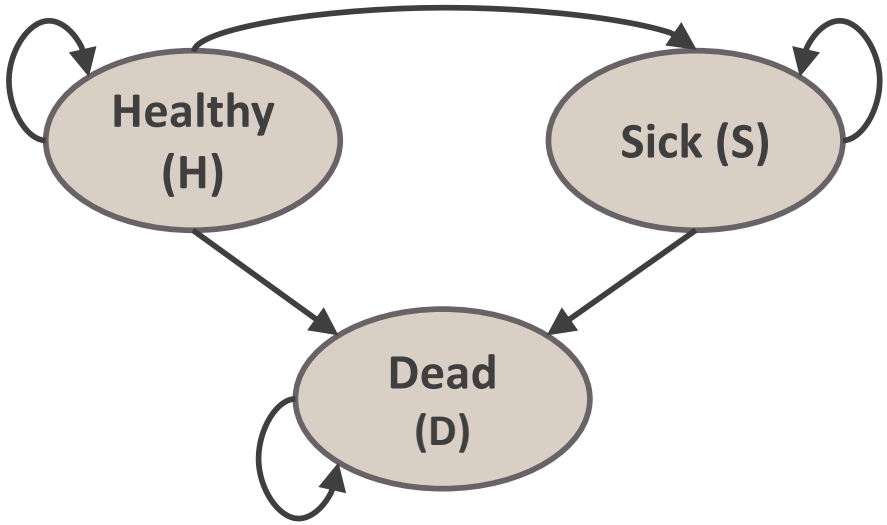
Matrix Algebra

DARTH workgroup

Why matrix algebra in decision analysis?

Calculations in decision analysis involve a lot of multiplying and adding one of the strengths of matrix algebra

State-transition model



An example that involves multiplication and addition is the estimation of the proportion of a cohort in a specific health states

people in Sick =

$(p_{Sick,Sick})$*Pr(*Sick*) +

$(p_{Healthy,\ Sick})$*Pr(*Healthy*) +

$(p_{Dead,\ sick})$*Pr(*Dead*)

Matrix Addition and Subtraction

- Matrix addition and subtraction are ***element-wise*** operations.
- Only matrices with the same dimensions can be added/subtracted

$$\begin{pmatrix} 1 & 1 \\ 5 & 8 \\ -2 & 3 \\ 4 & 0 \\ 1 & -6 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 9 & 1 \\ -2 & -3 \\ 3 & 1 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 14 & 9 \\ -4 & 0 \\ 7 & 1 \\ 8 & -4 \end{pmatrix}$$

In R

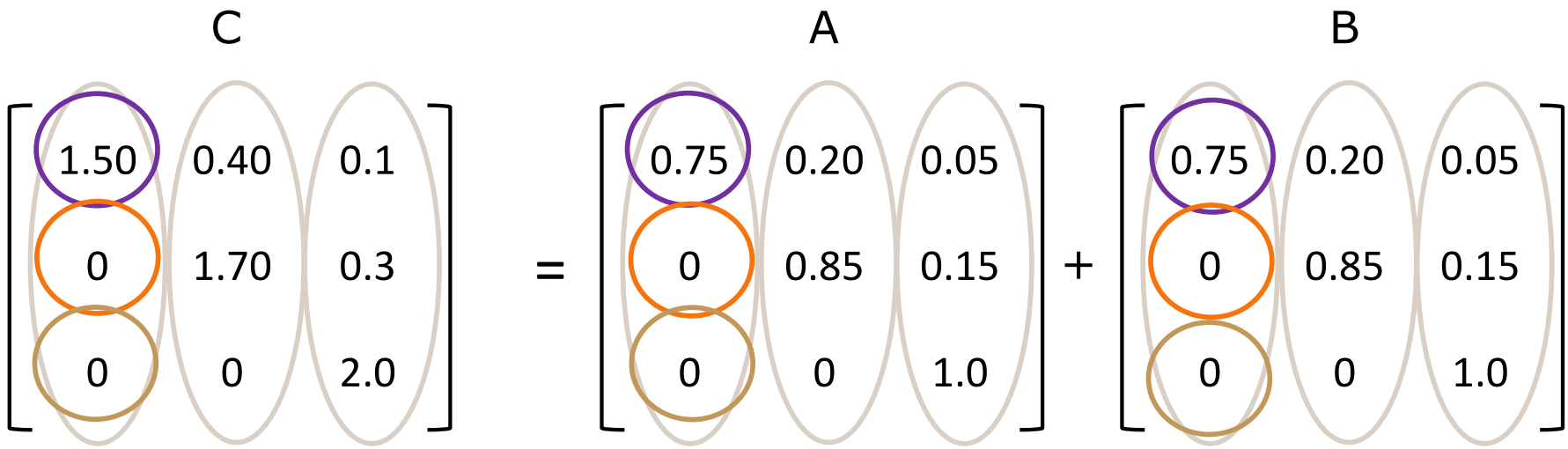
```
> matA <- matrix(rep(1.5, 6),
+                nrow = 2,
+                ncol = 3)
>
> matB <- matrix(1:10,
+                nrow = 2,
+                ncol = 5)
>
> matC <- matrix(rep(1:5, 2),
+                nrow = 2,
+                ncol = 5)
>
> matB + matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    2    6   10    9   13
[2,]    4    8    7   11   15
> matA + matB
Error in matA + matB : non-conformable arrays
```

