

# Using Nodal Domain Decomposition With Relaxation Method For Dynamic Characteristic Solving

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Finite element method (FEM) is necessary to analyze the dynamic characteristics of electromagnetic mechanisms. Parallelization by domain decomposition (DD) technology makes the solving of FEM more effective. In this paper, a new finite element solution technique called nodal domain decomposition with relaxation (NDDR) is used to solve the dynamic characteristics of the electromagnetic mechanism. This method can improve the parallelism to accelerate of dynamic characteristic solving. To further improve the solving efficiency of NDDR method, we supposed a new idea to accelerate the process.

**Index Terms**—Domain decomposition(DD), dynamic analysis, electromagnetic apparatus, finite element method(FEM), parallel algorithm

## I. INTRODUCTION

IN electromagnetic apparatus, electromagnets are often used as driving components to drive contact splitting or other load work. Although the static characteristic is usually employed to judge electromagnetic device in engineering design, it is the dynamic characteristic that determines the movement process of the electromagnets. Therefore, the analysis of dynamic characteristics plays an important role for the design of electromagnetic apparatus[1].

In the computation of dynamic characteristic, the electromagnetic field distribution is necessary to be calculated when the core at different positions, in order to solve the magnetic field force, and then solve the motion characteristics of the electromagnetic mechanism[2]. The finite element method (FEM) is generally used to calculate the electromagnetic field distribution. Many scholars have studied the acceleration of solving FEM in electromagnetic field. Recently, some scholars have proposed a new nodal domain decomposition with relaxation method (NDDR) for electromagnetic field parallel calculation of electromagnetic apparatus, which has achieved good acceleration effect. [4]

The computation of dynamic characteristic will consume a lot of computing resources, it is necessary to find way to accelerate the solving process. The NDDR method achieves a good acceleration effect in the calculation of the static electromagnetic field distribution, but few scholars pay attention to it's usage in dynamic characteristic solving. What's more, it is believed NDDR method's efficiency can be further improved. In this paper, we propose a algorithm to improve the efficiency of NDDR method, and use this method to accelerate the solution of the dynamic characteristic.

## II. NODAL DOMAIN DECOMPOSITION WITH RELAXATION METHOD

Both conventional domain decomposition (DD) method and NDDR method focus on accelerating the solution of FEM by parallelization. Conventional DD method is shown in Fig.1(a), the whole solution domain is decomposed into several subdomains and processed separately by multiple cores assigned to

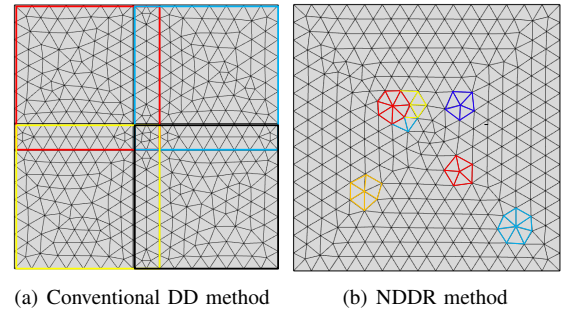


Fig. 1. Comparison between conventional DD method and NDDR method

the CPU. The fewer nodes and units that each core needs to process, the higher solution efficiency can obtain. Therefore, NDDR method propose a extreme idea, which means only one node in a subdomain, as shown in Fig. 1(b). In this way, the degree of parallelism is greatly increased. In the case where the number of cores is sufficient, each core only needs to solve one node unit, and the solving efficiency can be greatly improved.

Although NDDR method has achieved good acceleration in static electromagnetic solving, it's efficiency can still be improved. In the case study of NDDR method proposed by[4], thousands of GPU core are used to solve the electromagnetic distribution, but the acceleration effect obtained is only tens of times. What's more, with the number of nodes increases, each core need to process multiple nodes, and the solution efficiency is further reduced.

## III. DYNAMIC ANALYSIS WITH NDDR

To solve the dynamic characteristic of electromagnetic mechanisms, we need to calculate differential equations for electromagnetic filed distribution and dynamic properties. The solution of the electromagnetic field distribution is consistent with the solution of the static magnetic field. The differential equations of the electromagnetic filed are solved by FEM. It can be described as follows:

$$\begin{cases} \nabla \cdot A = B \\ \nabla \cdot (v \nabla A) = -J \end{cases} \quad (1)$$

Where  $A$  is magnetic filed potential vector,  $B$  is magnetic induction,  $\nu$  is magnetic permeability and  $J$  is current density.

After the cells of each sub-network are discretized by Galerkin's method[5], a set of equations in the unit can be obtained:

$$[S^e][A^e] = [F^e] \quad (2)$$

Where  $[S^e]$  relate to the node coordinates in the element,  $[A^e]$  represent magnetic filed potential vector of each node and  $[F^e]$  relate to the size of element and the current density in the element.

