft = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{\pi}{4(2\pi f)} = \frac{\pi}{4(2\pi \times 50)} = \frac{1}{400}$$

$$\Rightarrow t = 2.5 \text{ ms}$$

Q12

Let the equation of two waves are

$$y_1 = a \sin(\omega t - kx) \quad \dots(i)$$

$$\text{And } y_2 = a \sin(\omega t - kx + \phi) \quad \dots(ii)$$

When they superpose, the resultant wave is

$$y = y_1 + y_2$$

$$= a[\sin(\omega t - kx) + \sin(\omega t - kx + \phi)]$$

$$= a[2 \sin\left(\omega t - kx + \frac{\phi}{2}\right) \cos\left(-\frac{\phi}{2}\right)]$$

$$= 2a \sin\left(\omega t - kx + \frac{\phi}{2}\right) \cos \frac{\phi}{2}$$

$$= \left(2a \cos \frac{\phi}{2}\right) \sin\left(\omega t - kx + \frac{\phi}{2}\right) \dots (iii)$$

Comparing Eq. (iii) with (i) or (ii), we get

$$a = 2a \cos \frac{\phi}{2} \implies \cos \frac{\phi}{2} = \frac{1}{2}$$

$$\implies \cos \frac{\phi}{2} = \cos \frac{\pi}{3}$$

$$\therefore \cos \frac{\phi}{2} = \cos \frac{\pi}{3}$$

$$\therefore \frac{\phi}{2} = \frac{\pi}{3}$$

$$\text{or } \phi = \frac{2\pi}{3}$$

Q13

Given that,

$$B_0 = 10^{-4} \text{ T},$$

the speed of the electromagnetic wave is

$$c = \frac{E_0}{B_0}$$

$$(c = 3 \times 10^8 \text{ m s}^{-1})$$

$$E_0 = cB_0$$

$$E_0 = 3 \times 10^8 \times 10^{-4}$$

$$E_0 = 3 \times 10^4 \text{ V m}^{-1}$$

Q14

We know that, radius of Bohr orbit is $r_n = \left(\frac{n^2 h^2}{4\pi^2 m k Z e^2} \right)$

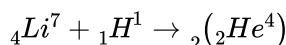
where, m is the reduced mass of the electron.

$$m = \frac{m_e M}{m_e + M}$$

where, m_e is the mass of the electron and M is the mass of the nucleus.

The mass of deuterium is more than that of hydrogen. So, the reduced mass of electron is more for deuterium than that for hydrogen.

Hence, the radius of first Bohr orbit of deuterium is less than that of hydrogen.

Q15

$$BE \text{ of products} = ((5.6 \text{ MeV}) \times 7) + 0$$

$$= 39.2 \text{ MeV}$$

$$E_i = -39.2 \text{ MeV}$$

$$BE \text{ of reactant} = (7.06) \times 4 \times 2$$

$$= 56.48 \text{ MeV}$$

$$E_f = -56.48 \text{ MeV}$$

As nuclear energy decreases, some energy will be released.

$$Q_{\text{release}} = E_i - E_f = (-39.2) - (-56.48) = 17.28 \text{ MeV}$$

Q16

$$\text{Band width} = 2f_m$$

$$\omega_m = 1.57 \times 10^8 = 2\pi f_m$$

$$BW = 2f_m = \frac{10^8}{2} \text{ Hz} = 50 \text{ MHz}$$

Q17

Hints and Solutions

Efficiency (η) of a Carnot engine is given by $\eta = 1 - \frac{T_2}{T_1}$, where T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 500$ K.

$$\therefore 0.5 = 1 - \frac{500}{T_1} \Rightarrow T_1 = 1000 \text{ K}$$

Now, $\eta' = 0.6 = 1 - \frac{T'_2}{1000}$ (T'_2 is the new sink temperature)

$$T'_2 = 400 \text{ K}$$

Q18

Here, distance between parallel plates $d = 4 \text{ mm} = 0.004 \text{ m}$, $K = 3$, thickness $t = 3 \text{ mm} = 0.003 \text{ m}$ and $d_1 = ?$

$$\therefore C = \frac{\epsilon_0 A}{d} \text{ and } C_1 = \frac{\epsilon_0 A}{d_1 - t \left(1 - \frac{1}{K}\right)}$$

since $C_1 = \frac{2}{3}C$ (given)

$$\therefore \frac{\epsilon_0 A}{d_1 - t \left(1 - \frac{1}{K}\right)} = \frac{2}{3} \frac{\epsilon_0 A}{d}$$

$$\frac{1}{d_1 - t \left(1 - \frac{1}{K}\right)} = \frac{2}{3d}$$

$$\frac{1}{d_1 - 0.003 \left(1 - \frac{1}{3}\right)} = \frac{2}{3 \times 0.004}$$

$$\frac{1}{d_1 - 0.003 \times \frac{2}{3}} = \frac{1}{0.006}$$

$$\frac{1}{d_1 - 0.002} = \frac{1}{0.006}$$

$$d_1 - 0.002 = 0.006$$

$$d_1 = 0.006 + 0.002 = 0.008 \text{ m} = 8 \text{ mm.}$$

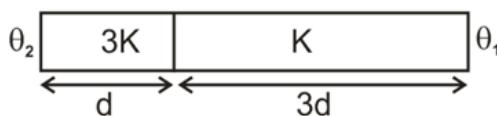
Q19

The magnetic field at a point along the axis at distance R from the centre of a circular coil of radius R carrying i is,

$$B = \frac{\mu_0 i R^2}{2(R^2 + R^2)^{3/2}}, \text{ by using formula magnetic field at axial point at a distance equal to the radius of coil.}$$

$$= \frac{\mu_0 i}{2\sqrt{8}R} = \frac{B}{\sqrt{8}} \left[B_{\text{centre}} = B = \frac{\mu_0 i}{2R} \right]$$

Q20



Let the temperature of the junction T $^{\circ}\text{C}$.

Rate of heat flow in Rod 1 = rate of heat flow in Rod 2

Hints and Solutions

$$\frac{3kA}{d}(\theta_2 - T) = \frac{kA}{3d}(T - \theta_1)$$

$$\Rightarrow 9(\theta_2 - T) = (T - \theta_1)$$

$$\Rightarrow 10T = 9\theta_2 + \theta_1$$

$$\Rightarrow T = \frac{9\theta_2 + \theta_1}{10} = \frac{\theta_1}{10} + \frac{9\theta_2}{10}$$

Q21

Pressure of a gas is given by $P = \frac{1}{3} \frac{mN}{V} (v_{rms})^2$.

Where, m = mass of the gas,

N = Number of gas molecules,

V = Volume of the vessel,

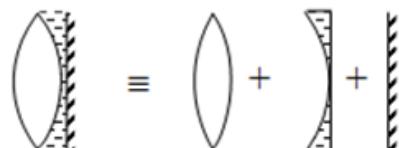
v_{rms} = RMS speed of gas molecules.

$$\text{So, } P_0 = \frac{1}{3} \frac{mN}{V} (v_{rms})^2.$$

If the mass of all the molecules are halved and their speed is doubled,

$$\begin{aligned} P &= \frac{1}{3} \frac{(m/2)N}{V} (2v_{rms})^2 \\ \Rightarrow P &= 2 \left[\frac{1}{3} \frac{mN}{V} (v_{rms})^2 \right] \\ \Rightarrow P &= 2P_0 \end{aligned}$$

Therefore, $n = 2$.

