

# An Improved Domain Decomposition with Relaxation Method Used for Dynamic Simulation of Electromagnetic Devices

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Finite element method (FEM) is necessary to analyze the dynamic characteristics of electromagnetic mechanisms. Parallel technology makes the computation of FEM more effective, such as domain decomposition (DD) method. In this paper, a new finite element solution technique called nodal domain decomposition with relaxation (NDDR) is used to solve the dynamic characteristics of the electromagnetic device. This method can improve the parallelism to accelerate the dynamic characteristic solving. To greatly improve the solving efficiency of NDDR method, we introduce Robin-type transmission condition to it.

**Index Terms**—Domain decomposition (DD), Dynamic analysis, Electromagnetic apparatus, Finite element method (FEM), Parallel algorithm

## I. INTRODUCTION

IN electromagnetic apparatus, electromagnetic devices are often used to drive contact separating or other load work. Although the static simulation is usually performed to judge the characteristic of the electromagnetic device in engineering, it is the dynamic characteristic that determines the movement process of the electromagnetic device. Therefore, analysis of dynamic characteristics plays an important part in the design of electromagnetic apparatus [1].

In the computation of the dynamic characteristic, the electromagnetic field distribution is necessary to be calculated when the armature moves at different positions [2]. The finite element method (FEM) is widely used to calculate the electromagnetic field distribution. Many scholars have studied the acceleration of FEM process in electromagnetic field. Recently, some scholars have proposed a new nodal domain decomposition with relaxation method (NDDR) for electromagnetic field parallel calculation, which has achieved excellent acceleration effect [3].

The computation of dynamic characteristic always consumes a lot of computing resources, so it is necessary to find a way to accelerate the solving process. The NDDR method achieves a good acceleration effect in the calculation of the static electromagnetic field distribution, however, few scholars pay attention to its use in dynamic characteristic solving. What's more, it is believed that NDDR method's efficiency need to be further improved. In this paper, we apply the Robin type condition in the interface of two adjacent domains to improve the convergence speed of NDDR method, and apply this method to accelerate the solution of the dynamic characteristic.

## II. IMPROVED NODAL DOMAIN DECOMPOSITION WITH RELAXATION METHOD

Both conventional domain decomposition (DD) method and NDDR method focus on accelerating the solution of FEM using parallel techniques. Conventional DD method is shown in Fig. 1(a), the whole solution domain is decomposed into several subdomains and processed separately by different cores of

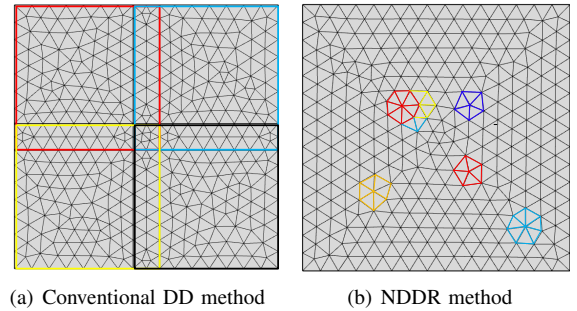


Fig. 1. Comparison between conventional DD method and NDDR method

CPU. The fewer nodes and elements that each core is required to process, the higher solution efficiency can obtain. Therefore, NDDR method proposes an extreme idea, which means only one node in a subdomain, as showed in Fig. 1(b). In this way, the degree of parallelism is greatly increased. In the case where the number of cores is sufficient, each core only needs to solve one node unit, and the solving efficiency can be significantly improved.

Although NDDR method has achieved good acceleration in static electromagnetic solving, its efficiency can still be improved. In the case study of NDDR method proposed by [3], thousands of GPU cores are used to solve the electromagnetic distribution, but the acceleration effect obtained is only tens of times. What's more, with the number of nodes increases, each core need to process multiple nodes, and the solution efficiency is further reduced.

In conventional DD method, using Robin-type transmission condition can obtain faster convergence speed than Dirichlet-type transmission condition [4]. Therefore, we introduce Robin-type transmission condition to NDDR method, to improve the solution efficiency of NDDR.

