KTH ROYAL INSTITUTE
OF TECHNOLOGY

Introduction to Robotics

DD2410

Lecture 2 - Manipulators, Kinematics

Schedule - Lectures

Aug 29 - 1. Intro, Course fundamentals, Topics, What is a Robot, History Applications.

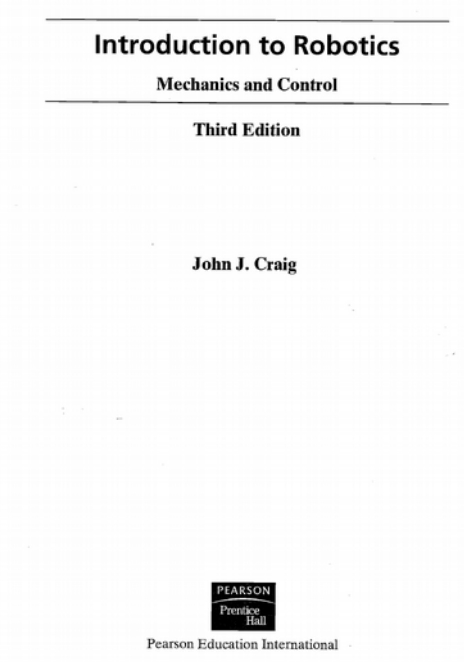
Aug 30 - 2 Manipulators, Kinematics
(Aug 31 - 3 ROS Introduction)
Sep 03 - 4. Differential kinematics, dynamics
Sep 04 - 5. Actuators, sensors I (force, torque, encoders, ...)
Sep 10 - 6. Grasping, Motion, Control

Sep 11 - 7. Behavior Trees and Task Switching
Sep 17 - 8. Planning (RRT, A*, ...)

Sep 18 - 9. Mobility and sensing II (distance, vision, radio, GPS, ...)
Sep 24 - 10. Localisation (where are we?)
Sep 27 - 11. Mapping (how to build the map to localise/navigate w.r.t.?)
Oct 01 - 12. Navigation (how do I get from A to B?)

Overview

- Manipulators
  - Examples
  - Types
- Kinematics
  - Forward Kinematics
  - DH, transfer frames
  - Inverse Kinematics

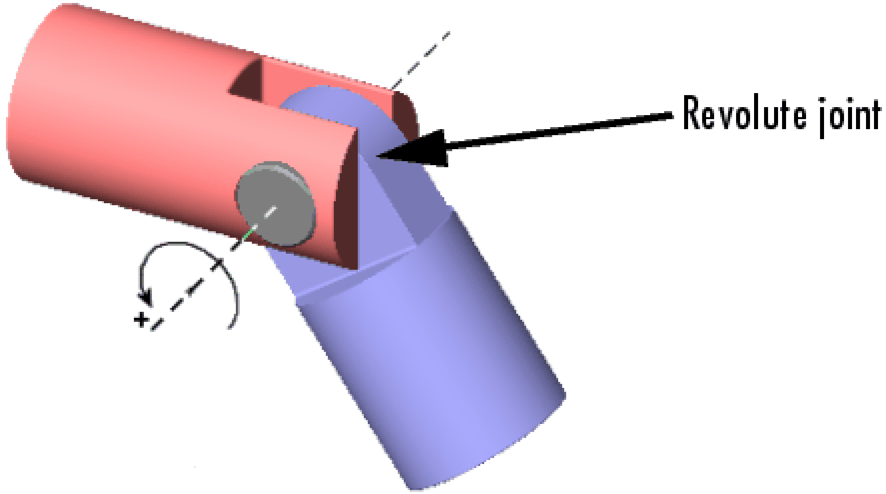

Introduction to Robotics
Mechanics and Control
Third Edition
John J. Craig
PEARSON
Prentice Hall
Pearson Education International

Manipulators

- Robot arms
  - (Mostly) rigid links connected by joints

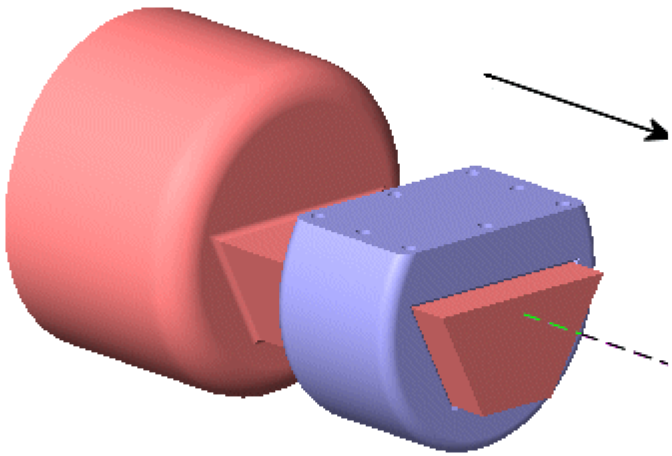
Components

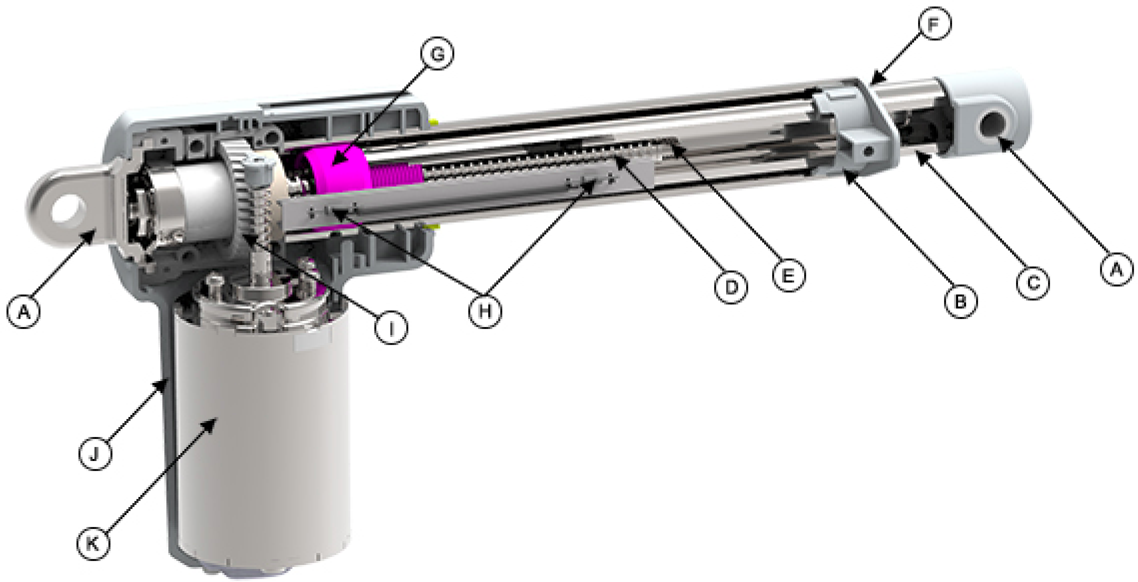
- Joint types:
- **R**otational



Components

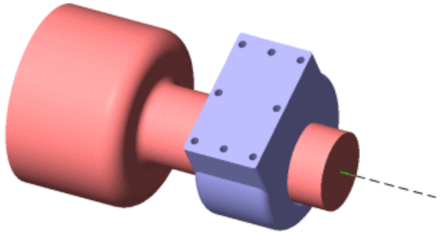
- Joint types:
- **P**rismatic

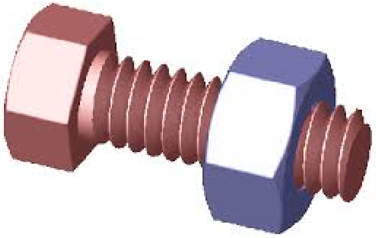




Kinematics

- Joint types:
- Ball, cylindrical, universal, screw, ...





