Answer Key

Q1 (1)	**Q2** (3)	**Q3** (1)	**Q4** (4)
Q5 (1)	**Q6** (2)	**Q7** (1)	**Q8** (3)
Q9 (1)	**Q10** (1)	**Q11** (1)	**Q12** (1)
Q13 (3)	**Q14** (2)	**Q15** (3)	**Q16** (1)
Q17 (4)	**Q18** (1)	**Q19** (2)	**Q20** (3)
Q21 (2)	**Q22** (3)	**Q23** (3)	**Q24** (2)
Q25 (1)	**Q26** (72)	**Q27** (75)	**Q28** (27)
Q29 (3)	**Q30** (24)	**Q31** (2)	**Q32** (3)
Q33 (2)	**Q34** (2)	**Q35** (3)	**Q36** (1)
Q37 (4)	**Q38** (1)	**Q39** (3)	**Q40** (2)
Q41 (2)	**Q42** (4)	**Q43** (3)	**Q44** (1)
Q45 (2)	**Q46** (4)	**Q47** (4)	**Q48** (1)
Q49 (1)	**Q50** (4)	**Q51** (3)	**Q52** (3)
Q53 (2)	**Q54** (1344)	**Q55** (6)	**Q56** (100)
Q57 (37)	**Q58** (7)	**Q59** (4)	**Q60** (23)
Q61 (3)	**Q62** (3)	**Q63** (4)	**Q64** (2)
Q65 (1)	**Q66** (3)	**Q67** (3)	**Q68** (2)

Q69 (3)	**Q70** (4)	**Q71** (1)	**Q72** (4)
Q73 (1)	**Q74** (1)	**Q75** (3)	**Q76** (3)
Q77 (2)	**Q78** (2)	**Q79** (2)	**Q80** (3)
Q81 (2)	**Q82** (3)	**Q83** (2)	**Q84** (6)
Q85 (4)	**Q86** (3)	**Q87** (8)	**Q88** (9)
Q89 (4)	**Q90** (96)		

Q1

For an isothermal process

$\mathrm{PV} = \text{constant} \therefore \mathrm{PV} = \mathrm{P_I 2V}$

$P_I = \frac{P}{2} \qquad \ldots(\mathrm{i})$

For an adiabatic process

$\mathrm{PV}^\gamma = \text{constant} \therefore \mathrm{PV}^\gamma = \mathrm{P_A}(2\mathrm{V})^\gamma$

or $\quad P_A = \frac{P}{2^\gamma} \qquad \ldots(\mathrm{ii})$

Divide (i) by (ii), we get

$\frac{P_I}{P_A} = \frac{2^\gamma}{2} = 2^{\gamma-1}$

Q2

$\left(\frac{\mathrm{I_T}}{\mathrm{I_C}}\right)^2 = 1 + \frac{\mathrm{m}_a^2}{2}$

$I_T = 8.88\mathrm{A},\ \ I_C = 8\mathrm{A};\ \ \mathrm{m}_a = ?$

$\left(\frac{8.88}{8}\right)^2 = 1 + \frac{m_a^2}{2}$

$1.232 = 1 + \frac{m_a^2}{2}$

$\frac{m_a^2}{2} = 1.232 - 1 = 0.232$

$\mathrm{m_a} = \sqrt{2 \times 0.232} = 0.68$ or 68%

Q3

Material A is paramagnetic and Material B is ferromagnetic.

The susceptibility of material B is larger than A at given magnetic field because ferromagnetic material gets strongly magnetised and hence produces a larger intensity of magnetization in comparison to paramagnetic substance, therefore it is strongly magnetised.

Q4

From the definition of bulk modulus,

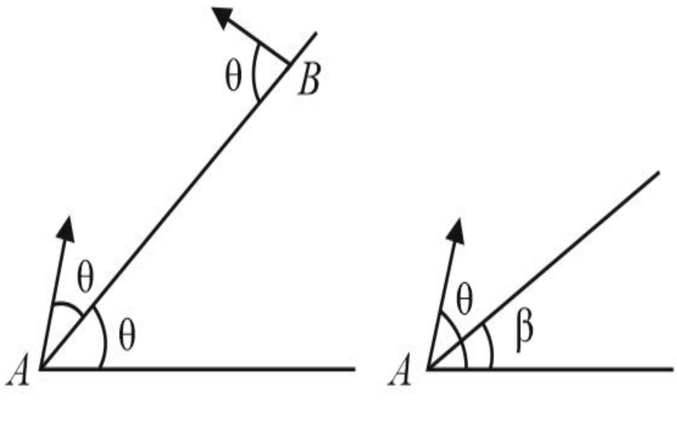
$\beta = -\frac{\Delta P}{\Delta V/V} = -V \cdot \frac{\Delta P}{\Delta V}$

Substituting the values, we have

$\beta=\frac{-(1.165-1.01)\times10^{5}}{-10}\times100=1.55\times10^{5}\ \mathrm{Pa}$

Q5

Here, $\alpha=2\theta$, $\beta=0$



(a) (b)

Time of flight of $\mathrm{T}_1=\frac{2\mathrm{u}\sin(\alpha-\beta)}{\mathrm{g}\cos\beta}$

$=\frac{2\mathrm{u}\sin(2\theta-\theta)}{\mathrm{g}\cos\theta}=\frac{2\mathrm{u}}{\mathrm{g}}\tan\theta$

Time of flight of $\mathrm{T}_2=\frac{2\mathrm{u}\sin\theta}{\mathrm{g}\cos\theta}=\frac{2\mathrm{u}}{\mathrm{g}}\tan\theta$

So, $T_1=T_2$. The acceleration of both the particles is g downwards. Therefore, relative acceleration between the two is zero or relative motion between the two is uniform. The relative velocity of A w.r.t. B is towards AB, therefore collision will take place between the two in mid air.

Q6

$\mathrm{T}=20\ \mathrm{N}$ and $\mathrm{kx}=20\ \mathrm{N}$

$\therefore\ \mathrm{x}=\frac{20}{40}=\frac{1}{2}\ \mathrm{m}$

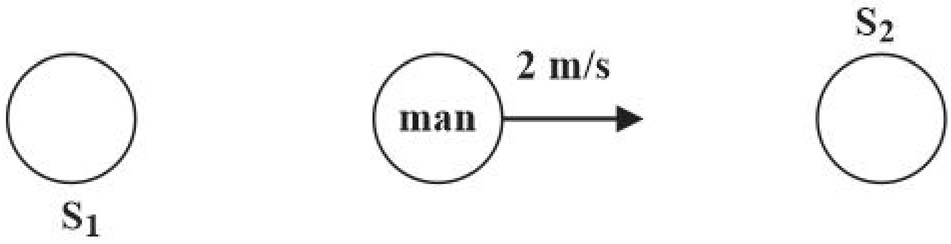
Now $\mathrm{mgx}=\frac{1}{2}\mathrm{mv}^2+\frac{1}{2}\mathrm{kx}^2$

$\Rightarrow\ 5\times10\times\frac{1}{2}=\frac{1}{2}\times5\mathrm{v}^2+\frac{1}{2}\times40\times\frac{1}{4}$

$\Rightarrow\quad 25=5+\frac{5}{2}\mathrm{v}^2$

$\Rightarrow \mathrm{v}=2\sqrt{2}\ \mathrm{m\ s}^{-1}$

Q7



$n_1=n_0\left\{\frac{v-v_0}{v-v_s}\right\}=800\left\{\frac{320-2}{320}\right\}=\frac{800\times318}{320}\ \mathrm{Hz}$

$n_2=n_0\left\{\frac{v-v_0}{v-v_s}\right\}=800\left\{\frac{320+2}{320}\right\}=\frac{800\times322}{320}\ \mathrm{Hz}$

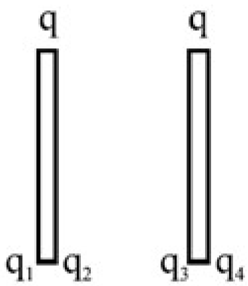
$\therefore$ Beat frequency $= n_2 - n_1$

$=\frac{800(322-318)}{320}$

$=\frac{800\times4}{320}=10\ \mathrm{Hz}$

Q8

Capacitance does not depend on charge on it, $C=\frac{\varepsilon_0 A}{d}$



In final charge distribution, Charge on outer plates $q_1=q_4=q$ and inner plates $q_2=q_3=0$

As charge on inner plates are zero so there is no field between the plates and no potential difference and no energy is stored.

Q9

According to the figure

$H = H_1 + H_2$

$\frac{3\mathrm{KA}(100-\mathrm{T})}{l}=\frac{2\mathrm{KA}(\mathrm{T}-50)}{l}+\frac{\mathrm{KA}(\mathrm{T}-0)}{l}$

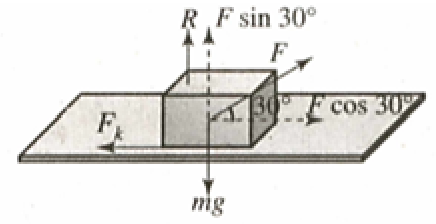
300 - 3T = 2T - 100 + T

6T = 400

Or $T=\frac{200}{3}\,^{o}\mathrm{C}$

Q10

FBD of the block is shown below



Kinetic friction acting on the body$=\mu_k R=0.2(Mg-F\sin 30^{\mathrm{o}})$
$=0.2\left(5\times 10-40\times\frac{1}{2}\right)=0.2(50-20)=6\ \mathrm{N}$

By Newton's second law of motion, acceleration of the block
$=\frac{net\ force}{mass}=\frac{F\cos 30^{\mathrm{o}}-kinetic\ friction}{mass}=\frac{40\times\frac{\sqrt{3}}{2}-6}{5}=5.73\ \mathrm{m/s^2}$

Q11

The equation for the given v - x graph is

$v=-\frac{v_0}{x_0}x+v_0$ ...(i)

Differentiating the above equation w.r.t. x we get

$\frac{dv}{dx}=-\frac{v_0}{x_0}$

Multiplying both sides of the above equation by v, we get

$v\frac{dv}{dx}=-\frac{v_0}{x_0}\times v=-\frac{v_0}{x_0}\left[-\frac{v_0}{x_0}x+v_0\right]$ From (i),

$\therefore a=\frac{v_0^2}{x_0^2}x-\frac{v_0^2}{x_0}$ ...(ii) $\left[\because a=v\frac{dv}{dx}\right]$

On comparing equation (ii) with equation of a straight line $y = mx + c$ where m is the slope of the line and c is its intercept on y-axis we get, $m = \frac{v_0^2}{x_0^2}$ [therefore, m will always be positive]

$\Rightarrow$ The slope m or $\tan\theta = 0$ or $'\theta'$ is an acute angle.

