

# Risk-Based Interest Rate Expectations

Robert Rogers\*

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\*London School of Economics. Email: [r.rogers@lse.ac.uk](mailto:r.rogers@lse.ac.uk). I thank Ian Martin, Dimitri Vayanos, Peter Kondor, Martin Oehmke, and seminar participants at the London School of Economics for their helpful comments and advice.

# Risk-based Interest Rate Expectations

## Abstract

I develop a new method to extract interest rate expectations from option markets that circumvents the challenges of non-stationary interest rate levels. The approach leverages a no-arbitrage relationship linking term premia to variance risk premia observable in swaptions markets. Under the assumption that investors' marginal utility has limited nonlinear exposure to interest rates, I can convert variance risk premia directly into term premium estimates without specifying the dynamics of interest rate levels. Using USD swaptions data from 2002 to 2023, I document three main findings. First, the unconditional term premium averaged 44 basis points annually on 10-year rates — historical excess bond returns reflected large, anticipated risk premia rather than systematic forecast errors. Second, term premia constructed from variance forecasts significantly outperform standard affine term structure models and survey expectations in predicting excess bond returns out-of-sample, achieving a Sharpe ratio of 0.73 versus -0.17 for a traditional term structure model. Third, the puzzling secular decline in long-term rates during FOMC announcement windows is accompanied by sharp, transitory compressions in risk-neutral variance that are difficult to reconcile with information revelation but consistent with temporary risk premium effects. The method provides a tractable, theory-based alternative to traditional term structure modeling.

*Keywords:* *term premium, term structure, interest rate risk, duration*

*JEL classification:* *G12, G21*

In January 2010 the yield curve sloped sharply upwards. A 1 year loan could be made with a yield of just 0.6%, but the same loan agreed to start in 1 year would pay 2%.<sup>1</sup> For central bankers and investors this situation posed urgent questions: do markets really expect interest rates to rise this quickly — perhaps they are expecting inflation to spike, or have misinterpreted central bank communications? Or was this just a big risk premium?

The finance profession has many tools to answer this question, from simple bond return regressions to full-fledged dynamic term structure models. What all of these approaches have in common is that they use a set of historical predictors to forecast interest rates, or, equivalently, bond returns, next period.

This paper proposes a different approach: measure risk premium from options markets. I start from an identity that relates the risk premium that an investor perceives on an interest rate forward to the risk premium she perceives on an interest rate *variance* contract — a contract that pays off squared changes in interest rates. I show that if the marginal investor's payoffs are not “too nonlinear” in rates, we can use option prices to convert this variance contract risk premium into interest rate expectations.

I then forecast the variance risk premium, derive the implied interest rate expectations, and show it has better out of sample predictive power than traditional forecasts. This measure can be interpreted as the expected interest rates that must be perceived by an unconstrained, rational investor with log utility who chooses to hold roughly linear exposure to interest rates.

Intuitively, if markets expect there are large returns to be had from long-term bonds, then market participants will bet heavily on bonds. The equilibrium price of insurance against large interest rate changes (the “variance risk premium”) will thus be expensive because of all of this risk-taking. I measure the price of this insurance, and use it to back out the implied interest rate expectations.

This approach may seem unnecessarily indirect, but it offers three key advantages over

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<sup>1</sup>The 1 year LIBOR swap rate on January 1st was 0.6%, and the 1y-in-1y swap forward rate was 2%

directly forecasting interest rates:

First, it is simply empirically easier to forecast variance and variance risk premium than levels and returns. Variance can be learned quickly from high frequency observations and simple forecasting tools are empirically successful across a range of asset classes (e.g. [Bollerslev, 1986](#); [Corsi, 2009](#)).

Second, interest rate variance is clearly stationary, whereas the level may not be. As of 2021, the 10 year interest rate had fallen almost continuously for 30 years. So who could say what long term rate it would converge to, or if it even does converge to one? As a result, direct forecast of rates are highly sensitive to specification choices around stationarity and long term behavior that the data says very little about ([Cochrane, 2007](#)). In contrast, the variance has clearly bounced around a mean with little long term trend, shown in figure 1.

Third, variance-based forecasts are more credible as a measure of subjective market expectations. Professional forecasters 2000-2021 consistently missed by forecasting rates that were too high. Because interest rate forecasts were so sensitive to assumptions, these may well have been rational forecasts based on a reasonable set of priors ([Nakamura and Steinsson, 2018](#)). So if we set up a model that, with the benefit of hindsight, predicts rates better than the forecasters, are we capturing true expectations, or just mistakes and differences in priors? In contrast, there is no reason why investors should be making systematic mistakes in forecasting variance. And it would be a surprising coincidence if they were to make mistakes forecasting both levels and variance that neatly coincided in the implied risk premium.

The risk-based estimate of term premium depends on one key unobservable quantity, which I will call the “residual coskew.” This term measures the risk neutral covariance of squared rate changes with the inverse SDF (i.e. an investor’s marginal utility). I will show that if we assume that this quantity is 0, the implied term premium is a strong out-of-sample predictor of interest rates. This amounts to an assumption that the variance risk premium primarily reflects directional interest rate risk (fear of rates rising or falling) instead of non-linear interest rate risk (fear of both high and low rate states).

I will argue that this assumption is not unreasonable, based on the data we can observe. For popular models of the SDF, covariance with squared changes in rates appears to be too low to easily explain the variance risk premium. And the time series of variance risk premium does not line up with conditional estimates of this covariance.

Ultimately, the best test of the model is whether it forecasts interest rates. My predicted interest rates are highly significant predictors that outperform traditional models. I forecast 10 year yields 2002-2023 over 1 month, 1 quarter, and 1 year horizons with lower squared errors than the most common dynamic term structure models or the expectations hypothesis (table 5). These differences are economically large. A simple trading strategy based on these forecasts yields a Sharpe ratio of 0.73, vs -0.17 for one based on a typical dynamic term structure model.

The power of the forecasts comes from the use of theory to derive the level of term premium. An alternative “atheoretical” approach of regressing the predictor onto changes in interest rates to calculate the level yields worse out of sample results.

I use these new forecasts to investigate two economic questions:

First, were bond investors in 2000-2024 expecting rates to rapidly bounce back to their long term means, as implied by professional forecaster surveys? Or were they expecting rates to stay low, and instead expecting large returns on long term bonds? I find that investors did not seem to expect rapid return of rates to historical means. Expected returns on long-term bonds were high, consistent with assets being priced by leveraged intermediaries with high portfolio durations.

Second, do investors learn their long-term interest rate expectations from the Fed? [Hillenbrand \(2025\)](#) documents a striking empirical finding: the entire decline in long-term interest rates over the past few decades occurred in the three-day window around FOMC announcements. This finding is interpreted as evidence that markets learn about the long run state of the economy from the Fed. I provide suggestive evidence that it may instead represent declining risk premia, with little change in expectations.

The identity employed in this paper implies that if the decline in interest rates comes from term premium, we should expect to see a sharp drop in risk neutral variance and variance risk premium that reverses after the announcements.

In fact we see a 3% drop in risk neutral variance per meeting that later reverses. This decline is far larger than what can be explained by a mechanical effect of uncertainty resolving after the meeting, and is potentially large enough that term premium could account for the entire FOMC-window decline in rates. While I cannot rule out that physical variance expectations also shift during these windows, the large and transitory nature of these moves points toward risk premium effects rather than fundamental reassessments of future rate volatility.

This pattern suggests a possible alternative interpretation in which the monetary policy consistently pushed down term premium, either through demand effects driven by the declines in short term rates or by communication.

## Related Literature

This paper contributes to three literatures.

First, it extends techniques from the literature on option-based expected returns ([Martin, 2017](#); [Kremens and Martin, 2019](#); [Chabi-Yo and Loudis, 2020](#); [Tetlock, McCoy, and Shah, 2024](#)) to apply to a broader range of asset classes.

So far this literature has taken the perspective of an equities investor to derive interest rate expectations. This is not possible for interest rates, because there are no derivative products that reveal the risk neutral covariance of rates with equities. I show that by instead taking the perspective of an investor with arbitrary linear exposures we can derive expectations for any asset class with options, including interest rates. The specifics of my method are particular close to [Tetlock et al. \(2024\)](#), who also calculate expected returns from variance risk premium and option implied skewness.

Second, it contributes to the extensive literature on term premium prediction and dy-

namic term structure models. The conditional forecast methodology presents an alternative to traditional term structure models as a forecasting tool. But the results can also be interpreted as supporting the use of shifting end-point models that allow for non-stationary interest rates e.g. [Kozicki and Tinsley \(2001\)](#); [van Dijk, Koopman, van der Wel, and Wright \(2014\)](#); [Bauer and Rudebusch \(2020\)](#), and as supporting evidence for models with unspanned stochastic volatility, e.g. [Collin-Dufresne and Goldstein \(2002\)](#); [Collin-Dufresne, Goldstein, and Jones \(2009\)](#).

Third it offers a novel perspective on the Fed's influence over long term rates, adding to a long literature including [Gürkaynak, Sack, and Swanson \(2005\)](#); [Campbell, Evans, Fisher, and Justiniano \(2012\)](#); [Hanson and Stein \(2015\)](#); [Nakamura and Steinsson \(2018\)](#); [Hillenbrand \(2025\)](#) and many others.

The empirical findings of this paper are also related to [Bauer and Chernov \(2024\)](#), who find that risk-neutral skewness predicts treasury yields, [Choi, Mueller, and Vedolin \(2017\)](#), who document that there is a durable interest rate variance risk premium, and [Trolle and Schwartz \(2014\)](#), who measure risk neutral yield moments from swaptions.

## Organization of the paper

Section 1 derives the key relationship between variance risk premia and term premia. Section 2 describes the swaptions data and construction of risk-neutral moments. Section 3 tests the unconditional implications, showing that historical term premia must have been large. Section 4 develops the conditional term premium measure and section 5 evaluates its forecasting performance. Section 6 validates the key identifying assumption of the method: that the residual coskew is small. Section 7 examines FOMC announcement windows, documenting the puzzling behavior of risk-neutral variance. Section 8 concludes. An appendix contains extensions to other maturities, robustness tests, and technical details.