Manipulators

2 DoF (PP)

4 DoF (RRRP)

Image: ITBotics, Fanuc

Manipulators

6 DoF (RRRRRR)

7 DoF (RRRRRRR)

Image: ABB

Mobile manipulators

Image: KUKA Roboter GmbH

Manipulators

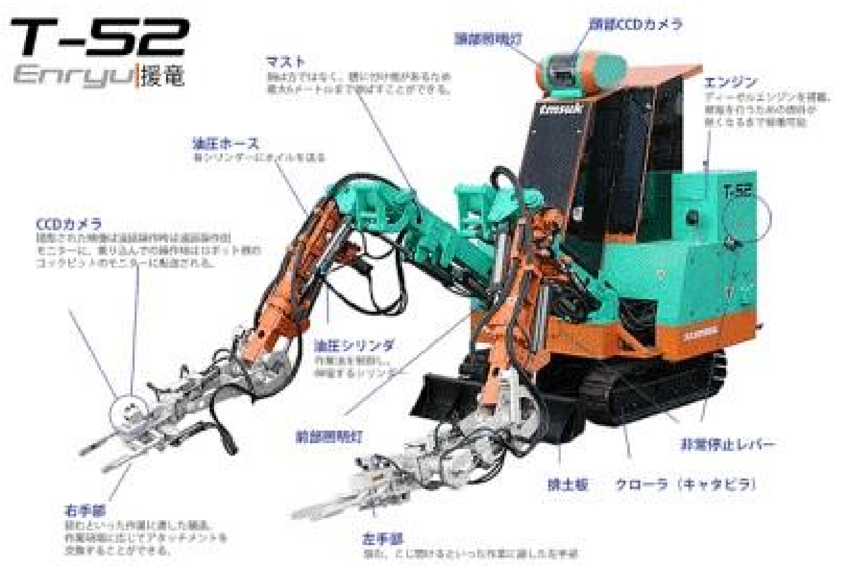


Image: TMSUK

Manipulators

Manipulators

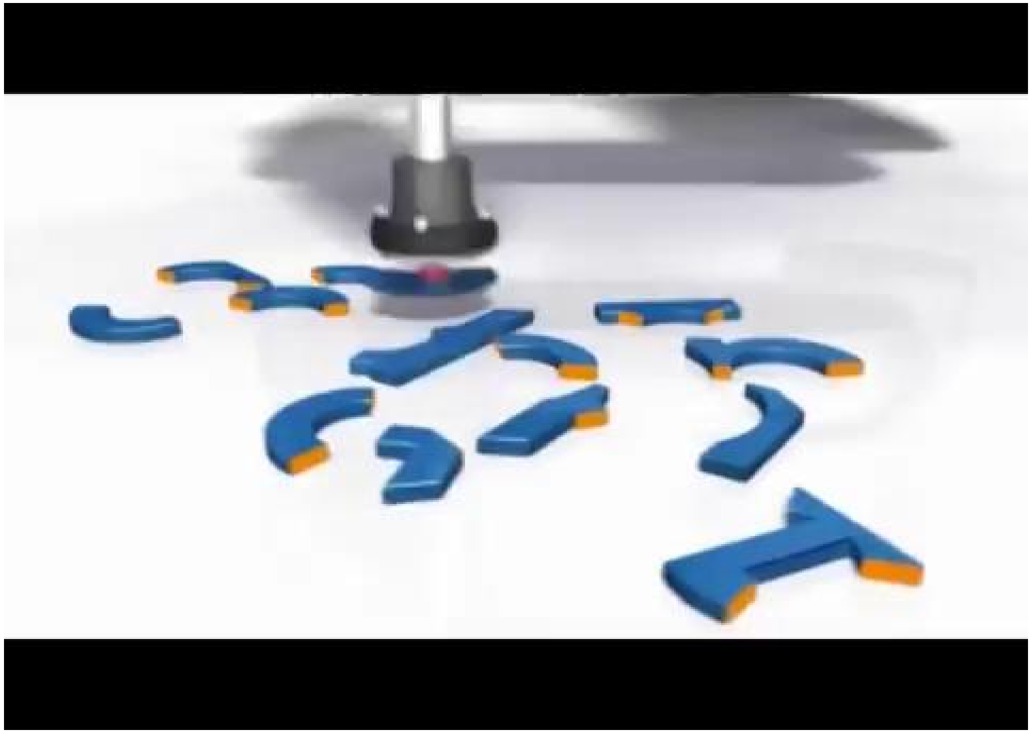
Image: Cyberdyne

Manipulator applications

- Pick and place
- Welding, painting
- Surgery
- Camera placement

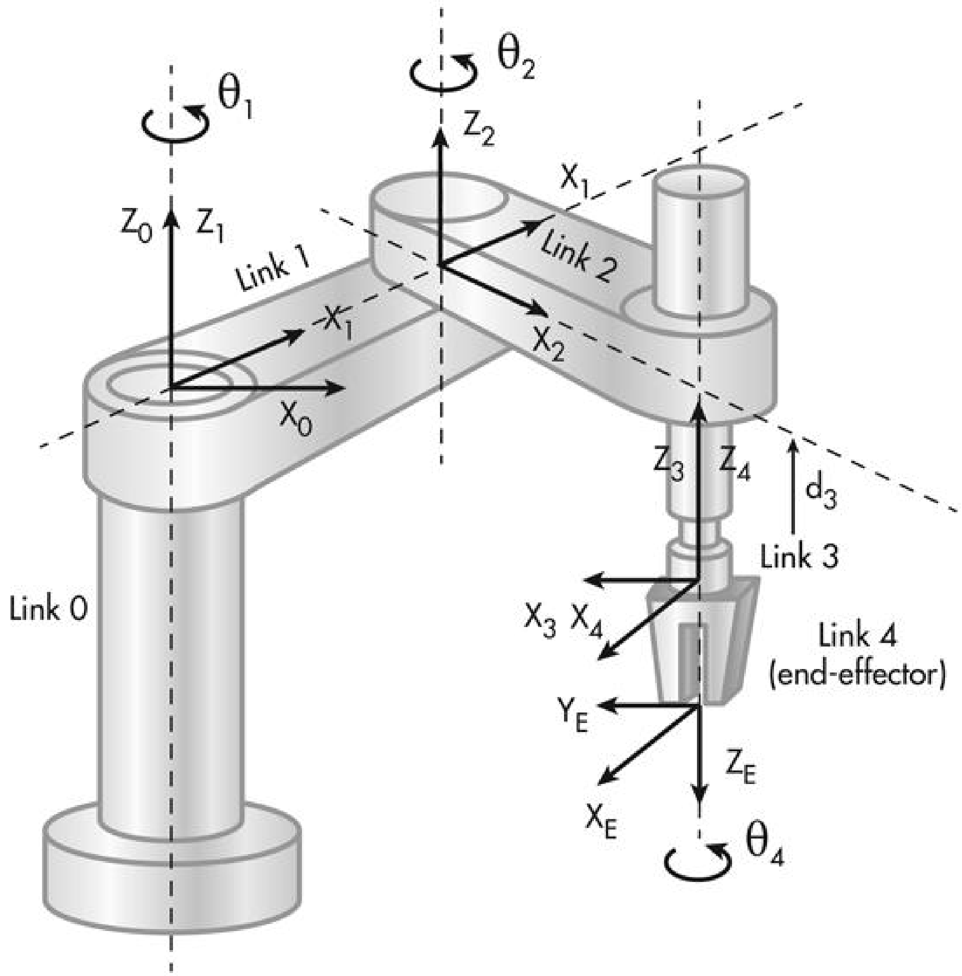
SCARA - Selective Compliance Articulated Robot Arm
(1981 - Sankyo Seiki, Pentel and NEC)

SCARA - Selective Compliance Articulated Robot Arm



Video: Omron Adept Technologies

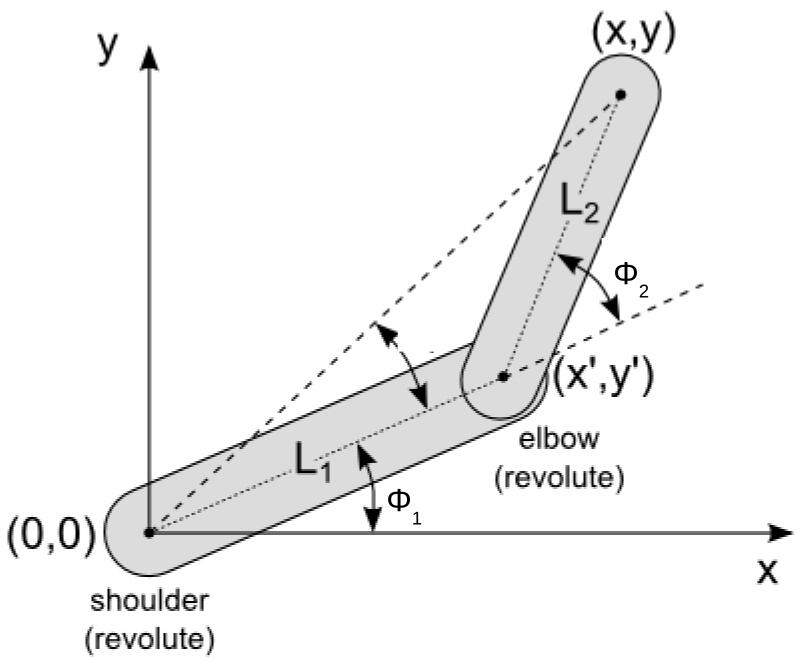
SCARA - Selective Compliance Articulated Robot Arm

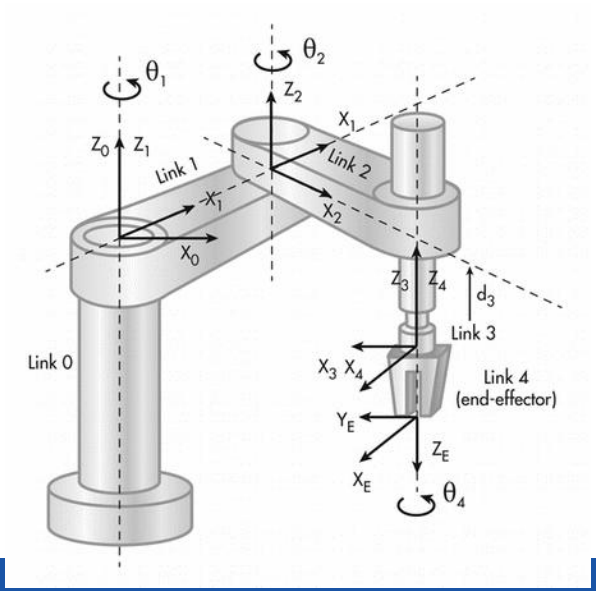


Kinematics

- Scara example

$$x = L_1\cos(\Phi_1) + L_2\cos(\Phi_1+\Phi_2)$$
$$y = L_1\sin(\Phi_1) + L_2\sin(\Phi_1+\Phi_2)$$
$$z = -d_3$$
$$\Theta = \Phi_1 + \Phi_2 + \Theta_4$$





kinematics

Forward Kinematics

1 Forward Kinematics

The Puma 560 can be seen in Figures 1 and 2.

The forward kinematics K_f can be formulated as:

$$\mathbf{X} = K_f(\Theta) \tag{1}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ z \\ p \\ t \\ a \end{bmatrix}, \quad \Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \tag{2}$$

we have:

$$\begin{aligned} x &= cos(\theta_1) * [a_2 cos(\theta_2) + a_3 cos(\theta_2 + \theta_3) - d_4 sin(\theta_2 + \theta_3)] - d_3 sin(\theta_1) \\ y &= sin(\theta_1) * [a_2 cos(\theta_2) + a_3 cos(\theta_2 + \theta_3) - d_4 sin(\theta_2 + \theta_3)] + d_3 cos(\theta_1) \\ z &= -a_3 sin(\theta_2 + \theta_3) + a_2 sin(\theta_2) - d_4 cos(\theta_2 + \theta_3) \\ p &= tan^{-1}\left(\frac{s1(c23c4s5+s23c5)-c1s4s5}{c1(c23c4s5+s23c5)+s1s4s5}\right) \\ t &= tan^{-1}\left(\frac{-s1(c23c4s5+s23c5)+c1s4s5}{sin(tan^{-1}(-s1(c23c4s5+s23c5)+c1s4s5/-c1(c23c4s5+s23c5)-s1s4s5))(s23c4s5-c23c5)}\right) \\ a &= tan^{-1}\left(\frac{-(s23(s4c6-c4c5s6)+c23s5s6)}{s23(s4s6-c4c5c6)-c23s5c6}\right) \end{aligned} \tag{3}$$

Where the latter uses shorthand. The full expression is:

$$\begin{aligned} p &= tan^{-1}\left(\frac{sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))-cos(\theta_1)sin(\theta_4)sin(\theta_5)}{cos(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+sin(\theta_1)sin(\theta_4)sin(\theta_5)}\right) \\ t &= tan^{-1}\left(\frac{-sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+cos(\theta_1)sin(\theta_4)sin(\theta_5)}{sin\left(tan^{-1}\left(\frac{-sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+cos(\theta_1)sin(\theta_4)sin(\theta_5)}{-cos(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))-sin(\theta_1)sin(\theta_4)sin(\theta_5)}\right)\right)(sin(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)-cos(\theta_2+\theta_3)cos(\theta_5))}\right) \\ a &= tan^{-1}\left(\frac{-(sin(\theta_2+\theta_3)(sin(\theta_4)cos(\theta_6)-cos(\theta_4)cos(\theta_5)sin(\theta_6))+cos(\theta_2+\theta_3)sin(\theta_5)sin(\theta_6))}{sin(\theta_2+\theta_3)(sin(\theta_4)sin(\theta_6)-cos(\theta_4)cos(\theta_5)cos(\theta_6))-cos(\theta_2+\theta_3)sin(\theta_5)cos(\theta_6)}\right) \end{aligned} \tag{4}$$

