

# CS-E5710 Bayesian Data Analysis

## Assignment 7

November 17, 2019

**Note:** Complete source code is given in the Appendix.

### 1 Linear model: drowning data with Stan

```
1 drowning_data = pd.read_fwf('./drowning.txt').values
2 years = drowning_data[:, 0]
3 drowning = drowning_data[:, 1]
4 plt.plot(years, drowning)
5 print("mean:", np.mean(drowning))
6 print("standard deviation:", np.std(drowning, ddof=1))
```

```
1 mean: 137.72222222222223
2 standard deviation: 26.612146647843897
```

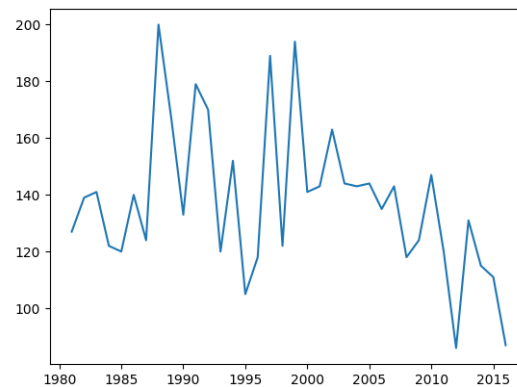


Figure 1: The number of people drown per year

We can see that the number of drown in Finland is decreasing.

## Stan model

```

1 stan_code = '''
2 data {
3   int<lower=0> N; // number of data points
4   vector[N] x;   // observation year
5   vector[N] y;   // observation number of drowned
6   real xpred;    // prediction year
7   real tau;
8 }
9 parameters {
10  real alpha;
11  real beta;
12  real<lower=0> sigma;
13 }
14 transformed parameters {
15  vector[N] mu;
16  mu = alpha + beta * x;
17 }
18 model {
19  beta ~ normal(0, tau * tau);
20  y ~ normal(mu, sigma);
21 }
22 generated quantities {
23  real ypred;
24  ypred = normal_rng(alpha + beta * xpred, sigma);
25 }
26 '''

```

**Fix 1** In parameters, sigma has no lower bound and should be fixed by

```

1 real<lower=0> sigma;

```

**Fix 2** In generated quantities, it aims to calculate for the prediction year and should be fixed by

```

1 ypred = normal_rng(alpha + beta * xpred, sigma);

```

The suitable numerical value presented for  $\tau = 26.6121$

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
alpha	1804.1	24.67	840.37	171.67	1250.4	1786.9	2335.3	3506.5	1161	1.0
beta	-0.83	0.01	0.42	-1.68	-1.1	-0.83	-0.56	-0.02	1161	1.0
sigma	26.4	0.08	3.33	20.89	24.07	26.07	28.34	34.04	1640	1.0
mu[1]	152.4	0.23	8.51	135.9	146.86	152.33	157.86	169.86	1393	1.0
mu[2]	151.57	0.22	8.16	135.74	146.28	151.49	156.84	168.27	1421	1.0
						...	...			
mu[36]	123.22	0.22	8.6	106.16	117.52	123.27	128.97	139.76	1485	1.0
ypred	121.0	0.46	27.14	68.54	102.4	120.68	139.68	174.54	3444	1.0
lp__	-131.8	0.04	1.2	-134.9	-132.4	-131.5	-130.9	-130.4	1072	1.0