# 1 Measuring expectations from risk

If investors are highly exposed to interest rate risk, the equilibrium price to insure against variance in rates will be high. The price of that insurance can therefore inform us about their interest rate exposure and the expected interest rate.

This section formalizes this intuition by deriving an identity relating the term premium to the variance risk premium and one unobservable quantity. I then discuss the key assumption needed to operationalize this relationship.

## 1.1 Defining interest rates and term premium

This paper aims to measure an expected interest rate next period:

$$E_t(y_{t+1})$$

The  $y_{t+1}$  could represent any rate or yield. When I implement the theory, I will use swap rates, focusing on the 10 year swap rate one quarter or one year in the future.

Consider a one-period linear interest rate forward. At time  $t + 1$  the buyer receives the level of the interest rate minus the pre-agreed forward price  $F_t$ . By definition, the price of this forward will be the risk neutral expectation of the interest rate. The buyer's payoff is equal to:

$$\Delta y_{t+1} = y_{t+1} - F_t = y_{t+1} - E_t^*(y_{t+1})$$

where the  $E_t^*$  operator represents risk neutral expectations.

If we can measure the expected payoff on the interest forward, then we know the expected interest rates next period. I will define the “term premium” as the expected payoff from selling the linear interest rate forward. This is equal to the difference between the forward rate and the expected rate:

$$TP_t = E_t^*(y_{t+1}) - E_t(y_{t+1}) = -E_t(\Delta y_{t+1})$$

Under the expectations hypothesis, the term premium is always zero, and forward rates tell us the expected rate in the future. In reality, of course, it turns out that forward rates are a very poor guide to the future path of rates, and so term premium must be large and variable (Fama and Bliss, 1987; Campbell and Shiller, 1991). This paper attempts to measure those premia, and thus the expected interest rate.

Note that my definition of “term premium” differs slightly from the terminology used in many other papers in which term premium refers to the difference between spot rates and the expected short rate over the life of a bond. My definition reflects the more general, colloquial sense of term premium as being compensation for interest rate risk.<sup>2</sup>

I use linear interest rate forwards as the basic unit of my analysis, whereas most work in this field uses excess returns on zero coupon bonds. While these two concepts are closely related, the forward framework allows us to work with any yield or interest rate, rather than just zero coupon bonds. In practice we observe options for swaps or coupon bonds and not zero coupon bonds, so this will prove convenient.

We do not observe linear forwards for most interest rates. However their value can be constructed from the observable yield curve, with a minuscule convexity adjustment described in sections 2 and appendix B.

## 1.2 Measuring term premium without assumptions on the rate process

I will assume that there is no arbitrage and the fundamental theorem of asset pricing holds throughout this paper. In that case, the following result allows us to learn about expectations without specifying an interest rate process:

**Proposition 1.** *If no arbitrage holds, the expectation of any payoff  $X$  is given by:*

$$E_t^*(X) - E_t(X_t) = -\frac{1}{R_{f,t}} \text{cov}_t^* \left( \frac{1}{M_{t+1}}, X \right)$$

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<sup>2</sup>E.g. “The term premium is meant to measure duration risk, or the risk inherent in owning a Treasury that matures in 10 years instead of one year, or five.” (Scaggs, 2025)

Where  $E^*$  represents risk neutral expectations. From [Martin and Wagner \(2019\)](#)<sup>3</sup>.

Applying this identity to the interest rate  $y_t$  yields:

$$TP_t = -\frac{1}{R_{f,t}} \text{cov}_t^* \left( \frac{1}{M_{t+1}}, y_{t+1} \right)$$

The term premium is revealed by the risk neutral covariance of interest rates with the SDF. This result resembles the more familiar asset pricing identity that relates expected returns with the *physical* covariance with the SDF. However, it has the advantage of working with directly observable quantities. Risk neutral covariances are potentially observable from asset prices.

Hence if we can make some assumptions about the nature of the SDF (i.e. what constitutes a good or bad state) and find the price of an asset whose payoffs are linked to interest rates but also this SDF, then we can calculate interest rate expectations.

### 1.3 Projecting the SDF onto yields

The existing literature that uses proposition 1 to measure expectations has taken the perspective of an equity investor for whom  $\frac{1}{M_{t+1}}$  is a function of stock market returns ([Martin, 2017](#); [Martin and Wagner, 2019](#); [Kremens and Martin, 2019](#); [Chabi-Yo and Loudis, 2020](#); [Tetlock et al., 2024](#)). In this case, expected returns are given by the risk neutral covariance with stock market returns.

This approach will not work for interest rates. We simply do not observe any liquid assets that reveal the risk neutral covariance of interest rates with stocks [Martin \(2025\)](#).

Additionally, even if such an asset were available, we might question whether equity returns are a reasonable proxy for fixed income investor wealth. The efforts to find joint risk factors across stocks and bonds have not always been successful.

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<sup>3</sup>The proposition can be proven simply by calculating the price of an asset with payoff  $Z = \left( \frac{1}{M_{t+1}} - R_{f,t} \right) X$  using both SDF and risk neutral pricing notation

I therefore propose a new approach that does not rely on equity returns. Instead I will consider the SDF as a function of interest rates and residual terms.

Without loss of generality, we can consider the projection of any SDF onto interest rates ( $\Delta y$ ) under the risk neutral measure:

**Definition 1.** *Projection of the inverse SDF*

$$\frac{1}{M_{t+1}} = R_{f,t+1} (1 - \lambda_t \Delta y_{t+1} + \varepsilon_{t+1}) \quad (1)$$

Where  $E_t^*(\varepsilon_{t+1}) = \text{cov}_t^*(\varepsilon_{t+1}, \Delta y_{t+1}) = 0$

This is simply a linear projection, and is a fully general statement for any model with no arbitrage. The intercept must be  $R_{f,t+1}$  because  $E_t^*\left(\frac{1}{M_{t+1}}\right) = R_{f,t+1}$  and  $E_t^*(\Delta y_{t+1}) = 0$ . The projection will be exact, and  $\varepsilon_{t+1} = 0$ , only in the case of a log investor with exactly linear exposures to the interest rate  $y_t$ , i.e. an investor who only holds the risk free rate and interest rate forwards.

In general, I will refer to  $\lambda_t$  as the investor's "exposure" to interest rates, because it captures how much the investor suffers from a rise in interest rates.

In the simple case of a log investor in bonds,  $\lambda_t$  is the duration of the investor's portfolio — i.e. how much the wealth decreases if rates increase. If the investor also has other asset exposures (e.g. equities), then  $\lambda_t$  will be the risk neutral beta of his assets to changes in the interest rate.

The approximation is not limited to log utility. If, for example we consider a bond investor with CRRA utility with risk aversion  $\gamma$ ,  $\lambda$  will be equal to portfolio duration multiplied by  $\gamma$  plus a small convexity adjustment. For example, with a CRRA of 2 and a linear exposure of duration 20,  $\lambda \approx 40$ .

## 1.4 Term premium as a function of exposure

Applying proposition 1 to measure interest rate expectations, and plugging in the projection for  $\frac{1}{M_{t+1}}$  immediately yields an expression for term premium

$$TP_t = \lambda_t var_t^*(\Delta y_{t+1}) \equiv \lambda_t \sigma_t^{*2} \quad (2)$$

In words, equation (2) tells us the term premium is equal to exposure times risk neutral variance. Loosely speaking, this is because in equilibrium, the amount of risk agents take should tell us both the rewards to taking interest rate risk (the term premium) and the price to insure against large changes in interest rates (the risk neutral variance).

Now we have an expression for term premium in terms of an observable variable. The risk neutral variance represents the price of a contract that pays out squared interest rate changes and can be easily measured from option prices, using the methodology described in section 2. It is the interest rate equivalent of the *SVIX*<sup>2</sup> equities index from [Martin \(2017\)](#), and is highly correlated with the well-known *VIX* “fear index” for equity markets or the *MOVE* index for bonds.

However, we still need to know our exposure term,  $\lambda_t$  before we know interest rate expectations. [Martin \(2017\)](#) derives a similar expression for equities and posits that the exposure should be near 1. But for interest rates, the sign and magnitude of  $\lambda$  is not obvious. Is the marginal investor long or short duration, and by how much?

## 1.5 Exposure as a function of variance

We want to use the value of  $\lambda_t$  to measure expected payoffs on interest rate forwards. But the same logic can be applied in reverse: if we can estimate any expected payoff, we can use it to learn about  $\lambda_t$ .

I will measure the expected payoff on a “variance contract” — i.e. a contract that pays out  $\Delta y_{t+1}^2$  — and use this expected payoff to learn about  $\lambda_t$ . This approach resembles the

strategy that [Tetlock et al. \(2024\)](#) apply to equity markets.

Variance has three important characteristics that make it an attractive payoff to forecast:

1. Interest rate variance does not display the same long-term non-stationary trend as the level (see figure 1)
2. Variance can be measured easily with high frequency data. For any function involving higher moments, this becomes more challenging due to volatility clustering.<sup>4</sup>
3. There is a long track record of empirical success with simple variance forecasting models (e.g. [Bollerslev, 1986](#); [Corsi, 2009](#))

Applying proposition 1 to the payoff  $\Delta y_{t+1}^2$  yields:

$$\sigma_t^{*2} - E_t(\Delta y_{t+1}^2) = \lambda_t E_t^*(\Delta y_{t+1}^3) - cov_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1}) \quad (3)$$

This equation tells us the expected profit from selling variance contracts is equal to exposure scaled by the risk neutral third moment minus an unobservable term that I will call the “residual coskew.”

The risk neutral third moment can be measured from option data in exactly the same way as the risk neutral variance. So if we can make an assumption the size of the residual coskew term, then a forecast of the expected squared changes in rates ( $E_t(\Delta y_{t+1}^2)$ ) will yield a value for the exposure and term premium.