Q22

focal length of convex mirror (f_1) = Rm

focal length of concave lens (f_2) = $-3Rm$

$$\text{effective lens } (f_{eq}) = \frac{f_1 f_2}{f_1 + f_2} = \frac{-3R}{-2} = \frac{3R}{2} = 1\text{m}$$

$$\frac{1}{f} = \frac{1}{f_M} - \frac{2}{f_1} = \frac{1}{\infty} - \frac{2}{1}$$

$$f = -\frac{1}{2}m$$

$$P = -2D$$

Q23

$$i = \frac{(12-8)}{(200+200)} A = \frac{4}{400} = 10^{-2} A$$

$$\text{Power loss in each diode} = (4)(10^{-2}) W = 40 \text{ mW}$$

Q24

$$I_{\max} = \frac{V}{R} = \frac{20 \text{ V}}{10 \text{ K}\Omega} = 2 \text{ mA}$$

For LR -decay circuit,

$$I = I_{\max} e^{-Rt/L}$$

$$I = 2 \text{ mA } e^{\frac{-10 \times 10^3 \times 1 \times 10^{-6}}{10 \times 10^{-3}}}$$

$$I = 2 \text{ mA } e^{-1}$$

$$I = 2 \times 0.37 \text{ mA}$$

$$I = \frac{74}{100} \text{ mA}$$

$$x = 74$$

Q25

$$n_1 \bar{X}_1 = n_2 \bar{X}_2$$

$$n_1 \frac{D\lambda_1}{d} = n_2 \frac{D\lambda_2}{d}$$

$$n_1 \lambda_1 = n_1 \lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} = \frac{4000}{5600} = \frac{40}{58} = \frac{5}{7}$$

$$\therefore \frac{n_2}{n_1} = \frac{5}{7} \Rightarrow \frac{X_1}{X_2} = \frac{7}{5}$$

$$\therefore y = 7 \bar{X}_1 = \frac{7D\lambda_1}{d}$$

Q26

Given,

Energy of two photons are $E_1 = 4 \text{ eV}$ and $E_2 = 2.5 \text{ eV}$

The ratio of maximum speeds of the photoelectrons emitted in the two cases is $\frac{v_1}{v_2} = 2$

Using Einstein equation of photoelectric effect,

$$KE_{max} = \frac{1}{2}mv^2 = E - \phi \dots (1)$$

Where, ϕ is the work function of metal and E is the energy of photon

Now using equation for both the cases we get,

$$\frac{1}{2}mv_1^2 = 4 - \phi \dots (2)$$

$$\frac{1}{2}mv_2^2 = 2.5 - \phi \dots (3)$$

Dividing equation (2) and (3) and substitute given values, we get,

$$\frac{v_1^2}{v_2^2} = \frac{4-\phi}{2.5-\phi} = (2)^2$$

$$\Rightarrow 3\phi = 6$$

$$\Rightarrow \phi = 2 \text{ eV}$$

Q27

The ball, B , follows horizontal and angular projectile and the ball A follows only horizontal projectile,

the height of the tower is, $h = 490 \text{ m}$, and both the particle follows the same range,

now for particle A ,

$$R = u\sqrt{\frac{2h}{g}} = 10 \times \sqrt{\frac{2 \times 490}{9.8}} = 100 \text{ m}$$

and for oblique projectile,

$$R = 100 \text{ m} = u\cos\theta \times t + u\cos\theta \times \left(490 + \frac{u\sin^2\theta}{2g} \right)$$

it means,

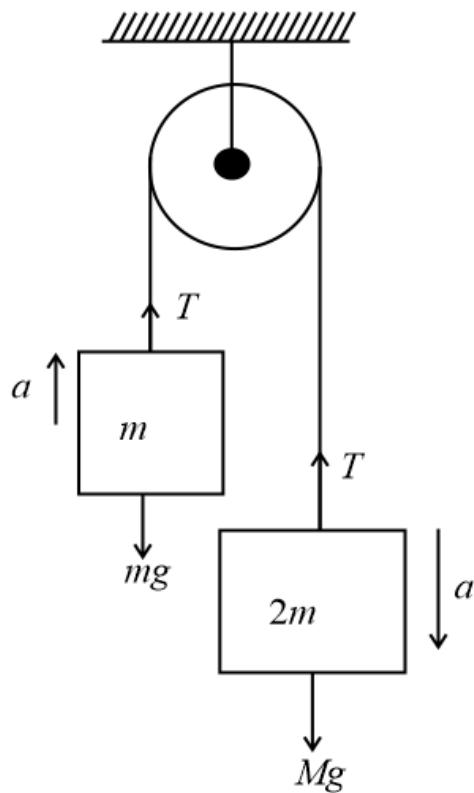
$$\begin{aligned} R &= u\cos\theta \times \frac{u\sin\theta}{g} + u\cos\theta \times \sqrt{\frac{\left(490 + \frac{u^2\sin^2\theta}{2g} \right)}{g}} \\ \Rightarrow R &= 100 = u\cos 30^\circ \times \frac{u\sin 30^\circ}{9.8} + u\cos 30^\circ \times \sqrt{\frac{\left(490 + \frac{u^2\sin^2 30^\circ}{2 \times 9.8} \right)}{9.8}} \\ \Rightarrow u &= 10.9 \text{ m s}^{-1} \end{aligned}$$

Q28

Given, $m = 0.36 \text{ kg}$ and $M = 2m = 0.72 \text{ kg}$

Let a be the acceleration when the system is released.

Forces on m and M are shown in figure.



From the figure, we have

$$T - mg = ma \dots (1)$$

$$\text{and } Mg - T = Ma \dots (2)$$

Adding above two equations,

$$g(M - m) = (M + m)a$$

$$\Rightarrow a = \frac{g(M - m)}{(M + m)}$$

$$a = \frac{g(0.72 - 0.36)}{(0.72 + 0.36)} = \frac{g \times 0.36}{1.08} = \frac{g}{3}$$

Putting these values in (1),

$$T = \left(m \times \frac{g}{3} \right) + (m \times g)$$

$$T = \frac{4mg}{3}$$

Now, displacement of block is $s = ut + \frac{1}{2}at^2$

Here, initial velocity $u = 0$, then $s = \frac{1}{2}at^2$.

Work done by the string on the block is $W = T \times s = T \times \frac{1}{2}at^2$

$$= \frac{4mg}{3} \times \frac{1}{2} \times \frac{g}{3} \times t^2$$

$$= \frac{4 \times 0.36 \times 10}{3} \times \frac{1}{2} \times \frac{10}{3} \times 1$$

$$W = 8 \text{ J}$$

Q29

The given Wheat stone's bridge is in a balanced condition

$$\frac{P}{Q} = \frac{R}{S}$$

$$\Rightarrow \frac{100}{l_1} = \frac{\left(\frac{100x}{100+x}\right)}{l_2}$$

$$\therefore \frac{l_1}{l_2} = 2 \text{ So, } \frac{100}{\left(\frac{100x}{100+x}\right)} = 2$$

\Rightarrow The unknown resistance is $x = 100 \Omega$

Q30

$$\vec{r} = (4 - 1)\hat{i} + (3 - 2)\hat{j} + (-1 - 1)\hat{k}$$

$$= 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\tau = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \hat{i}(7) - \hat{j}(11) + \hat{k}(5) = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

$$= \sqrt{49 + 121 + 25} = \sqrt{195}$$

Q31

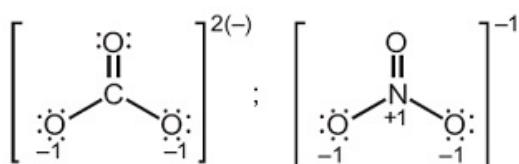
Octet rule: Central atom should have eight electrons in its valence shell in the bonded state.

- Around N in NO_2 has 7 electrons in valence shell
- Around S in SF_4 has 10 electrons in valence shell
- Around Cl in ClO_3^- has 12 electrons in the valence shell

Q32

Latex is a colloidal solution of rubber particles which are negatively charged not positively charged.