Also, the comparison gives -

$c = \frac{-v_0^2}{x_0}$ or the intercept will be negative.

These conditions are met only in this graph.

Q12

For X: energy $E_1 = 200 \times 7.4 = 1480 MeV$

For A: energy $E_2 = 110 \times 8.2 = 902 MeV$

For B: energy $E_3 = 80 \times 8.1 = 648 MeV$

Energy Released $E =$ -$\mathrm{E}_1 + E_2 + E_3$

$E = (902 + 648) - 1480 = 70 MeV$

Q13

Let $\mathrm{M} \propto \left[\mathrm{F^a L^b T^c}\right]$

Writing dimensions on both sides and using the principle of homogeneity of dimensions we have,

$\left[\mathrm{M^1 L^0 T^0}\right] = \mathrm{k}\left[\mathrm{M\,L\,T^{-2}}\right]^{\mathrm{a}}[\mathrm{L}]^{\mathrm{b}}[\mathrm{T}]^{\mathrm{c}}$

On comparing the power on both sides we get $a = 1, a + b = 0$ and $-2a + c = 0$

On solving we have $b = -1, c = 2, a = 1$

$\therefore$ units of mass is $\left[\mathrm{F\,L^{-1}T^2}\right]$

Q14

In figure A , two springs are connected in parallel. The effective spring constant is, $\mathrm{k_{eff}} = \mathrm{k_1} + \mathrm{k_2}$

$\therefore \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{\mathrm{k_1+k_2}}}; \mathrm{A} - \mathrm{s}$

In figure B, two identical spring are connected in parallel. The effective spring constant is

$\mathrm{k_{eff}} = \mathrm{k} + \mathrm{k} = 2\mathrm{k} \therefore \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m}}{2\mathrm{k}}}; \mathrm{B} - \mathrm{r}$

In figure C, two identical spring are connected in series. The effective spring constant is

$\frac{1}{\mathrm{k_{eff}}} = \frac{1}{\mathrm{k_1}} + \frac{1}{\mathrm{k_2}} = \frac{\mathrm{k_2+k_1}}{\mathrm{k_1k_2}} \text{or } \mathrm{k_{eff}} = \frac{\mathrm{k_1k_2}}{\mathrm{k_1+k_2}}$

$\therefore \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{m(k_1+k_2)}}{\mathrm{k_1k_2}}}; \mathrm{C-p}$

In figure D, two identical spring are connected in series. The effective spring constant is

$\mathrm{k_{eff}} = \frac{\mathrm{(k)(k)}}{\mathrm{k+k}} = \frac{\mathrm{k}}{2} \therefore \mathrm{T} = 2\pi\sqrt{\frac{\mathrm{2m}}{\mathrm{k}}}; \mathrm{D-q}$

Q15

Assertion is correct because $EMF \ \propto \cos\theta$

Reason is wrong because field is steady.

$Induced\ EMF\ in\ AC\ generator\ is\ given\ by\ E = BAN\omega\cos(\omega t)$

$where\ B - uniform\ magnetic\ field\ strength$

$A - area\ of\ the\ coil$

$N - \ number\ of\ turns\ of\ the\ coil$

$\omega - angular\ velocity\ of\ the\ rotating\ coil$

$t - instantaneous\ time$

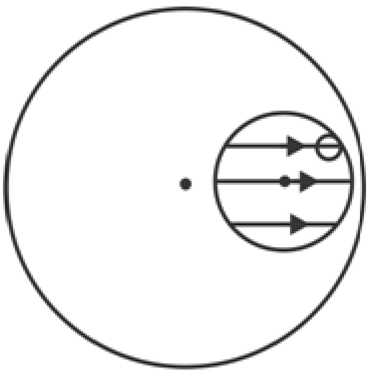
$\theta = \omega t,\ angle\ between\ plane\ of\ the\ coil\ and\ magnetic\ field$

$If\ \theta\ is\ zero,\ induced\ \ emf\ is\ maximum$

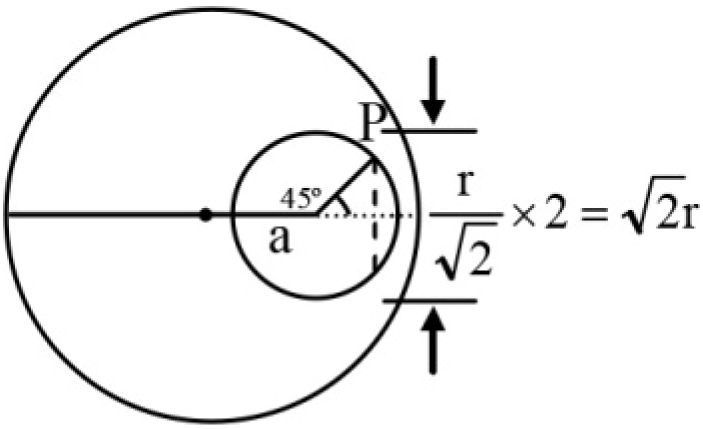
Q16

$\mathrm{E} \ = \ \frac{\rho \mathrm{a}}{3\varepsilon_0} \ \mathrm{F} \ = \ \mathrm{eE} \ = \ \frac{\mathrm{e}\rho\mathrm{a}}{3\mathrm{m}\varepsilon_0}$

$\text{Acceleration} \ = \ \frac{\rho \mathrm{ea}}{3\mathrm{m}\varepsilon_0}$



&



$= \ \frac{1}{2} \times \frac{\rho \mathrm{ea}}{3\mathrm{m}\varepsilon_0} \times \mathrm{t}^2 \ = \ \sqrt{2}\mathrm{r}$

Electron will move opposite to electronic field with constant accleration along shown horizontal line of length $= \sqrt{2}r$

Q17

If the loop current is coming out it should be taken as positive and if it is going in it is taken as negative.

i_3 is coming out and again going in so its net contribution will be zero.

According to Ampere's law,

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0(\Sigma\, i_{inside})$$
$$= \mu_0(i_1 + i_2 + i_3 - i_3)$$
$$= \mu_0(i_1 + i_2)$$

Q18

In parallel combination of electrical appliances,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

As voltage (V) across each appliance is same, so

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^2}{R_3}$$
$$P = P_1 + P_2 + P_3$$

Thus, both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

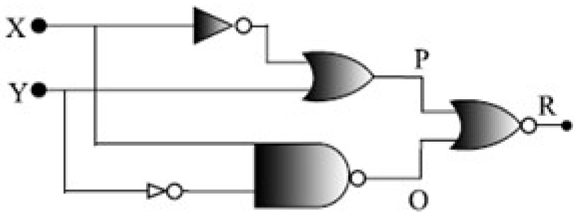
Q19

The oil drop experiment was to measure the specific charge of an electron.

Davisson & Germer experiment proved the wave nature of electrons.

Rutherford experiment proved the presence of a nucleus, while the Franck-Hertz experiment established the Quantisation of energy levels.

Q20



From figure

$\left[\overline{\left(\overline{X}+Y\right)+X\overline{Y}}\right]=R$

$R=\left(\overline{\overline{X}+Y}\right)\cdot\left(\overline{X\overline{Y}}\right)$

$R=\left(\overline{X}\overline{Y}\right)\left(\overline{X}+\overline{Y}\right)$

$R=X\overline{Y}\left(\overline{X}+Y\right)$

$R=X\overline{X}\overline{Y}+X\overline{Y}$

$R=X\overline{Y}$

i.e., when $\left.\begin{matrix}X=1\\Y=0\end{matrix}\right]\Rightarrow R=1$

Q21

Average K.E. per molecule of a gas at the given temperature (T) is given as,

$\frac{1}{2}mv^2=\frac{3}{2}kT$

where, m= molecular mass of the gas

$\therefore\quad mv=\sqrt{3mkT}$

But, $\lambda=\frac{h}{mv}=\frac{h}{\sqrt{3mkT}}$

$\therefore\quad \frac{\lambda_{11}}{\lambda_{11}}=\sqrt{\frac{m_{H_1}T_{H_2}}{m_{11}T_{11}}}=\sqrt{\frac{(4)(273+327)}{(2)(273+27)}}=\sqrt{4}=2$

Q22

From lens maker's equation,

$$P=\frac{1}{f}=\left(\mu-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For plano - convex lens,

$$\mathrm{R}_1=\infty, \mathrm{R}_2=-\mathrm{R}$$

$$\therefore \;\; P_1=(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{(\mu_1-1)}{R}$$

For plano-concave lens,

$$\mathrm{R}_1=-\mathrm{R}, \mathrm{R}_2=\infty$$

$$\therefore \;\; \mathrm{P}_2=(\mu_2-1)\left(\frac{1}{-\mathrm{R}}-\frac{1}{\infty}\right)=\frac{-(\mu_2-1)}{\mathrm{R}}$$