## 1.6 The residual coskew assumption

$\varepsilon_t$  represents the portion of the inverse SDF that is uncorrelated with interest rates under the risk neutral measure. The inverse SDF, loosely speaking, captures how “good” states

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<sup>4</sup>Intuitively, the daily autocorrelation of changes in rates should be low. Otherwise there would be large potential trading profits. But the daily autocorrelation of squared changes in rates, or the last period’s change in rates with next periods squared change may be very high. Variance is persistent. Hence we cannot find a good estimate for third or higher moments by simply dividing up the sample into small pieces and calculating the moments of the short observations.

are for the investor ( $1/\lambda_t$  marginal utility). So the residual coskew term,  $\text{cov}_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1})$  represents the extent to which the investor tends to be better or worse off in states of the world with large interest rate changes, regardless of their sign.

This quantity is unobservable. We do not observe  $\varepsilon$ , and even if we did, we would not observe the option prices that would reveal the relevant risk neutral moment.

Producing an estimate of term premium therefore requires us to take an assumption on the size of this covariance. The simplest assumption that is most consistent with the existing literature is simply that the value is zero. This assumption will approximately hold, for example in all standard dynamic term structure models, because they assume that the log SDF is linear in interest rate factors (making the inverse SDF nearly linear over short horizons).

Setting the residual coskew to zero amounts to an assumption that the risk premium on variance contracts reflects directional interest rate risk by the investor (i.e.  $\lambda_t$ ) rather than a non-linear exposure to rates. In other words, the reason why the investor does not choose to sell variance contracts despite their positive payoff is because of his level of interest rate risk. This does not impose that other asset classes (e.g. equities) are not important to the investor, but only that the directional exposure of those assets to interest rate risk is more important than exposure to squared changes.

I will take this as my main benchmark in developing a measure of interest rate expectations. It turns out that despite the simplicity of this assumption, it produces forecasts that perform very well out of sample and line up with the variables we expect to forecast term premium (e.g. the slope of the yield curve), as shown in section 5.

In section 6 I will measure the closest observable data and argue the assumption is empirically reasonable. I calculate the physical covariance of  $\varepsilon$  with squared changes in rates under a few common specifications for the SDF and find they are an order of magnitude too small to explain the size variance risk premium. I further show that they do not help to explain the time series of variance risk premium vs a simple no-residual-coskew model.

## 1.7 Identifying the term premium

Imposing the no-residual-coskew assumption, we can rewrite equation (3) in terms of more easily measurable items as:

$$\sigma_t^{*2} - \sigma_t^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \sigma_t^{*4} \lambda_t^2 \quad (4)$$

Where I have here used the fact that:

$$E_t(\Delta y_{t+1}^2) = \sigma_t^2 + TP_t^2 = \sigma_t^2 + \sigma_t^{*4} \lambda_t^2$$

I will call the left hand side of (4) — the difference between the risk neutral and physical variance — the “variance risk premium” (VRP), consistent with the existing literature (e.g. [Bollerslev, Tauchen, and Zhou, 2009](#); [Drechsler and Yaron, 2011](#)).

We now have an equation with only two items that cannot be directly read from asset prices: the physical variance  $\sigma_t^2$ , and the exposure  $\lambda_t$ . So an estimate of the physical variance will identify the value of  $\lambda_t$ .

The risk neutral third moment ( $E_t^*(\Delta y_{t+1}^3)$ ) tells us how directional exposure translates into variance risk premium. If the distribution is highly right skewed (i.e. the third moment is high), then the variance contract will mostly pay off in high-rate states of the world where the investor is poor and hence will be highly valuable. If on the other hand the risk neutral third moment is very low, it will take a large amount of exposure to deliver any variance risk premium. The identification of  $\lambda_t$  will therefore be stronger in periods when the risk neutral third moment is high.

## 2 Data and risk neutral moment construction

I extract risk premia from the prices of interest rate swaps and swaptions. This section describes the data sources and how I construct risk-neutral moments from option prices.

## 2.1 Data

I measure realized variance from daily interest rate swap rates obtained from Refinitiv, and risk neutral moments from swaptions quotes obtained from Bloomberg. The sample covers USD LIBOR swaps from December 2001 to June 2023, when LIBOR ceased publication. I track six tenors (1, 2, 5, 10, 20, and 30 years) at three forecast horizons (3 months, 1 year, and 5 years). I construct swap forwards from swap rates and treasury yields using the procedure described in appendix [B](#).

The main results of these paper focus on the 10 year swap rate, the most liquid tenor. The methodology can be readily applied to other tenors of swap rate with similar results, although a few extra complications are introduced by the zero lower bound for short tenors. Appendix [A](#) shows the results from forecasts of the 1, 2, 5, 20, and 30y term premium.

The results of this paper should be interpreted as term premium on swap rates, rather than treasury rates. Arguably this is the more relevant rate for many market participants who are more likely to borrow at rates linked to swap rates or hedge using swaps.

The differences are unlikely to be large. The realized quarterly term premium on 10 year swaps vs treasuries during my sample period differs by just 0.7 basis points. The spread between 10 year swaps and the equivalent (off the run) treasury yield averages just 12 basis points during the sample period and it explains only 3% of the variation of the 10 year yield. Since 2008 this spread is often negative and is thought to be related to leverage constraints and capital requirements rather than bank credit risk ([Boyarchenko, Gupta, Steele, and Yen, 2018](#)).

I use swaptions to calculate risk neutral interest rate moments. A swaption is a contract that gives its holder the right to enter an interest rate swap at a predetermined rate. Swaption markets represent the most liquid interest option market, with extensive participation of both buy-side and inter-dealer trading ([ISDA, 2014](#); [Barnes, 2024](#)). While swaptions trade over-the-counter rather than on exchanges, collateralization through initial and variation margin is standard practice and limits counterparty credit risk.

Before 2011 only at-the-money swaption quotes are available. Out-of-the-money quotes are available for 10 year treasuries from CME and for closely-related LIBOR caps and floors from Bloomberg. I therefore estimate pre-2011 risk neutral variance and skewness using the parameters from a regression on at-the-money variance, the 10 year treasury risk neutral skewness (calculated by [Bauer and Chernov, 2024](#)), and the risk neutral skewness of caps and floors. These regressions have an  $R^2$  of 0.99 for variance and 0.7 for skewness. Data before 2007 is only available on a weekly basis.

Treasury options data from CME offers an alternative source of interest rate option data. While this data source has a longer history than swaptions, it covers fewer tenors, has lower trading volume, and trades as American options at a single, short and time-varying horizon of 1-3 months.

## 2.2 Constructing risk-neutral moments

I follow [Carr and Madan \(1998\)](#) and [Martin \(2017\)](#) to extract model-free measures of risk-neutral moments from option prices. The key insight is that a portfolio of options with strikes spanning all possible outcomes has a payoff equal to the squared rate change. The price of this portfolio therefore equals the risk-neutral variance.

This result holds exactly when we can trade options on the underlying rate itself. For swaptions, which are written on swap rates rather than yields, I need a technical adjustment. Swaption payoffs depend on annuity factors that vary with rates. I handle this by assuming changes in annuity yields approximate changes in swap yields—an approximation that introduces negligible error for the rate movements in my sample (see appendix [B](#) for the derivation and calculation steps).

The third moment follows similarly. A portfolio of options weighted appropriately produces a payoff of  $\Delta y_{t+1}^3$ , and its price gives the risk-neutral third moment.

Table [1](#) presents summary statistics for these risk-neutral moments, and figure [8](#) in appendix [C](#) plots the time series. Variance is high during and after the financial crisis,

during the 2022 inflation, and, more surprisingly, in 2002-4. The high option-implied interest rate volatility of the early 2000s as noted at the time as puzzling by central banks (Fornari, 2005; ECB, 2005).

These risk-neutral moments form the foundation of my analysis. Combined with forecasts of physical variance, they allow me to extract term premia without taking a stand on the long-run level of interest rates.

### 3 A simple risk-based test of term premium

Over my sample from 2002 to 2023, 10-year swap rates exceeded the forward rate-implied prediction by an average of 34 basis points per year, leading to large excess returns for long term bond investors. But were these excess returns anticipated? Or were markets repeatedly surprised, expecting mean reversion that failed to materialized, as implied by the Survey of Professional forecasters and the Bluechip survey? I test this by inferring investor's beliefs from variance risk premium.

If we take as an assumption that the “residual coskew” term is 0 (discussed in greater detail in section 6), equation (4) allows us to calculate the investor's exposure to rates as a function of the physical variance:

$$\sigma_t^{*2} - \sigma^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \sigma^{*4} \lambda_t^2$$

To estimate the exposure, we can simply replace the physical variance with the realized variance from daily data, plus an error term.

$$\sigma_t^{*2} - RV_{t \rightarrow t+1} = \lambda_t E_t^*(\Delta y_{t+1}^3) + \sigma^{*4} \lambda_t^2 + \eta_{t+1} \quad (5)$$

Now all terms in this equation are observable except for  $\lambda_t$  the error term  $\eta_t$ . And if the conditional expectation of realized variance is equal to variance, then this error term

should be uncorrelated with the other terms in the equation

As a simple test of the size of term premium, I solve for the constant  $\lambda$  that minimizes the sum of squared errors  $\eta$ . This is a straightforward non-linear least squares estimate. The results are shown in table 2.

I can soundly reject the expectations hypothesis that  $\lambda = 0$  with a t-statistic of over 5. Instead I find that  $\lambda = 0.41$  (standard error 0.06), implying an average term premium of 11 bp, or 44 bp annualized.

On average, the expected term premium is similar to the 34 bp of realized term premium. Markets seem to have been demanding compensation for bearing interest rate risk, rather than making systematic mistakes. Interpreting the realized term premium as a mistake would require that investors also made equivalent-sized mistakes in forecasting variance.

Since I denominate  $\Delta y_t$  in percentage points, this exposure estimate corresponds to a duration of 42 years for a log fixed income investor (with a confidence interval from 0.26 to 0.58), or approximately 21 years for a fixed income investor with CRRA utility and a risk aversion coefficient of 2.

This high sensitivity to interest rate changes could be consistent with models where highly leveraged financial intermediaries are the marginal investors in fixed income markets. For example [Kekre, Lenel, and Mainardi \(2024\)](#) suggest durations of 10 to 30 years for fixed income arbitrageurs.