## III. DYNAMIC ANALYSIS WITH NDDR

To solve the dynamic characteristic of an electromagnetic device, we need to calculate differential equations for electromagnetic field distribution. The solution of the electromagnetic

field distribution is consistent with the solution of the static magnetic field. The differential equations of the electromagnetic field are solved by FEM. It can be described as follows:

$$\begin{cases} \nabla \cdot A = B \\ \nabla \cdot (v \nabla A) = -J \end{cases} \quad (1)$$

where  $A$  is the magnetic field potential vector,  $B$  is the magnetic induction,  $v$  is the magnetic permeability and  $J$  is the current density.

After each sub-network are discretized by Galerkin's method [5], a set of equations in the element can be obtained:

$$[S^e][A^e] = [F^e] \quad (2)$$

where  $[S^e]$  relates to the node coordinates in the element,  $[A^e]$  represents magnetic field potential vector of each node and  $[F^e]$  relate to the size of element and the current density in the element.

During the usual finite element solution process, each element needs to be assembled after element analysis. Then we need to solve a nonlinear large sparse symmetric positive definite (SPD) system. The finite element solution of the dynamic characteristic in this paper is achieved by improved NDDR method, which avoids the assembly process of the matrix and improves the efficiency of the solution through parallelization.

In the dynamic characteristic electromagnetic field solving process, the position of the armature in each time step is different, and the whole domain needs to be re-meshed and re-solved. Considering that the electromagnetic field at the same position does not change much, we can use the final value in the previous solution at the same position as the initial value of the next solution to achieve the iterative acceleration effect.

The differential equations of dynamic characteristics include circuit equations and motion equations:

$$\begin{cases} u = iR + \frac{d\psi}{dt} \\ m \frac{d^2x}{dt^2} = F - F_f \end{cases} \quad (3)$$

where  $u$ ,  $i$ ,  $R$  represent the voltage, current and resistance of the coil,  $\psi$  is the flux linkage,  $m$  is the mass of armature,  $F$  is the electromagnetic force and  $F_f$  is the load reaction force.

To simplify the solving process, converting (3) to the form of the first derivative equations as shown:

$$\begin{cases} \frac{di}{dt} = \frac{u - iR}{L} - \frac{i}{L} \frac{dL}{dt} \\ \frac{dv}{dt} = \frac{1}{m} (F - F_f) \\ \frac{dx}{dt} = v \end{cases} \quad (4)$$

where  $L$  is the magnetic system inductance.

In this paper, we use the weak coupling method to solve the dynamic characteristics of the electromagnetic system. The solving process is shown in Fig. 2. The difference equation in this process can be solved by fourth-order Runge-Kutta method.

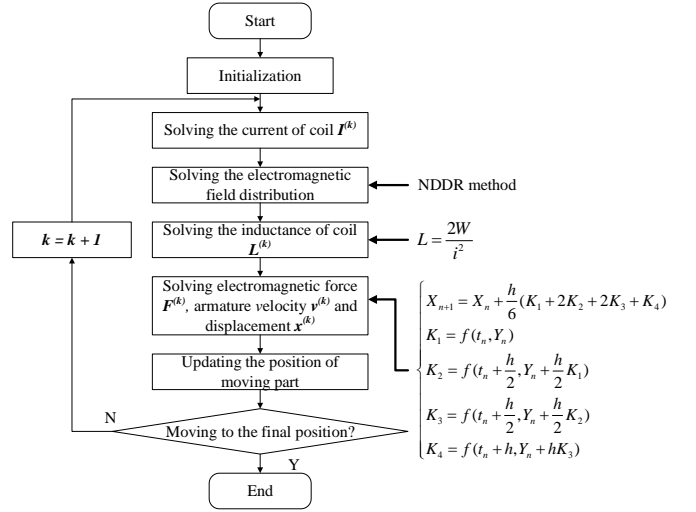


Fig. 2. Dynamic solving process with NDDR

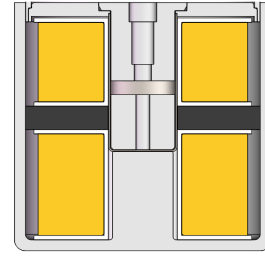


Fig. 3. The model of solenoid contactor

#### IV. CASE STUDY

In order to verify that the NDDR method can accelerate the solution of electromagnetic field dynamics, this paper uses a dynamic model of the solenoid contactor to calculate, and finally compares with the solution efficiency and solution accuracy of COMSOL. The model that needs to be solved is shown in Fig. 3.

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