Project Report

PH 556 : Astrophysics

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Aim

Study an RR-Lyrae variable and calculate luminosity, distance, temperature and spectral class.

Variability period - Using images across varying brightness.

Total Luminosity from band-wise period-luminosity relations.

Distance by comparing luminosity and apparent brightness from the images.

Radius using a radius-period relation [1].

Temperature using total luminosity and radius.

Spectral Class using temperature co-relations.

Assumptions and Givens

We assumed that the star is an RR-Lyrae variable and follows known period-luminosity relations and radius-period relations.

Over the course of the project, we realized we also needed to assume that the metallicity takes a representative value, and that we required mean apparent magnitudes from catalogues for curve-fitting due to limited phase range of the data.

Verification

Compared obtained values to the ones from existing literature: [1] [2] [3] [4]

Target Star

We have chosen the target star UZ UMa (Simbad link), which is an RRab variable[2].

Magnitude and Coordinates

RA = 08°18' 53".9127454200, **DEC** = +73°05' 47".926302000

Apparent Magnitudes : [3] **g** 14.7030 **r** 14.1269 **i** 14.2903 **z** 14.3070

Methods

Photometry

We had 35 images across bands: 'z': 8, 'i': 8, 'g': 9, 'r': 9. Of these, one of the images in the z band had flaring artifacts, possibly due to a bright source like the moon. We discarded this image. We performed aperture photometry and PSF photometry and used the results from the latter for further calculations.

Source Selection

For both zero-point corrections and PSF photometry, we need to pick sources in the image which are within the bounds of the image and have significant magnitudes so as to avoid background domination in their measurements without being so bright as to saturate the camera. We also require the sources to not have blended with artifacts.

We queried the Panstarrs-1 catalogue from Vizier for the known magnitudes of the sources, and we used SExtractor to select good sources from our image. We limited the maximum magnitude to 18 and selected for point-like sources by limiting the FWHM to 2 arcseconds.

Then we compared these and selected the common sources for further steps.

PSF Photometry

Using the package PSFex, we fit various point-spread functions to the good sources selected earlier. The standard Moffat distribution is satisfactory for our purposes, so we use that.

We then use the best-fit PSF on our good sources to obtain instrumental magnitudes and error bars for each of them. Comparing with Panstarrs-1 magnitudes, we find the expected linear relation. But this line does not pass through the origin as we have not yet performed zero-point correction.

Zero-point corrections

For aperture photometry, we manually selected 10 good stars for zero-point correction. For PSF photometry, our cross-matching of good sources from SExtractor and Panstarrs-1 greatly simplifies and improves our correction procedure. As with our target, we use the best-fit PSF to determine

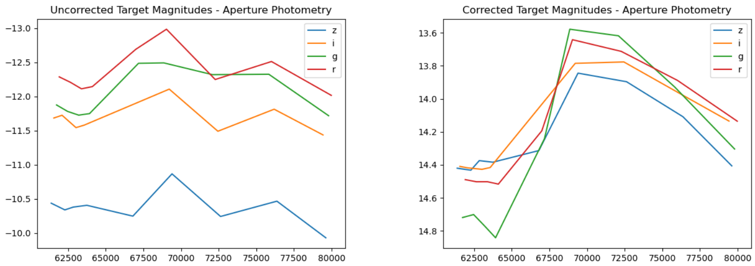
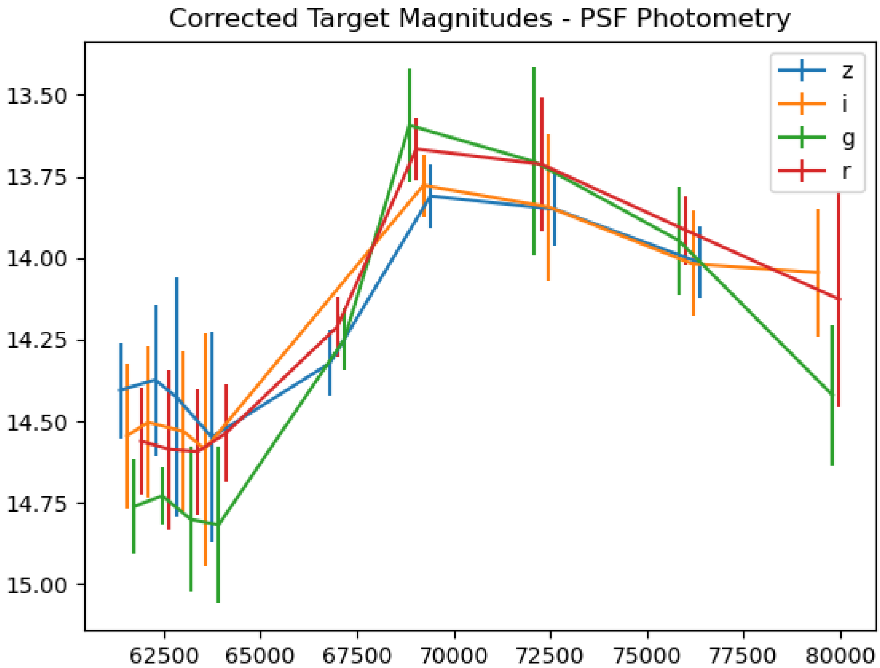