## 4 Constructing a conditional term premium forecast

The simple tests from section 3 establish that historical term premia were large. But for practical applications like trading or policy analysis we need conditional estimates. What is the term premium today? How much room does the Fed have to lower long rates through market actions that don't affect expectations? When is duration risk particularly expensive or cheap?

This section develops a conditional term premium estimator. I start from the same relationship as in section 3:

$$\sigma_t^{*2} - \sigma_t^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \lambda_t^2 \sigma_t^{*4} \quad (6)$$

where I continue to assume the residual coskew term is small (discussed in Section 6). I then add structure in two ways. First I parameterize  $\lambda_t$  as a function of yield curve factors. Second, I use conditional forecasts of physical variance based on past realized variance.

These assumptions add power to the estimator, but their form is not critical. The appendix contains versions that drop each of these assumptions.

#### 4.1 Parameterizing the marginal investor's exposure

I take a theory-agnostic approach and simply parameterize  $\lambda_t$  as a function of the first three principal components of the yield curve and the two risk neutral moments used to construct my estimates (variance and skewness):

$$\lambda_t = \lambda_0 + \lambda_1 f_{1,t} + \lambda_2 f_{2,t} + \lambda_3 f_{3,t} + \lambda_4 \sigma_t^{*2} + \lambda_5 skew_t^* \quad (7)$$

where  $f_1$ ,  $f_2$ , and  $f_3$  are the first three principal components of yields, which capture the level, slope, and curvature of the yield curve.

This can be interpreted as an assumption that investors choose their level of exposure depending on the shape of the yield curve and risk neutral distribution of future yields, or that the shape of the yield curve responds to investor choices and exposure.

The first three principle components explain  $> 99\%$  of variation in the shape of the yield curve and are empirically well known to be related to term premium. It has been standard practice in term structure modeling since Duffee (2002) to assume that the price of risk is a function of yield curve factors.

I include the risk neutral moments for the sake of completeness because they are used

in construction of the estimator and because empirical work has suggested they contain additional information on term premium that is not included in the yield curve ([Joslin and Konchitchki, 2018](#); [Bauer and Chernov, 2024](#)). Ultimately I find these are not significant explainers of the variance risk premium (shown in table 7 in section 5), but I allow my estimator to determine this rather than assuming anything a priori.

This paper's methodology is flexible, and more state variables could easily be added in the future to improve prediction.

## 4.2 Forecasting physical variance

To measure the variance risk premium  $\sigma_t^{*2} - \sigma_t^2$ , I need a forecast of physical variance. I employ the heterogeneous autoregressive realized variance (HAR-RV) model of [Corsi \(2009\)](#), widely used in the variance risk premium literature (e.g., [Bollerslev et al., 2009](#); [Drechsler and Yaron, 2011](#)):

$$RV_{t \rightarrow t+H} = \beta_0 + \beta_1 RV_{t-5d \rightarrow t} + \beta_2 RV_{t-21d \rightarrow t} + \beta_3 RV_{t-63d \rightarrow t} + \varepsilon_{t+H} \quad (8)$$

Where  $RV_{t \rightarrow t+H}$  is the realized variance calculated from daily data from period  $t$  to  $t+H$ . I use weekly, monthly, and quarterly lags rather than the standard daily, weekly, and monthly to better suit the longer forecast horizons in my application, and because I do not have intra-day swap rate data. The model performs remarkably well out-of-sample, achieving  $R^2$  values exceeding 20% even at quarterly horizons (table 3). This predictability confirms that it is easier to learn about variance than means, one of the key advantages of the risk-based forecasting approach.

Results are robust to including interest rate levels or allowing variance to depend on rate levels through a constant elasticity of variance specification.

### 4.3 Estimation

Substituting the parameterization (7) and the variance forecast into (6) yields:

$$\sigma_t^{*2} - \hat{\sigma}_t^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \lambda_t^2 \sigma_t^{*4} + \eta_t \quad (9)$$

Where:

$$\lambda_t = \lambda_0 + \lambda_1 f_{1,t} + \lambda_2 f_{2,t} + \lambda_3 f_{3,t} + \lambda_4 \sigma_t^{*2} + \lambda_5 skew_t^*$$

The error term  $\eta_t$  captures three potential sources: (i) forecast errors in physical variance, (ii) time variation in  $\lambda_t$  orthogonal to the principal components, and (iii) the residual coskew term.

I estimate the parameters  $\{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4\}$  using nonlinear least squares. To address heteroskedasticity and reduce the influence of a few high-variance states, I weight observations by the inverse of risk-neutral variance. This is equivalent to dividing equation (4.3) through by  $\sigma_t^*$  and improves the stability of estimates, reducing the excess kurtosis of the left hand side variable from 4.2 to 0.1.

Standard errors are calculated using GMM with Newey-West corrections, using lags equal to the forecast horizon. Some caution is warranted in interpreting these standard errors as the appropriate autocorrelation structure for  $\eta_t$  depends on the relative importance of its three components.

### 4.4 Non-parametric specifications

Equations 7 and 8 impose structure that enables estimation but may miss important variation in  $\lambda_t$  or physical variance. In the appendix D, I consider a lambda-parameter-free approach estimates  $\lambda_t$  period-by-period without functional form assumptions. This provides maximum flexibility but suffers from poor identification when skewness is low. During the high-skew periods, the results closely match the main results from the parameterized version.

## 5 Risk-based conditional term premium results

The conditional estimator from Section 4 produces term premium forecasts purely from variance data, without any direct use of yield curve information. Does this variance-based approach actually work? In this section I validate the measure’s predictive power, and explore what it reveals about the economic forces driving term premia.

First, I show the forecasts are powerful predictors of interest rate changes from 1 month to 1 year. The variance-based approach significantly outperforms the expectations hypothesis, survey expectations, and standard stationary dynamic term structure models (DTSMs), with large improvements in  $R^2$ , and implied trading profits. The approach also outperforms the simple random walk forecasts, but without statistical significance. The results of this section focus on the 10 year swap rate, but appendix A shows the predictive power is also high across other tenors.

Second, I compare my the time series of my term premium estimate to the traditional DTSM estimates. My measure captures some of the same familiar patterns: term premia rise when the yield curve steepens and the level increases. But the key difference is that I do not forecast large negative term premium when interest rates are low. Stationary models imply investors were expecting rates to pop back up to average levels quickly, whereas I find investors never seem to expect rates to rise much faster than forwards. As a result, the average value of my estimated term premia are larger and align more closely with realized excess returns.

Third, I show that the underlying exposure of the marginal investor to interest rates has been gradually declining over time. Some of this change can be related to changes in the stock bond correlation, but much remains a puzzle.

### 5.1 Forecasting performance

The most direct test of any term premium measure is whether it predicts changes in interest rates. If my variance-based estimates truly capture term premia, then high estimated term

premia today should predict that rates will rise relative to forward-rate expectations.

Table 4 reports regressions of realized rate changes on predicted term premia at various horizons:

$$\Delta y_{t,t+h} = \alpha + \beta \times \widehat{\text{TP}}_{t,h} + \varepsilon_{t+h} \quad (10)$$

where  $\widehat{\text{TP}}_{t,h}$  denotes the predicted term premium from time  $t$  to  $t+h$ .

The predictions are highly significant at monthly (t-statistic 2.9), quarterly (t-statistic 2.9), and annual (t-statistic 2.4) horizons. More importantly, I cannot reject  $\beta = 1$  or  $\alpha = 0$  at any horizon. This suggests the measure captures term premia accurately in levels, not just direction. The variance-based approach, using no information from the yield curve itself, recovers the compensation investors demand for bearing duration risk.

The  $R^2$  values are high even at short horizons (4% monthly, and 6% quarterly), where term structure models have traditionally struggled. The short term performance allows for high trading profits as demonstrated in section 5.1.1.

This predictive power is greater than that of professional forecasters or traditional term structure models. Table 5 reports out of sample relative  $R^2$  statistics of the risk-based model vs these other models, calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the error when the risk-based model is used to forecast changes in the yield and  $\nu_t$  is the error when the alternative model is used. This represents the improvement in forecast accuracy from using the risk-based measure versus alternatives.

I compare my forecasts of the 10 year yield to the expectations hypothesis, the survey of professional forecasters (SPF), the two most commonly used DTSMS ([Adrian, Crump, and Moench, 2013](#); [Kim and Wright, 2005](#)), the [Bauer and Rudebusch \(2020\)](#) non-stationary “observed shifting endpoint” model and a simple forecast of no change in rates (i.e. “random walk”). The SPF and [Bauer and Rudebusch \(2020\)](#) data are produced quarterly, and [Bauer and Rudebusch \(2020\)](#) forecasts are only produced up to 2018. Details on the construction of the alternative benchmarks are included in the appendix.

The risk-based interest rate expectation measure delivers more accurate forecasts than all alternative benchmarks at all horizons. Improvements are highly statistically significant vs the expectations hypothesis and the stationary DTSMs at the 1 month and 1 year horizons. The improvements versus the [Bauer and Rudebusch \(2020\)](#) model are weaker (9% improvement in annual  $R^2$ ), and with just 18 years of data, the difference is not statistically significant.

The power of the risk-based forecasts comes from the use of theory to derive the level of term premium. If instead we were to take our predictor and regress it on changes in interest rates in each period, we would get a substantially worse out of sample performance, as the coefficient loadings swing around based on past performance. The last row of table 5 compares the out of sample  $R^2$  from the risk based measure to a regression-based forecast constructed in this fashion. Using theory to derive the levels of the forecasts delivers a large (although only borderline significant) improvement in  $R^2$ .

The random walk forecast (i.e. the current rate) remains surprisingly competitive, beating all benchmarks except for my risk-based forecast. This reminds us that assuming no mean reversion was, ex post, a successful strategy during this period. However, ex-ante it had no obvious justification.

### 5.1.1 Economic significance

To demonstrate that the  $R^2$  from the predictive regressions in section 5.1 are large, figure 7 shows cumulative returns from trading strategies based on different term premium estimates.