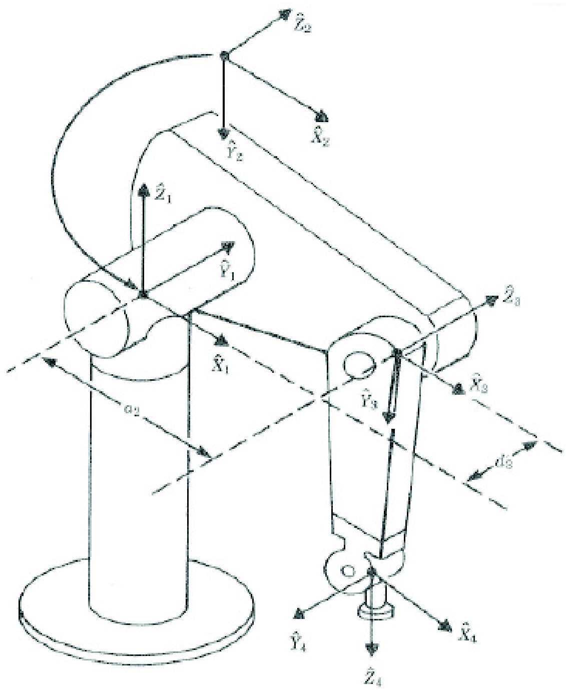


Figure 1: The puma 560

A systematic approach for forward kinematics

- How do we find the forward kinematics for an articulated system in general?

Transformations

- Consider a vector

$$\bar{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

If two coordinate systems are rotated w.r.t. each other we can write:

$$\bar{x}^1 = R\bar{x}^2 \tag{2}$$

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{pmatrix} \tag{3}$$

$$R_y = \begin{pmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{pmatrix} \tag{4}$$

$$R_z = \begin{pmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

R_x, R_y, R_z give example for rotation around x,y,z-axis

Transformations cont'd

- For pure translation we have

$$\bar{x}^1 = \bar{x}^2 + t \tag{6}$$

$$t = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \tag{7}$$

Homogeneous coordinates

- Translation and rotation gives

$$\bar{x}^1 = R\bar{x}^2 + t \tag{8}$$

- Introduce homegenous coordinates

$$\bar{X} = \begin{pmatrix} \bar{x} \\ 1 \end{pmatrix} \tag{10}$$

- Which allows us to write

$$\bar{X}^1 = T\bar{X}^2 \tag{11}$$

$$T = \begin{pmatrix} R & t \\ 0 & 1 \end{pmatrix} \tag{12}$$

- We can now describe the (stationary) transform between coordinate systems **A** and **B** with a matrix ${}^{A}\mathbf{T}_{B}$
- Any vector $\mathbf{x}_B$ in coordinate system **B** can be transformed to the corresponding vector $\mathbf{x}_A$ in coordinate system **A** by multiplication

$$\mathbf{x}_A = {}^{A}\mathbf{T}_{B}\mathbf{x}_B$$

Coordinate transforms (A1.2 in Handbook of Robotics)

- If we have a sequence of transformations, ${}^{A}\mathbf{T}_{B}$, ${}^{B}\mathbf{T}_{C}$, ${}^{C}\mathbf{T}_{D}$
- Any vector $\mathbf{x}_{D}$ in coordinate system **D** can be transformed to the corresponding vector $\mathbf{x}_{A}$ in coordinate system **A** by multiplication

$$\mathbf{x}_{A} = {}^{A}\mathbf{T}_{B}\,{}^{B}\mathbf{T}_{C}\,{}^{C}\mathbf{T}_{D}\,\mathbf{x}_{D} = {}^{A}\mathbf{T}_{D}\,\mathbf{x}_{D}$$

- Inversely, we have that

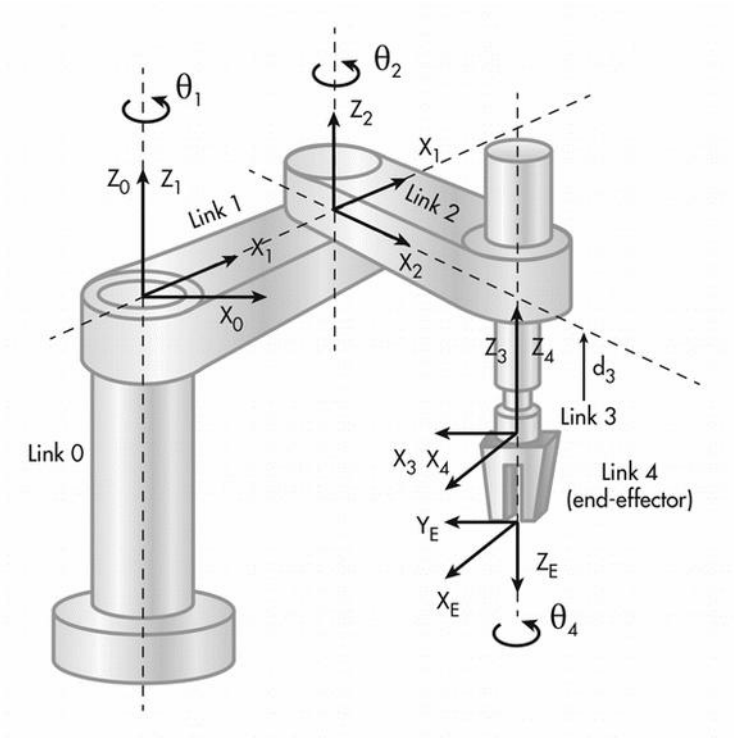
$$\mathbf{x}_{D} = ({}^{A}\mathbf{T}_{D})^{-1}\mathbf{x}_{A} = {}^{D}\mathbf{T}_{A}\,\mathbf{x}_{A}$$

Coordinate transforms - main idea

- If we find the individual transforms from one link to the next, we can multiply these together and find the transform to the world frame from the end effector frame.

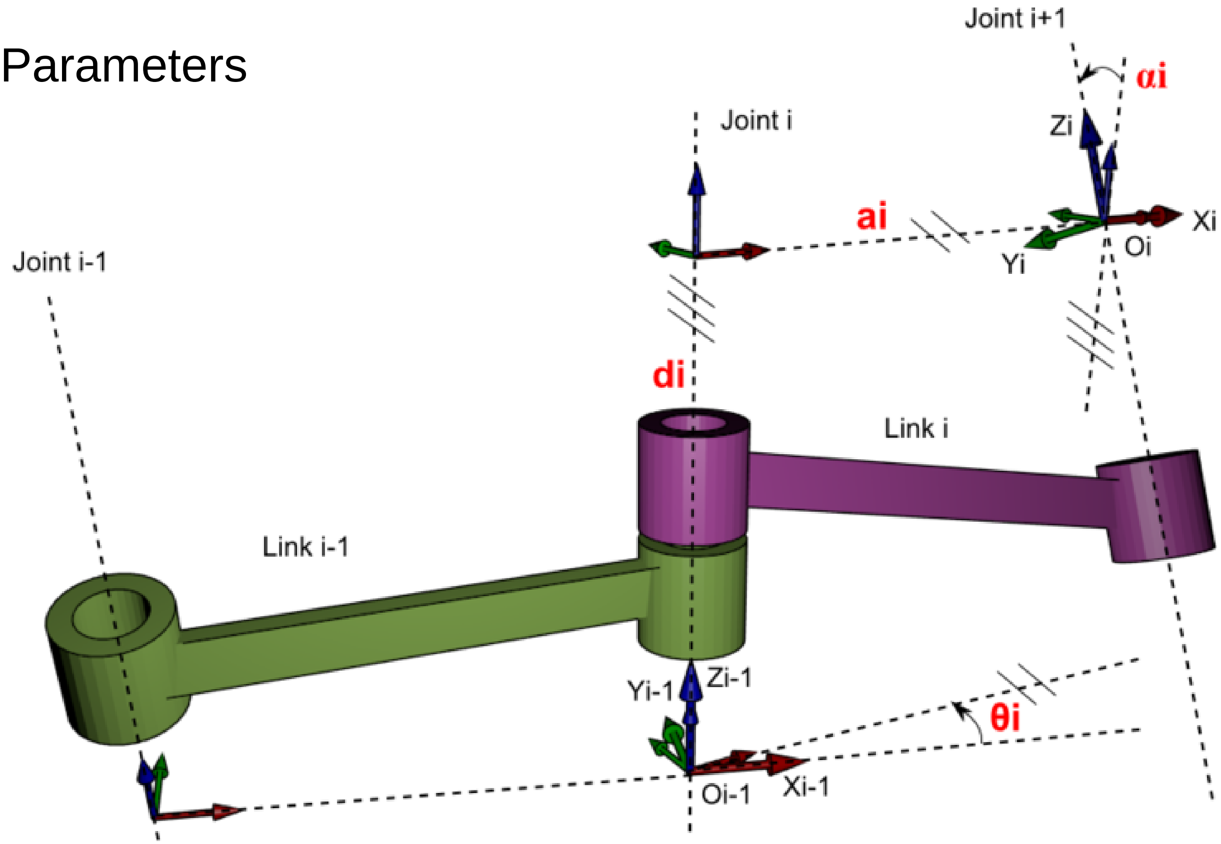
Forward kinematics

- Transform ${}^{B}\mathbf{T}_{E}$ from end effector to base frame is dependant on configuration $\mathbf{\Theta}$
- The function that generates ${}^{B}\mathbf{T}_{E}$ given $\mathbf{\Theta}$ is called forward kinematics $\mathbf{K}(\mathbf{\Theta})$.
- Commonly, we define $\mathbf{K}(\mathbf{\Theta})$ to output the pose vector $\mathbf{r} = [x\ y\ z\ \alpha\ \beta\ \gamma]^T$, where $\alpha\ \beta\ \gamma$ are the *Euler Angles*



- Minimal description for serial links: DH parameters
- Defined by Jacques Denavit and Richard Hartenberg
- Introduced in 1955

