高等代数 (荣誉) I 作业模板

请输入姓名

Sunday 25th May, 2025

0 说明

可以将作业中遇到的问题标注在此. 如有, 请补充.

目录

1 Problem Set for 19-May and 22-May

1.1 Exercise 2

Let J_A and J_B denote the Jordan form of A and B respectively (all matrices are in $\mathbb{F}^{n\times n}$).

1. Write down the Jordan form of the linear transformation $X \mapsto AXA^T$;
2. Write down the Jordan form of the linear transformation $X \mapsto AXA$;
3. Write down the Jordan form of the linear transformation $X \mapsto AX - XA$;
4. Write down the Jordan form of the linear transformation $X \mapsto AX - XA^T$;
5. Write down the Jordan form of the linear transformation $X \mapsto AXB$.

解答

1. $A\otimes A = (P^{-1}J_AP)\otimes(P^{-1}J_AP) = (P^{-1}\otimes P^{-1})(J_A\otimes J_A)(P\otimes P) = (P\otimes P)^{-1}(J_A\otimes J_A)(P\otimes P)$
2. $J_A \otimes J_A^T$
3. $J_A \otimes J_A \otimes I - I \otimes A^T$
4. $J_A \otimes J_A \otimes I - I \otimes A$
5. $J_A \otimes J_B^T$

1.2 Exercise 3

Show that $\dim\ker[X \mapsto (AX - XA^T)] \geq n$, and explain when the equality holds.

解答 考虑 $A \otimes I - I \otimes A$, 每个块 $J_i \otimes I - J_A$ 中一定有一个列为 0, 通过初等变换至少有 $n_i - 1$ 个新的列为 0, 如果存在特征值相同为 0 的列更多.

故 $\dim\ker[X \mapsto (AX - XA^T)] \geq n$ 取等条件为 A 的特征值各不相同或可对角化

1.3 Exercise 4

Show that there exists $\{0,1\}$-matrices A and B such that $A \otimes B$ and $B \otimes A$ are not similar.

解答 Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$,

$$A \otimes B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, B \otimes A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$