From the above table the the ypred is the prediction for 2019 which is 121

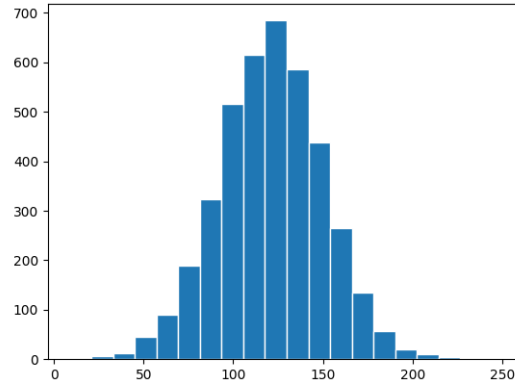


Figure 2: The number of people drowned per year

## 2 Hierarchical model: factory data with Stan

### 2.1 Separate model

```

1 stan_code_separate = '''
2 data {
3     int<lower=0> N;           // number of data points
4     int<lower=0> K;           // number of groups
5     int<lower=1,upper=K> x[N]; // group indicator
6     vector[N] y;
7 }
8 parameters {
9     vector[K] mu;           // group means
10    vector<lower=0>[K] sigma; // group stds
11 }
12 model {
13     y ~ normal(mu[x], sigma[x]);
14 }
15 generated quantities {
16     real ypred;
17     ypred = normal_rng(mu[6], sigma[6]);
18 }
19 '''

```

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
mu[1]	76.62	0.7	19.01	43.84	68.49	76.47	83.97	110.09	745	1.0
mu[2]	106.19	0.24	9.29	87.8	101.66	106.24	110.57	126.56	1557	1.0
mu[3]	87.24	0.42	10.69	65.71	82.54	87.72	92.73	106.98	645	1.01
mu[4]	111.75	0.18	6.44	99.6	108.51	111.57	114.55	125.91	1252	1.0
mu[5]	90.12	0.28	9.26	70.87	85.78	90.14	94.41	109.23	1116	1.0
mu[6]	86.1	0.36	14.32	56.76	78.74	86.31	93.5	115.39	1583	1.0
sigma[1]	32.31	0.86	24.94	12.65	19.46	25.7	36.74	95.36	837	1.0

9	sigma[2]	18.43	0.34	11.8	7.64	11.67	15.03	21.25	51.02	1212	1.0
10	sigma[3]	20.43	0.49	13.09	8.35	12.66	16.81	23.5	57.44	720	1.0
11	sigma[4]	12.35	0.27	8.78	4.99	7.52	10.0	14.04	33.83	1094	1.0
12	sigma[5]	17.5	0.35	11.64	7.14	10.76	14.15	20.01	48.24	1086	1.0
13	sigma[6]	29.34	0.47	17.23	12.54	18.86	24.72	34.16	74.79	1347	1.0
14	ypred	86.96	0.65	36.82	10.78	69.4	87.07	104.4	161.39	3187	1.0
15	lp__	-81.31	0.11	3.19	-88.67	-83.14	-80.92	-78.99	-76.24	899	1.0

i) The posterior distribution of the mean of the quality measurements of the sixth machine.

$$\mu_6 = 86.1$$

```

1 fit_separate = model_separate.sampling(data=data_separate, n_jobs=-1)
2 mu_data_separate = fit_separate.extract()['mu']

```

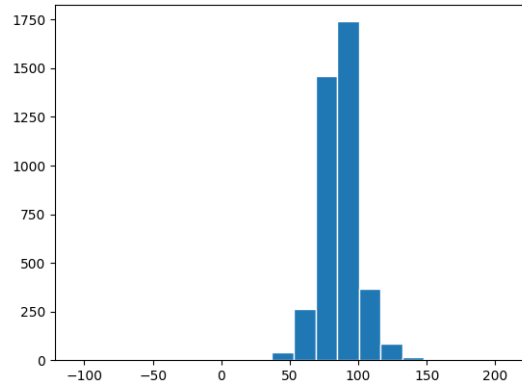


Figure 3:  $\mu$  histogram

ii) The predictive distribution for another quality measurement of the sixth machine.

```

1 fit_separate = model_separate.sampling(data=data_separate, n_jobs=-1)
2 y_pred_separate = fit_separate.extract()['ypred']

```

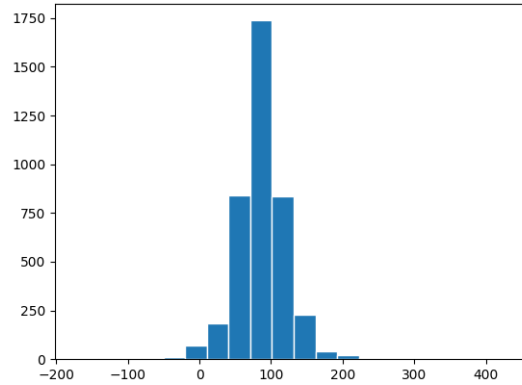


Figure 4: Prediction histogram

iii) The posterior distribution of the mean of the quality measurements of the seventh machine.