D-H Parameters



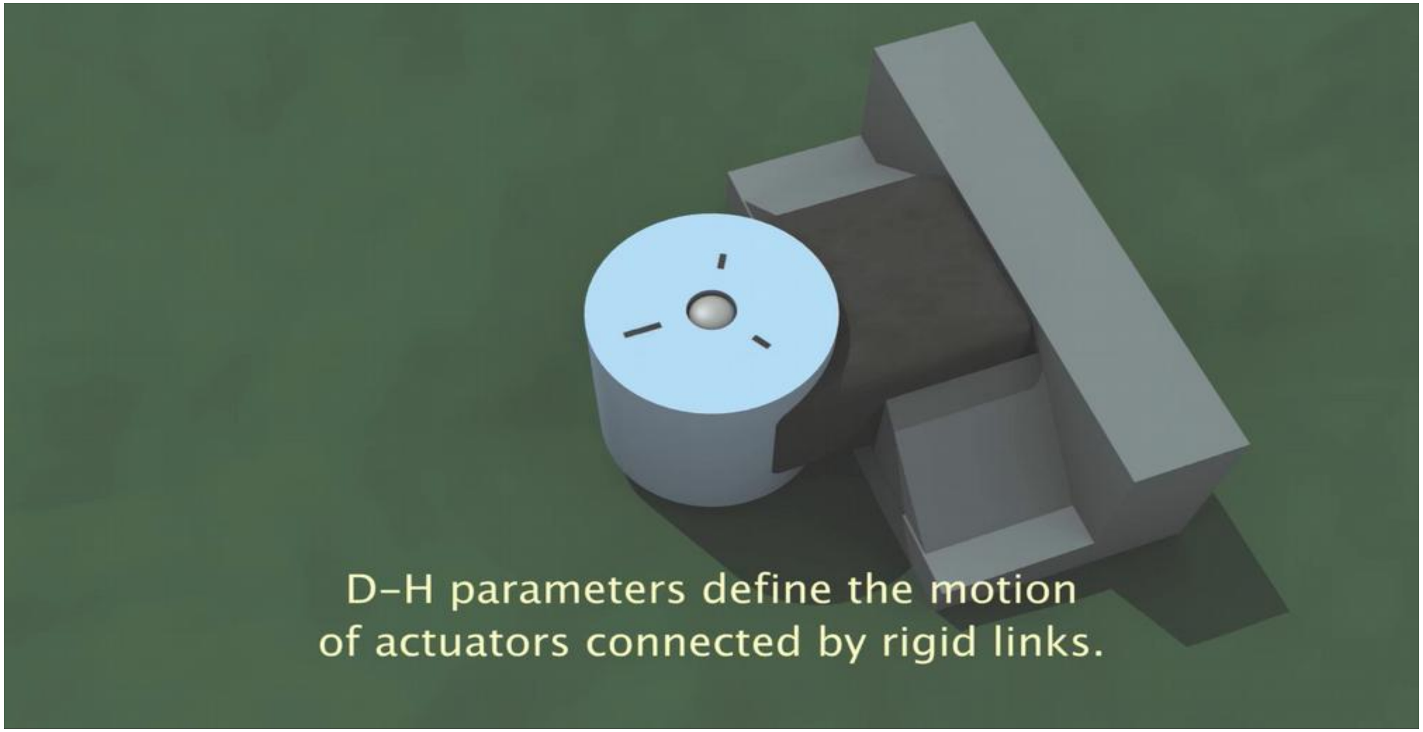
d: offset along previous *z* to the common normal

θ: angle about previous *z*, from old *x* to new *x*.

r: length of the common normal. For revolute joints, this is the radius about previous z.

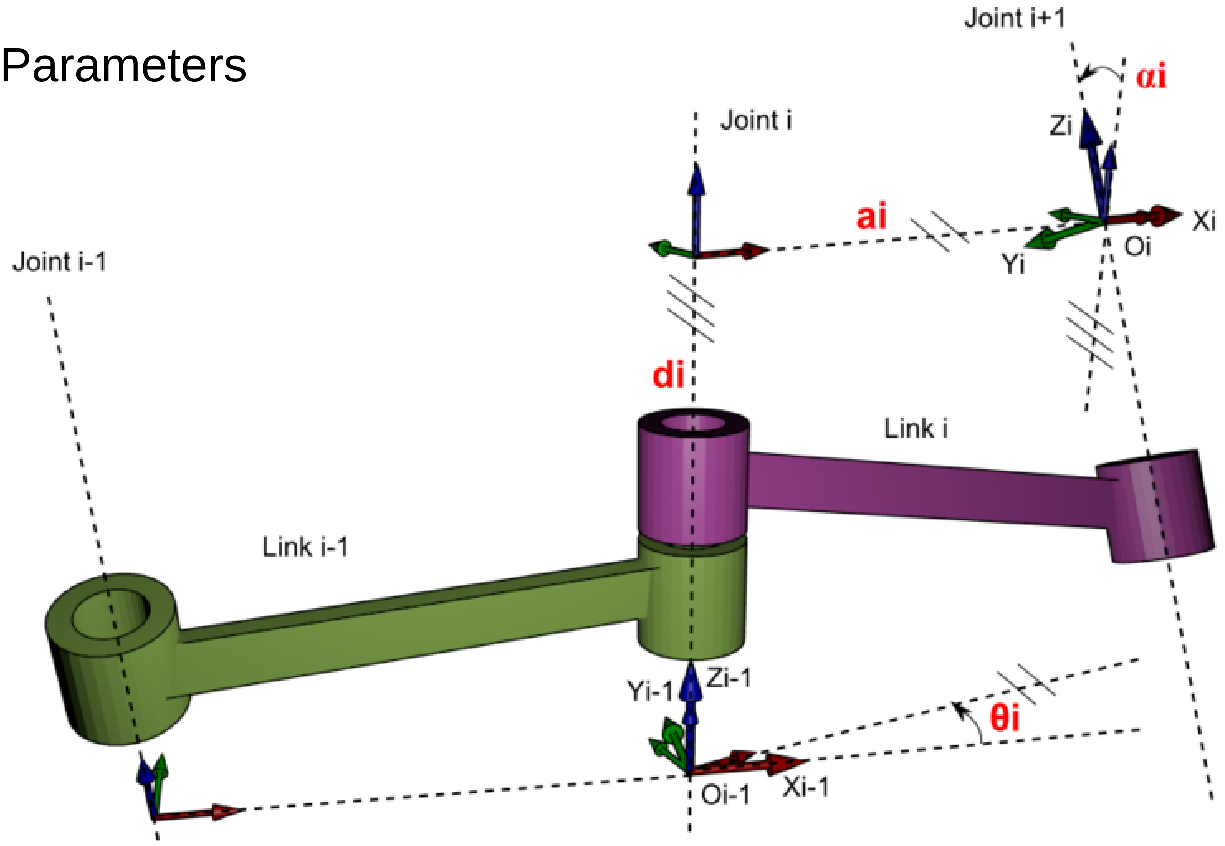
α: angle about common normal, from old z-axis to new z-axis

D-H Video



Video courtesy of Tekkotsu, open source project created & maintained at Carnegie Mellon University

D-H Parameters



$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

D-H to forward kinematics

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

D-H to forward kinematics

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

D-H to forward kinematics

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathrm{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

D-H to forward kinematics

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

D-H to forward kinematics

$$^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n)$$

$$\mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathrm{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\mathrm{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathrm{Rot}_{x_n}(\alpha_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

D-H to forward kinematics

$$ {}^{n-1}T_n = \mathrm{Trans}_{z_{n-1}}(d_n) \cdot \mathrm{Rot}_{z_{n-1}}(\theta_n) \cdot \mathrm{Trans}_{x_n}(r_n) \cdot \mathrm{Rot}_{x_n}(\alpha_n) $$

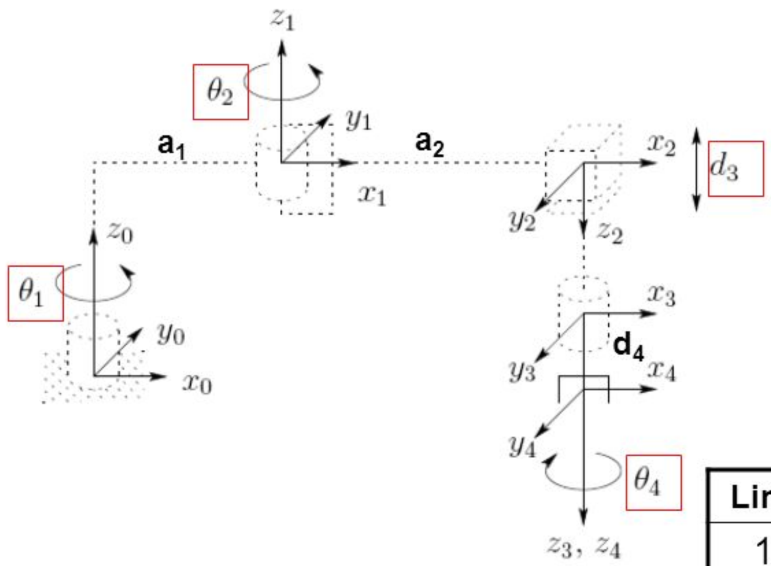
$$ \mathrm{Trans}_{z_{n-1}}(d_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathrm{Rot}_{z_{n-1}}(\theta_n) = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n & 0 & 0 \\ \sin\theta_n & \cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

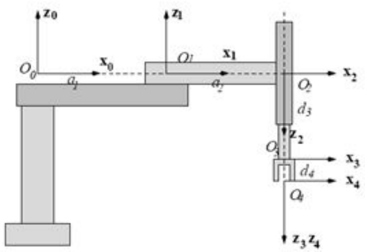
$$ \mathrm{Trans}_{x_n}(r_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & r_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \qquad \mathrm{Rot}_{x_n}(\alpha_n) = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_n & -\sin\alpha_n & 0 \\ 0 & \sin\alpha_n & \cos\alpha_n & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

$$ {}^{n-1}T_n = \left[\begin{array}{ccc|c} \cos\theta_n & -\sin\theta_n\cos\alpha_n & \sin\theta_n\sin\alpha_n & r_n\cos\theta_n \\ \sin\theta_n & \cos\theta_n\cos\alpha_n & -\cos\theta_n\sin\alpha_n & r_n\sin\theta_n \\ 0 & \sin\alpha_n & \cos\alpha_n & d_n \\ \hline 0 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right] $$

DH example

SCARA MANIPULATOR:



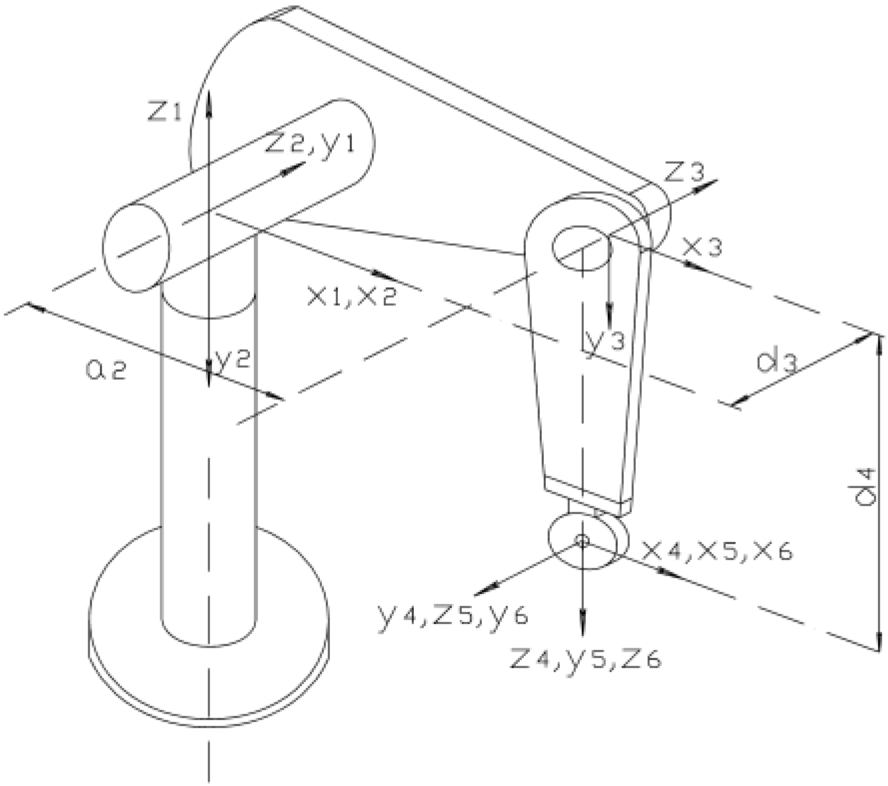


i	a_i	d_i	α_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	π	θ_2
3	0	d_3	0	0
4	0	d_4	0	θ_4

Figure 4.22: The SCARA robot.