Given that,

$$\mathrm{P}_2=2\mathrm{P}_1$$

$$\therefore \;\; \frac{-(\mu_2-1)}{\mathrm{R}}=\frac{2(\mu_1-1)}{\mathrm{R}}$$

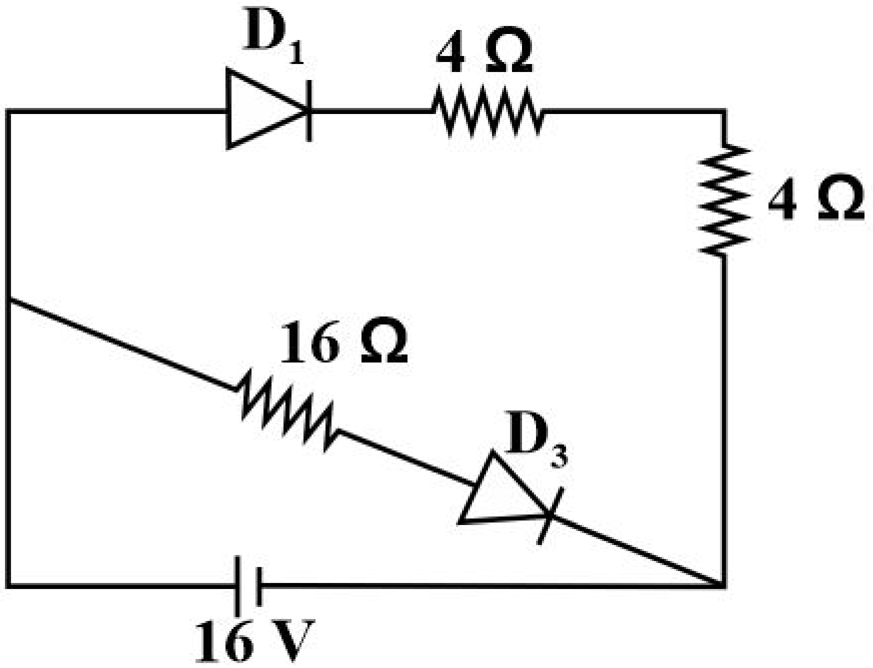
$$\therefore \;\; -\mu_2+1=2\mu_1-2$$

$$\therefore \;\; 2\mu_1+\mu_2=3$$

$$\therefore \;\; \mu_1+\frac{\mu_2}{2}=1.5$$

Q23

In the given circuit diode D_2 is reverse biased, so it will not conduct. Diode D_1 and D_3 are forward biased, so they will conduct, Thus the equivalent circuit is



$\therefore \mathrm{R_{net}} = \frac{(4+4)\times16}{(4+4)+16} = \frac{8\times16}{24} = \frac{16}{3}\ \Omega$

$\therefore$ current through the battery,

$\mathrm{I} = \frac{\mathrm{V}}{\mathrm{R_{net}}} = \frac{16}{\frac{16}{3}} = 3\ \mathrm{A}$

Q24

$$\mathrm{K_f} - \mathrm{K_i} = \int \mathrm{P}\cdot \mathrm{dt}$$
$$\frac{1}{2}\mathrm{mv}^2 - 0 = \int_0^2 \left(\frac{3}{2}\mathrm{t}^2\right)\mathrm{dt}$$
$$\frac{1}{2}\left(2\right)\mathrm{v}^2 = \frac{3}{2}\left[\frac{\mathrm{t}^3}{3}\right]_0^2 = 4$$
$$\mathrm{v} = 2\ \mathrm{m\ s^{-1}}$$

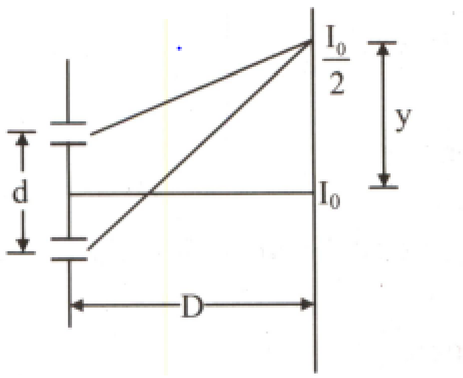
Q25

First Recall the formula of power factor, $\cos\phi = \frac{R}{Z} \Rightarrow \cos\phi_{\text{initial}} = \frac{R}{Z} = \frac{R}{\sqrt{(X_{\mathrm{L}})^2 + R^2}} = \frac{R}{\sqrt{9R^2+R^2}} = \frac{1}{\sqrt{10}}$.

$$\Rightarrow \cos\phi_{final} = \frac{R}{Z} = \frac{R}{\sqrt{(X_{\mathrm{L}} - X_C)^2 + R^2}} = \frac{R}{\sqrt{R^2+R^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\cos\phi_{final}}{\cos\phi_{\text{initial}}} = \frac{\sqrt{5}}{x} \Rightarrow x = 1$$

Q26



$\mathrm{I} = \mathrm{I}_0 \cos^2 \frac{\phi}{2}$

$\Rightarrow \frac{I_0}{2} = I_0 \cos^2 \frac{\phi}{2}$

$\because \cos \frac{\phi}{2} = \frac{1}{\sqrt{2}}$

$\therefore \ \frac{\phi}{2} = \frac{\pi}{4}$

$\therefore \ \phi = \frac{\pi}{2}$

since $\frac{2\pi}{\lambda}\left(\frac{yd}{D}\right) = \frac{\pi}{2} \quad \ldots . \left(\text{Path difference } = \frac{yd}{D}\right)$

$\therefore \ y = \frac{\lambda D}{4d}$

$= \frac{6000 \times 10^{-10} \times 120 \times 10^{-2}}{4 \times 0.25 \times 10^{-2}}$

$= 7.2 \times 10^{-5}\mathrm{m}$

$= 72\mu\mathrm{m}$

Q27

de-Broglie wavelength is

$\lambda = \frac{\mathrm{h}}{\mathrm{p}} = \frac{\mathrm{h}}{\sqrt{2\,\mathrm{mE}}}$

When K.E. is increased by 15 times,

$$\mathbf{E}' = \mathbf{E} + 15\mathbf{E} = 16\mathbf{E}$$

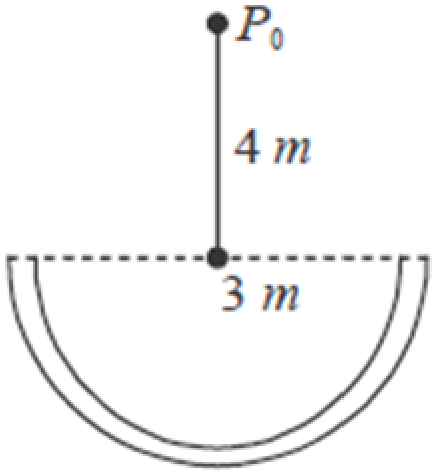
$$\therefore \quad \lambda' = \frac{\mathrm{h}}{\sqrt{2\mathrm{m}(16\mathrm{E})}} = \frac{1}{4}\frac{\mathrm{h}}{\sqrt{2\,\mathrm{mE}}} = \frac{\lambda}{4} = 0.25\lambda$$

$$\therefore \ \%change = \lambda - \lambda' = \lambda - 0.25\lambda$$

$$= 0.75\lambda \Rightarrow 75\%$$

Note: There is a word increased by and not increased to. Most students get confused by the language of question. Similar thing is given in PYQs as well

Q28



Total power emitted by shell due to radiation

$$P_{\text{out}} = \sigma\left(2\pi R^2 + \pi R^2\right)T^4 = 3\pi R^2\sigma T^4$$

Power gained by the shell from P_0 :

$$P_{\text{in}} = P_0\frac{2\pi r^2\left(1-\cos 37^{\circ}\right)}{4\pi r^2} = \frac{P_0}{10}$$

Equating: $\frac{P_0}{10} = 3\pi R^2\sigma T^4$

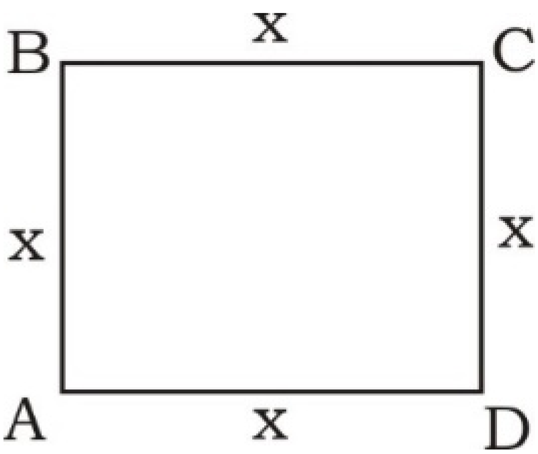
Gives $T^4 = \frac{P_0}{270\pi\sigma}$

$n=\frac{270}{10}=27$

Q29

Let the equivalent resistance of one infinite ladder be x. Then the complete network reduces to

$\therefore\ R_{\mathrm{AB}}=\frac{x \times 3 x}{x+3 x}=\frac{3}{4} x$ and $R_{\mathrm{AC}}=\frac{2 x \times 2 x}{2 x+2 x}=x$;

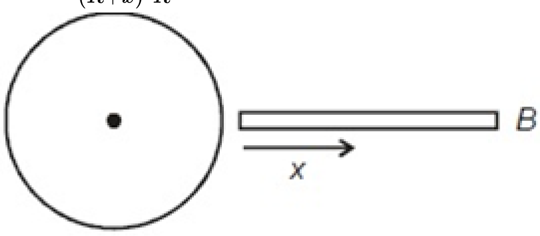


Hence $\frac{R_{\mathrm{AB}}}{R_{\mathrm{AC}}}=\frac{3}{4}$

Q30

Net force on the rod is equal to centripetal force.

$f_g=\frac{G M m}{(R+x)^2 R} d x$



$=\frac{G M m}{R} \int_0^R \frac{d x}{(R+x)^2}$

$f_g=\frac{G M m}{2 R^2}$

Q31

$\pi_1 V_1 + \pi_2 V_2 = \pi_R (V_1 + V_2)$

$1 \times 1 + 3.5V = 2.5(1 + V)$

$1 + 3.5V = 2.5 + 2.5V$

or $V = 1.5$ L

Q32

(I) $CH_3 - \ddot{\underset{\cdot\cdot}{O}} - \overset{\oplus}{C}H_2$ is more stable than $CH_3 - \overset{\oplus}{C}H_2$, becausse of $+$M-effect of $CH_3 - \ddot{\underset{\cdot\cdot}{O}} -$.

(II) $Me_3\overset{\oplus}{C}H$ is more stable than $CH_3 - CH_2 - \overset{\oplus}{C}H_2$,

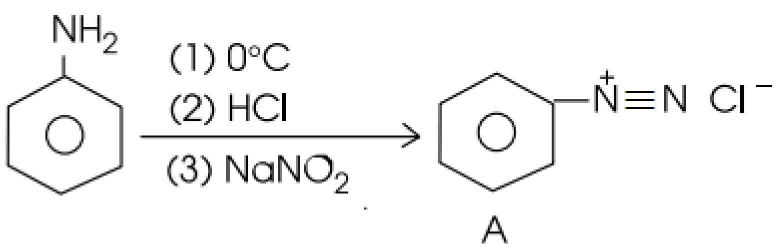
because $Me_3C^{\oplus}$ have more 2-hydrogen atoms than $CH_3 - CH_2 - CH_2^{\oplus}$, so, it is more stable, due to greater hyperconjugation.