In the separate model we treat each machine separately. Since we don't have information about the seventh machine. Thus we cannot tell its posterior distribution.

## 2.2 pooled model

```

1 stan_code_pooled = '''
2 data {
3   int<lower=0> N;          // number of data points
4   vector[N] y;           //
5 }
6 parameters {
7   real mu;                // group means
8   real<lower=0> sigma;    // common std
9 }
10 model {
11   y ~ normal(mu, sigma);
12 }
13 generated quantities {
14   real ypred;
15   ypred = normal_rng(mu, sigma);
16 }
17 '''

```

	mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
mu	92.99	0.06	3.53	86.23	90.54	93.01	95.32	100.04	2953	1.0
sigma	18.83	0.05	2.54	14.6	17.06	18.55	20.36	24.63	3023	1.0
ypred	93.43	0.31	19.54	55.75	80.25	93.81	106.42	130.8	3953	1.0
lp__	-99.34	0.02	1.01	-102.0	-99.73	-99.05	-98.62	-98.34	1806	1.0

i) The posterior distribution of the mean of the quality measurements of the sixth machine.

For the pooled model, all the machines are considered as an entity, thus all the measurements are combined into one and performed prediction on the whole data.  $\mu$  will be the same for all the machines.

```
1 fit_pooled = model_pooled.sampling(data=data_pooled)
2 mu = fit_pooled.extract()['mu']
```

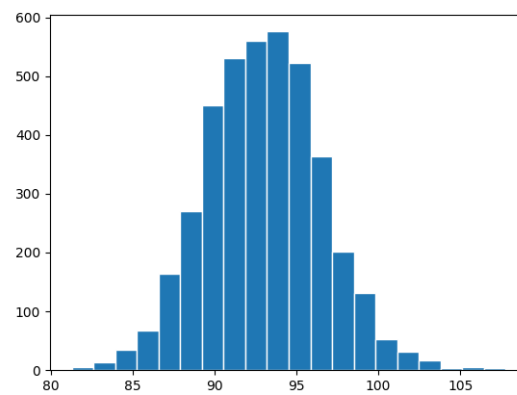


Figure 5: Prediction histogram

ii) The predictive distribution for another quality measurement of the sixth machine.

```
1 fit_pooled = model_pooled.sampling(data=data_pooled)
2 y_pred_pooled = fit_pooled.extract()['ypred']
```

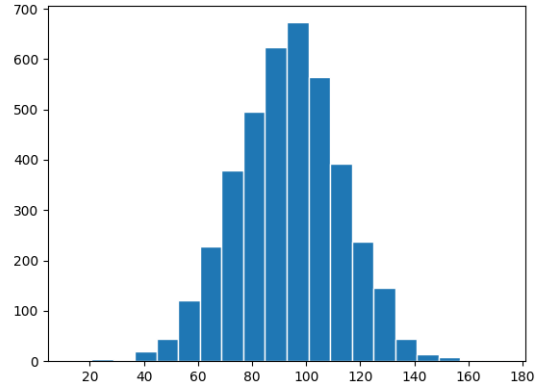


Figure 6: Prediction histogram

iii) The posterior distribution of the mean of the quality measurements of the seventh machine.

For the pooled model, all the machines are considered as an entity, thus all the measurements are combined into one and performed prediction on the whole data. The posterior distribution of the mean of the quality measurements of the seventh machine is equal to that of the sixth machine.