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	180	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

DH Example

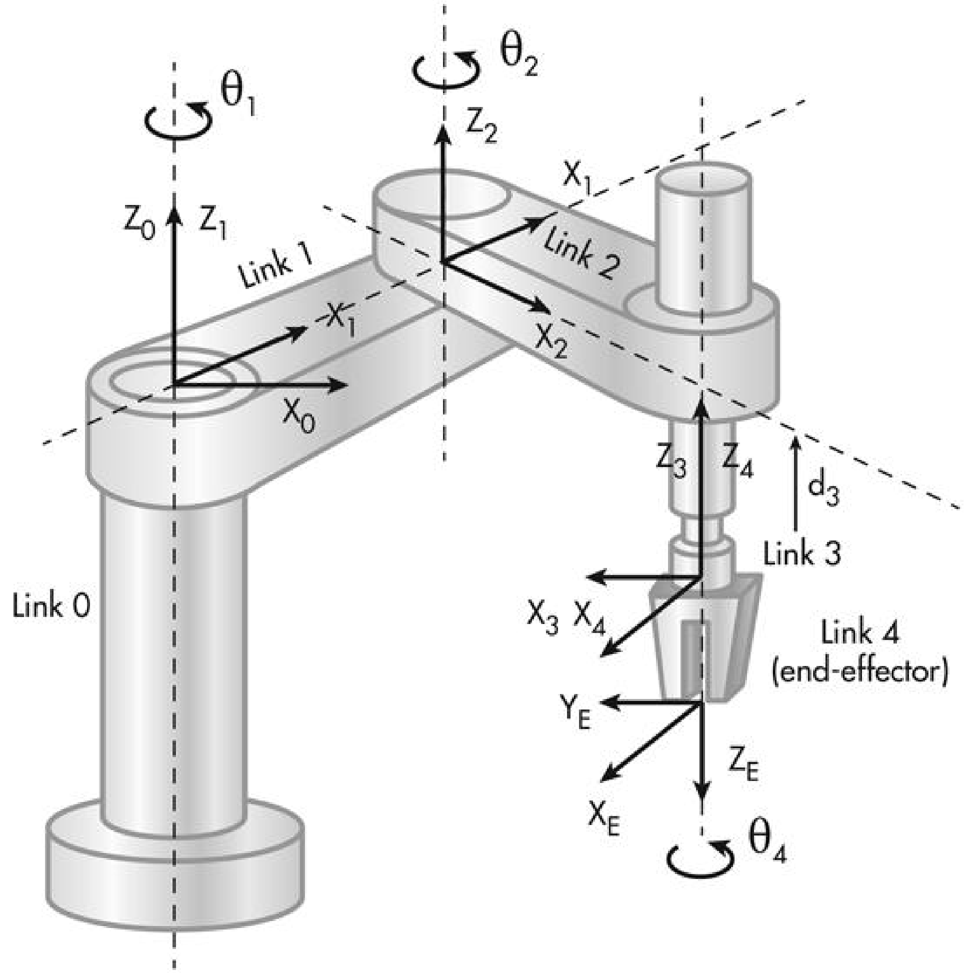


Picture 1: Robotic manipulator PUMA560 with assigned link parameters according to J.J. Craig

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	-90°	0	0	θ_2
3	0°	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

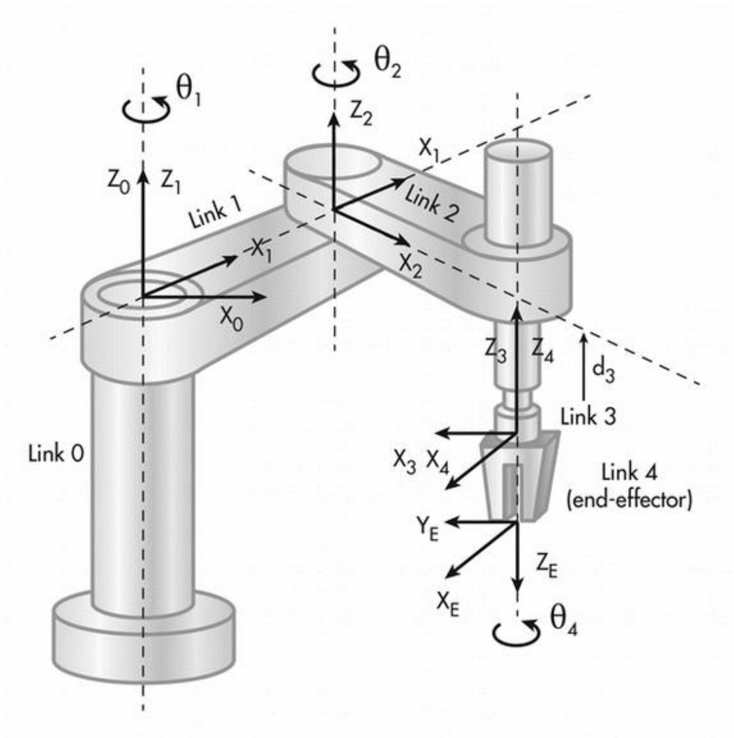
Table 1: Link parameters for PUMA 560 robotic manipulator

SCARA - Selective Compliance Articulated Robot Arm



SCARA - Selective Compliance Articulated Robot Arm

- Transform ${}^{B}\mathbf{T}_{E}$ from end effector to base frame is dependant on configuration $\boldsymbol{\Theta}$
- The function that generates $\boldsymbol{\Theta}$ given ${}^{B}\mathbf{T}_{E}$ (or $\mathbf{r}$) is called inverse kinematics $\mathbf{K}^{-1}$.

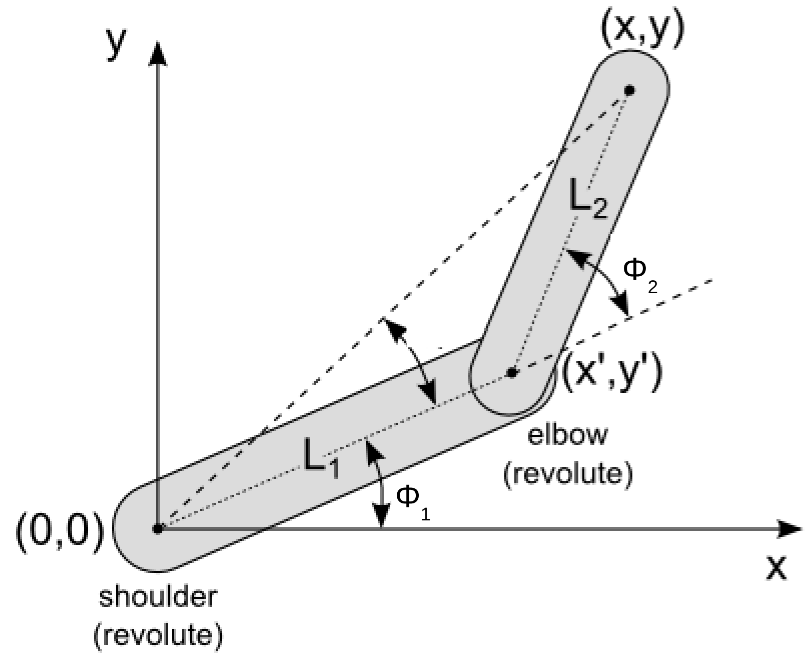


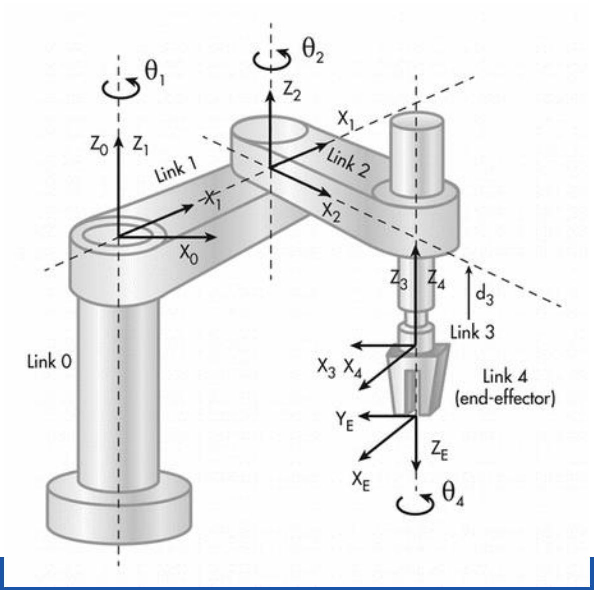
Inverse kinematics

$$x = L_1\cos(\Phi_1) + L_2\cos(\Phi_1+\Phi_2)$$
$$y = L_1\sin(\Phi_1) + L_2\sin(\Phi_1+\Phi_2)$$
$$z = -d_3$$
$$\Theta = \Phi_1 + \Phi_2 + \Theta_4$$

Solve for Φ_1, Φ_2, d_3 and Θ_4:

Assignment 2





Inverse Kinematics

- Is target reachable?