During the usual finite element solution process, each unit need to be assembled after unit analyze. It means we finally solve a large sparse symmetric positive definite (SPD) system. The finite element solution of the dynamic characteristic in this paper is achieved by NDDR method, which omits the assembly process of the matrix and improves the efficiency of the solution through parallelization.

In the dynamic characteristic electromagnetic field solving process, the moving part of each time step is different, and the whole domain needs to be re-meshed and solved. Considering that the electromagnetic field at the same position does not change much, we will use the final value in the previous solution at the same position as the initial value of the next solution to achieve the iterative acceleration effect.

The differential equations of dynamic characteristics include circuit equations and motion equations.

$$\begin{cases} u = iR + \frac{d\psi}{dt} \\ m \frac{d^2x}{dt^2} = F - F_f \end{cases} \quad (3)$$

Where  $u$ ,  $i$ ,  $R$  represent the voltage, current and resistance of the coil,  $\psi$  is the flux linkage,  $m$  is the armature mass,  $F$  is the electromagnetic attraction and  $F_f$  is the load reaction force.

To simplify the solving process, converting (3) to the form of the first derivative equations as shown:

$$\frac{di}{dt} = \frac{u - iR}{L} - \frac{i}{L} \frac{dL}{dt} \quad (4)$$

$$\frac{dv}{dt} = \frac{1}{m} (F - F_f) \quad (5)$$

$$\frac{dx}{dt} = v \quad (6)$$

Where  $L$  is the magnetic system inductance.

In this paper, we use the weak coupling method to solve the dynamic characteristics of the electromagnetic system. The solution flowchart is shown in Fig. 2. The difference equation in this process can be solved by the fourth-order Runge-Kutta method.

#### IV. SCREW CONTACTOR MODEL

In order to verify that the NDDR method can accelerate the solution of electromagnetic field dynamics, this paper uses a dynamic model of the solenoid contactor to calculate, and finally compares with the solution efficiency and solution accuracy of COMSOL. The model that needs to be solved is shown in Fig. 3.

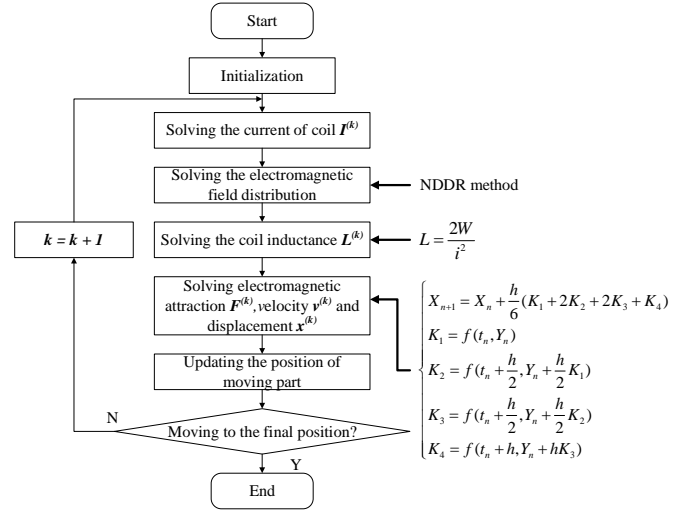


Fig. 2.

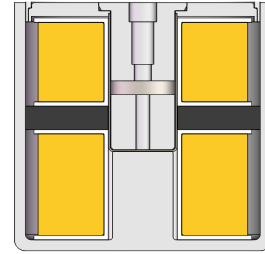


Fig. 3. The model of solenoid contactor

#### REFERENCES

- [1] S.- Wang, T. Miyanao, and M. Hubbard, "Electromagnetic field analysis and dynamic simulation of a two-valve solenoid actuator," IEEE Transactions on Magnetics, vol. 29, no. 2, pp. 1741–1746, Mar. 1993.
- [2] Y. Jiaxin, L. Huimin, M. Guangcheng, C. Shuqing, and C. Zhaowen, "Research on the dynamic calculation model for a dc solenoid electromagnetic contactor and its contact characteristics in break process," in 2015 IEEE 61st Holm Conference on Electrical Contacts (Holm), 2015, pp. 191–194.
- [3] A. Dziekonski, A. Lamecki, and M. Mrozowski, "GPU-accelerated finite element method," in 2016 IEEE MTT-S International Conference on Numerical Electromagnetic and Multiphysics Modeling and Optimization (NEMO), 2016, pp. 1–2.
- [4] P. Liu and V. Dinavahi, "Matrix-Free Nodal Domain Decomposition With Relaxation For Massively Parallel Finite-Element Computation of EM Apparatus," IEEE Transactions on Magnetics, vol. 54, no. 9, pp. 1–7, Sep. 2018.
- [5] Rao, S. S. . The Finite Element Method in Engineering. Pergamon Press, 1982, pp. 194-198.