The basic strategy takes positions in interest rate forwards proportional to expected term premia: buy duration when term premia are high, sell when they are low. Panel (b) shows returns using simple monthly position sizes equal to the expected term premium in percentage points. Panel (a) scales positions by the inverse of risk-neutral variance, investing more aggressively when uncertainty is low, mimicking the behavior of the implied log investor.

The risk-based strategy generates substantial cumulative returns over the sample period, with a Sharpe ratio of 0.75. Using variance-weighted positions, a dollar invested in 2002 grows to over \$100 by 2023, compared to <\$5 using SPF forecasts or ACM. Much of the outperformance comes from 2013–2021, when stationary models consistently predicted rising rates that did not arrive.

The large cumulative returns on the variance-weighted strategy underscores that the log investor benchmark may be more aggressive than the typical institutional investor. If we instead consider an investor with CRRA coefficient 2 with the same  $\lambda_t$ , he will take positions approximately one half as large and have a cumulative return of around 10 instead of  $> 100$ . This is roughly in line with the cumulative return of stock indices during this period.

These results should be interpreted cautiously. The strategies exhibit high volatility, and transaction costs, though likely small in liquid swap markets, would reduce returns. Nevertheless, the magnitude of outperformance suggests that the statistical improvements in forecasting are economically meaningful.

## 5.2 The time series of term premium

The time series of term premium is shown in figure 3. We see two large peaks in 2002-4 and 2009-11. Term premium is then low 2012–2022 except for a brief spike during the “taper tantrum” in 2013.

To contextualize the size of these term premia, figure 4 compares expectations hypothesis versus risk based forecasts over time. The dark line plots the time series of the 10 year swap rate, while at each quarter the faint gray and red lines plot the forward rates and the expected rate (using this paper’s estimates) one year ahead. In the early 2000s and in 2010 a naive observer might look at the forward rates and assume the market was expecting rapid increase in rates (light gray). But the risk adjusted expectations measure from this paper shows those were just periods of very high risk aversion and the true expectations were for the 10y rates to remain roughly constant over the following year.

This time series shows many similarities to well known term premium models. Table 6 compares the forecast to various alternative benchmarks including the Survey of Professional Forecasters (SPF) and the widely used [Adrian et al. \(2013\)](#) (ACM) affine term structure model, and figure 5 plots the ACM series term premium versus my estimates. The SPF and ACM forecasts both show > 50% correlation with the risk based estimate at the 1y horizon, despite the different underlying methods and sources of information.

The key difference lies in the behavior at low rates. The stationary assumption build into standard DTSMs meant that they always forecast rates to return to their long term averages eventually. So when current and forward rates were low, as in 2016–2020, the ACM model forecast a very large negative term premiums. At their lowest, these forecasts implied investors were expecting to lose -1.5% on 10 year bonds in a single quarter.<sup>5</sup>

In contrast, my estimates make no assumption on stationarity of interest rate levels, and end up forecasting almost no term premium in this period. These forecasts are more in line with the suggestive evidence from surveys of institutional investors who mostly do not seem to report negative expected excess returns on bonds ([Dahlquist and Ibert, 2024](#); [Couts, Gonçalves, and Loudis, 2023](#)).

### 5.3 Variation in investor exposure to rates

Besides a term premium estimate, the variance based approach delivers an estimate of  $\lambda_t$ : the exposure of the marginal investor's wealth to interest rate increases. Figure 6 shows the evolution of the estimated (in-sample)  $\lambda_t$  over time. The series shows a gradual decline from 0.8 to approximately 0 in 2020, followed by an increase to 0.35 in 2023.

The gradual pace of changes suggests that the simple example of the log investor with constant exposure (discussed in section 1) works fairly well over short horizons. The  $\lambda$  parameter is relatively stable over the course of a month, with changes in risk neutral variance explaining 77% of the variance in 1 month changes in 1 quarter term premium.<sup>6</sup>

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<sup>5</sup>Assuming duration of 8, and 10y-in-1q term premium of -0.2

<sup>6</sup>Variance of 4-week changes in log risk neutral variance is 5.25, and of changes in log term premium is 6.75

However, over the long run, the drop is substantial. To provide suggestive evidence on the causes, I repeat the conditional term premium estimation exercise (i.e. equation 4.3) allowing  $\lambda_t$  to be a function of economically-motivated variables instead of yield curve principal components. Table 7 reports coefficients from these alternative specifications. I find two significant relationships in column 5.

First,  $\lambda_t$  increases 1:1 with the sensitivity of equity markets to interest rate changes. The coefficient on equity beta (the sensitivity of stock returns to yield changes) is 0.97, significant at the 1% level. This relationship is consistent with a log investor fully invested in equities.<sup>7</sup>

Second,  $\lambda_t$  appears to rise following increases in aggregate bond supply, although the statistical significance is weak. A 1% increase in aggregate bond duration (scaled by GDP) raises  $\lambda$  by 0.004, or approximately 1%. While the effect is only marginally significant, it aligns with models where slow-moving capital forces specialized arbitrageurs to absorb supply shocks, increasing their exposure and the compensation they demand. So the long term decline in exposure could be related to the long term growth of arbitrageur capital relative to the aggregate bond duration.

In general, economically-motivated variables do a poor job explaining the exposure variable  $\lambda$  relative to the simple principal components, with none adding more than 2 ppt of  $R^2$  over the constant-only model. The underlying causes of the reduction in sensitivity of investors to interest rates remains an open question.

## 6 Validating the residual coskew assumption

The main theoretical result of this paper is that the term premium is a function of the variance risk premium and an unobservable term, the “residual coskew,” that captures the correlation of squared rate changes rates with the inverse SDF (i.e. an investor’s marginal

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<sup>7</sup>It is also consistent with an arbitrageur whose equity value is correlated with the overall market (e.g. a bank). For example, the rolling sensitivity to interest rate changes of stock returns of the primary dealers from the [He, Kelly, and Manela \(2017\)](#) closely resembles that of the overall market.

utility).

In section 5 I show that if we assume this residual coskew term is 0, the implied term premium is a strong out-of-sample predictor of interest rates.

In this section I argue that this is a reasonable assumption based on the data that we can observe. I show:

- For common models of the SDF, the equivalent *physical* moment ( $cov_t(\Delta y_{t+1}^2, \varepsilon_{t+1})$ ) is an order of magnitude smaller than what would be needed to explain the observed variance risk premium.
- In the time series the physical residual coskewness and other related measures do not help explain the variance risk premium versus term-premium-only model

In the appendix, further show that allowing the investor to have CRRA utility and linear exposures to rates cannot explain the average observed VRP, and that the levels of bond market convexity typically observed in the market are far smaller than the convexity that an investor would require. I also recalculate my estimated term premium after allowing for the risk neutral residual coskew to equal physical.

## 6.1 Physical residual coskew under different SDF models

The average realized quarterly variance risk premium from Section 3 is 0.04 percentage points squared. To fully explain this through the residual coskew term alone would require  $cov_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1}) = -0.04$ .

Measuring the residual coskew would require observing a set of options connecting interest rates with the parts of the SDF that are uncorrelated with interest rates. Since these options do not exist in reality, the best we can do is measure the physical variance under various common models of the SDF.

In table 8 I calculate the residual coskew term under te SDF implied by the log-equity-investor model of Martin (2017), a linear Fama French three factor model, and a typical

dynamic term structure model. In all cases, the covariance is at least one order of magnitude smaller than the variance risk premium.

To estimate these figures I first regress daily observations of the SDF onto daily changes in the 10 year rate and collect the residuals:  $\varepsilon_t$ . I then calculate quarterly covariance of epsilon with squared interest rate changes, using this daily data<sup>8</sup>

For the Fama French three factor model I use the price of risk coefficients that match the observed average returns on the three factors. For the dynamic term structure model SDF, I calibrate the terms by fitting the 5 factor model of [Adrian et al. \(2013\)](#).

## 6.2 Time series evidence

Equation (4) tells us that the variance risk premium is a function of exposure multiplied by the risk neutral third moment as well as the residual coskew. If the residual coskew plays an important role, then we should find that it helps explain the time series of variance risk premium. If it doesn't, then we should find the third moment alone explains the time series well.

In section 5, I showed that the third moment explains a large share of the time series variance in estimated variance risk premium, even with a simplistic constant exposure assumption (see column 1 of table 7).

Do measures of residual coskew improve upon this fit? To answer this question I run simple regressions of VRP on the risk neutral third moment alone, as well as the risk neutral third moment plus variables that might plausibly be associated with residual coskew. This is a simplified version of the nonlinear estimation from section 4, where we ignore the  $TP^2$  term and assume  $\lambda_t$  is constant.

As variables that might be correlated with conditional coskew, I include rolling 1 year estimates of the physical residual coskew under the log equity investor ( $M_t = \frac{1}{R_m}$ ) model<sup>9</sup>, risk

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<sup>8</sup>Since there are 63 business days in the quarter, this is calculated as  $63 \text{ cov}(\varepsilon_t, \Delta y_t^2) + \sum_{j=1}^{62} (63 - j) (\text{cov}(\varepsilon_{t-j}, \Delta y_t^2) + \text{cov}(\varepsilon_t, \Delta y_{t-j}^2))$

<sup>9</sup>I do not include the Fama French 3 factor estimate because the two time series of coskewness is 99%

neutral equity volatility (*SVIX*), aggregate bond market convexity (from the Bloomberg “Agg” index), and the [He et al. \(2017\)](#) intermediary capital ratio.

The results are reported in table 9. In all cases the other third moment is highly significant with a t-statistic of over 7, while the alternative explanatory variables are at best marginally significant. Most of the coefficients also have the wrong sign from what we would expect for a residual-covariance based explanation. For example, we would expect periods of negative residual coskew are associated with high variance risk premium, but we find the opposite (although insignificant). Low intermediary capital ratios are also associated with lower variance risk premium, surprisingly.

In short, measures of conditional residual coskewness do not improve on the simple zero-residual-coskew constant-lambda model at explaining the time series of VRP.