(III) The positive charge is delocalised in $CH_2 = CH - \overset{\oplus}{C}H_2$, so more stable than, $CH_3 - CH_2 - CH_2^{\oplus}$.

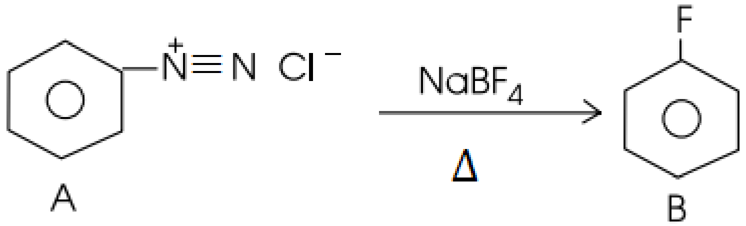
(IV) $CH_2 = \overset{\oplus}{C}H$ will be less stable than $CH_3 - \overset{\oplus}{C}H_2$ as positive charge is less stable on double bonded 'C' than on single bonded 'C'.

Q33

Benzenediazonium chloride is prepared by the reaction of aniline with nitrous acid at $273 - 278$ K. Nitrous acid is produced in the reaction mixture by the reaction of sodium nitrite with hydrochloric acid.



When benzene diazonium chloride is treated with sodium fluoroborate, benzene diazonium fluoroborate is precipitated which on heating decomposes to yield benzyl fluoride.



Q34

$\alpha=\frac{\Lambda_{\mathrm{m}}}{\Lambda_{\mathrm{m}}^{\infty}}=\frac{5.2}{390.7}=0.0133$

Q35

This problem includes conceptual mixing of sweetening agent and their sweetening capacity

Sweetening agent: The substances other than sugar, used for sweetening of food and enhancing odour and flavour are known as sweetening agents. Commonly used sweetening agents are Saccharin, Aspartame, Alitame etc.

Comparative study of sweetening agent:

Sweetening capacity of saccharin = 500$\times$ sweetening capacity of sugar

Sweetening capacity of aspartame = 180$\times$ sweetening capacity of sugar

Sweetening capacity of alitame = 2000$\times$ sweetening capacity of sugar

Hence, the correct order of sweetening capacity is represented by choice (III > I > II).

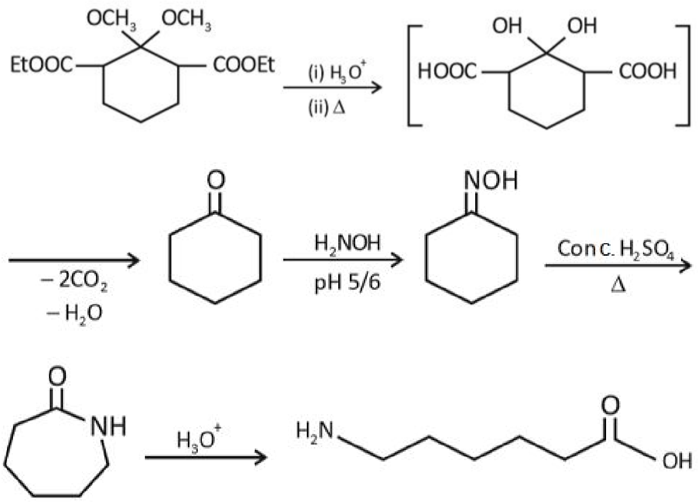
Q36

$2\mathrm{CrO}_4^{2-}+2\mathrm{H}^{+}\longrightarrow\mathrm{Cr}_2\mathrm{O}_7^{2-}+\mathrm{H}_2\mathrm{O}$ [where x= pH = 4 - 5]

$\mathrm{Cr}_2\mathrm{O}_7^{2-}+2\mathrm{OH}^{-}\longrightarrow 2\mathrm{CrO}_4^{2-}+\mathrm{H}_2\mathrm{O}$ [where y =pH = 9 - 10]

so pH value is 5 and 9 respectively

Q37

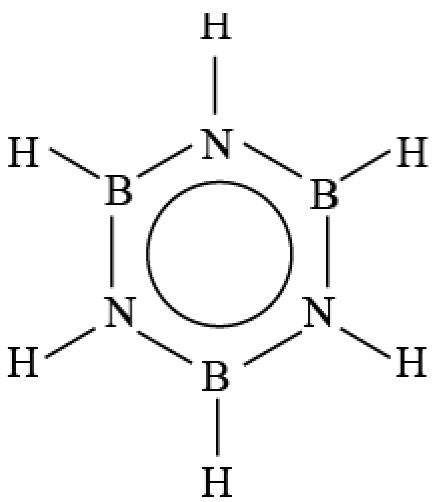


Q38

The ions that cause hardness in water are Ca^{+2} and Mg^{+2}. They form salts with HCO_3, Cl^-, $\mathrm{SO_4}^{-2}$ions which when dissolved in water, cause the hardness of the water.

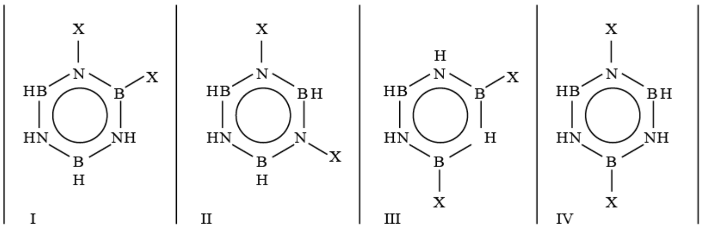
Q39

I. Borazine is aromatic - True



Each Nitrogen donates two electrons of its lone pair to the pi electron cloud. Hence with 6π electron, it follows Huckels Rule of anomorich $(4n+2)\pi$ electron in ring.

II. There are four isomeric distributed borazine molecules $\mathrm{B}_3\ \mathrm{N}_3\mathrm{H}_4X_2$ - True.

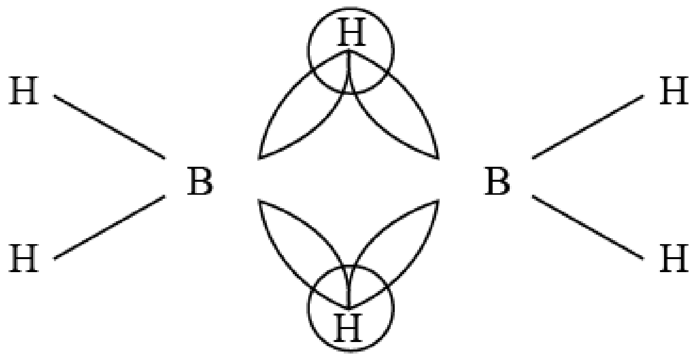


4 possible structures

III. Bonazine is more reactive towards addition reactions than benzene.

Benzene (C_6H_6) contains $C-C$ bonds of a π saturated system. Where Bonazine contains $B-N$ bond with electronegativity difference which makes the π-electron cloud to be more localized on N atoms rather the B. This makes bonozire more prone to addition reactions.

IV. Banana bonds in. B_2H_6 are longer but stronger than normal B-H bonds - True



Banana bonds are longer due to the $3c-2e^{-}$involvement. But they are stronger than normal $B-H$ bonds.

Option C is correct - I, II, III, IV

Q40

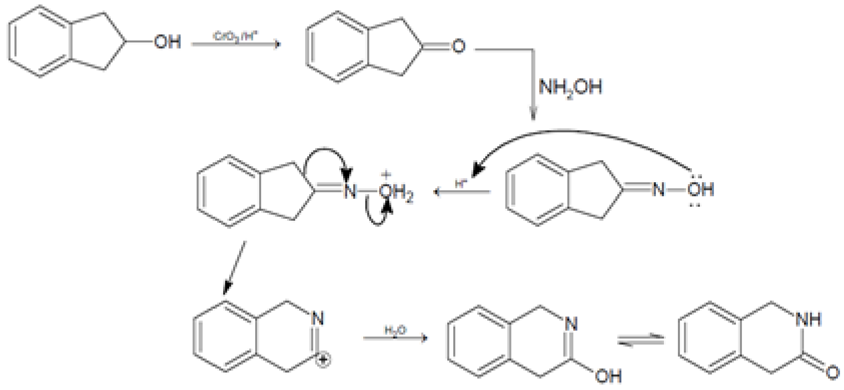
Amino acids are monomers of proteins. These are substituted methanes and has a variable group R attached to the carbon. Due to presence of R group various types of amino acids are formed. Cysteine and Methionine are Sulphur containing amino acids. Tyrosine are amino acids with benzene ring. Glutamic acid is acidic and that means have acidic group. Serine has alcoholic group.

Q41

(i) Stability ↑ as size of cation ↑, ionic character ↑

(ii) Solubility in water ↓

Q42



Q43

Photochemical smog is formed on the chemical reaction of sunlight, nitrogen oxides and volatile organic compounds in the atmosphere.

$NO_2(g) \xrightarrow{hv} NO(g) + O(g)$

Oxygen atom (nascent) is very reactive and combines with O_2 in air to produce ozone.

$O(g) + O_2(g) \rightarrow O_3(g)$

Q44

$\because M \times V(ml) = m \text{ mol}$

$10 \text{ m mol } H_2SO_4 = 20 \text{ m mol of } NH_3$

$[H_2SO_4 + 2NH_3 \rightarrow (NH_4)_2SO_4]$

1 mol NH_3 contains 14 g nitrogen

20×10^{-3} mol NH_3 contains $14 \times 20 \times 10^{-3}$ nitrogen

0.75 g of sample contains

$\% \text{ Nitrogen} = \frac{14 \times 20 \times 10^{-3}}{0.75} \times 100 = 37.33\%$

Q45

Arrhenius equation;

$d(\ln k) = \frac{E_a}{RT^2} dT$

$\frac{d(\ln k)}{dT} = \frac{E_a}{RT^2}$

Hence, when $dT =$ same, the change in $d(\ln K)$ wil be highest when E_a is highest.

So, if temperature increases, then rate of reaction having more E_a increases sharply and the reaction becomes more sensitive to temperature changes.