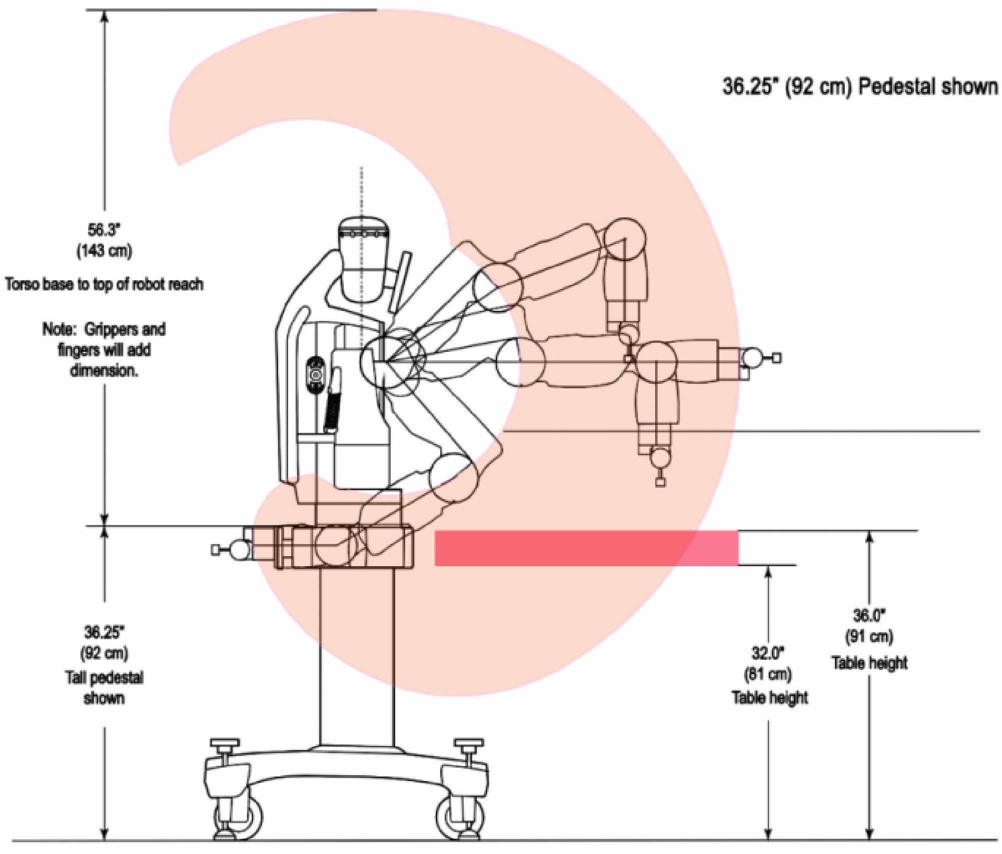


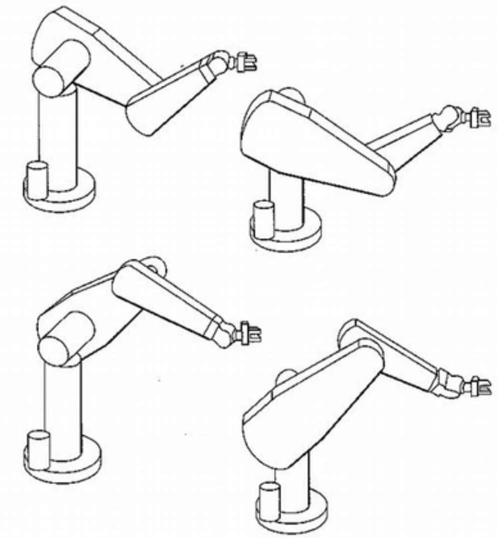
Image: Rethink Robotics

- Is target reachable?
- Is solution unique?

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	-90°	0	0	θ_2
3	0°	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Table 1: Link parameters for PUMA 560 robotic manipulator

Multiple solutions

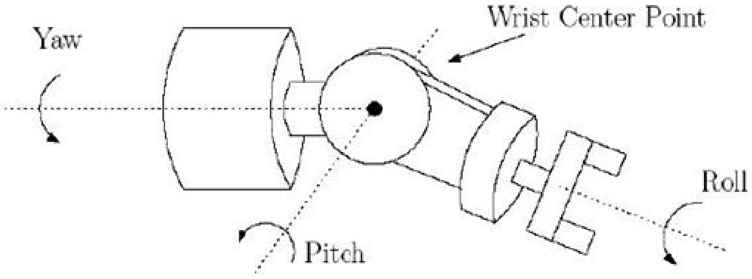


Direct & Inverse Kinematics 76

a_i	Number of solutions
$a_1 = a_3 = a_5 = 0$	$\leqslant 4$
$a_3 = a_5 = 0$	$\leqslant 8$
$a_3 = 0$	$\leqslant 16$
All $a_i \neq 0$	$\leqslant 16$

FIGURE 4.5: Number of solutions vs. nonzero a_i.

- Is target reachable?
- Is solution unique?
- Does closed form solution exist?



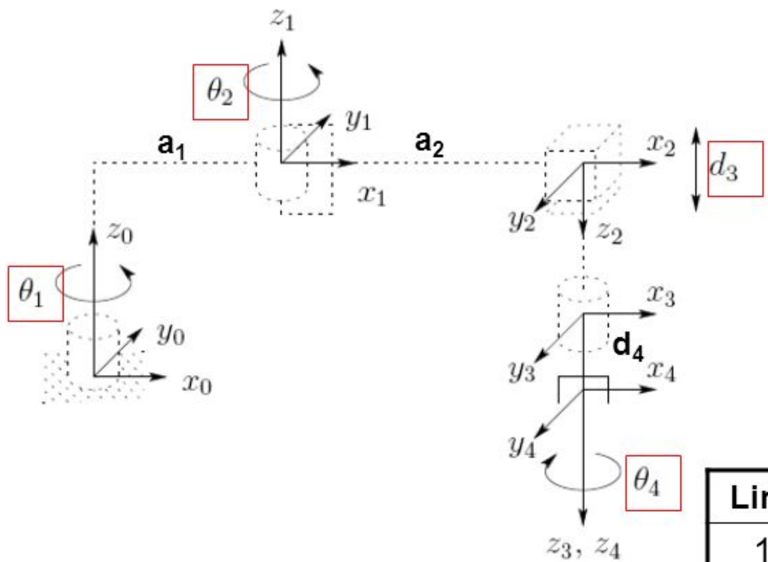
- Sufficient requirement for chain of 6 rotational joints:

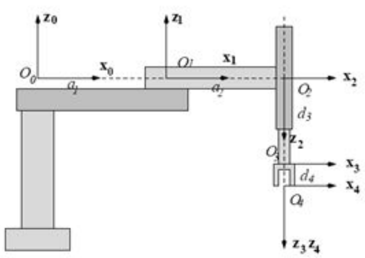
All α integer multiple of $\pi/2$ and three consecutive axis intersect

Image: J. J. Craig

DH example

SCARA MANIPULATOR:



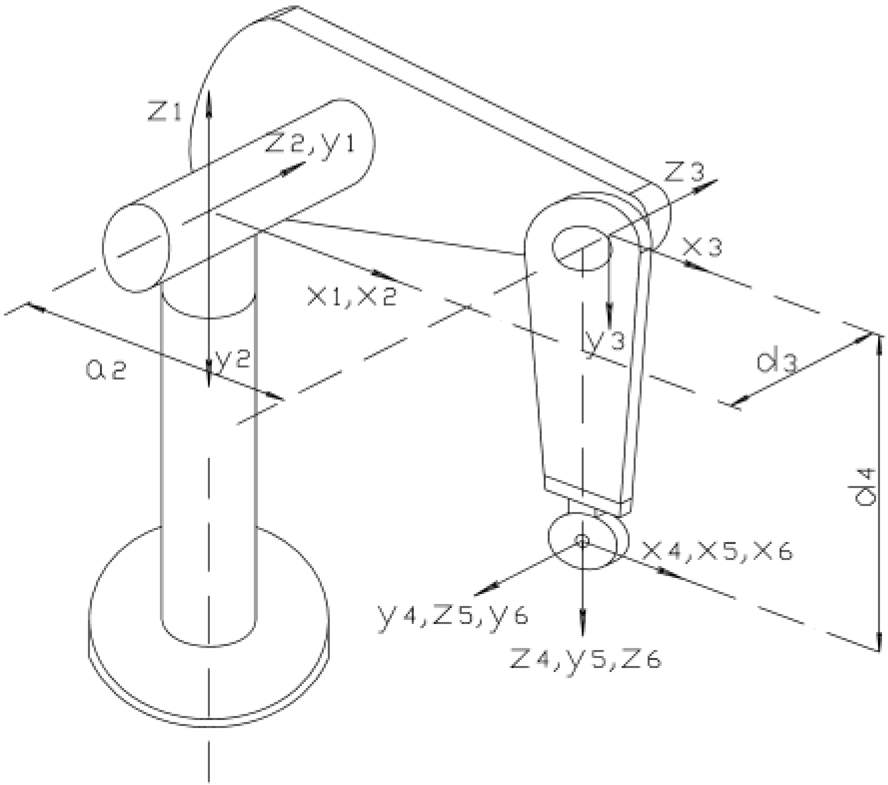


i	a_i	d_i	α_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	π	θ_2
3	0	d_3	0	0
4	0	d_4	0	θ_4

Figure 4.22: The SCARA robot.

Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	180	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

DH Example, as solved by J.J. Craig



Picture 1: Robotic manipulator PUMA560 with assigned link parameters according to J.J. Craig

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0°	0	0	θ_1
2	-90°	0	0	θ_2
3	0°	a_2	d_3	θ_3
4	-90°	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

Table 1: Link parameters for PUMA 560 robotic manipulator

Forward Kinematics

1 Forward Kinematics

The Puma 560 can be seen in Figures 1 and 2.