Q33



Molecules contain the same number of electrons are known as isoelectronic species and those have same shape known as isostructural. Above two compounds, Both have 32 electrons with trigonal planar structure.

Q34

- (a) Concentration of Ag is performed by leaching with dilute NaCN solution
 (b) Pig iron is formed in blast furnace
 (c) Blister Cu is produced in Bessemer converter
 (d) Froth floatation method is used for sulphide ores.

Note : During extraction of Cu reverberatory furnace is involved.

Q35

Relative abundance:

It is the existence of a naturally occurring element in a percentage of atoms with a particular atomic weight or molar mass.

There are three isotopes of hydrogen:

Protium (H), deuterium (D) and tritium (T).

Order of abundance: H > D > T

Order of density: H < D < T

Order of boiling point: H < D < T

Order of atomic mass: H < D < T

Q36

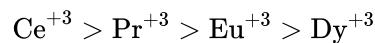
As we know that hydration power decreases on moving down the group hence among alkali metals Li has excessive hydration & hence it has low mobility in aqueous solution.

Q37

$[\text{Ni}(\text{en})_3]^{2+}$ contains chelating ligand whereas $[\text{Ni}(\text{NH}_3)_6]^{2+}$ contains ammonia molecule as ligand which can't form chelate. Due to this former is more stable than the latter.

Q38

Due to increase in effective nuclear charge on moving left to right in periodic table, size decreases.



Q39

The standard cell potential can be calculated by using this formula,

$$E_{\text{cell}}^{\circ} = E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ}$$

Given, $E^{\circ}_{\text{Cr}^{3+}/\text{Cr}} = -0.74 \text{ V}$ and $E^{\circ}_{\text{Cd}^{2+}/\text{Cd}} = -0.40 \text{ V}$

$$E_{\text{cell}}^{\circ} = E_{\text{Cd}^{2+}/\text{Cd}}^{\circ} - E_{\text{Cr}^{3+}/\text{Cr}}^{\circ}$$

$$= -0.40 - (-0.74) = +0.34 \text{ V}$$

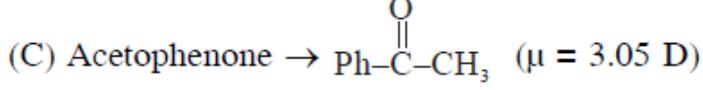
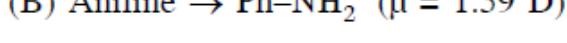
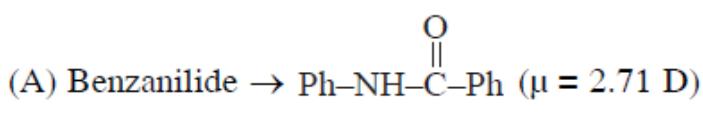
Q40

Eutrophication refers to excessive increases in minerals and nutrients in aquatic system, resulting in excess of algae growth and reduction in dissolved oxygen.

Q41

A dipole moment arises in any system in which there is a separation of charge. They can, therefore, arise in ionic bonds as well as in covalent bonds. Dipole moments occur due to the difference in electronegativity between two chemically bonded atoms.

A bond dipole moment is a measure of the polarity of a chemical bond between two atoms in a molecule. It involves the concept of electric dipole moment, which is a measure of the separation of negative and positive charges in a system.



Dipole moment : C > A > B

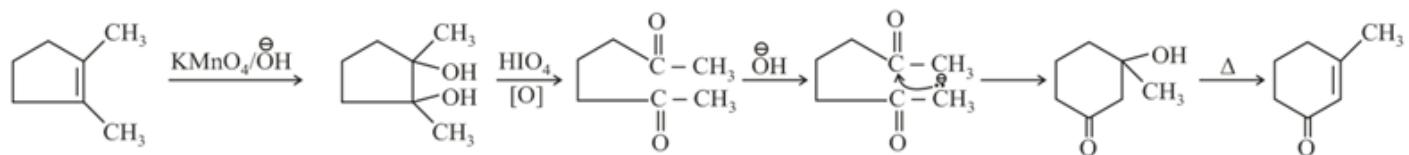
Hence the sequence of obtained compounds is (C), (A) and (B)

Q42

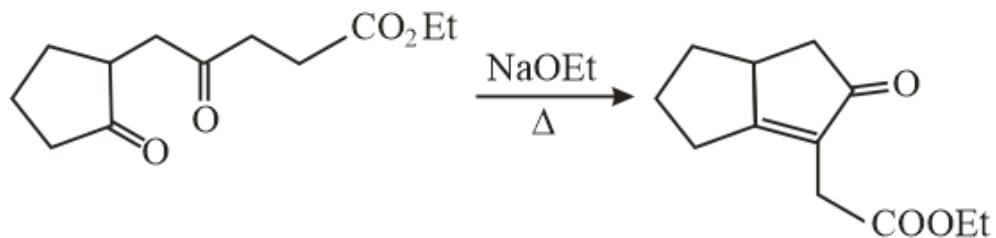
- Baeyer's reagent, named after the German organic chemist Adolf von Baeyer, is used in organic chemistry as a qualitative test for the presence of unsaturation, such as double bonds.
- While phenol is reacted with NaNO₂ and concentrated H₂SO₄, it provides a deep green or blue colour which changes to red on dilution with water. While generated alkaline along with NaOH original green or blue colour is restored. This reaction is termed as Liebermann's nitroso reaction and is employed as a test of phenol.
- Fructose is a reducing sugar, it can reduce both Tollen's reagent and Fehling's solution

Q43

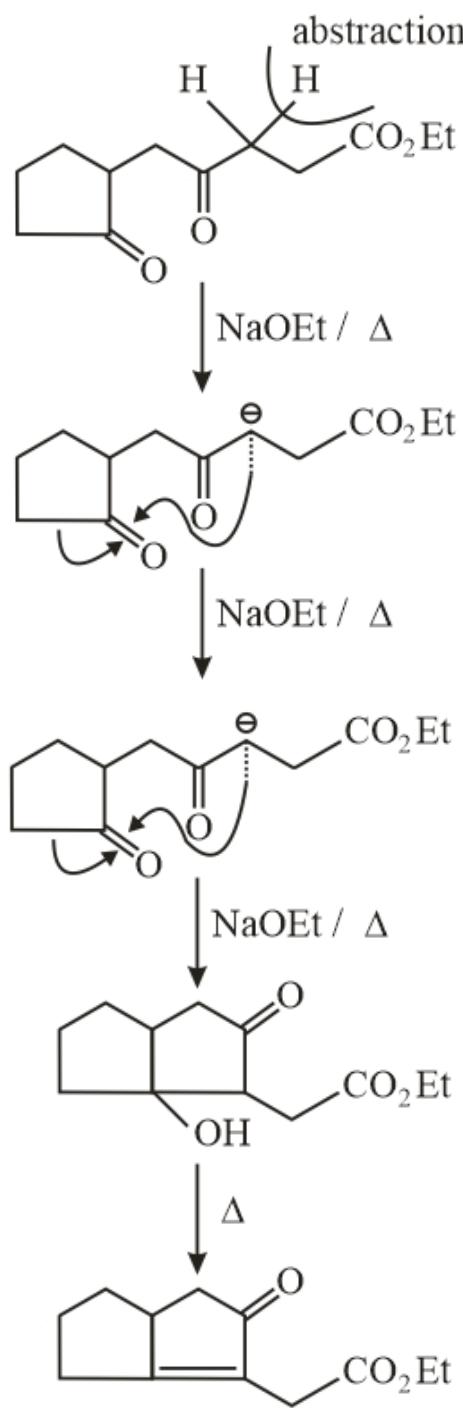
Initially potassium permanganate forms diol which are oxidised by periodic acid and forms diketone. This diketone later shows intramolecular aldol condensation reaction with hydroxide ion and later removes water to form alkene in presence of heat as shown below:



