CSC 249/449 Machine Vision: Homework 3

Term: Spring 2020
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Due Date: Mar 19, 2020 11:59pm

Constraints: This assignment is to be carried out independently.

Problem 1 (10 pt) Prove the simple, but extremely useful, result that the perpendicular distance from a point (u, v) to a line $ax + by + c = 0$ is given by $|au + bv + c|$ if $a^2 + b^2 = 1$.

Problem 2 (30 pt) The Hough transform was discussed in class as a way to detection lines in images by voting in a transformed parameter space. For the case of lines, this is a 2D parameter space.
Now, consider different possible parameterizations of boundaries of interest. In particular, consider a circle or a quadratic curve. These examples may be used in various applications. In this problem, we will use shoe-print images as examples, shown in Fig. 1 and Fig. 2.

Figure 1: circle shoe-print

Figure 2: curve shoe-print

1. Select either circle or quadratic curve and derive the Hough transform for that case.
2. Implement your own Python routine for the boundary shape you chose and apply it to the appropriate image in shoeprint/ (based on the image name). You may need to think behind simple Canny edge detection to provide sufficient input to the Hough transform.
3. Are you able to get it to work? Discuss the benefits and drawbacks of the Hough transform in the context of the boundary shape you chose and the image domain you worked with. Supplement the discussion with other images if needed. Max 1 page.

Problem 3 (60 pt): See https://github.com/jshi31/csc249tracking

Problem 4 (20 pt): Least square linear fitting is one of the most fundamental fitting model. In the class, we have learned 2-D line fitting. Now let's extend it to 1) high dimensional points; 2) fitting with L2 weight penalty, which is also known as ridge regression. Now we have
- Data: $(\mathbf{x_n}, \mathbf{y_n}), n = 1, 2, ..., N$, $\mathbf{x_n} \in \mathbb{N}^d$, $\mathbf{y_n} \in \mathbb{N}^k$.
- Model: $\mathbf{y} = W\mathbf{x}$, where $W \in \mathbb{N}^{k \times d}$ is the weight of the linear model.
- Objective: find optimal W to minimize

$$E = ||Y - WX||^2 + \lambda ||W||^2, \tag{1}$$

where $Y = [\mathbf{y_1}, \mathbf{y_2}, ..., \mathbf{y_N}]$, $X = [\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_N}]$, $||\cdot||$ is Frobenius Norm, which means $||X|| = \sqrt{\text{Tr}(X^T X)}$.
Try to prove that the optimal W is the

$$W = YX^T(XX^T + \lambda I)^{-1} \tag{2}$$

Submission Process: Please follow the submission guideline in https://github.com/jshi31/csc249tracking.