This works similar for subtractions

Matrix Addition and Subtraction (2)

n x m m x p

3 x 3 3 x 3



In R: `m_A + m_B`

This works similar for subtractions

Matrix Multiplication

Multiple a matrix by a number
Each element in the matrix multiplied with that number

$$2 \times \begin{pmatrix} 8 & 0 & 2 & 2 \\ -2 & 5 & 3 & -5 \\ 5 & 7 & -3 & 0 \\ 3 & -1 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 16 & 0 & 4 & 4 \\ -4 & 10 & 6 & -10 \\ 10 & 14 & -6 & 0 \\ 6 & -2 & 2 & -4 \end{pmatrix}$$

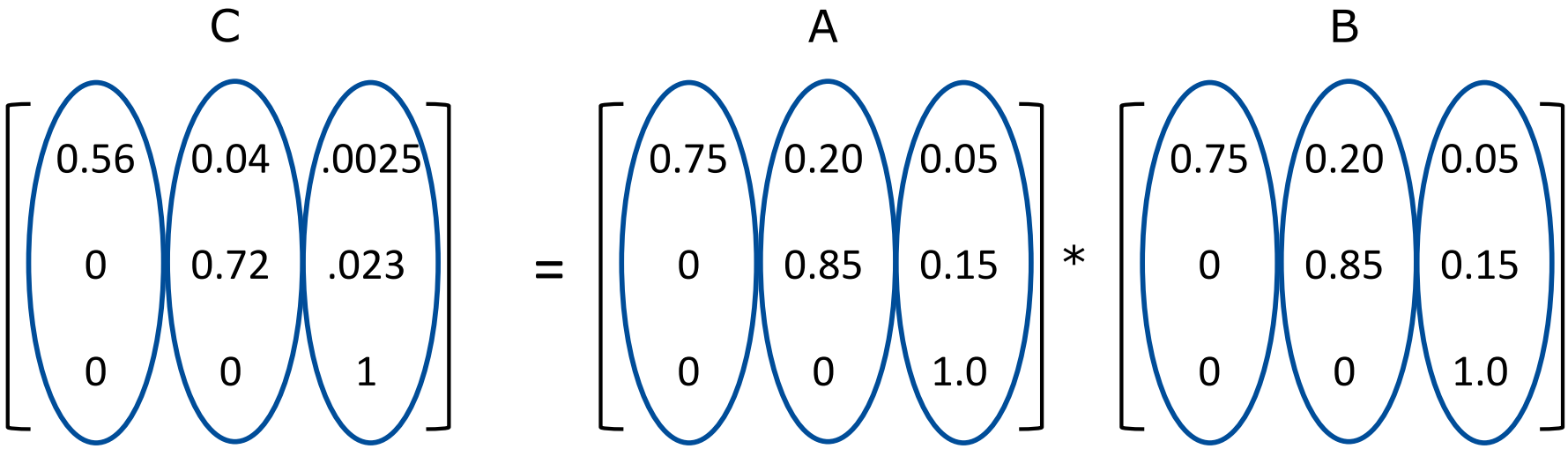
In R

```
> matB <- matrix(1:10, nrow = 2,ncol = 5)
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
```

```
> 2 * matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    2    6   10   14   18
[2,]    4    8   12   16   20
```