Q44

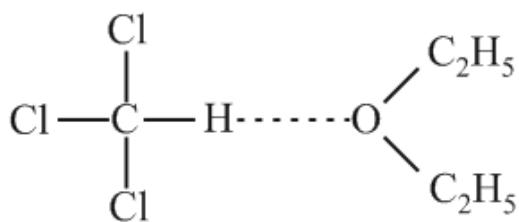


Intramolecular aldol condensation

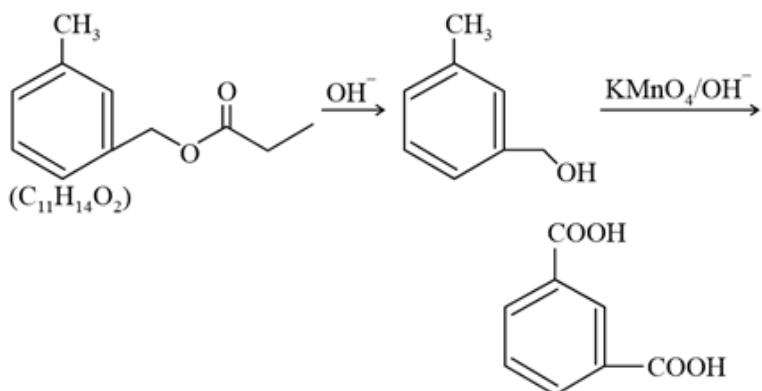


Q45

Azeotropic mixtures with higher boiling point show negative deviation. There occurs H-bonding between chloroform and diethyl ether in the solution, hence the escaping tendency of either component becomes less.

**Q46**

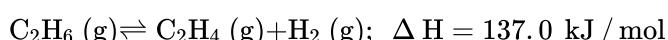
Reaction is Reimer Tiemann .Overall CHO will get substituted in place ortho position and resultant compound is called salicylaldehyde. Difference in molecular mass between phenol and salicylaldehyde is due to carbon and oxygen which is equal to $12+16=28$

Q47

- 1) First reaction is hydrolysis of ester in basic medium. Ester on hydrolysis gives Carboxylic acid and Alcohol.
- 2) Second reaction is side chain Oxidation of Benzene in presence of KMnO_4 to give Carboxylic acid group.

Q48

Alitame is an artificial sweetener that has 1000 times the sweetness value of cane sugar.

Q49

$$\Delta n_g = (1 + 1) - 2 = -1 \text{ (Negative)}$$

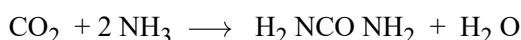
As ΔH is positive, increasing temperature will shift reaction forward (right) direction and as Δn_g is negative, increasing the volume of container or decreasing pressure can shift reaction forward.

Q50

The strength of cellulose is improved by acetylation and then used for making packing material. Rayon (semisynthetic polymer of cellulose) has superior properties than natural cotton.

Q51

Urea manufacture



$$44 \text{ g} \quad 60 \text{ g}$$

For 1000 Kg urea CO_2 required is

$$\frac{44}{60} \times 1000 \times 10^3 = 733.33 \text{ g} \times 10^3$$

$$\text{C}_n\text{H}_{2n+2} \quad \left(n + \frac{2n+2}{4}\right) \text{ O}_2 \rightarrow n \text{ CO}_2 + (n+1) \text{ H}_2\text{O } (\ell)$$

$$n = \frac{733.33}{44} \times 10^3 \text{ moles}$$

Apply POAC to C

$$n \left(\frac{236.1 \times 10^3}{12n+2n+2} \right) = \frac{733.33}{44} \times 10^3 \times 1$$

$$\frac{n}{14n+2} \times 236.1 = 16.67$$

$$236.1 n = 233.38 n + 33.34$$

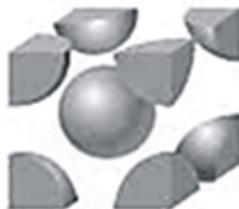
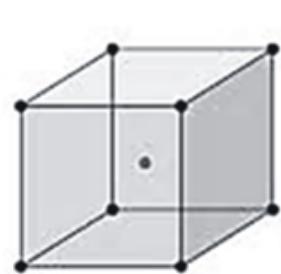
$$2.72 n = 33.34$$

$$n = \frac{33.34}{2.72} \cong 12$$

Q52

Given,

Edge length (a) for BCC unit cell for cube 1 = 4 pm



Body-centered cubic structure

In BCC, number of atoms per unit cell = 2

(atoms occupy corner of the cube as well as the body centre of cube).

Volume of cube = a^3

Volume for cube 1 when edge length (a) = 4 pm

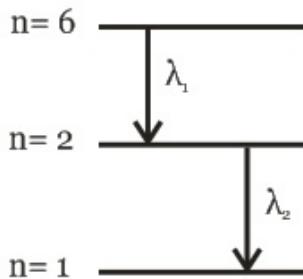
$$\text{Volume} = (a)^3 = (4)^3 = 64 \text{ pm}^3$$

Now atom at the body centre of cube 1 can be viewed to be lying on the corner of another cube 2,

$$\text{Side length of common volume} = \frac{a}{2} = \frac{4}{2} = 2 \text{ pm}$$

$$\text{Now, the volume common to cube 1 and cube 2} = \left(\frac{a}{2}\right)^3 = (2)^3 = 8 \text{ pm}^3$$

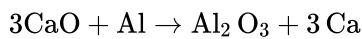
Q53



It first comes to energy level no. 2 before de-excitation.

Q54

Given reaction:



$$\text{Now, } \Delta_r H^\circ = \sum \Delta_f H_{\text{Products}}^\circ - \sum \Delta_f H_{\text{Reactants}}^\circ$$

$$= [1 \times (-1675) + 3 \times 0] - [3 \times (-635) + 2 \times 0]$$

$$= +230 \text{ kJ mol}^{-1}$$

Q55

Hints and Solutions

$$U_{rms} = \sqrt{\frac{3RT}{M}}$$

$$U_{mp} = \sqrt{\frac{2RT}{M}}$$

X represents most probable velocity.

If we divide rms value with mp speed.

We get the ratio as shown below:

$$U_{rms} = 1.2248 U_{mp}$$

$$\frac{4.89}{1.2248} = 4 \text{ ms}^{-1}$$

Q56

For acetic acid and sodium acetate, Normality = Molarity (as n-factor is 1)

$$n_{\text{CH}_3\text{COONa}} = \frac{M}{10} \times V = V/20 \text{ mmoles}$$

$$n_{\text{CH}_3\text{COOH}} = \frac{M}{10} \times 250 = 25 \text{ mmoles}$$

Now, the total volume = $(250 + V)$ mL

$$\text{Concentration of } \text{CH}_3\text{COONa} = \frac{V/20}{(250+V)}$$

$$\text{Concentration of } \text{CH}_3\text{COOH} = \frac{25}{(250+V)}$$

Here in the question, an acidic buffer solution is forming. Now, for the equilibrium, $\text{CH}_3\text{COOH} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}^+$

$$K_a = \frac{[\text{CH}_3\text{COO}^-][\text{H}^+]}{[\text{CH}_3\text{COOH}]}$$

$$1.8 \times 10^{-5} = \frac{\frac{V}{20} \times 10^{-5}}{25}$$

$$\Rightarrow 1.8 = \frac{V}{500}$$

$$\Rightarrow V = 1.8 \times 500 = 900 \text{ mL}$$

Q57

To produce one mole of ClO_4^- , 2 mole of electron are required.