## 2.3 Hierarchical model

```

1 stan_code_hierarchical = '''
2 data {
3   int<lower=0> N;           // number of data points
4   int<lower=0> K;           // number of groups
5   int<lower=1,upper=K> x[N]; // group indicator
6   vector[N] y;
7 }
8 parameters {
9   real mu0;                // prior mean
10  real<lower=0> sigma0;     // prior std
11  vector[K] mu;            // group means
12  real<lower=0> sigma;      // common std
13 }
14 model {
15   mu ~ normal(mu0, sigma0);
16   y ~ normal(mu[x], sigma);
17 }
18 generated quantities {
19   real ypred6;
20   real mu7;
21   ypred6 = normal_rng(mu[6], sigma);
22   mu7 = normal_rng(mu0, sigma0);

```

```

23 }
24 . . .

```

		mean	se_mean	sd	2.5%	25%	50%	75%	97.5%	n_eff	Rhat
1	mu0	92.96	0.27	9.11	77.19	88.49	92.97	97.37	109.84	1122	1.0
2	sigma0	16.2	0.33	10.13	4.18	10.21	14.02	19.23	41.65	928	1.0
3	mu[1]	79.85	0.18	6.91	66.07	75.28	79.8	84.49	92.89	1467	1.0
4	mu[2]	103.08	0.26	6.77	88.38	98.76	103.18	107.48	116.39	677	1.0
5	mu[3]	89.05	0.11	6.17	76.57	85.04	89.2	93.19	101.16	3223	1.0
6	mu[4]	107.11	0.29	7.13	91.57	102.47	107.58	111.89	120.77	607	1.0
7	mu[5]	90.6	0.1	5.95	78.46	86.91	90.6	94.42	102.4	3606	1.0
8	mu[6]	87.54	0.11	6.14	75.3	83.45	87.76	91.68	99.85	3297	1.0
9	sigma	15.24	0.08	2.39	11.36	13.54	14.93	16.65	20.54	941	1.0
10	ypred6	87.32	0.26	16.48	54.75	76.32	87.2	98.16	120.36	4099	1.0
11	mu7	92.7	0.43	22.23	51.61	82.94	92.81	102.74	133.11	2670	1.0
12	lp__	-108.8	0.07	2.49	-114.6	-110.2	-108.4	-107.0	-105.2	1377	1.0

i) The posterior distribution of the mean of the quality measurements of the sixth machine.

The hierarchical model not only treats every machine separately, but also computes the combination of all the machines as one entity.  $\mu_6 = 87.54$

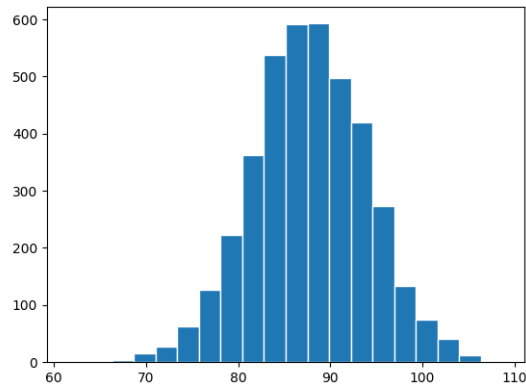


Figure 7: Prediction histogram

ii) The predictive distribution for another quality measurement of the sixth machine.

```

1 fit_hierarchical = model_hierarchical.sampling(data=data_hierarchical, n_jobs=-1)
2 mu_data_hierarchical = fit_hierarchical.extract()['mu']

```



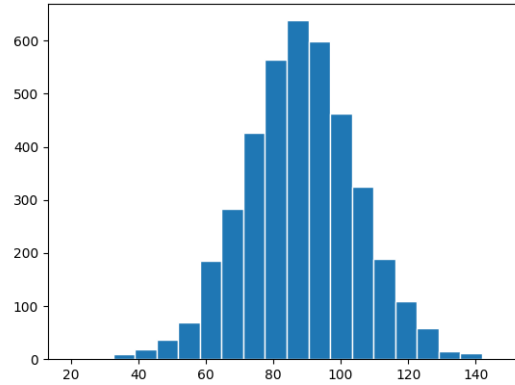


Figure 8: Prediction histogram

iii) The posterior distribution of the mean of the quality measurements of the seventh machine.