Figure 1: Uncorrected and corrected magnitudes from aperture photometry showcasing the importance of zero-point correction

instrumental magnitudes of all of the good sources, subtract from the catalog magnitudes, and perform 3-σ clipping to obtain the mean and error of the zero-point (which we add in quadrature to instrumental error for total error).

Performing this procedure for all 34 images across bands allowed us to plot the time-series data for magnitudes with error bars, which clearly shows the behavior of an RR-Lyrae light curve.

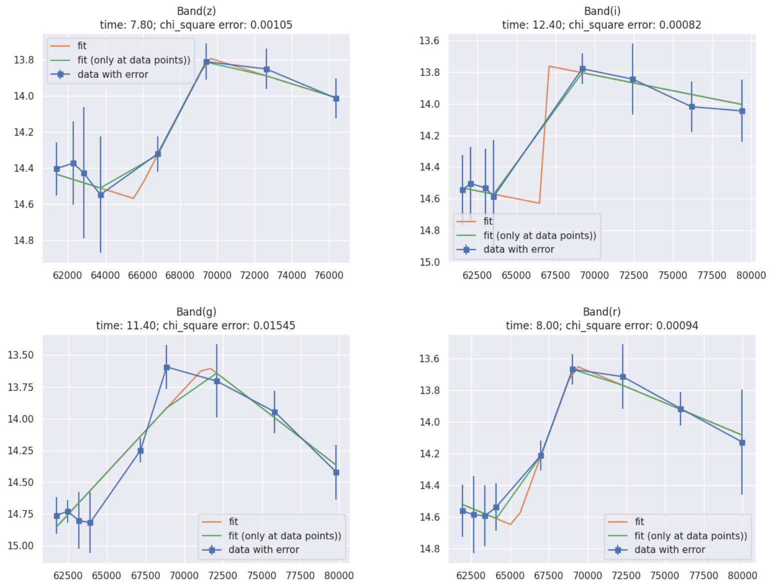


Period Estimation

For each attempt in this section, we have worked with the bands individually.

Template Matching

This approach can be attributed to this work by James Long[5]. The code takes in our data points for the light curve, and by using previously sampled data, tries to construct a statistical fit to our points.



This gave pleasing but volatile results but very high errors, so we instead attempted curve-fitting.

Curve Fitting

We have used the `scipy.optimize.curve_fit`[6] to fit these functions to our data. To prevent over-fitting due to a large number of parameters and a small number of data-points, we have had to iterate over time period values and minimise the χ^2 minimization[7] error.

Most RR Lyrae periods range from a few hours to a few days, so we narrow the range and decrease the step size in the loop to get a more precise estimate.
We also fixed the values of A_0 (y-shifting constant/zero point of the wave) with the same motivation. This used different approaches for each curve.

Sawtooth signals

Here we used the `scipy.signal.sawtooth` function[8], parametrized as:

$$m = A_0 + A_1 \text{sawtooth}(\omega t + \phi, w) \tag{1}$$

where m is the apparent magnitude, A_i are the scaling/amplitude constants, ω, ϕ are the frequency, phase and w is the ratio of width of the sawtooth(distance of peak from a trough) and the wavelength.

We fix the value of A_0 to be the approximate mean of the minima and maxima since the final fit would be symmetric about this value.