Therefore, number of Faraday for production of 1 mole of NaClO_4 = 2 F

Q58

$A \rightarrow \text{Products}$

From graph \rightarrow

$$\log t_{\frac{1}{2}} \propto \tan 45 (= \log a)$$

$$\log t_{\frac{1}{2}} \propto \log a$$

$$t_{\frac{1}{2}} \propto a \text{ (Initial Condition)}$$

For zero-order reaction

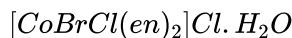
$$\therefore t_{\frac{1}{2}} \propto (a)^{1-n}$$

Q59

In any complex compounds, only the group present outside the coordination sphere as counter ions show their tests. Thus,

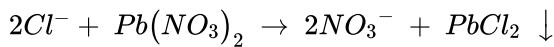
1. Since the compound is showing loss in weight with dehydrating agent, it must have water molecule outside the coordination sphere.
2. Since the complex is giving white precipitate with AgNO_3 , it must have atleast one Cl^- ion outside the coordination sphere.
3. Co has a coordination number six, Thus the total sum of dentencies of all ligands inside the coordination sphere should be six.

The formula that fits into all above criterias is-



Clearly, one mole of compound will give one mole chloride ion on dissolving in water.

Now, two moles of chlroide ions on reaction with one mole of $\text{Pb}(\text{NO}_3)_2$ gives one mole of PbCl_2 .



Thus, with one mole of chloride ion, we will get only 0.5 moles of lead(II) chloride.

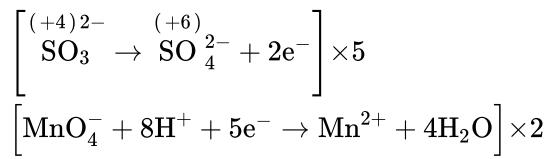
Therefore, $X = 0.5$

i.e. $2X = 1$

Q60

In acidic medium, every molecule of KMnO_4 accepts 5 - electron and SO_2 is oxidized to SO_4^{2-} by releasing two electron per SO_2 molecular.

In acidic medium, KMnO_4 oxidises sulphite into sulphate



5 moles of $\text{SO}_3^{2-} \equiv 2$ moles of KMnO_4

1 mole of $\text{SO}_3^{2-} \equiv \frac{2}{5}$ moles of KMnO_4

Q61

Given, $R = \{(x, y) \in W \times W : \text{the word } x \text{ and } y \text{ have atleast one letter in common}\}$

Let, $W = \{\text{cat, toy, you, ...}\}$

Clearly, R is reflexive and symmetric but not transitive.

[Since, $\text{cat} R \text{toy, toy} R \text{you} \Rightarrow \text{cat} R \text{you}$]

Q62

$f(x) = x^3 + ax^2 + bx + c$ is divisible by $x^2 + 1$ or $(x+i)(x-i)$

So, $f(i) = 0$ and $f(-i) = 0$

$$f(i) = 0 \Rightarrow i^3 + ai^2 + bi + c = 0$$

$$\Rightarrow -i - a + bi + c = 0$$

$$\Rightarrow (c-a) + i(b-1) = 0 \Rightarrow c = a, b = 1$$

$$f(-i) = 0 \Rightarrow (-i)^3 + a(-i)^2 + b(-i) + c = 0$$

$$\Rightarrow i - a - ib + c = 0$$

$$\Rightarrow (c-a) - i(b-1) = 0 \Rightarrow c = a, b = 1$$

$b = 1$ and $c = a$

$\Rightarrow 10$ polynomials are possible

Q63

Group of two persons can be selected in 4C_2 ways. Let P_1P_2 are together and P_3P_4 are together. Let x_1 be the number of seats vacant at the left end, x_2 seats are between two pairs and x_3 seats are vacant at the right end.

$$\underline{x_1} \quad P_1P_2 \quad \underline{x_2} \quad P_3P_4 \quad \underline{x_3}$$

We have, $x_1 + x_2 + x_3 = 6$, where $x_1, x_3 \geq 0$ and $x_2 \geq 1$

$$\Rightarrow x_1 + (x_2 + 1) + x_3 = 6, \text{ where } x_1, x_2, x_3 \geq 0$$

$$\Rightarrow x_1 + x_2 + x_3 = 5$$

$$\text{Number of ways of selecting seats} = {}^{n+r-1}C_{r-1} = {}^{3+5-1}C_{3-1} = {}^7C_2$$

Persons can interchange their seats in $2! \times 2!$ ways

$$\Rightarrow \text{Required ways} = {}^4C_2 \times {}^7C_2 \times 2! \times 2! = {}^4P_2 \times {}^7P_2$$

Q64

Last three digits of 17^{256}

$$= (289)^{128} = (-1 + 290)^{128}$$

$$= {}^{128}C_0(-1)^{128} - {}^{128}C_1 \cdot 290 + {}^{128}C_2 \cdot (290)^2 + \dots$$

$$= 1 - 128 \times 290 + 64 \times 127 \times 29^2 \times 100 + \dots$$

$$= 1 - \dots - 120 + \dots - 800 + \dots$$

$$= 681$$

$$\text{Hence, } \frac{681}{100} = 6.81$$

Q65

$$a + b + c = 25; 2a = 2 + b; c^2 = 18b$$

$$\Rightarrow 2a + 2b + 2c = 50$$

$$\Rightarrow 2 + b + 2b + 2c = 50$$

$$\Rightarrow 3b + 2c = 48 \Rightarrow \frac{c^2}{6} + 2c = 48$$

$$\Rightarrow c^2 + 12c - 48 \times 6 = 0 \Rightarrow c^2 + 12c - 24 \times 12 = 0$$

$$\Rightarrow (c + 24)(c - 12) = 0$$

$$\Rightarrow c = 12, -24 \Rightarrow c = 12 \text{ (between 2 and 18)}$$

$$\Rightarrow b = \frac{c^2}{18} = \frac{144}{18} = 8$$

$$\Rightarrow a = \frac{b+2}{2} = 5$$

$$\Rightarrow a = 5, b = 8 \text{ and } c = 12$$

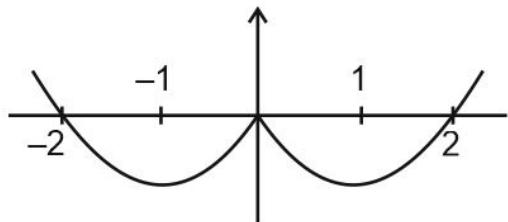
$$\Rightarrow c - a = 7$$

Q66

Hints and Solutions

Here $f(x) = \begin{cases} x^2 + 2x, & x < 0 \\ 0, & x = 0 \\ x^2 - 2x, & x > 0 \end{cases}$

graph of $f(x)$



Case I : $-2 \leq x < -1$, $f(x)$ decreases $\Rightarrow g(x) = x^2 + 2x$

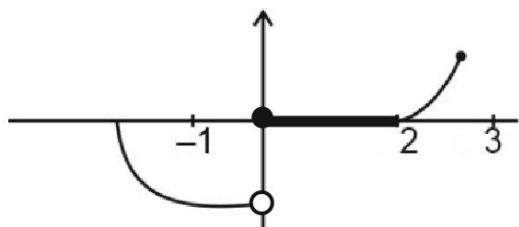
Case II : $-1 \leq x < 0$, $f(x)$ increases $\Rightarrow g(x) = -1$

Case III : $0 \leq x < 1$, $f(x)$ decreases $\Rightarrow g(x) = 0$

Case IV : $1 \leq x < 2$, $f(x)$ increases but $f(x) < 0 \Rightarrow g(x) = f(0) = 0$

Case V : $2 \leq x \leq 3$, $f(x)$ increases and $f(x) \geq 0 \forall x \in [2, 3] \Rightarrow g(x) = f(x) = x^2 - 2x, 2 \leq x \leq 3$

graph of $g(x)$



Therefore $g(x) = \begin{cases} x^2 + 2x, & -2 \leq x < -1 \\ -1, & -1 \leq x < 0 \\ 0, & 0 \leq x < 2 \\ x^2 - 2x, & 2 \leq x \leq 3 \end{cases}$

Clearly $g(x)$ is continuous everywhere except at $x = 0$.