## 7 Term premium during FOMC announcement windows

[Hillenbrand \(2025\)](#) documents an intriguing fact: the entire secular decline in long-term rates from the 1990s to 2020 occurred during three-day windows around FOMC meeting. This has been interpreted as evidence that markets primarily learn about long-run interest rate levels from the Federal Reserve. I provide suggestive evidence that, instead, this FOMC-window decline may reflect falling term premia.

Standard dynamic term structure models are not very useful to decompose announcement effects. They either inherently assume constant long term expectations (e.g. [Adrian et al., 2013](#)) or update long-run expectations at lower frequencies than daily (e.g. [Bauer and Rudebusch, 2020](#)).

However, this paper gives us a tool to learn about term premium from daily observations of option prices. If the decline in interest rates comes from term premium, we should expect to see a sharp drop in risk neutral variance and variance risk premium that reverses after the

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correlated

announcements. Whereas if the decline is a result of learning about long term expectations from the fed, we should see no increase in variance risk premium.

In this section I first document that there is a large decline in risk neutral variance around FOMC meetings that appears to be consistent with a change in risk premium. I then show that the size of the declines is large enough to be potentially consistent with full FOMC-window effect being driven by term premium rather than expectations. The fact that the FOMC-window declines match the magnitude of overall declines in rates could simply represent a coincidence.

## 7.1 The FOMC-window variance decline

Table 10 shows that risk neutral variance of interest rates declines across tenors and maturities during FOMC windows. The proportional declines are similar whether we look at short or long tenors, 3 month or 1 year horizons. Even five-year variance—capturing uncertainty about rates far in the future—shows substantial FOMC-window declines.

This decline is much larger than the net change in physical or risk neutral variance during the sample period. For example, over the full sample, five year risk-neutral variance for the 10-year rate actually increased from 3.0 to 4.2 percentage points squared. Yet during FOMC windows alone, it declined by a cumulative 12.2 percentage points squared. For variance to end up higher despite falling dramatically during FOMC meetings requires an offsetting increase of 13.4 percentage points squared between meetings.

The evidence for the idea that the fed primarily learns about the long term from the fed was the fact that the size FOMC-window and aggregate declines in long term forward rates coincide. This is not the case for the declines in long term risk neutral variance. If there is information in the FOMC announcement windows about variance, it must be systematically countered by other information received outside the FOMC windows.

It seems likely that instead, these FOMC-window declines in risk neutral variance are a result of declines in risk premia that reverse outside the windows. This would be consistent

with demand-based mechanisms by which declines in short rates are believed to push down long-term term premium. For example, reaching for yield (e.g. [Hanson and Stein, 2015](#)), improvements in intermediary capital (e.g. [Kekre et al., 2024](#)), or MBS or bank deposit duration hedging effects (e.g. [Hanson, 2014](#); [Rogers, 2024](#)).

In appendix F I show two further pieces of evidence to support the idea that these declines come from variance risk premium rather than physical variance. First, they are too large to be explained by simple “mechanical effects” from the high-variance FOMC window dropping out of the forecast period. Second, using the information from these changes does not improve on the regression-based forecast of interest rate variance.

## 7.2 Measuring the implied term premium decline

I use the risk-based term premium identity from Section 1 to quantify how much of the forward rate decline could come from term premium rather than changing expectations, if the declines in risk neutral variance come mostly from variance risk premium instead of changes in expectations.

If we combine equations (2) and (4) to term premium in terms of variance risk premium, and then differentiate the result for small changes, we get:

$$dTP_t \approx \frac{d(\sigma_t^{*2} - \sigma_t^2)}{\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}} + 2TP_t} \quad (11)$$

Change in term premium is revealed by change in variance risk premium, scaled by skewness and term premium.<sup>10</sup>

This equation naturally suggests an estimator for the average FOMC-window decline in rates. Simply plug in the fitted values for term premium and physical variance from

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<sup>10</sup>I omit a term involving changes in skewness,  $-E_t(\Delta y_{t+1})d\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}}$ , which is empirically negligible.

section 5:

$$E(\Delta TP_t) \approx \frac{(\Delta\sigma_t^{*2} - \Delta\hat{\sigma}_t^2)}{\left(\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}} + 2\widehat{TP}_t\right)} + \eta_t$$

Then solve for the average change in term premium that minimizes the sum of the squared errors  $\eta_t$ .

An alternative would be to simply measure the daily changes in the conditional term premium estimates from section 5. However, we might be concerned that the functional form we have imposed for  $\lambda_t$  does not accurately capture small changes in the relationship between term premium and the yield curve during announcements.

To understand how much term premium decline could be implied by the declines in variance risk premium, I will find the result if we assume there is no change in the physical variance during the estimation windows, i.e.  $\Delta\sigma_t^2 = 0$ . This is roughly what is implied by the regression-based variance stimates from section 5.

From 2007-2018 (the end of the sample period from Hillenbrand, 2025), the average FOMC-window decline in the 10y-in-1y forward rates is 2.8 bp per meeting. The estimation procedure above yields a decline in 10y-in-1y term premium of **3.4 bp**, with a standard error of **1.2 bp**, using daily data with standard errors clustered by meeting.

In other words, the entire FOMC-window decline in long term rates could be attributable to declines in risk premia that are expected to be realized over the course of a year, with no change in long term expectations at all.

Of course to the extent that physical variance declined during these windows as well, the decline in term premium could be smaller. It is likely that the true decline involved some combination of term premium and expectations.

## 8 Conclusion

This paper introduces a new approach to measuring interest rate expectations and term premia. By shifting focus from the non-stationary level of interest rates to their stationary variance, we can extract expectations from options data and test hypotheses about interest rate dynamics in a new way.

The core theoretical result links the term premium perceived by an investor to the variance risk premium perceived by the same investor and an unobserved “residual coskew” quantity related to the non-linearity of the his marginal utility with respect to interest rates. If we assume, as a benchmark, that this residual coskew term is 0 (an assumption that does not appear unreasonable, based on the data), we can derive the exact perceived variance risk premium.

The empirical findings are threefold. First, the historical term premium is large. The strong average bond returns over the past 25 years appear to be mostly due to term premium rather than forecasting errors.

Second, I construct a conditional measure of the term premium that outperforms standard dynamic term structure models and survey-based measures in forecasting excess returns at short horizons, with a trading strategy based on the measure generating large economic profits.

Finally, I find that the secular decline in long-term rates concentrated in FOMC announcement windows may be better explained by a compression of risk premia than by learning about the long-run level of rates. Risk-neutral variance falls by a large amount during these windows, a pattern more consistent with temporary changes in risk appetite or exposure than with information updates.

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## Figures

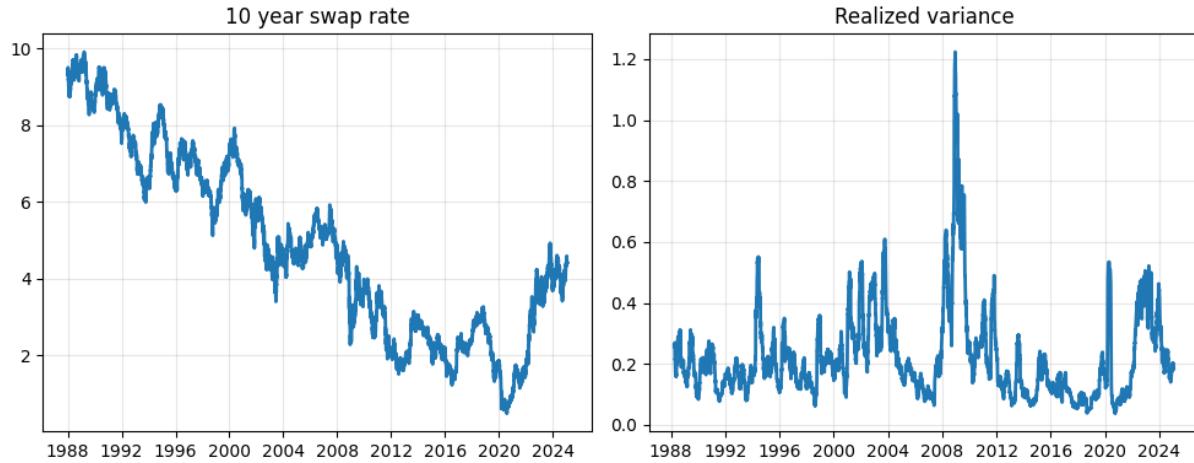


Figure 1: 10 year swap rate (left hand side) and quarterly realized variance on 10 year swap rate (right hand side), 1988-2025. The time series of interest rate levels shows strong signs of non-stationarity (e.g. Dickey Fuller test does not reject unit root), whereas the time series of variance does not (e.g. Dicky Fuller test rejects unit root with p value <0.0001). Swap rates are taken from Refinitiv, and realized variance is calculated from daily data.

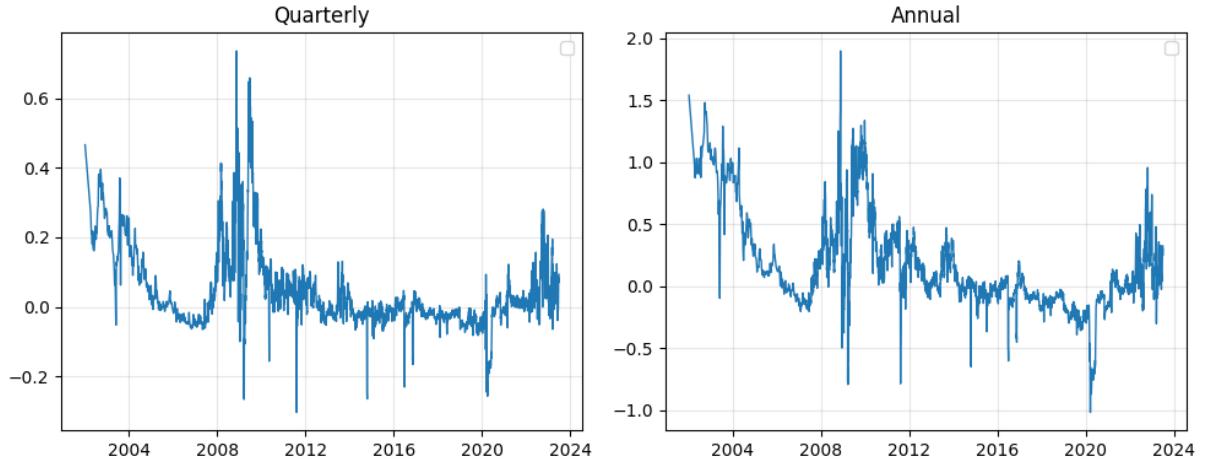


Figure 2: Conditional variance risk premium for 10-year swap rate at quarterly and annual horizons. These figures plot the difference between risk-neutral variance and forecasted physical variance, 2002–June 2023. Positive values indicate that investors are willing to pay a premium for protection against large changes in interest rates. Methodology is described in section 4, and units are percentage point squared.