The forward kinematics K_f can be formulated as:

$$\mathbf{X} = K_f(\Theta) \tag{1}$$

where

$$\mathbf{X} = \begin{bmatrix} x \\ y \\ z \\ p \\ t \\ a \end{bmatrix}, \quad \Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \tag{2}$$

we have:

$$\begin{aligned}
x &= cos(\theta_1) * [a_2 cos(\theta_2) + a_3 cos(\theta_2+\theta_3) - d_4 sin(\theta_2+\theta_3)] - d_3 sin(\theta_1) \\
y &= sin(\theta_1) * [a_2 cos(\theta_2) + a_3 cos(\theta_2+\theta_3) - d_4 sin(\theta_2+\theta_3)] + d_3 cos(\theta_1) \\
z &= -a_3 sin(\theta_2+\theta_3) + a_2 sin(\theta_2) - d_4 cos(\theta_2+\theta_3) \\
p &= tan^{-1}\left(\frac{s1(c23c4s5+s23c5)-c1s4s5}{c1(c23c4s5+s23c5)+s1s4s5}\right) \\
t &= tan^{-1}\left(\frac{-s1(c23c4s5+s23c5)+c1s4s5}{sin(tan^{-1}(-s1(c23c4s5+s23c5)+c1s4s5/-c1(c23c4s5+s23c5)-s1s4s5))(s23c4s5-c23c5)}\right) \\
a &= tan^{-1}\left(\frac{-(s23(s4c6-c4c5s6)+c23s5s6)}{s23(s4s6-c4c5c6)-c23s5c6}\right)
\end{aligned} \tag{3}$$

Where the latter uses shorthand. The full expression is:

$$\begin{aligned}
p &= tan^{-1}\left(\frac{sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))-cos(\theta_1)sin(\theta_4)sin(\theta_5)}{cos(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+sin(\theta_1)sin(\theta_4)sin(\theta_5)}\right) \\
t &= tan^{-1}\left(\frac{-sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+cos(\theta_1)sin(\theta_4)sin(\theta_5)}{sin\left(tan^{-1}\left(\frac{-sin(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))+cos(\theta_1)sin(\theta_4)sin(\theta_5)}{-cos(\theta_1)(cos(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)+sin(\theta_2+\theta_3)cos(\theta_5))-sin(\theta_1)sin(\theta_4)sin(\theta_5)}\right)\right)(sin(\theta_2+\theta_3)cos(\theta_4)sin(\theta_5)-cos(\theta_2+\theta_3)cos(\theta_5))}\right) \\
a &= tan^{-1}\left(\frac{-(sin(\theta_2+\theta_3)(sin(\theta_4)cos(\theta_6)-cos(\theta_4)cos(\theta_5)sin(\theta_6))+cos(\theta_2+\theta_3)sin(\theta_5)sin(\theta_6))}{sin(\theta_2+\theta_3)(sin(\theta_4)sin(\theta_6)-cos(\theta_4)cos(\theta_5)cos(\theta_6))-cos(\theta_2+\theta_3)sin(\theta_5)cos(\theta_6)}\right)
\end{aligned} \tag{4}$$

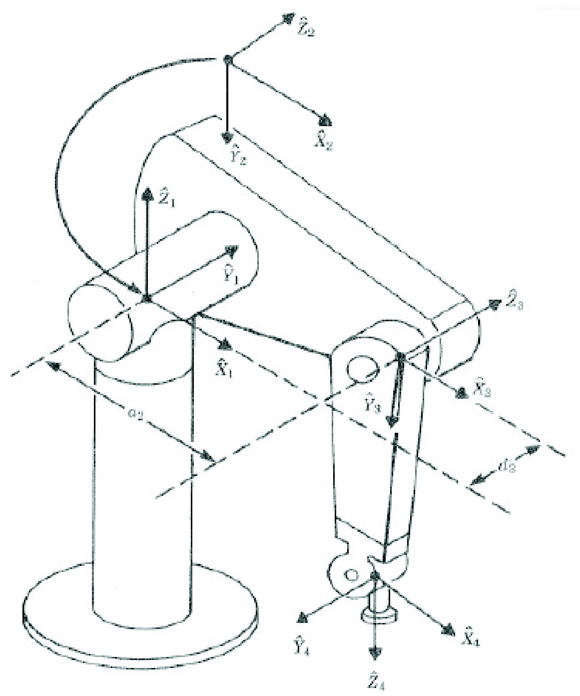


Figure 1: The puma 560

$$\theta_1 = tan^{-1}(\frac{y}{x}) - tan^{-1}\left(\frac{d_3}{\pm\sqrt{x^2+y^2-d_3^2}}\right) \tag{6}$$

$$\theta_3 = tan^{-1}(\frac{a_3}{d_4}) - tan^{-1}\left(\frac{\frac{x^2+y^2+z^2-a_2^2-a_3-d_3^2-d_4^2}{2a_2}}{\pm\sqrt{a_3^2+d_4^2-\left(\frac{x^2+y^2+z^2-a_2^2-a_3-d_3^2-d_4^2}{2a_2}\right)^2}}\right) \tag{7}$$

In order to maintain a reasonable size, we use the variables θ_1 and θ_3 in the following expression

$$\theta_2 = tan^{-1}\left(\frac{(-a_3 - a_2cos(\theta_3))z - (cos(\theta_1)x + sin(\theta_1)y)(d_4 - a_2sin(\theta_3))}{(a_2sin(\theta_3) - d_4)z - (a_3 + a_2cos(\theta_3))(cos(\theta_1)x + sin(\theta_1)y)}\right) - \theta_3 \tag{8}$$

In order to maintain a reasonable size, we use the variables θ_1 to θ_3 in the following expression

$$\theta_4 = tan^{-1}\left(\frac{-cos(p)sin(t)sin(\theta_1) + sin(p)sin(t)cos(\theta_1)}{-cos(p)sin(t)cos(\theta_1)cos(\theta_2+\theta_3) - sin(p)sin(t)sin(\theta_1)cos(\theta_2+\theta_3) + cos(t)sin(\theta_2+\theta_3)}\right) \tag{9}$$

In order to maintain a reasonable size, we use the variables θ_1 to θ_4 in the following expression

$$\theta_5 = tan^{-1}\left(\frac{cos(t)(sin(\theta_2+\theta_3)cos(\theta_4)) - cos(p)sin(t)(cos(\theta_1)cos(\theta_2+\theta_3)cos(\theta_4) + sin(\theta_1)sin(\theta_4)) - sin(p)sin(t)(sin(\theta_1)cos(\theta_2+\theta_3)cos(\theta_4) - cos(\theta_1)sin(\theta_4))}{-cos(p)sin(t)(cos(\theta_1)sin(\theta_2+\theta_3)) - sin(p)sin(t)(sin(\theta_1)sin(\theta_2+\theta_3)) - cos(t)(cos(\theta_2+\theta_3))}\right) \tag{10}$$

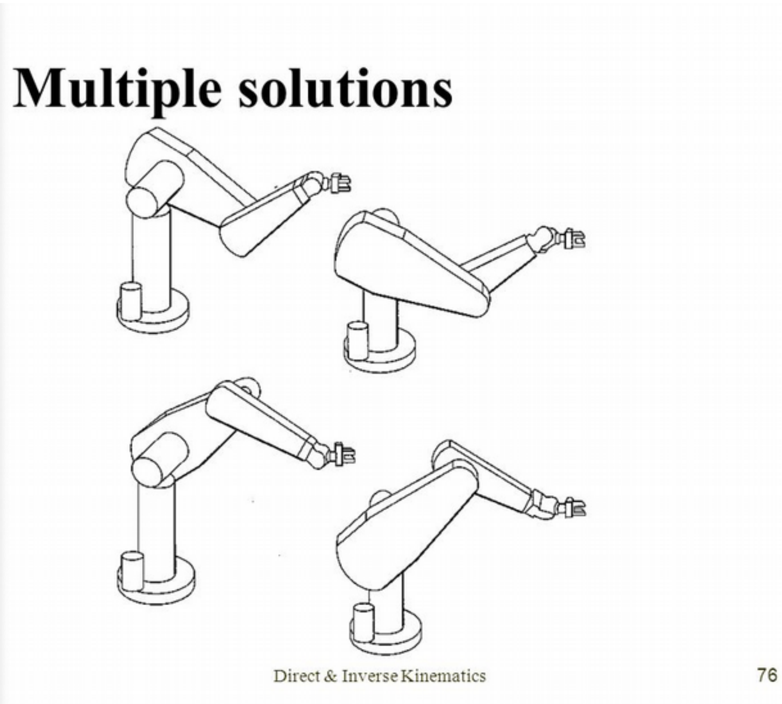
In order to fit the last equation on the page, we need to use some shorthand notation. Note that the r_{xx} terms have names from the rotation matrix they come from.

$$\begin{aligned}
c_i &= cos(\theta_i) \\
s_i &= sin(\theta_i) \\
c_{ij} &= cos(\theta_i + \theta_j) \\
s_{ij} &= cos(\theta_i + \theta_j) \\
r_{13} &= cos(p)sin(t) \\
r_{23} &= sin(p)sin(t) \\
r_{33} &= cos(t) \\
r_{12} &= -cos(p)cos(t)sin(a) - sin(p)cos(a) \\
r_{22} &= cos(p)cos(a) - sin(p)cos(t)sin(a) \\
r_{32} &= sin(t)sin(a)
\end{aligned}$$

We can now write:

$$\theta_6 = tan^{-1}\left(\frac{-(r_{22}r_{31} - r_{21}r_{32})(c_1c_{23}s_4 - s_1c_4) - (r_{32}r_{11} - r_{12}r_{31})(s_1c_{23}s_4 + c_1c_4) + (r_{12}r_{21} - r_{11}r_{22})(s_{23}s_4)}{(r_{22}r_{31} - r_{21}r_{32})((c_1c_{23}c_4 + s_1s_4)c_5 - c_1s_{23}s_5) + (r_{32}r_{11} - r_{12}r_{31})((s_1c_{23}c_4 - c_1s_4)c_5 - s_1s_{23}s_5) - (r_{12}r_{21} - r_{11}r_{22})(s_{23}c_4c_5 + c_{23}s_5)}\right) \tag{11}$$

Inverse kinematics, multiple solutions



How do we choose the best configuration?

Task-dependent

Redundant manipulators

- Solution for inverse kinematics in general case: Numerical solvers.
- Available in ROS: KDL, TRAC-IK
- Typically requires differential kinematics
- See next lecture, and assigment 2!

Further Kinematics: Parallell robots

Forward kinematics typically more difficult than inverse.
Better rigidity than serial robots -> faster, stronger
Less flexibility -> smaller workspace