The hierarchical model not only treats every machine separately, but also computes the combination of all the machines as one entity. It can predict measurements for the machines even without data. we can plot the histogram:  $\mu_7 = 92.7$

```
1 fit_hierarchical = model_hierarchical.sampling(data=data_hierarchical, n_jobs=-1)
2 mu_data_hierarchical_7 = fit_hierarchical.extract()['mu7']
```

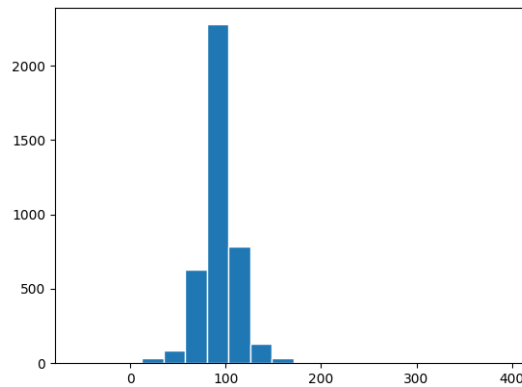


Figure 9: Prediction histogram

## A Exercise 1

```
1 import matplotlib
2 from scipy.stats import norm
3 import matplotlib.pyplot as plt
4 import numpy as np
5 import pandas as pd
6 import pystan
7
8 drowning_data = pd.read_fwf('./drowning.txt').values
9 years = drowning_data[:, 0]
10 drowning = drowning_data[:, 1]
11
12 print("mean:", np.mean(drowning))
13 print("standard deviation:", np.std(drowning, ddof=1))
14
15 plt.plot(years, drowning)
16 plt.savefig('./trend.png')
17 plt.show()
18
19 stan_code = '''
20 data {
21     int<lower=0> N; // number of data points
22     vector[N] x;   // observation year
23     vector[N] y;   // observation number of drowned
24     real xpred;    // prediction year
25     real tau;
26 }
27 parameters {
28     real alpha;
29     real beta;
30     real<lower=0> sigma;
31 }
32 transformed parameters {
33     vector[N] mu;
34     mu = alpha + beta * x;
35 }
36 model {
37     beta ~ normal(0, tau*tau);
38     y ~ normal(mu, sigma);
39 }
40 generated quantities {
41     real ypred;
42     ypred = normal_rng(alpha + beta * xpred, sigma);
43 }
44 '''
45
46 stan_model = pystan.StanModel(model_code=stan_code)
47
48 data = dict(
49     N=len(years),
50     x=years,
51     y=drowning,
52     xpred=2019,
53     tau=26.612146647843897,
54 )
```

```

55
56 fit = stan_model.sampling(data=data)
57 print(fit)
58
59 y_pred = fit.extract()['ypred']
60 plt.hist(y_pred, bins=20, ec='white')
61 plt.savefig('./hist.png')
62 plt.show()

```

## B Exercise 2

```

1  import matplotlib
2  from scipy.stats import norm
3  import matplotlib.pyplot as plt
4  import numpy as np
5  import pandas as pd
6  import pystan
7
8  machines = pd.read_fwf('./factory.txt', header=None).values
9  machines_transposed = machines.T
10
11
12  stan_code_separate = '''
13  data {
14      int<lower=0> N;                // number of data points
15      int<lower=0> K;                // number of groups
16      int<lower=1,upper=K> x[N];    // group indicator
17      vector[N] y;
18  }
19  parameters {
20      vector[K] mu;                 // group means
21      vector<lower=0>[K] sigma;     // group stds
22  }
23  model {
24      y ~ normal(mu[x], sigma[x]);
25  }
26  generated quantities {
27      real ypred;
28      ypred = normal_rng(mu[6], sigma[6]);
29  }
30  '''
31
32  model_separate = pystan.StanModel(model_code=stan_code_separate)
33  data_separate = dict(
34      N=machines_transposed.size,
35      K=6,
36      x=[
37          1, 1, 1, 1, 1,
38          2, 2, 2, 2, 2,
39          3, 3, 3, 3, 3,
40          4, 4, 4, 4, 4,
41          5, 5, 5, 5, 5,
42          6, 6, 6, 6, 6,