Q46

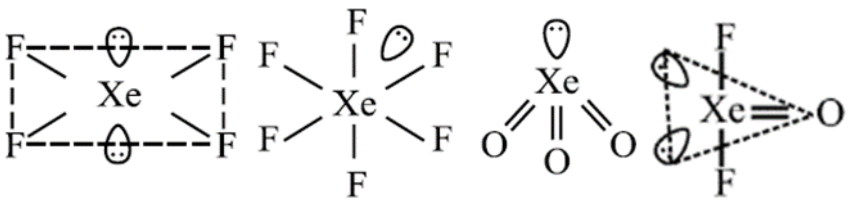
Ag_2S and $CuFeS_2$ are sulphide ores and froth flotation is used for the concentration of the ore.

Q47

$Cl_2 + NaOH \xrightarrow{\text{Cold\&dil}} NaCl + NaOCl$

$Cl_2 + NaOH \xrightarrow{\text{Hot\&Conc.}} NaCl + NaOCl_3$

Q48



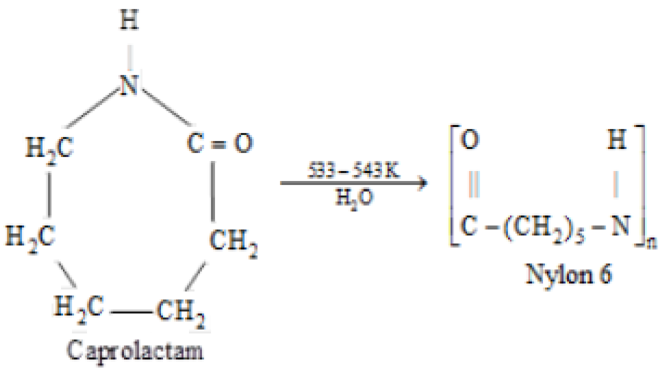
Hence the correct match is $1 - D, 2 - C, 3 - A, 4 - B$

Q49

When a system undergoes a change under the condition that there is no exchange of heat between the system and surroundings, then the process is called an adiabatic process. The leaking air of balloon undergoes adiabatic expansion. Air cools down due to adiabatic expansion as air has to do work against the external pressure at the cost of its internal energy. Due to work done against external pressure, the internal energy of air reduces. Thus, it becomes cooler.

Q50

(i) Nylon-6



(ii) Nylon-6, 6

$$\mathrm{n\,H_2\,N\,(CH_2)_6\,NH_2 + \,n\,HOOC\,(CH_2)_4\,COOH \rightarrow \underset{Nylon\,6,\,6}{—[NH\,(CH_2)_6\,NHCO\,(NH_2)_4\,CO]_n—} + nH_2O}$$

(iii) Polythene

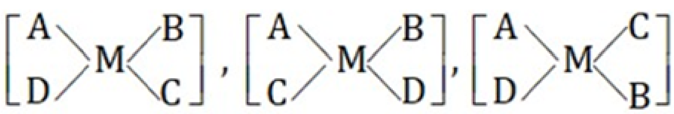
$$\underset{\text{Ethene}}{\mathrm{n\,CH_2 = CH_2}} \xrightarrow{\text{Polymerisation}} \underset{\text{Polythene}}{\mathrm{—[\ CH_2 - CH_2]_n—}}$$

Q51

Points (ii), (iii) and (iv) are correct regarding physisorption.

Q52

Complex is square planar



Q53

In Williamson's synthesis,

i. 1° alkyl halide and 3° alkoxide give best yield of corresponding ethers.

ii. In case of, 3° alkyl halide, elimination competes over substitution and alkene is obtained as a product.

iii. Aryl halides do not undergo nucleophilic substitution unless it is activated by the presence of electron withdrawing group. Thus, $(C_6H_5)_2O$ and $(CH_3)_3C-O-C(CH_3)_3$ cannot be prepared by Williamson's synthesis.

Q54

Meq of H_2O_2 = Meq of I_2 = Meq of $Na_2S_2O_3$.

If N is normality of H_2O_2, then

N x 25 = 0.3 x 20, $N_{H_2O_2}=0.24N$

$M_{H_2O_2}=0.12\ M$

N = 0.24

Volume strength = 0.12×11.2

=1.344 Vol

Final answer $=1.344\times 1000=1344$

Q55

$$\begin{array}{lcc} & N_2O_4(g) \rightleftharpoons & 2NO_2(g) \\ \text{at } t=0 & 1 & - \\ \text{at } t=1 & 1-\alpha & 2\alpha \\ \text{mole fraction} & \dfrac{1-\alpha}{1+\alpha} & \dfrac{2\alpha}{1+\alpha} \end{array}$$

$K_p=\frac{[P_{NO_2}]^2}{P_{N_2O_4}}$

$K_p=\frac{4\alpha^2}{1-\alpha^2}p$

$\Rightarrow K_p=\frac{4\times(0.2)^2}{1-(0.2)^2}=\frac{0.16}{0.96}$

for $2NO_2 \rightleftharpoons N_2O_4$; $K_p=\frac{0.96}{0.16}=6$

Q56

$\mathrm{v_{H2}} = \mathrm{v_{O2}}$
$\sqrt{\frac{3RT_{H2}}{MH_2}} = \sqrt{\frac{3RT_{O2}}{MO_2}}$
$\mathrm{So}, \sqrt{\mathrm{M_{O2}T_{H2}}} = \sqrt{\mathrm{M_{H2}T_{O2}}}$

$32 \times \mathrm{T_{H2}} = 2 \times 1600$

$\mathrm{T_{H2}} = 2 \times \frac{1600}{32} = 100$

Q57

Amount of CO_2 in one liter of solution $= 4.4\ . = 0.1$ Mole

$\mathrm{pH} = 1/2\{\mathrm{pKa} - \log\mathrm{C}\}$

For a weak acid solution

$\mathrm{pH} = 1/2\{6.4 + 1\} = 3.7$

Q58

Number of atoms of A at corners $= 7$ (one 'A' is missing)

$\therefore$Contribution of atoms of 'A' to the unit cell

$= 7 \times \frac{1}{8} = \frac{7}{8}$

Total number of atoms 'B' at face $= 6$

$\therefore$ Contribution of atom 'B' to the unit cell

$= 6 \times \frac{1}{2} = 3$

$A : B = \frac{7}{8} : 3 = 7 : 24$

Hence, formula $= \mathrm{A_7B_{24}}$

$\therefore$The value of x is 7

Q59

Energy of light $= \frac{1240}{300} = 4.13\mathrm{eV}$, so the metal having work function lower than 4.13eV will emit photo-electron, so the following 4 metals will show photoelectric effect

Metal	Li	Na	K	Mg
Ø(eV)	2.4	2.3	2.2	3.7

Note: For photoelectric effect to occur, the energy of a photon must be greater than the work function of the metal is the explanation

Q60

	$2N_2O_5 \xrightarrow{\Delta}$	$2NO_2(g)$	$+\ O_2(g)$
t = 0	1 atm	0	0
t = t	(1-P)	P	$\frac{P}{2}$
t = ∝	0	1 atm	0.5 atm

$\mathrm{P_0 = 1\ atm,\ P_t = 1\ .45\ \ atm,\ P_{\propto} = 1\ .5\ \ atm}$

From the unit of $\mathrm{K} = 5 \times 10^{-4}\mathrm{s}^{-1}$,

The reaction is first-order

$t = \frac{2.303}{2\times5\times10^{-1}}\log\left(\frac{1.5-1}{1.5-1.45}\right)$

$\mathrm{t} = 2.303 \times 10^3 = \mathrm{y} \times 10^3$

$\therefore y = 2.303$

After round off: $\mathrm{y} = 2.3$

Q61

$\because \quad f(2x) = f(x)$ replace x by $\frac{x}{2}$

$\therefore \quad f(x) = f\left(\frac{x}{2}\right) = f\left(\frac{x}{2^2}\right) = \ldots\ldots\ldots = f\left(\frac{x}{2^n}\right)$

$\Rightarrow \quad \lim_{n\to\infty} f(x) = \lim_{n\to\infty} f\left(\frac{x}{2^n}\right) \Rightarrow f(x) = f(0) = \text{ constant}$

$\because \quad f(1) = 3$

$\therefore \quad f(x) = 3 \forall x \in R$

$\therefore \quad \int_{-1}^{1} f(f(x)) dx = \int_{-1}^{1} 3 dx = 6.$

Q62

$$\sin x \cdot \tan 4x = \cos x$$
$$\sin x \cdot \sin 4x = \cos x \cdot \cos 4x$$
$$\cos 4x \cdot \cos x - \sin x \cdot \sin 4x = 0$$
$$\cos(4x + x) = 0$$
$$\cos 5x = 0$$
$$5x = (2n-1)\frac{\pi}{2} \quad (n \in I)$$
$$x = (2n-1)\frac{\pi}{10}$$
$$\therefore \quad x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$$

Q63

$$\lim_{x\to 0} |\cos x + \sin 2x + \sin 3x|^{\cot x} = e^m$$
$$= e^{\lim_{x\to 0}(|\cos x + \sin 2x + \sin 3x| - 1)\cot x} = e^m$$
$$\Rightarrow \quad m = \lim_{x\to 0}(|\cos x + \sin 2x + \sin 3x| - 1)\cot x$$
$$= \lim_{x\to 0}(\cos x + \sin 2x + \sin 3x - 1)\cot x$$
$$= \lim_{x\to 0} \frac{\cos x - 1 + \sin 2x + \sin 3x}{x \cdot \frac{\tan x}{x}}$$
$$= \lim_{x\to 0}\left(\frac{\cos x - 1}{x} + \frac{\sin 2x}{x} + \frac{\sin 3x}{x}\right) = 2 + 3 = 5$$

Q64

Matrices value of whose determinant is zero.

$$\underbrace{\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}}_{2}\underbrace{\begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}}_{2}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \rightarrow 6$$

From the remaining 10 matrices, 5 have negative values of their determinant and 5 have positive values of their determinant.