Matrix Multiplication (2)

Matrix Element-wise Multiplication



In R: `m_A * m_B`

This works similar for divisions

Matrix Multiplication (3)

Multiple a matrix by a matrix

$$\begin{pmatrix} 8 & 0 & 2 & 2 \\ -2 & 5 & 3 & 1 \end{pmatrix} \times \begin{pmatrix} 8 & 0 & 2 \\ -2 & 5 & 3 \\ 5 & 7 & -3 \\ 3 & -1 & 1 \end{pmatrix} = \begin{pmatrix} 80 & 12 & 12 \\ -8 & 45 & 3 \end{pmatrix}$$

2 x 4 4 x 3 2 x 3

Matrix multiplication requires the first matrix to have the same number of *columns* and the number of *rows* in the second matrix.

■ = 8 x 8 + 0 x (-2) + 2 x 5 + 2 x 3 = 80

■ = 8 x 0 + 0 x 5 + 2 x 7 + 2 x (-1) = 12

Matrix Multiplication in R

- The standard multiplication operator in R * gives *element-wise* multiplication

```
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
> matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    2    4
[2,]    2    4    1    3    5
```

```
> matB * matC
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    9   25   14   36
[2,]    4   16    6   24   50
```

- Matrix multiplication is achieved using the %*% operator

```
> matD <- matrix(c(8, -2, 0, 5, 2, 3, 2, 1), nrow = 2, ncol = 4)
> matE <- matrix(c(8, -2, 5, 3, 0, 5, 7, -1, 2, 3, -3, 1), nrow = 4, ncol = 3)
> matD %*% matE
     [,1] [,2] [,3]
[1,]   80   12   12
[2,]   -8   45    3
```

Matrix Transpose

- The transpose of a matrix is where the first *row* of the original matrix becomes the first *column* of the transposed matrix
- This is sometimes required to match the dimensions of matrices for calculations

$$B = \begin{pmatrix} 1 & 1 \\ 5 & 8 \\ -2 & 3 \\ 4 & 0 \\ 1 & -6 \end{pmatrix}$$

$$B^T = \begin{pmatrix} 1 & 5 & -2 & 4 & 1 \\ 1 & 8 & 3 & 0 & -6 \end{pmatrix}$$

In R

```
> matB
     [,1] [,2] [,3] [,4] [,5]
[1,]    1    3    5    7    9
[2,]    2    4    6    8   10
> t(matB)
     [,1] [,2]
[1,]    1    2
[2,]    3    4
[3,]    5    6
[4,]    7    8
[5,]    9   10
```

DARTH Workgroup

Fernando Alarid-Escudero, PhD[1]
Eva A. Enns, MS, PhD[2]
M.G. Myriam Hunink, MD, PhD[3,4]
Hawre J. Jalal, MD, PhD[5]
Eline M. Krijkamp, MSc[3]
Petros Pechlivanoglou, PhD[6]

In collaboration of:
1 Division of Public Administration, Center for Research and Teaching in Economics (CIDE), Aguascalientes, Mexico
2 University of Minnesota School of Public Health, Minneapolis, MN, USA
3 Erasmus MC, Rotterdam, The Netherlands
4 Harvard T.H. Chan School of Public Health, Boston, USA
5 University of Ottawa, Ottawa, Canada
6 The Hospital for Sick Children, Toronto and University of Toronto, Toronto ON, Canada

www.darthworkgroup.com

CIDE

SCHOOL OF PUBLIC HEALTH
UNIVERSITY OF MINNESOTA

Erasmus MC
Netherlands Institute for Health Sciences

University of Pittsburgh
Graduate School of Public Health

SickKids