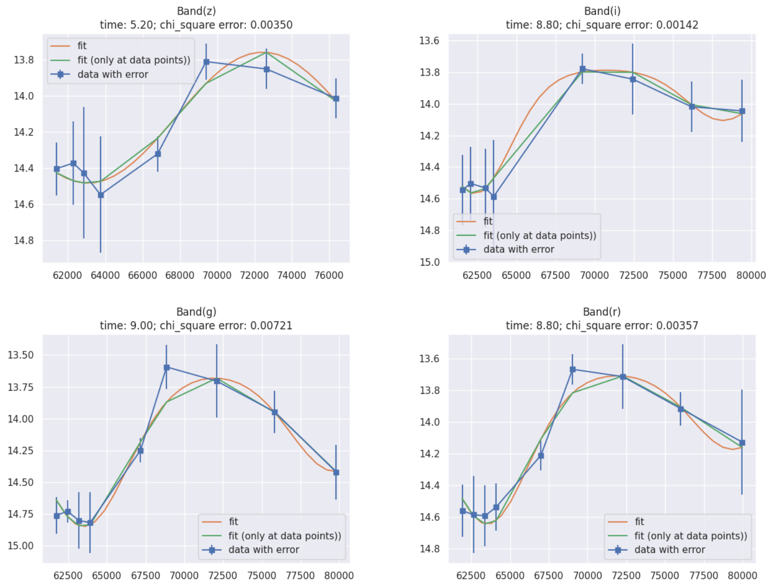
Periodic Gaussian signals

In this method, we use the below function in order to fit our curve

$$m = A_0 + \sum_{i=0}^{N-1} A_{i+1} \frac{e^{\kappa[2\pi(x - \frac{i}{N})]}}{2\pi I_0(\kappa)} \tag{2}$$

[9] where $I_0(k)$ is the modified Bessel function of order zero. We truncate the summation to two terms for the second harmonic and only the first term for the first harmonic. We also truncate the expansion of $I_0(\kappa)$ two the first 3-4 terms.

$$I_0(x) = \sum_{m=0}^{\text{inf}} \frac{x^{2m}}{2^{2m}.(m!)^2} \tag{3}$$



And as expected we get a better fit for the second harmonic than the first, with plots being as follows:

Sinusoidal signals

We tried fitting the data with a superposition of first and second harmonics:

$$m = A_0 + A_1 \sin(\omega t + \phi_1) + A_0 \sin(2\omega t + \phi_2) \tag{4}$$

Since our data only extends over 5 hours while the period is 11 hours, and our data does not capture the low brightness region of the light curve, curve fitting leads to an incorrect mean magnitude and hence bad results. Expecting this, we took the values of A_0 to be the mean apparent magnitudes from the Panstarrs-1 catalogs.[3]

Working with this, we get the following plots for each band:

We decided to pick the sinusoidal curve fitting for our results due to the following considerations:

1. The periodic Gaussian signals lead to over-fitting to our data. Since our data does not cover one full cycle of the star, we need to be very careful about curves that have a high curvature in the low brightness region, which in the full cycle would be preceded by a flat region.