$\therefore \quad$ Required probability $=\frac{5}{11}$

Q65

$$\begin{aligned}
&\text{Let} \quad z=x+iy \quad \therefore \quad \bar{z}=x-iy \\
&\therefore \quad (2iy)^2=12\left(x^2+y^2\right)-4 \Rightarrow 12x^2+16y^2=4
\end{aligned}$$

$$3x^2+4y^2=1 \quad \Rightarrow \quad \frac{x^2}{\frac{1}{3}}+\frac{y^2}{\frac{1}{4}}=1$$

$$\begin{aligned}
&\therefore \quad x=\sqrt{\frac{1}{3}}\cos\theta, y=\sqrt{\frac{1}{4}}\sin\theta \\
&\therefore \quad 3\sqrt{3}\operatorname{Re}(z)+8\operatorname{Im}(z)=3\cos\theta+4\sin\theta \\
&\therefore \quad \max=5
\end{aligned}$$

Q66

$$\begin{aligned}
&\text{As, } \left(\alpha^2+\sqrt{3}\right)=-(3)^{1/4}\cdot\alpha \\
&\Rightarrow \left(\alpha^4+2\sqrt{3}\alpha^2+3\right)=\sqrt{3}\alpha^2 \text{ (On squaring)} \\
&\therefore \left(\alpha^4+3\right)=(-)\sqrt{3}\alpha^2 \\
&\Rightarrow \alpha^8+6\alpha^4+9=3\alpha^4 \quad \text{(Again squaring)} \\
&\therefore \alpha^8+3\alpha^4+9=0 \\
&\Rightarrow \alpha^8=-9-3\alpha^4
\end{aligned}$$

(Multiply by α^4)

So, $\alpha^{12}=-9\alpha^4-3\alpha^8$

$$\begin{aligned}
&\therefore \quad \alpha^{12}=-9\alpha^4-3\left(-9-3\alpha^4\right) \\
&\Rightarrow \alpha^{12}=-9\alpha^4+27+9\alpha^4
\end{aligned}$$

Hence, $\alpha^{12}=(27)^2$

$$\begin{aligned}
&\Rightarrow \left(\alpha^{12}\right)^8=(27)^8 \\
&\Rightarrow \alpha^{96}=(3)^{24}
\end{aligned}$$

Similarly $\beta^{96}=(3)^{24}$

$\therefore \alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)=(3)^{24}\times 52$

$\Rightarrow$ Option (3) is correct.

Q67

$|[\vec{\mathbf{a}}\,\vec{\mathbf{b}}\,\vec{\mathbf{c}}]|=30$

$|abc\sin\theta\cos\phi|=30\Rightarrow\theta=\dfrac{\pi}{2},\phi=0\Rightarrow\vec{\mathbf{a}},\vec{\mathbf{b}},\vec{\mathbf{c}}$ are mutually perpendicular

$$(2\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})\cdot[(\vec{\mathbf{a}}\times\vec{\mathbf{c}})\times(\vec{\mathbf{a}}-\vec{\mathbf{c}})+\vec{\mathbf{b}}]$$
$$=(2\vec{\mathbf{a}}+\vec{\mathbf{b}}+\vec{\mathbf{c}})\cdot\left[(\vec{\mathbf{a}}\cdot\vec{\mathbf{a}})\vec{\mathbf{c}}+c^2\vec{\mathbf{a}}+\vec{\mathbf{b}}\right)$$
$$=2a^2c^2+b^2+a^2c^2=3a^2c^2+b^2=300+9=309$$
$$\therefore\quad \frac{k}{103}=\frac{309}{103}=3$$

Q68

$P(x,y),Q(1,0)$

$PQ=\sqrt{(x-1)^2+y^2}$

$PQ^2=x^2+1-2x+y^2$

$PQ^2=x^2-2x+1+2x^3-3x^2+9$

$PQ^2=2x^3-2x^2-2x+10$

$\frac{d\left(PQ^2\right)}{dx}=6x^2-4x-2=0$

$6x^2-4x-2=0$

$3x^2-2x-1=0$

$3x^2-3x+x-1=0$

$3x(x-1)+(x-1)=0$

$(x-1)(3x+1)=0$

$x=1,x=-\frac{1}{3}$

$\frac{d^2\left(PQ^2\right)}{dx^2}=12x-4$

So,PQ^2 is minimum at $\mathrm{x}=1 y^2=2+9-3=8$

$\left(\mathrm{PQ}\right)_{\text{minimum}}=\sqrt{0+8}=2\sqrt{2}$

Q69

$\det.(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}A))))=|\mathrm{A}|^{(3-1)^4}$

$=|\mathrm{A}|^{16}=4^8.5^{16}$

$\Rightarrow|\mathrm{A}|=\pm 10$

$|\mathrm{A}|=\begin{vmatrix} \mathrm{x} & \mathrm{y} & -\mathrm{z} \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{vmatrix}=\mathrm{x}+\mathrm{y}+\mathrm{z}=\pm 10$

$\because \mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{N} \Rightarrow \mathrm{x}+\mathrm{y}+\mathrm{z}=-10$ (not possible)

Hence, $\mathrm{x}+\mathrm{y}+\mathrm{z}=10$

The number of such matrices $={ }^9\mathrm{C}_2$

$=36$

Q70

Let the general equation of circle be

$\mathrm{x}^2+\mathrm{y}^2+2\mathrm{gx}+2\mathrm{fy}+\mathrm{c}=0$ .... (i)

It cuts the circle $\mathrm{x}^2+\mathrm{y}^2-\ 20\mathrm{x}+4=0$ orthogonally

$\therefore 2\ (-10\mathrm{g}+0\ \times \mathrm{f}\ )=\ \mathrm{c}+4\ \Rightarrow -20\ \mathrm{g}=\mathrm{c}+4$ ....(ii)

$\because$ circle (i) touches $\mathrm{x}=2$

therefore, perpendicular distance from centre to the tangent to the circle=radius

$\Rightarrow\left|\frac{-\mathrm{g}+0-2}{\sqrt{1^2+0^2}}\right|=\sqrt{\mathrm{g}^2+\mathrm{f}^2-\mathrm{c}}$

$\Rightarrow(\mathrm{g}+2)^2=\ \mathrm{g}^2+\mathrm{f}^2-\mathrm{c}$

$\Rightarrow \mathrm{g}^2+4+4\mathrm{g}=\mathrm{g}^2+\mathrm{f}^2-\mathrm{c} \Rightarrow\ 4\mathrm{g}+4=\mathrm{f}^2-\mathrm{c}$ ... (iii)

on eliminating c from (ii) and (iii) we get

$-16\mathrm{g}+4=\mathrm{f}^2+4 \Rightarrow \mathrm{f}^2\ +16\mathrm{g}=0$

Hence, locus of $(-\mathrm{g},\ -\mathrm{f})$ is,

$\mathrm{y}^2-\ 16\mathrm{x}=0$ (replacing $-\mathrm{f}$ & $-\mathrm{g}$ by x & y)

Q71

Let $\mathrm{a}=2\mathrm{x}+1,\ \mathrm{b}=2\mathrm{y}+1, \mathrm{c}=2\mathrm{z}+2$, where $\mathrm{x}, \mathrm{y}, \mathrm{z} \in$ whole number

$\therefore \quad \mathrm{a}+\mathrm{b}+\mathrm{c}=16$

$\Rightarrow \quad 2\mathrm{x}+1+2\mathrm{y}+1+2\mathrm{z}+2=16$

$\Rightarrow \quad \mathrm{x}+\mathrm{y}+\mathrm{z}=6$

$\Rightarrow$ Number of integral solutions

$={}^{6+3-1}\mathrm{C}_{3-1}={}^{8}\mathrm{C}_{2}=28$

Q72

$T_{r+1}={}^{n}C_{r}2^{r}x^{r}$

$a_{k}={}^{n}C_{k}2^{k}$

$$\sum_{k=0}^{n}(3k+1)^{n}C_{k}\cdot a_{k}=3\sum_{k=0}^{n}k\cdot{}^{n}C_{k}\cdot 2^{k}+\sum_{k=0}^{n}{}^{n}C_{k}\cdot 2^{k}=3\sum_{k=0}^{n}{}^{n\cdot n-1}C_{k-1}\cdot 2^{k}+(1+2)^{n}$$

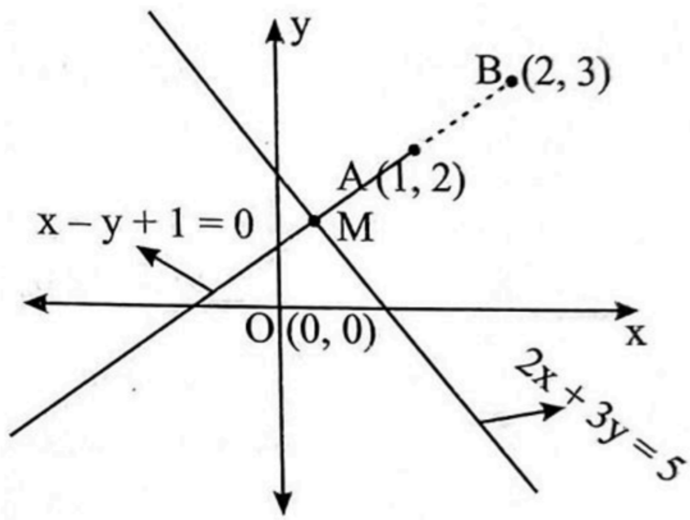
$$=3n\sum_{k=0}^{n}{}^{n-1}C_{k-1}\cdot 2^{k+1}+3^{n}=2\cdot 3n(1+2)^{n-1}+3^{n}=2n\cdot 3^{n}+3^{n}=(2n+1)3^{n}$$

$\therefore p=2, q=1, r=3$

Q73

As, $|MA-MB| \leq AB$

$\therefore \quad |MA-MB|_{\text{maximum}}=AB$, which is possible when M is the point of intersection of line joining



$A(1,2), B(2,3)$ and $2x+3y=5$

. So, $M\left(x=\frac{2}{5}, y=\frac{7}{5}\right)$

Q74

The system of equations has a non-trivial solution if and only if $\begin{vmatrix} \sin 3\theta & -2 & 3 \\ \cos 2\theta & 8 & -7 \\ 2 & 14 & -11 \end{vmatrix} = 0$

Applying $\mathrm{R}_2 \rightarrow \mathrm{R}_2 + 4\mathrm{R}_1,\ \mathrm{R}_3 \rightarrow \mathrm{R}_3 + 7\mathrm{R}_1$, we get,

$\begin{vmatrix} \sin 3\theta & -2 & 3 \\ \cos 2\theta + 4\sin 3\theta & 0 & 5 \\ 2 + 7\sin 3\theta & 0 & 10 \end{vmatrix} = 0$

Expanding along C_2, we get,

$2(\cos 2\theta + 4\sin 3\theta) - (2 + 7\sin 3\theta) = 0$

$\Rightarrow 2 - 2\cos 2\theta - \sin 3\theta = 0$

$\Rightarrow 4\sin^2\theta - \left(3\sin\theta - 4\sin^3\theta\right) = 0$

$\Rightarrow \sin\theta\left(4\sin^2\theta + 4\sin\theta - 3\right) = 0$

$\Rightarrow \sin\theta\,(2\sin\theta - 1)\,(2\sin\theta + 3) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = 1/2$.