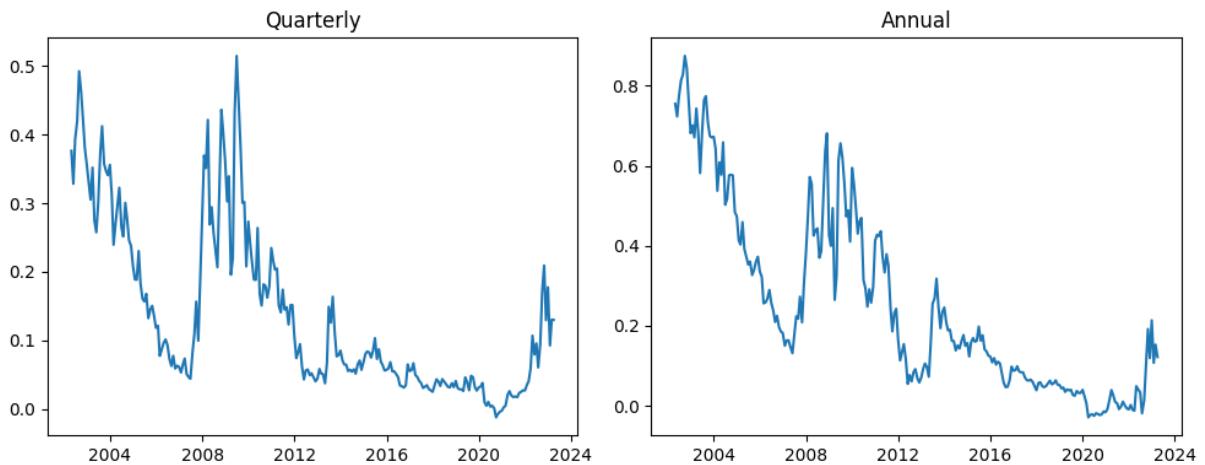


Figure 3: Risk-based term premium estimates for 10-year swap rates. The figure displays quarterly (left hand side) and annual (right hand side) estimates of the difference between the 10y forward swap rate and the expected 10 year swap rate. Estimates are calculated using the methodology described in section 4 and expressed in percentage points.



Figure 4: Expectations hypothesis vs risk-based forecasts of the 10 year swap rate. The black line plots the 10 year swap rate from 2002 to 2023. The light gray lines each quarter plot the 1q and 1y forward rate (i.e. the forecasts of the swap rate if there is no risk premium). The red lines plot the 1q and 1y expected interest rates calculated using the methodology from this paper. Units are percentage points

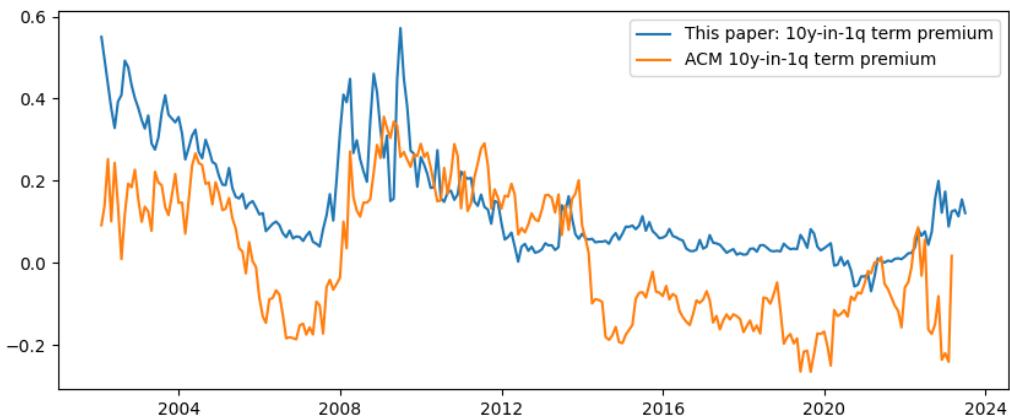


Figure 5: Out of sample 10y-in-1q term premium estimates from this paper versus the [Adrian et al. \(2013\)](#) dynamic term structure model. These figures represent the difference between 10y-in-1q forward rates, and the 10y rate 1 quarter from now. Term premia are expressed in percentage points. Methodology is described in section 4

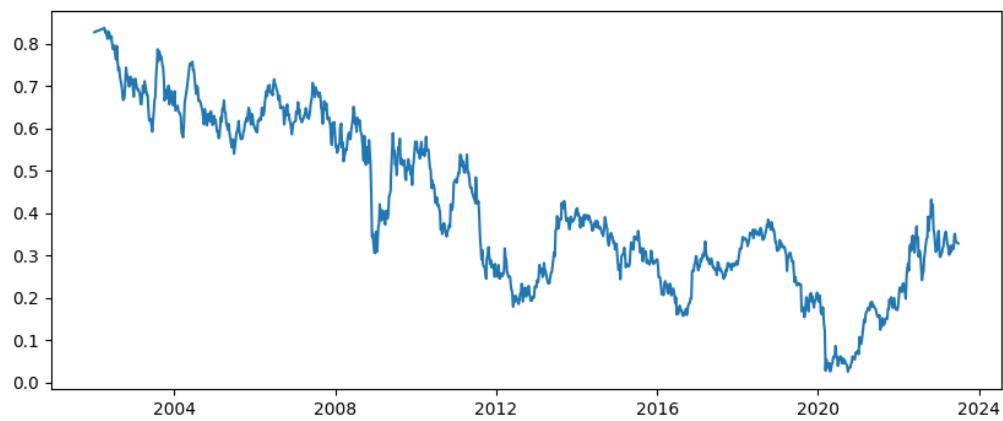


Figure 6: The investor's exposure to interest rate risk. The figure plots the estimated exposure parameter  $\lambda_t$  (blue line), which captures the sensitivity to interest rate increases of the log investor with linear interest rate exposures. The exposure parameter is estimated on the whole sample 2002-2023 using the methodology described in section 5. Interest rate changes are measured in percentage points, so an exposure of 0.5 is consistent with a portfolio duration of 50 for the log investor.

Panel (a) Panel (b)

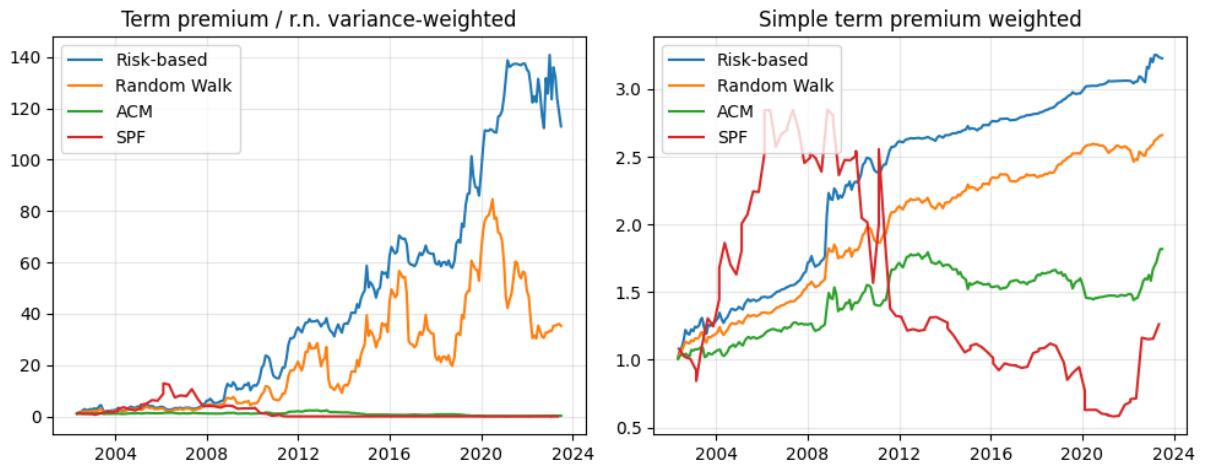


Figure 7: Cumulative returns from term premium-based trading strategies. The figure shows cumulative dollar returns from strategies that take positions in 1-month interest rate forwards proportional to expected term premia from various models. Initial investment is normalized to \$1 in April 2002. Panel (b) uses position sizes equal to the expected monthly term premium. Panel (a) scales positions by the inverse of risk-neutral variance, investing more aggressively when uncertainty is low, consistent with the behavior of the implied log marginal investor. Strategies are implemented using the risk-based measure, Survey of Professional Forecasters expectations, the [Adrian et al. \(2013\)](#) affine model , and a random walk assumption. Returns are calculated before transaction costs. The survey of professional forecasters strategy uses quarterly forwards instead of monthly due to the quarterly frequency of the surveys.

## Tables

			Quarterly		Annual
		Mean	Std Dev	Mean	Std Dev
Data	10 year rate ( $y_t$ )	3.18	1.36	3.18	1.36
	Change in 10y rate ( $\Delta y_{t+1}$ )	-0.09	0.48	-0.33	0.94
	R.n variance ( $\sigma_t^{*2}$ )	0.27	0.19	1.07	0.57
	R.n skewness ( $skew_t^*$ )	0.27	0.21	0.36	0.20
	R.n third moment ( $E_t(\Delta y_{t+1}^3)$ )	0.06	0.09	0.52	0.56
	Realized variance ( $RV_t$ )	0.23	0.18	0.92	0.59
Outputs	Conditional variance	0.22	0.10	0.89	0.29
	Variance risk prem.	0.05	0.11	0.18	0.39
	Term premium	0.13	0.13	0.25	0.23

Table 1: Summary statistics. The table reports summary means and standard deviations for the key model inputs and outputs. All statistics are quoted for January 2002 - June 2023 based on weekly data. Interest rates are reported in  $ppt$ , variances in  $ppt^2$ , and the third moment in  $ppt^3$ . Risk-neutral moments are extracted from swaption prices using the model-free approach of [Carr and Madan \(1998\)](#).