```

```

43     ],
44     y=machines_transposed.flatten()
45 )
46
47 fit_separate = model_separate.sampling(data=data_separate, n_jobs=-1)
48 print(fit_separate)
49
50 y_pred_separate = fit_separate.extract()['ypred']
51 plt.hist(y_pred_separate, bins=20, ec='white')
52 plt.savefig('./separate_hist.png')
53 plt.show()
54
55 mu_data_separate = fit_separate.extract()['mu']
56 plt.hist(mu_data_separate[:, 5], bins=20, ec='white')
57 plt.savefig('./separate_hist_mu_six.png')
58 plt.show()
59
60 stan_code_pooled = '''
61 data {
62     int<lower=0> N;          // number of data points
63     vector[N] y;           //
64 }
65 parameters {
66     real mu;                // group means
67     real<lower=0> sigma;    // common std
68 }
69 model {
70     y ~ normal(mu, sigma);
71 }
72 generated quantities {
73     real ypred;
74     ypred = normal_rng(mu, sigma);
75 }
76 '''
77
78 machines_pooled = machines.flatten()
79 model_pooled = pystan.StanModel(model_code=stan_code_pooled)
80 data_pooled = dict(
81     N=machines_pooled.size,
82     y=machines_pooled
83 )
84
85 fit_pooled = model_pooled.sampling(data=data_pooled)
86 print(fit_pooled)
87
88 y_pred_pooled = fit_pooled.extract()['ypred']
89 plt.hist(y_pred_pooled, bins=20, ec='white')
90 plt.savefig('./pooled_hist.png')
91 plt.show()
92
93 mu = fit_pooled.extract()['mu']
94 plt.hist(mu, bins=20, ec='white')
95 plt.savefig('./pooled_hist_mu.png')
96 plt.show()
97
98 stan_code_hierarchical = '''
99 data {

```

```

100     int<lower=0> N;           // number of data points
101     int<lower=0> K;           // number of groups
102     int<lower=1,upper=K> x[N]; // group indicator
103     vector[N] y;
104 }
105 parameters {
106     real mu0;                // prior mean
107     real<lower=0> sigma0;     // prior std
108     vector[K] mu;            // group means
109     real<lower=0> sigma;     // common std
110 }
111 model {
112     mu ~ normal(mu0, sigma0);
113     y ~ normal(mu[x], sigma);
114 }
115 generated quantities {
116     real ypred6;
117     real mu7;
118     ypred6 = normal_rng(mu[6], sigma);
119     mu7 = normal_rng(mu0, sigma0);
120 }
121 '''
122
123 model_hierarchical = pystan.StanModel(model_code=stan_code_hierarchical)
124 data_hierarchical = dict(
125     N=machines_transposed.size,
126     K=6,
127     x=[
128         1, 1, 1, 1, 1,
129         2, 2, 2, 2, 2,
130         3, 3, 3, 3, 3,
131         4, 4, 4, 4, 4,
132         5, 5, 5, 5, 5,
133         6, 6, 6, 6, 6,
134     ],
135     y=machines_transposed.flatten()
136 )
137
138 fit_hierarchical = model_hierarchical.sampling(data=data_hierarchical, n_jobs=-1)
139 print(fit_hierarchical)
140
141 mu_data_hierarchical = fit_hierarchical.extract()['mu']
142 plt.hist(mu_data_hierarchical[:, 5], bins=20, ec='white')
143 plt.savefig('./hierarchical_hist_mu_six.png')
144 plt.show()
145
146 y_pred_hierarchical = fit_hierarchical.extract()['ypred6']
147 plt.hist(y_pred_hierarchical, bins=20, ec='white')
148 plt.savefig('./hierarchical_hist.png')
149 plt.show()
150
151 mu_data_hierarchical_7 = fit_hierarchical.extract()['mu7']
152 plt.hist(mu_data_hierarchical_7, bins=20, ec='white')
153 plt.savefig('./hierarchical_hist_mu_seven.png')
154 plt.show()

```