Also $g(x)$ is non differentiable at $x = 0$ and 2 .

Q67

Rewrite the integral as

$$I_2 = \int_0^1 \left(\frac{x}{5+x} \right)^{7/2} \left(\frac{1-x}{5+x} \right)^{9/2} \frac{dx}{(5+x)^2}$$

and do the substitution $\frac{x}{5+x} = t$, so that $\frac{dx}{(5+x)^2} = \frac{dt}{5}$ and the integral becomes $\frac{1}{(5)^{11/2}} \int_0^{1/6} (t)^{7/2} (1-6t)^{9/2} dt$ and now from here do the substitution $6t = u$ and we simply obtain $I_2 = \frac{1}{5^{9/2} \times 6^{7/2}} I_1$ and we conclude $a = 30$.

Q68

Using King

$$I = \int_0^\pi \frac{\sin x(1+\sin x)e^{\sin x - \cos x}}{e^{-\cos x} + 1} dx$$

$$I = \int_0^\pi \frac{\sin x(1+\sin x)e^{\sin x}}{e^{\cos x} + 1} dx \dots(i)$$

$$\text{Add } 2I = \int_0^\pi \frac{\sin x(1+\sin x)e^{\sin x}(e^{\cos x}+1)}{e^{\cos x}+1} dx$$

$$I = \frac{1}{2} \int_0^\pi \sin x(1 + \sin x)e^{\sin x} dx$$

$$I = \frac{1}{2} \int_0^\pi e^{\sin x} (\sin x + 1 - \cos^2 x) dx$$

$$I = \frac{1}{2} \int_0^\pi e^{\sin x} dx + \frac{1}{2} \int_0^\pi e^{\sin x} (\sin x - \cos^2 x) dx$$

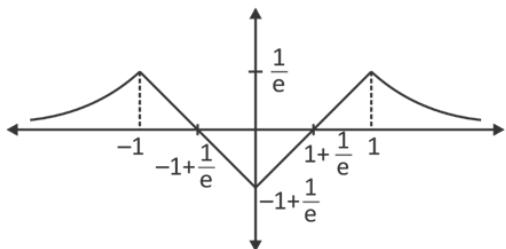
$$I = \frac{1}{2} \int_0^\pi e^{\sin x} dx + \frac{1}{2} [e^{\sin x}(-\cos x)]_0^\pi$$

$$I = \frac{1}{2} \int_0^\pi e^{\sin x} dx + \frac{1}{2}[1 + 1]$$

$$I = 1 + \frac{1}{2} \int_0^\pi e^{\sin x} dx$$

$$\therefore 100 \left(1 + \frac{1}{4}\right) = 125$$

Q69



\therefore Graph of $f(x)$ is symmetric about x -axis, hence,

$$\begin{aligned} \text{Area} &= 2 \int_0^\infty |f(x)| dx \\ &= 2 \left[\frac{1}{2} \times \left(1 - \frac{1}{e}\right) \left(1 - \frac{1}{e}\right) + \frac{1}{2} \times \frac{1}{e} \times \frac{1}{e} + \int_1^\infty e^{-x} dx \right] \\ &= 2 \left[\frac{1}{2} \left[1 - \frac{2}{e} + \frac{2}{e^2} \right] + [-e^{-x}]_1^\infty \right] \\ &= \left[1 - \frac{2}{e} + \frac{2}{e^2} + \frac{2}{e} \right] = 1 + \frac{2}{e^2} \text{ sq. units} \end{aligned}$$

Q70

$$\text{Given equation is } \frac{1}{\sqrt{y}} \frac{dy}{dx} + \frac{x}{1-x^2} \sqrt{y} = x$$

$$\text{Let, } 2\sqrt{y} = \nu$$

$$\Rightarrow \frac{1}{\sqrt{y}} \frac{dy}{dx} = \frac{d\nu}{dx}$$

Thus, we have

$$\frac{d\nu}{dx} + \frac{x}{2(1-x^2)} \nu = x$$

$$\therefore \text{I.F.} = e^{\int \frac{x}{2(1-x^2)} dx}$$

Hints and Solutions

$$\begin{aligned}
 &= e^{\int \frac{-d(1-x^2)}{4(1-x^2)}} \\
 &= e^{-\frac{1}{4} \ln(1-x^2)} \\
 &= (1-x^2)^{-\frac{1}{4}}
 \end{aligned}$$

Thus, the solution is $\nu(1-x^2)^{-\frac{1}{4}} = \int x(1-x^2)^{-\frac{1}{4}} dx$

$$\text{or } \nu \cdot (1-x^2)^{-\frac{1}{4}} = -\frac{2}{3}(1-x^2)^{\frac{3}{4}} + C'$$

$$\text{or } 2\sqrt{y} = -\frac{2}{3}(1-x^2) + C'(1-x^2)^{\frac{1}{4}}$$

$$\Rightarrow \sqrt{y} = -\frac{(1-x^2)}{3} + C(1-x^2)^{\frac{1}{4}}$$

$$\Rightarrow f(x) = 1 - x^2$$

$$\Rightarrow f\left(-\frac{1}{2}\right) = 1 - \frac{1}{4} = \frac{3}{4}$$

Q71

Applying LMVT in $x \in [0, t]$ for $f(x)$, we get

$$f'(c) = \frac{f(t) - f(0)}{t - 0}$$

$$\Rightarrow |f'(c)| = \left| \frac{f(t)}{t} \right| \leq 4$$

$$\Rightarrow |f(t)| \leq 4t$$

$$\text{As } t \in [0, 4]$$

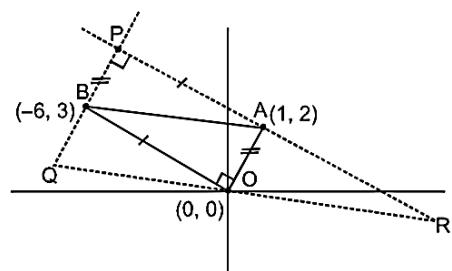
$$\therefore |f(t)| \leq 16$$

Q72

Line joining the midpoints of two sides is \parallel to the third side and half of it.

$$\angle O = 90^\circ \text{ (as } AO \perp OB)$$

Hence P will be the orthocentre

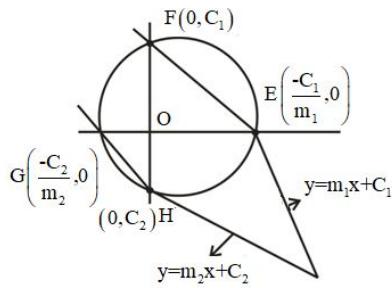


$AOBP$ forms a rectangle.