2. The sawtooth function leads to underfitting, visible in the large variation in periods obtained across bands.

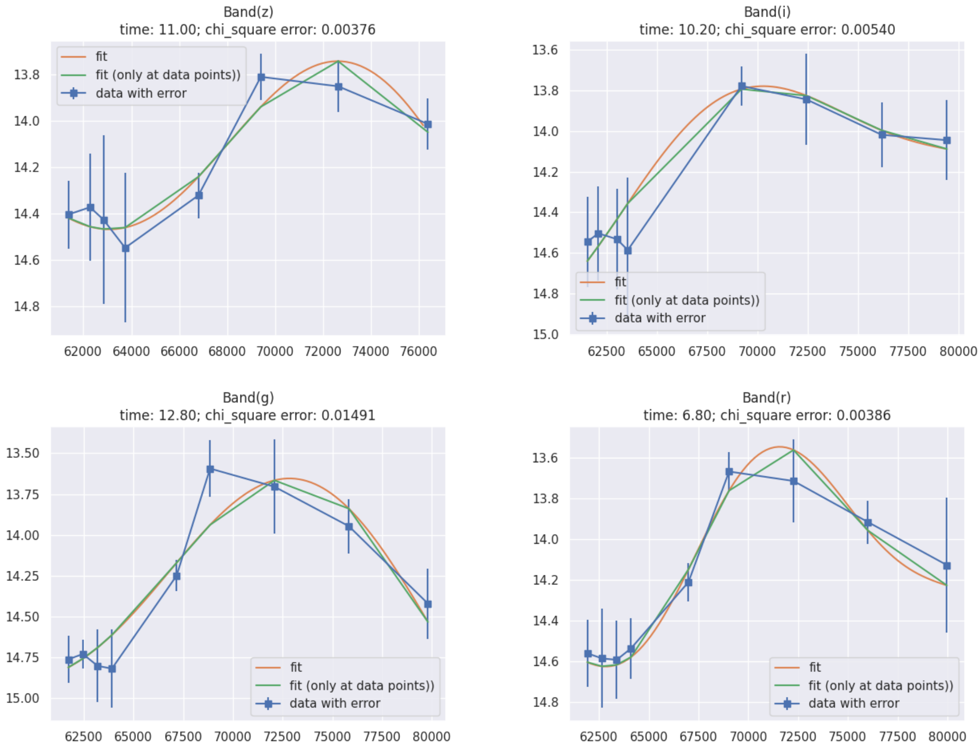
The sinusoidal curve leads to consistent results across bands with very low fitting errors, with the exception of the r band. Since our parameter calculation starting from luminosity is done on the i-band due to availability constraints of PL relations in literature, we go ahead with the period as **10.2 hours** or **0.425 days** obtained from the i band with sinusoidal fitting.

Calculation of parameters

Once we had the period, calculating all other parameters in a cascading manner was fairly easy, considering we already had all the relations from known literature.

The values retrieved are as below. Some of these could be verified from catalogs, with respective sources cited, others are deducible. Mass is speculative.

$$M_I = 0.4711 - 1.1318 log_{10}(P) + 0.2053 log_{10}(Z) \tag{5}$$



From [10]. We use the I band since it has a significant overlap with the i band. Period is in days. Z, the metallicity, is taken as 0.002 as the representative value.[11]

$$log_{10}(\frac{R}{R_\odot}) = 0.774 + 0.580log_{10}(P) - 0.035log_{10}(Z) \tag{6}$$

[1]

For the distance, we have:

$$m - M = 5\log d - 5 \tag{7}$$

And temperature can be calculated by:

$$L = 4\pi R^2 \sigma T^4 \tag{8}$$

Results

Table 1: UZ UMa Parameters: i Band

Parameter	**Calculated Value**	**Known Value**	**Error**
Period	0.425 days	0.467 days	8.93%
Mean Absolute Magnitude	0.3376	-	-
Distance	6201.29 pc	5701.25 pc	8.77%
Luminosity	$62.36L_\odot$	-	-
Radius	$4.498\ R_\odot$	-	-
Temperature	7657.9 K	8260 K	7.86%
Spectral Class	A8	A5	-

Discussion

We faced a series of challenges along the course of the project. Some of them are listed below:

- Certain images had flaring artifacts due to some external bright source. One of them had to be discarded, and the others had large errors in the zero-point due to lack of good sources, so were not as useful.
- The images were not evenly distributed throughout the cycle, but rather a lot of them were bunched together in a particular phase region, and the complete set of images also only spanned half of the complete cycle. This made period-fitting a challenge and we had to try many different curves.
- The non-uniformity of a typical RR Lyrae curve makes it harder to fit with any standard periodic curve if we do not have lot of datapoints.
- Various empirical relations for absolute magnitude and radius exist, but all of them take metallicity into account, and we had to use a representative value for metallicity since that cannot be obtained without spectroscopy.

The results, which had corresponding values in the existing literature, corroborated those values, with errors at roughly 8 percent. This can be deemed to be satisfactory, considering the sparse data we have been working with.

References

[1] In: (). URL: https://adsabs.harvard.edu/full/2004ASPC..310..502M.
[2] In: (). URL: https://arxiv.org/abs/astro-ph/0601432.
[3] In: (). URL: https://ui.adsabs.harvard.edu/abs/2016arXiv161205560C/abstract.
[4] In: (). URL: https://articles.adsabs.harvard.edu/pdf/1960ApJ...131..632H.
[5] In: (). URL: https://github.com/longjp/rr-templates.
[6] In: (). URL: https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.curve_fit.html.
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