[$\sin\theta = -3/2$ is not possible]

$\therefore$ For, $\theta = \mathrm{n}\pi$ the system of equations has a non-trivial solution.

Q75

$\text{Mean} = 6 = \frac{a+b+8+5+10}{5} \Rightarrow a + b = 7$

$\text{Variance} = 6.8 = \frac{a^2+b^2+64+25+100}{5} - 36$

$\Rightarrow a^2 + b^2 = 25$

$2ab = (a+b)^2 - \left(a^2 + b^2\right) \Rightarrow ab = 12$

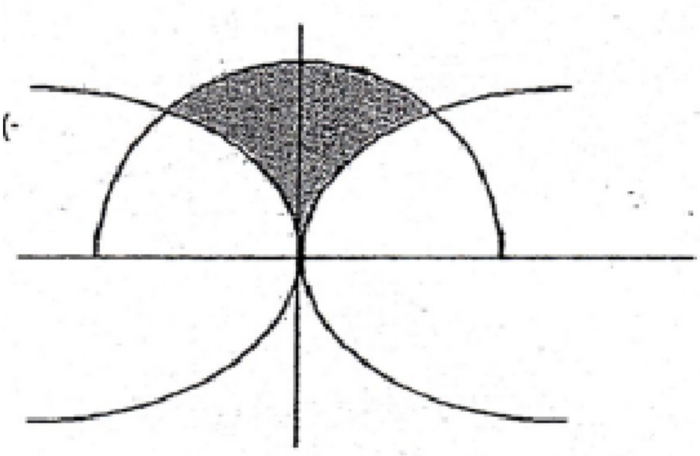
Now, $\left(a^3 + b^3\right) = (a+b)^3 - 3ab(a+b)$

$= 343 - 3 \times 12 \times 7$

$= 91$

Q76

Required area $= 2\int_0^1 \left(\sqrt{4-x^2}-\sqrt{3}x\right)dx$



$$= 2\left(\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)-\frac{\sqrt{3}\cdot 2x^{1/2}}{3}\right)_0^1$$

$$= \frac{2\pi-\sqrt{3}}{3}$$

Q77

Given,

$$xdy = \left(\sqrt{x^2+y^2}+y\right)dx$$

$$\Rightarrow xdy - ydx = \sqrt{x^2+y^2}dx$$

$$\Rightarrow \frac{xdy-ydx}{x^2} = \sqrt{1+\frac{y^2}{x^2}}\cdot\frac{dx}{x}$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$$

Now integrating both side we get,

$$\Rightarrow \int \frac{d\left(\frac{y}{x}\right)}{\sqrt{1+\left(\frac{y}{z}\right)^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \ln\left(\frac{y}{x}+\sqrt{\left(\frac{y}{x}\right)^2+1}\right) = \ln x + \ln c$$

$$\Rightarrow \frac{y+\sqrt{y^2+x^2}}{x} = cx$$

$$\Rightarrow y+\sqrt{y^2+x^2} = cx^2$$

Now given when $x=1, y=0 \Rightarrow 0+1=c \Rightarrow c=1$

So equation of curve is $y+\sqrt{x^2+y^2}=x^2$

Q78

(A) $p \vee(p \Rightarrow q)$ is false only if p is false and $(p \Rightarrow q)$ is false which is never possible simultaneously. Hence, $p \vee(p \Rightarrow q)$ is a tautology

(C) $p \Rightarrow(p \vee q)$ is false only if p is true and $(p \vee q)$ is false which is never possible simultaneously. Hence, $p \Rightarrow(p \vee q)$ is a tautology

(B) $p \Rightarrow(p \wedge q)$ is false only if p is true & $p \wedge q$ is false which is possible when p is true & q is false. Hence, $p \Rightarrow(p \wedge q)$ is not a tautology

Q79

The required number is of the form $2(2K+1)$, i.e. it will contain exactly one power of 2.

Also, we have to exclude 2 as $K \in N$.

Number of required divisors

$=1 \cdot 5 \cdot 11 \cdot 7-1=385-1=384$

Q80

Here, a,b,c are positive real numbers, so we can use AM GM inequality

$$\frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{2}+\frac{b}{2}+c}{6} \geq\left(\frac{a^3}{27} \cdot \frac{b^2}{4} \cdot c\right)^{1/6}$$

$$\Rightarrow\left(\frac{a^3}{27} \cdot \frac{b^2}{4} \cdot c\right) \leq 1 \Rightarrow a^3 b^2 c \leq 3^3 \cdot 2^2$$

Q81

Making the substitution $x \mapsto \pi-x$ yields $I(a)=\int_0^\pi \log\left(1+2a\cos x+a^2\right)dx=I(-a)$ so that

$I(a)=I(-a). \quad (\dagger)$

Then, consider

$$\begin{aligned} I(a)+I(-a) &= \int_0^\pi \log\left(\left(1-2a\cos x+a^2\right)\left(1+2a\cos x+a^2\right)\right)dx \\ &= \int_0^\pi \log\left(\left(1+a^2\right)^2-(2a\cos x)^2\right)dx \end{aligned}$$

Using double angle formulae produces

$$\begin{aligned} I(a)+I(-a) &= \int_0^\pi \log\left(1+2a^2+a^4-2a^2(1+\cos 2x)\right)dx \\ &= \int_0^\pi \log\left(1-2a^2\cos 2x+a^4\right)dx, \end{aligned}$$

so we may let $x \mapsto \frac{1}{2}x$ to give

$I(a)+I(-a) = \frac{1}{2}\int_0^{2x} \log\left(1-2a^2\cos x+a^4\right)dx$

We can then split the integral at π and set $x \mapsto 2\pi - x$ for the second integral:

$$\begin{aligned} I(a)+I(-a) &= \frac{1}{2}I\left(a^2\right)+\frac{1}{2}\int_\pi^{2\pi} \log\left(1-2a^2\cos x+a^4\right)dx \\ &= \frac{1}{2}I\left(a^2\right)+\frac{1}{2}\int_0^{\pi} \log\left(1-2a^2\cos x+a^4\right)dx \\ &= I\left(a^2\right). \end{aligned}$$

We thus have (applying (†))

$I(a) = \frac{1}{2}I\left(a^2\right)$.

Q82

Let

$$\begin{aligned} f(x) &= x^5-x^3+x-2 \\ f'(x) &= 5x^4-3x^2+1>0 \forall x \in R \end{aligned}$$

$\therefore f(x)$ is increasing $\Rightarrow$ only one real root.

$f(1)=-1, f(2)=24 \Rightarrow 1<\alpha<2$

Since, α is a root of $x^5-x^3+x-2=0$

$$\begin{aligned} &\Rightarrow \alpha^5-\alpha^3+\alpha=2 \\ &\alpha^4-\alpha^2+1=\frac{2}{\alpha} \\ &\left(\alpha^2+1\right)\left(\alpha^4-\alpha^2+1\right)=\frac{2}{\alpha}\left(\alpha^2+1\right) \end{aligned}$$

$$\alpha^6+1=2\alpha+\frac{2}{\alpha} \Rightarrow \alpha^6=2\alpha+\frac{2}{\alpha}-1$$

$$g(\alpha)=2\alpha+\frac{2}{\alpha}-1$$

$$g'(\alpha)=2-\frac{2}{\alpha^2}=\frac{2}{\alpha^2}\left(\alpha^2-1\right)=\frac{2}{\alpha^2}(\alpha-1)(\alpha+1)$$

g is increasing for $\alpha>1$.

$$g(1)<g(\alpha)<g(2)$$

$$3<g(\alpha)<4$$

$$3<\alpha^6<4$$

$$\left[\alpha^6\right]=3$$

$$\therefore \quad \left[\alpha^6\right]=3$$

Q83

As $(3053)^{456}-(2417)^{333}=(339\times 9+2)^{456}-(269\times 9-4)^{333}$

Remainder of given number is same as remainder of $2^{456}+4^{333}$ and $2^{456}+4^{333}=(64)^{76}+(64)^{111}$

$=(1+63)^{76}+(1+63)^{111}=(1+9\times 7)^{76}+(1+9\times 7)^{111}$

Hence the remainder is 2 .

Q84

$S_n=1!+2!+3!+4!+5!+6!+7!+\ldots\ldots$

$S_n=1+2+6+24+120+720+7I$ ...(1)