$\therefore P = A + B - O$ (using the concept that diagonals bisect each other)

$$P(x, y) = \begin{bmatrix} x = -6 + 1 - 0 = -5 \\ y = 2 + 3 - 0 = 5 \end{bmatrix}$$

Q73



Let the equation of tangents are

$$y = m_1 x + C_1, \text{ and}$$

$$y = m_2 x + C_2$$

which cuts the coordinate axes at E, F, G, H as shown in the figure

Now, $OE \times OG = OF \times OH$

$$\Rightarrow \left(\frac{-C_1}{m_1}\right) \times \left(\frac{+C_2}{m_2}\right) = (C_1)(-C_2) \Rightarrow m_1 m_2 = 1$$

Let P be (h, k) and equation of the tangent through P on the hyperbola is

$$y = mx \pm \sqrt{a^2 m^2 - b^2}$$

$$\Rightarrow (k^2 - mh)^2 = (a^2 m^2 - b^2)$$

$$\Rightarrow (h^2 - a^2)m^2 - 2khm + k^2 + b^2 = 0$$

$$\text{whose roots are } m_1 \text{ and } m_2 \Rightarrow m_1 m_2 = \frac{k^2 + b^2}{h^2 - a^2} = 1$$

$$\Rightarrow \text{locus is } x^2 - y^2 = a^2 + b^2$$

Q74

$$\text{Let, } \vec{A} = \sqrt{3} (\vec{a} \times \vec{b})$$

$$\vec{B} = \vec{b} - (\vec{a} \cdot \vec{b}) \vec{a}$$

$$\vec{A} \cdot \vec{B} = 0 \Rightarrow \vec{A} \text{ is perpendicular to } \vec{B}$$

$$|\vec{A}| = \sqrt{3} |\vec{a} \times \vec{b}| = \sqrt{3} |\vec{a}| |\vec{b}| \sin \theta = \sqrt{3} |\vec{b}| \sin \theta$$

$$|\vec{B}|^2 = \left| \vec{b} - (\vec{a} \cdot \vec{b}) \vec{a} \right|^2$$

$$= \left| \vec{b} \right|^2 + \left(\vec{a} \cdot \vec{b} \right)^2 |\vec{a}|^2 - 2 (\vec{a} \cdot \vec{b})^2$$

$$= \left| \vec{b} \right|^2 + \left| \vec{b} \right|^2 \cos^2 \theta - 2 \left| \vec{b} \right|^2 \cos^2 \theta$$

$$= \left| \vec{b} \right|^2 \sin^2 \theta$$

$$\Rightarrow |\vec{B}| = |\vec{b}| \sin \theta$$

$$\frac{|\vec{A}|}{|\vec{B}|} = \frac{\sqrt{3}}{1} \Rightarrow \text{other two angles are } \frac{\pi}{6}, \frac{\pi}{3}$$

Q75

Equation of tangent at (2,4) on the parabola $y^2 = 8x$ is

$$y(4) = 8\left(\frac{x+2}{2}\right) \Rightarrow y = x + 2$$

Let the equation of the circle touching line $y = x + 2$ at (2,4) is

$$(x - 2)^2 + (y - 4)^2 + \lambda(x - y + 2) = 0 \text{ which passes through (0, 4)}$$

$$\Rightarrow 4 + 0 + \lambda(0 - 4 + 2) \Rightarrow \lambda = 2$$

$$\Rightarrow \text{Required circle is } x^2 + y^2 - 2x - 10y + 24 = 0$$

$$\Rightarrow (x - 1)^2 + (y - 5)^2 = 2$$

If x and y are integers, then

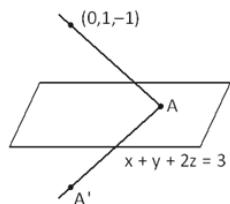
$$(x - 1)^2 = 1 = (y - 5)^2$$

$$\Rightarrow x = 0, 2 \text{ and } y = 4, 6$$

\Rightarrow 4 integral points lie on the circle

Q76

Any general point on the line is $(2\lambda, 5\lambda + 1, 3\lambda - 1)$



On satisfying this point on the plane, we get,

$$2\lambda + 5\lambda + 1 + 6\lambda - 2 = 3$$

$$13\lambda = 4 \Rightarrow \lambda = \frac{4}{13}$$

So, coordinates of the point are $\left(\frac{8}{13}, \frac{33}{13}, \frac{-1}{13}\right)$

This point also lies on the image of the line

Image of point $(0,1,-1)$ also lies on the image of the line

$$\frac{x-0}{1} = \frac{y-1}{1} = \frac{z+1}{2} = -2 \frac{(-4)}{6}$$

Hints and Solutions

$$x = \frac{4}{3}, y = \frac{7}{3}, z = \frac{5}{3}$$

Point is $\left(\frac{4}{3}, \frac{7}{3}, \frac{5}{3}\right)$

$$\text{Equation of image of the line is } \frac{x - \frac{4}{3}}{28} = \frac{y - \frac{7}{3}}{-8} = \frac{z - \frac{5}{3}}{68}$$

For xz -plane, putting $y = 0$, we get,

$$\begin{aligned} \frac{x - \frac{4}{3}}{28} &= \frac{7}{24} = \frac{z - \frac{5}{3}}{68} \\ \Rightarrow z &= \frac{129}{6} \end{aligned}$$

Q77

$$A = \{1, 2, 3, 4, 5, 6, 7\}$$

Case-I : When exactly 4 values follows $f(i) = i$

$${}^7C_4 \times 3! \left(\frac{1}{2!} - \frac{1}{3!} \right) = 70$$

Case-II : When exactly 5 values follows $f(i) = i$

$${}^7C_5 \times 1 = 21$$

Case-III : When all 7 values follows $f(i) = i$ number of function = 1

Total functions = $70 + 21 + 1 = 92$

Q78

$$\frac{\sum_{i=1}^n (x_i+1)^2}{n} = 7 \text{ (given)}$$

$$\Rightarrow \sum x_i^2 + 2 \sum x_i + n = 7n$$

$$\Rightarrow \sum x_i^2 + 2 \sum x_i = 6n \dots (1)$$

$$\text{Also, } \frac{\sum (x_i-1)^2}{n} = 3 \text{ (given)}$$

$$\Rightarrow \sum x_i^2 - 2 \sum x_i + n = 3n$$

$$\Rightarrow \sum x_i^2 - 2 \sum x_i = 2n \dots (2)$$

From (1) and (2)

$$\sum x_i^2 = 4n, \quad \sum x_i = n$$

$$\text{Standard deviation} = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

$$= \sqrt{\frac{4n}{n} - 1} = \sqrt{3}$$

Q79

$$\begin{aligned} \sim(p \wedge (q \rightarrow \sim r)) &= \sim p \vee (\sim(q \rightarrow \sim r)) \\ (\because \sim(p \wedge q) = \sim p \vee \sim q \ \& \ \sim(p \rightarrow q) = p \wedge \sim q) \\ &= \sim p \vee (q \wedge \sim(\sim r)) \\ &= \sim p \vee (q \wedge r) \end{aligned}$$

Q80If, $A + B = 45^\circ$

$$\tan(A + B) = 1$$

$$\therefore \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$$

$$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$$

LHS

$$= [(1 + \tan 1^\circ)(1 + \tan 44^\circ)][(1 + \tan 2^\circ)(1 + \tan 43^\circ)] \dots [(1 + \tan 45^\circ)] \left[\text{for each } (1 + \tan \theta) \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) = 2 \right] \right]$$

$$= 2^{22} (1 + 1)$$

$$= 2^{23}$$

$$= 2^\lambda$$

then, $\lambda = 23$.Hence the sum of digits of λ is $2 + 3$

$$= 5$$

Q81Any general point on the line $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ is $(3\lambda + 1, \lambda + 2, 2\lambda + 3)$ For the point of intersection, this point must satisfy the line $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

$$\Rightarrow \frac{3\lambda - 2}{1} = \frac{\lambda + 1}{2} = \frac{2\lambda + 1}{3} \Rightarrow \lambda = 1$$

So, point of intersection is (4,3,5)

The required equation of plane is

$$2(x - 4) + 3(y - 3) + 1(z - 5) = 0$$

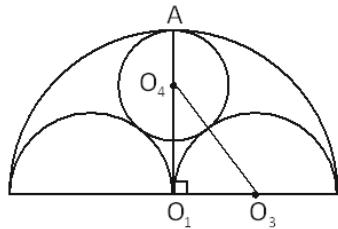
$$\Rightarrow 2x + 3y + z - 22 = 0$$

Hence, the distance of the plane from (1, 1, 3) = $\left| \frac{2+3+3-22}{\sqrt{14}} \right| = \sqrt{14} = k$