Video: ABB

Further kinematics: Snake robots

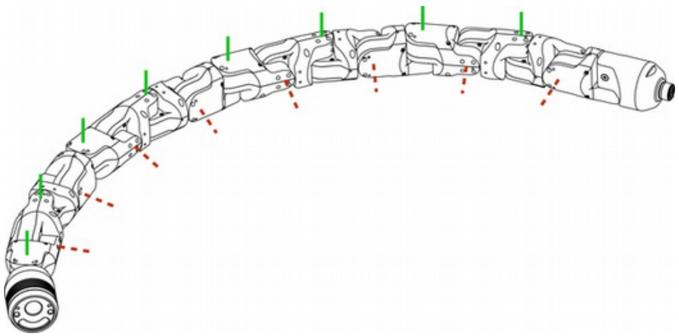
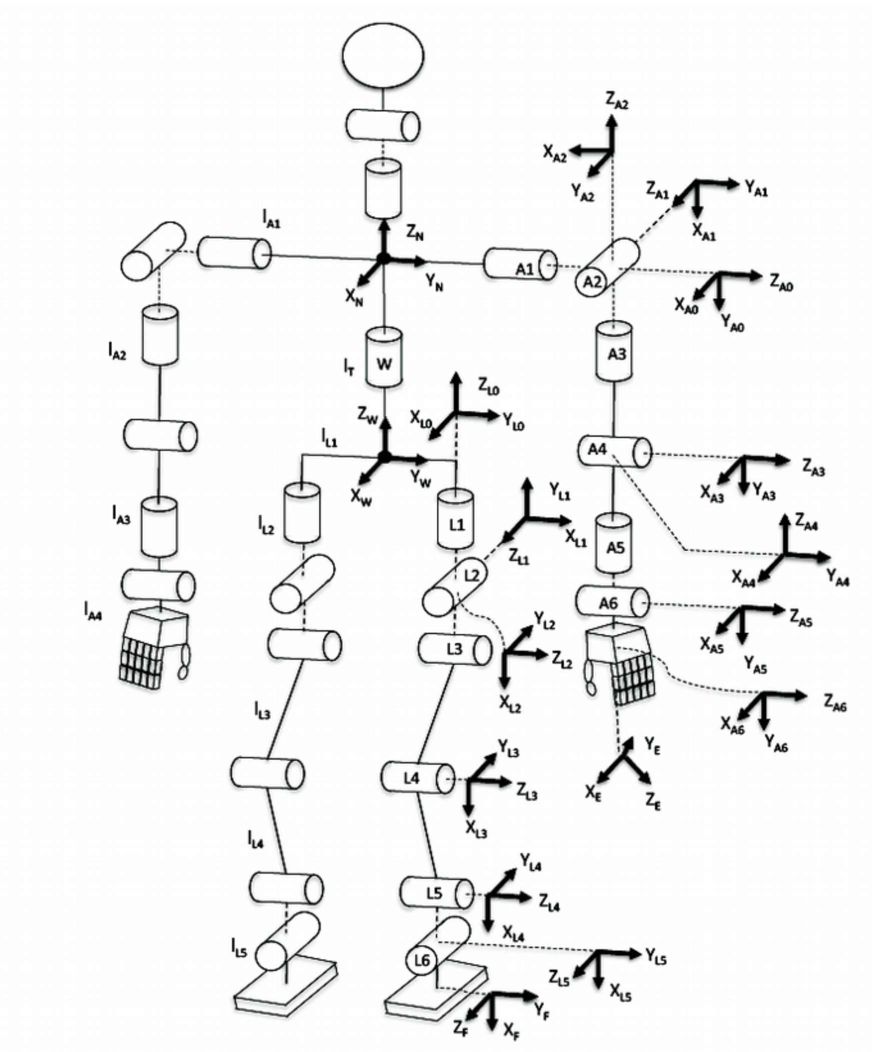


Image: CMU

Manipulators - branching chains

Manipulators - branching chains



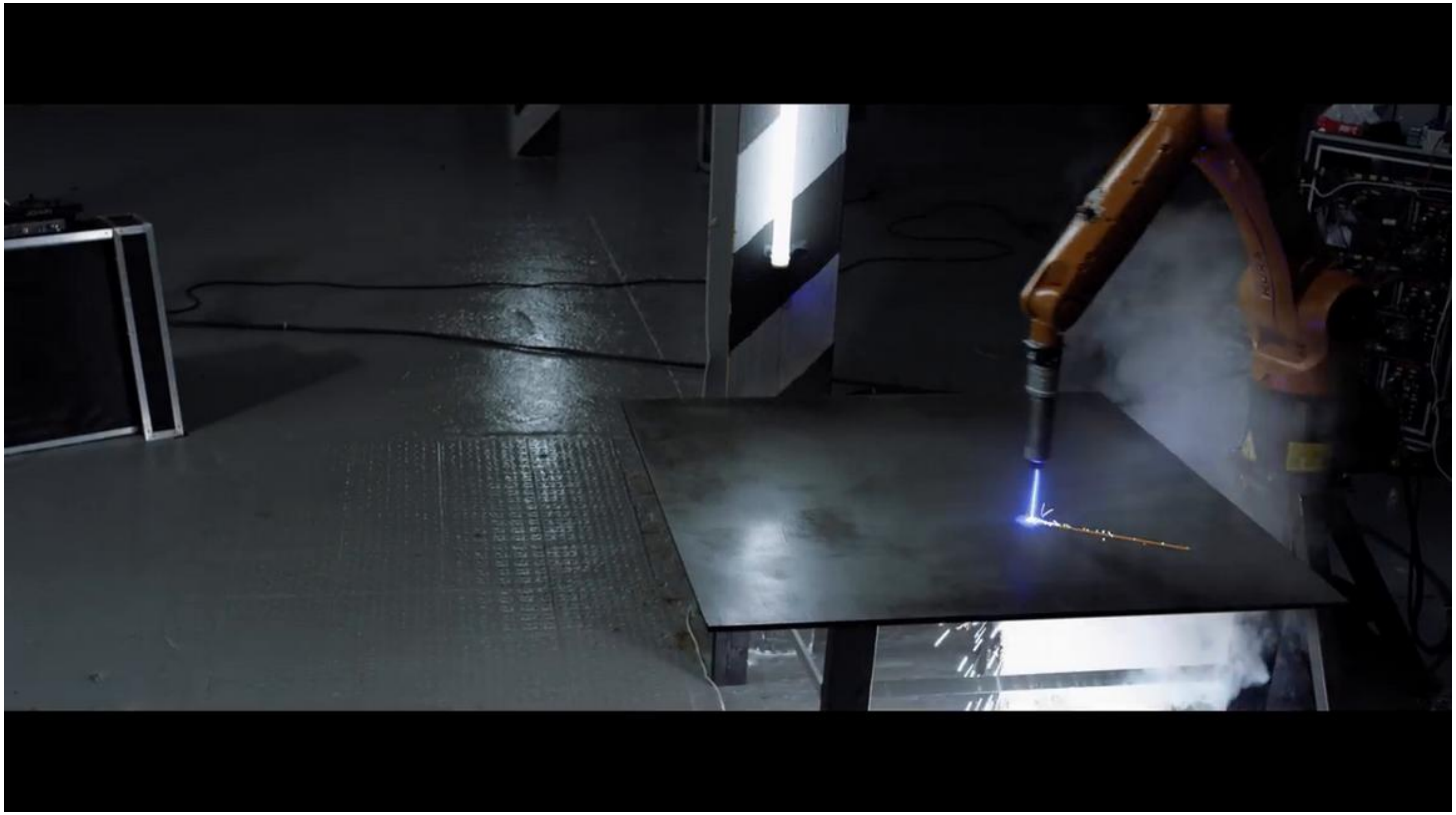
Rowland O'Flaherty, Peter Vieira, Michael Grey,Paul Oh, Aaron Bobick, Magnus Egerstedt, and Mike Stilman, "Kinematics and Inverse Kinematics for the Humanoid Robot HUBO2+" Georgia Institute of Technology, Technical Report, GT-GOLEM-2013-001, 2013

Complete kinematics break-down?

Image: ETHZ

At least some robots are simple...

Manipulator applications



Nigel Stanford:
Automatica

Title

- Example applications
  - Pickit 3D (Francisco)
  - Kinema Systems (Karl)

Kinema Systems

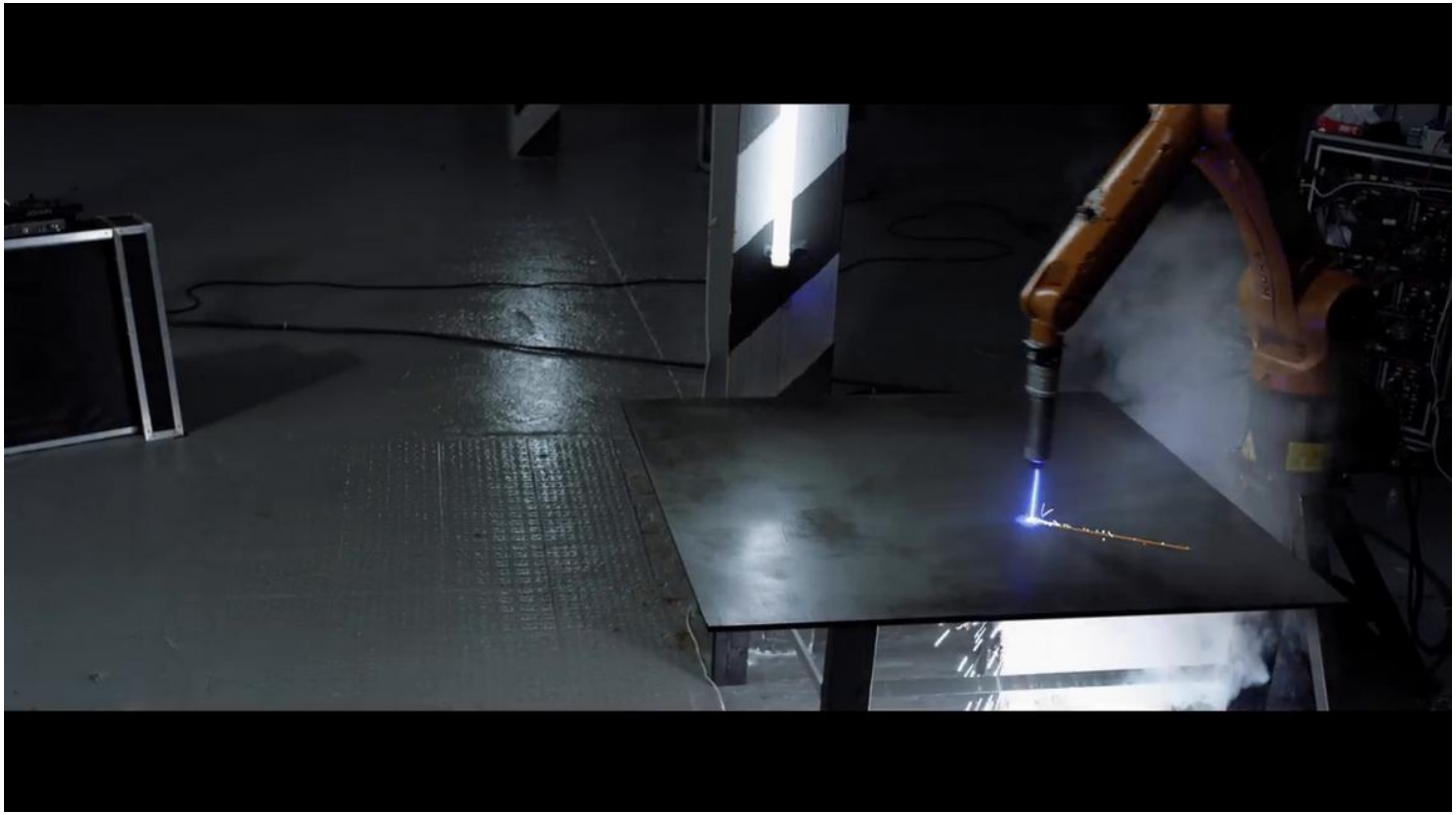
Pick-it 3D

Manipulator applications

- Camera placement

Bot and Dolly: Box

Manipulator applications



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