$S_n=873+7I$

$$\frac{S_n}{7}=\frac{873}{7}+I$$

$$\left[\frac{S_n}{7}\right]=124+I \quad \text{... (2)}$$

$$7\left[\frac{S_n}{7}\right]=868+7I$$

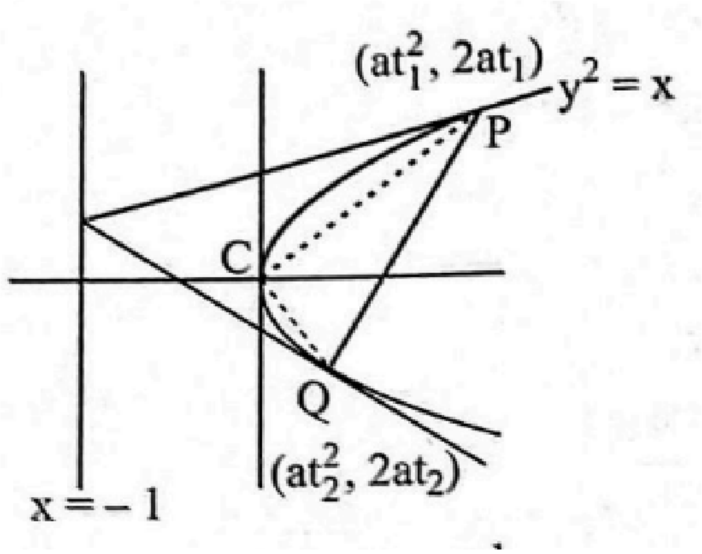
Eqn. (1) - Eqn. (2) $=5$

$$T=\sin^{-1}(\sin 5)=5-2\pi$$

$$\int_0^1 \frac{5-2\pi}{\sqrt{1-x^2}}dx=\frac{(5-2\pi)\pi}{2}=\frac{5\pi}{2}-\pi^2$$

$$\left(\frac{b}{c}+a\right)=\frac{2}{2}+5=6$$

Q85



$$at_1t_2 = -1 \Rightarrow t_1t_2 = \frac{-1}{a} = -4$$

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix} = \frac{1}{2}\left(2a^2t_1^2t_2 - 2a^2t_1t_2^2\right)$$

$$\Delta = \left|a^2t_1t_2\left(t_1 - t_2\right)\right| = \frac{1}{4}|t_1 - t_2|$$

$$\Delta = \frac{1}{4}\left(t_1 + \frac{4}{t_1}\right) = \frac{1}{4}\left[\left(\sqrt{t_1} - \frac{2}{\sqrt{t_1}}\right)^2 + 4\right]$$

$$\Delta_{\text{min.}} = 1 = M$$
$$4M = 4$$

Q86

$$S = \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{2}{25}\right) + \tan^{-1}\left(\frac{1}{18}\right) + \ldots\ldots\infty$$
$$= \tan^{-1}\left(\frac{2}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{2}{16}\right) + \tan^{-1}\left(\frac{2}{25}\right) + \tan^{-1}\left(\frac{2}{36}\right) + \ldots\ldots\infty$$

$$T_n = \tan^{-1}\left(\frac{2}{(n+1)^2}\right)$$

$$S = \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2}{r^2+2r+1}\right) = \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{(r+2)-r}{1+r(r+2)}\right) = \sum_{r=1}^{\infty}\tan^{-1}(r+2) - \tan^{-1}(r)$$

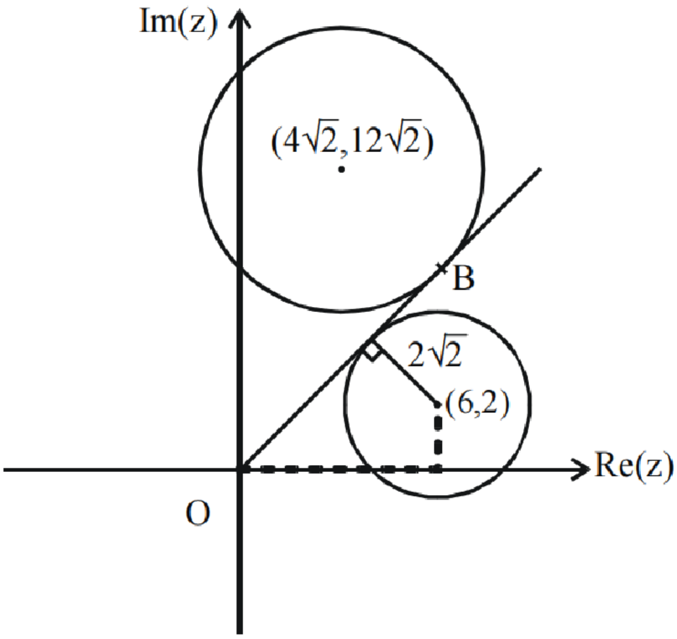
$$S = \lim_{n\to\infty}\tan^{-1}\left(\frac{3n^2+7n}{n^2+9n+10}\right) = \tan^{-1}(3)$$

(3) Let

Hence, $\tan S = \tan\left(\tan^{-1} 3\right) = 3$

Q87

Let the equation of OAB be $y = mx$



$$\Rightarrow \left|\frac{6\,m-2}{\sqrt{1+m^2}}\right| = 2\sqrt{2} \Rightarrow \frac{(3\,m-1)^2}{(1+m^2)} = 2$$

$$\Rightarrow \frac{9\,m^2-6\,m+1}{1+m^2} = 2 \Rightarrow 7\,m^2-6\,m-1=0$$

$$\Rightarrow 7\,m^2-7\,m+m-1=0$$

$$7\,m(m-1)+(m-1)=0 \Rightarrow m=1$$

$\Rightarrow y = x$ must be tangent to the second circle

$$\Rightarrow \left|\frac{12\sqrt{2}-4\sqrt{2}}{\sqrt{2}}\right| = k \Rightarrow k = 8$$

Q88

$$f(x) = x + \sin x - [x + \sin x] + [x - \sin x] + [x]$$

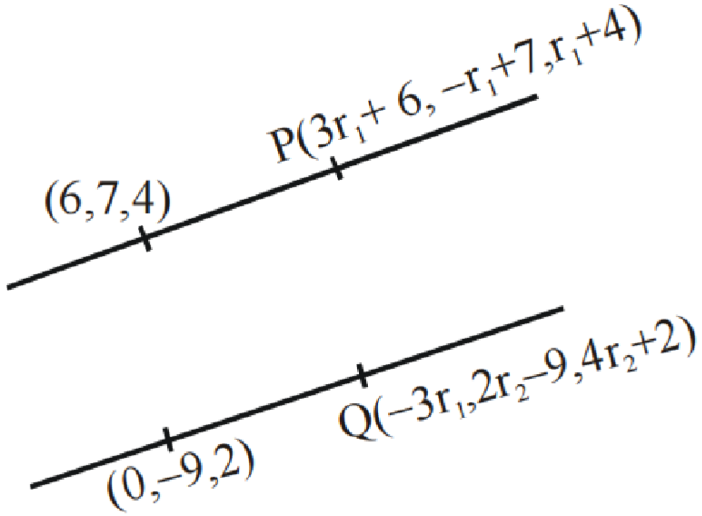
$x + \sin x = 0, 1, 2, 3 \Rightarrow x = 0, \alpha_1, \alpha_2, \alpha_3$

$x - \sin x = 0, 1, 2, 3 \Rightarrow x = 0, \beta_1, \beta_2, \beta_3$

f is continuous at $x = 0$, but discontinuous at $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, 1, 2, 3$

$\therefore$ Number of points of discontinuity $= 9$.

Q89



direction ratio of

$\mathrm{PQ} \equiv (-3r_2 - 3r_1 - 6, 2r_2 + r_1 - 16, 4r_2 - r_1 - 2)$

$\text{Now, } -3(-3r_2 - 3r_1 - 6) + 2(2r_2, r_1 - 16)$

$+4(4r_2 - r_1 - 2) = 0$

$7r_1 + 29r_2 = 22 \quad \ldots(1)$

$3(-3r_2 - 3r_1 - 6) - (2r_2 + r_1 - 16) + (4r_2 - r_1 - 2) = 0$

$7r_1 + 29r_2 = 22 \quad \ldots(4r_2 - r_1 - 2) = 0$

$3(-3r_2 - 3r_1 - 6) - (2r_2 + r_1 - 16) + (2)$

$11r_1 + 7r_2 = -4$

On solving (1) & (2)

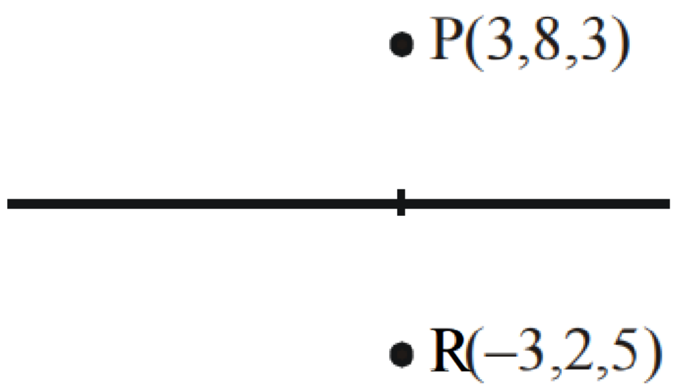
$\mathrm{r}_1 = -1, \mathrm{r}_2 = 1$

so $\mathrm{P}(3, 8, 3)$

Image of $\mathrm{P}(3, 8, 3)$ w.r.t Plane $3\mathrm{x} + 3\mathrm{y} - \mathrm{z} = 11$

$\mathrm{R}\ (-3, 2, 5)$

$\mathrm{a} + \mathrm{b} + \mathrm{c} = 4$



Q90

$T_1=(\overrightarrow{\mathbf{a}}\times\overrightarrow{\mathbf{b}})\times(\overrightarrow{\mathbf{c}}\times\overrightarrow{\mathbf{d}})$

$T_2=(\overrightarrow{\mathbf{a}}\times\overrightarrow{\mathbf{c}})\times(\overrightarrow{\mathbf{d}}\times\overrightarrow{\mathbf{b}})$

$T_3=(\overrightarrow{\mathbf{a}}\times\overrightarrow{\mathbf{d}})\times(\overrightarrow{\mathbf{b}}\times\overrightarrow{\mathbf{c}})$

$$\begin{aligned} T_1&=\left[\overrightarrow{\mathbf{c}}\quad\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{a}}\right]\overrightarrow{\mathbf{b}}-\left[\overrightarrow{\mathbf{c}}\quad\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{b}}\right]\overrightarrow{\mathbf{a}}\\ \text{Then, } T_2&=\left[\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{b}}\quad\overrightarrow{\mathbf{a}}\right]\overrightarrow{\mathbf{c}}-\left[\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{b}}\quad\overrightarrow{\mathbf{c}}\right]\overrightarrow{\mathbf{a}}\\ T_3&=\left[\overrightarrow{\mathbf{a}}\quad\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{c}}\right]\overrightarrow{\mathbf{b}}-\left[\overrightarrow{\mathbf{a}}\quad\overrightarrow{\mathbf{d}}\quad\overrightarrow{\mathbf{b}}\right]\overrightarrow{\mathbf{c}} \end{aligned}$$

$\text{Sum }=-2\left[\overrightarrow{\mathbf{b}}\quad\overrightarrow{\mathbf{c}}\quad\overrightarrow{\mathbf{d}}\right]\overrightarrow{\mathbf{a}}+k\overrightarrow{\mathbf{a}}=0$

$\left[\text{ Given}\left[\vec{b}\quad\overrightarrow{\mathbf{c}}\quad\vec{d}\,\right]=48\right]$

Hence, $\quad k=96$