Q82

$$\begin{aligned} & [\vec{a}_1 \quad 2\vec{a}_2 \quad 3\vec{a}_3] [\vec{b}_1 + \vec{b}_2 \quad \vec{b}_2 + \vec{b}_3 \quad \vec{b}_3 + \vec{b}_1] \\ &= 6 [\vec{a}_1 \quad \vec{a}_2 \quad \vec{a}_3] 2 [\vec{b}_1 \quad \vec{b}_2 \quad \vec{b}_3] \\ &= 12 \begin{vmatrix} \vec{a}_1 \cdot \vec{b}_1 & \vec{a}_1 \cdot \vec{b}_2 & \vec{a}_1 \cdot \vec{b}_3 \\ \vec{a}_2 \cdot \vec{b}_1 & \vec{a}_2 \cdot \vec{b}_2 & \vec{a}_2 \cdot \vec{b}_3 \\ \vec{a}_3 \cdot \vec{b}_1 & \vec{a}_3 \cdot \vec{b}_2 & \vec{a}_3 \cdot \vec{b}_3 \end{vmatrix} \\ &= 12 \begin{vmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 12 \times 64 = 768 \end{aligned}$$

Q83



Let the point of contact of C_4 & C_1 is A , center of C_4 is O_4 & radius is equal to r

$$\Rightarrow AO_1 = 12 \Rightarrow O_1O_4 = 12 - r$$

Also, $O_4O_3 = r + 6$ and $O_1O_3 = 6$

$$\Rightarrow (r + 6)^2 = (12 - r)^2 + 36$$

$$\Rightarrow 36r = 144 \Rightarrow r = 4 \Rightarrow A = 16\pi$$

$$\Rightarrow \frac{16\pi}{A} = \frac{16\pi}{16\pi} = 1$$

Q84

$$\text{Let, } L = \lim_{x \rightarrow \frac{\pi}{2}} \left(1^{\frac{1}{\cos^2 x}} + 2^{\frac{1}{\cos^2 x}} + \dots + 10^{\frac{1}{\cos^2 x}} \right)^{\cos^2 x}$$

Putting $\frac{1}{\cos^2 x} = y$, we get,

$$L = \lim_{y \rightarrow \infty} (1^y + 2^y + \dots + 10^y)^{\frac{1}{y}}$$

Hints and Solutions

$$= \lim_{y \rightarrow \infty} 10 \left[\left(\frac{1}{10} \right)^y + \left(\frac{2}{10} \right)^y + \dots + \left(\frac{10}{10} \right)^y \right]^{\frac{1}{y}}$$

$$= 10(0 + 0 + \dots + 1)^0 = 10$$

Q85

$$a_k = 2a_{k-1} - a_{k-2}$$

$\Rightarrow a_1, a_2, \dots, a_{11}$ are in AP with let common difference be d

$$\therefore \frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = \frac{11a^2 + 35 \times 11d^2 + 110ad}{11} = 90$$

$$\Rightarrow 225 + 35d^2 + 150d = 90$$

$$35d^2 + 150d + 135 = 0$$

$$\Rightarrow d = -3, -\frac{9}{7}$$

$$\text{Given, } a_2 < \frac{27}{2} \therefore d = -3 \text{ and } d \neq -\frac{9}{7}$$

$$\text{Hence, } a_1 + a_2 + \dots + a_{11} = \frac{11}{2}[2 \times 15 + 10(-3)] = 0$$

Q86

For non-trivial solutions,

$$\begin{vmatrix} a & 1 & b \\ b & 1 & a \\ a & b & ab \end{vmatrix} = 0$$

$$\Rightarrow (a-b)(a-b^2) = 0$$

$$\Rightarrow \text{Either } a = b \text{ or } a = b^2$$

When $a = b$, then ordered pairs are $(0,0), (1,1), (2,2), (3,3), (4,4)$

When $a = b^2$, then $(4, 2)$

Hence, number of ordered pairs are 6

Q87

$$\det(\text{adj}(\text{adj}(\text{adj}(\text{adj}(\text{adj}A)))) = |A|^{(3-1)^4}$$

$$= |A|^{16} = 4^8 \cdot 5^{16}$$

$$\Rightarrow |A| = \pm 10$$

$$|A| = \begin{vmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix} = x + y + z = \pm 10$$

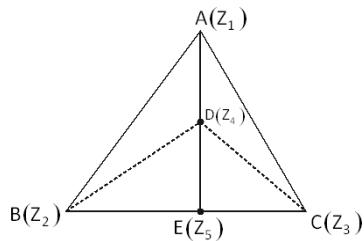
$$\because x, y, z \in N \Rightarrow x + y + z = -10 \text{ (not possible)}$$

$$\text{Hence, } x + y + z = 10$$

The number of such matrices = 9C_2

= 36

Q88



$$\text{Here, } Z_4 = \frac{\left(Z_1 + \frac{Z_2 + Z_3}{2}\right)}{2}$$

Let, midpoint of the line joining B & C is E(Z5), where $Z_5 = \frac{Z_2 + Z_3}{2}$, then

Z_4 is midpoint of Z_1 & Z_5

$$\Rightarrow \frac{\text{Area of } \triangle BDC}{\text{Area of } \triangle BAC} = \frac{1}{2} = 0.5$$

Q89

$$f(xf(y)) = x^p y^4, \text{ put } x = \frac{1}{f(y)}$$

$$\therefore f(1) = \left(\frac{1}{f(y)}\right)^p y^4 = \frac{y^4}{(f(y))^p}$$

$$\text{For } y = 1, \quad f(1) = \frac{1}{(f(1))^p} \Rightarrow f(1) = 1$$

$$\therefore f(y) = y^{4/p} \dots (1)$$

$$\therefore f(xy^{4/p}) = x^p y^4$$

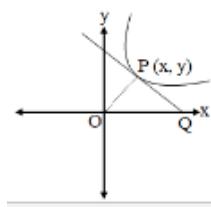
$$\text{Put } y = z^{p/4}$$

$$\therefore f(xz) = x^p z^p \Rightarrow f(x) = x^p \dots (2)$$

$$\frac{4}{p} = p \Rightarrow p = 2$$

Q90

$$(Y-y) = \frac{dy}{dx} (X-x)$$



$$\text{Thus meet x axis at } \left(x - y \frac{dy}{dx}, 0\right)$$

$$\begin{aligned}
 OP=OQ & \sqrt{x^2 + y^2} = x - y \frac{dy}{dx} \\
 -\frac{ydx + xdy}{\sqrt{x^2 + y^2}} &= dy \\
 \frac{ydx - xdy}{y^2 \sqrt{1+x^2/y^2}} &= \frac{-1}{y} dy \\
 \frac{1}{\sqrt{1+\left(\frac{x}{y}\right)^2}} d\left(\frac{x}{y}\right) &= -\frac{1}{y} dy \\
 \log \left\{ \frac{x}{y} + \sqrt{1 + \left(\frac{x}{y}\right)^2} \right\} &= -\log y + \log k \\
 \frac{x}{y} + \sqrt{1 + \frac{x^2}{y^2}} &= \frac{k}{y} \\
 x + \sqrt{x^2 + y^2} &= k
 \end{aligned}$$

passes through (1,0)

$k=2$

$$x^2 + y^2 = (2-x)^2$$

$$y^2 = 4 - 4x$$

$$y^2 = -4(x-1)$$

vertex (1, 0)

tangent at vertex

$$x=1 \Rightarrow a=1$$