Constant $\lambda$	Implied term premium
-0.41***	0.11***
(0.06)	(0.02)

Table 2: Unconditional test of quarterly term premium. Column (1) reports the estimated constant “exposure” ( $\lambda$ ) parameter that best explains the observed quarterly variance risk premium on the 10 year swap rate. Column (2) reports the quarterly term premium (i.e. difference between expected rates and forward rates) implied by this lambda. Monthly data from 2002–2023. Standard errors calculated using Newey-West with 3 lags.

	EH	RW
Monthly	0.22	0.29
Quarterly	0.30	0.17
Yearly	0.26	0.33

Table 3: Variance forecasting performance vs alternative benchmarks. The table reports out-of-sample improvement in  $R^2$  values for forecasts of realized variance of 10-year swap rates using the HAR-RV model described in section 4 compared to two benchmarks: risk-neutral variance from swaptions and lagged realized variance (random walk). Relative  $R^2$  is calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the regression-based forecaster and  $\nu_t$  is the forecast error from the alternative. The regression model regresses realized variance on lagged weekly, monthly, and quarterly realized variance. Forecasts are calculated using an expanding window starting in 1988. Sample period is 2002–2023.

	1m	1q	1y
Expected change in rates	1.8341*** (0.64)	0.9702*** (0.34)	1.4803** (0.61)
Intercept	0.0109 (0.02)	0.0362 (0.06)	0.0413 (0.25)
R2	0.0414	0.0602	0.1439
Observations	1064.0	1064.0	244.0
Frequency	Weekly	Weekly	Monthly
NW lags	5	13	12

Table 4: Term premium forecasting regressions. The table reports results from regressions of realized changes in 10-year swap rates on predicted term premia:  $\Delta y_{t,t+h} = \alpha - \beta \times \widehat{TP}_{t,h} + \varepsilon_{t+h}$ , where  $\widehat{TP}_{t,h}$  is the risk-based term premium estimate. Under the null hypothesis that the measure correctly captures term premia,  $\alpha = 0$  and  $\beta = 1$ . Sample period is March 2002– June 2023. Standard errors calculated using Newey-West with lags equal to the forecast horizon.

		Monthly	Quarterly	Annual
Expectations Hypothesis	R2	0.065	0.081	0.230
	p-value	0.014	0.160	0.010
Survey of Prof Forecasters	R2		0.275	0.271
	p-value		0.005	0.015
<a href="#">Adrian et al. (2013)</a>	R2	0.070	0.095	0.184
	p-value	0.003	0.077	0.203
<a href="#">Kim and Wright (2005)</a>	R2	0.100	0.150	0.497
	p-value	0.006	0.066	0.005
<a href="#">Bauer and Rudebusch (2020)</a>	R2		0.045	0.087
	p-value		0.406	0.311
Random Walk	R2	0.020	0.022	0.043
	p-value	0.073	0.347	0.287
Regression-rescaled	R2	0.049	0.136	0.216
	p-value	0.096	0.100	0.091

Table 5: Improvement in interest rate forecast performance vs alternative benchmarks. The table reports the improvement in  $R^2$  from using the risk-based term premium measure versus alternative models. Relative  $R^2$  is calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the risk-based model and  $\nu_t$  is the forecast error from the alternative. [Adrian et al. \(2013\)](#) estimates calculate the 10-year par-yield forecasts with rolling out of sample windows. [Kim and Wright \(2005\)](#) uses the 1y instantaneous term premium provided by the Federal Reserve Board. The Board provides a "newer" and "older" calibration. I use the older calibration up to 2019 to ensure estimates are out of sample. [Bauer and Rudebusch \(2020\)](#) uses the 8y spot yield forecasts from the [Bauer and Rudebusch \(2020\)](#) observed-shifting-endpoints model up to 2018, calculated using the authors replication code. "Regresssion-rescaled" denotes forecasts made from expanding window regressions of my risk-based expectation measure on changes interest rates, instead of using the theoretically-motivated magnitude. P-values are from one-sided Diebold-Mariano tests for an improvement in forecast accuracy. To increase power without using long overlaps, I use weekly frequencies for the monthly forecast R2, and monthly frequencies for the quarterly and annual forecast. Monthly frequencies are not available for the SPF or [Bauer and Rudebusch \(2020\)](#). Sample period is March 2002–June 2023.

	Mean	Quarterly Correlation	Mean	Yearly Correlation
Realized	0.08		0.34	
Risk-based estimate	0.13		0.26	
Expectations Hypothesis	0.00		0.00	
Survey of Prof Forecasters	-0.07	0.38	-0.12	0.60
Adrian et al. (2013)	0.02	0.65	0.06	0.74
Kim and Wright (2005)			-0.36	0.38
Bauer and Rudebusch (2020)	-0.06	-0.12	0.08	0.10
Random Walk	0.07	0.63	0.29	0.74

Table 6: Comparison of term premium estimates. The table reports average term premia from different models for 10-year swap rates, and their correlation with the risk-based estimates from this paper. The left two columns show the results for quarterly forecasts, and the right two columns for annual forecasts. Realized shows the ex post change in rates vs forward-implied rates. Risk-based uses the variance risk premium method developed in this paper. Other models are as described in figure 5. Monthly data from 2002–2023. All values in percentage points per period.

	(1) Constant	(2) PCs only	(3) Main	(4) $\Delta$ Agg duration	(5) Equity $\beta$	(6): 3 & 4
Intercept	0.51 (0.04)	-0.06 (0.07)	-0.02 (0.11)	0.50 (0.04)	0.61 (0.05)	0.62 (0.04)
PC 1		0.04 (0.01)	0.04 (0.01)			
PC 2		-0.06 (0.01)	-0.06 (0.01)			
PC 3		0.04 (0.03)	0.04 (0.03)			
Var		0.01 (0.09)	0.01 (0.09)			
Skew		-0.07 (0.25)				
$\Delta$ Dur			0.22 (0.26)		0.42 (0.24)	
Equity $\beta$				-0.80 (0.28)	-0.97 (0.31)	
R2	0.54	0.65	0.65	0.54	0.56	0.57

Table 7: Determinants of marginal investor exposure. The table reports coefficients from nonlinear least squares regressions where the exposure parameter  $\lambda_t$  is specified as a linear function of economic variables.  $\lambda_t$  captures the marginal investor's sensitivity to interest rate increases. PC1, PC2, and PC3 are the first three principal components of the yield curve. Skew is the risk neutral quarterly skewness of the 10 year rate.  $\Delta$ Dur is log changes in aggregate bond duration (from the Bloomberg Agg index) to GDP. Equity beta is sensitivity of S&P 500 returns to 10-year rate changes (calculated using physical 1 year correlation and risk neutral volatilities). Quarterly data from 2002–2023. Standard errors in parentheses calculated using GMM with Newey-West corrections with four lags.

Model	Residual Physical Covariance
Log equity investor	-0.004
Fama-French 3 factor	-0.001
Dynamic term structure model	-0.001

Table 8: Estimations of average physical quarterly residual coskew under different models of the SDF. Residual coskew is the covariance of squared changes in yields with the residual portion of the inverse SDF left after regressing on changes in yields. See section 6 for details.

	Dependent variable: VRP					
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-0.010 (0.010)	-0.008 (0.010)	0.044 (0.032)	-0.008 (0.009)	-0.069** (0.031)	-0.018 (0.060)
R.N. third moment	0.893*** (0.094)	1.013*** (0.148)	0.968*** (0.098)	0.862*** (0.095)	0.941*** (0.089)	0.966*** (0.117)
Residual coskew est.		3.360 (3.161)				0.728 (2.465)
R.N. equity vol			-0.306* (0.179)			-0.159 (0.220)
Bond mkt convexity				-5.859* (3.016)		-4.388 (3.280)
Intermediary capital					0.840** (0.402)	0.538 (0.477)
Observations	81	81	81	81	81	81
$R^2$	0.677	0.689	0.700	0.697	0.709	0.726
Adjusted $R^2$	0.673	0.681	0.692	0.689	0.701	0.708
Residual Std. Error	0.067	0.066	0.065	0.065	0.064	0.063
F Statistic	91.077***	43.311***	49.473***	47.405***	56.905***	29.394***

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 9: Measures associated with residual coskew do not seem to explain the variance risk premium well. This tables shows the results from regressing estimated conditional variance risk premium onto the risk neutral third moment alone (column 1) or the risk neutral third moment and other possible explanatory variables (columns 2-6). VRP being fully explained by the third moment is consistent with the mechanism in this paper by which VRP is a result of directional interest rate risk. The other variables could potentially be associated with non-linear interest rate risk. The residual coskew estimate uses the log-equity investor model, calculated on a rolling 1y basis (see section 6 for more detail). Risk neutral equity vol uses the *VIX* index rescaled by a regression onto the *SVIX* index from [Martin \(2017\)](#). Bond convexity is the convexity of the Bloomberg Barclays Aggregate Index. Intermediary capital is from [He et al. \(2017\)](#). Data is quarterly and standard errors in parentheses calculated using Newey-West with 4 lags. Sample period is 2002–2023

	Quarter	Year	5 Year
1Y	-2.5% (3.4%)	-4.1% (1.3%)	-1.6% (0.6%)
2Y	-4.3% (1.7%)	-3.4% (1.3%)	-1.8% (0.5%)
5Y	-3.2% (1.3%)	-2.8% (0.8%)	-1.5% (0.6%)
10Y	-2.6% (1.0%)	-2.3% (0.7%)	-1.4% (0.6%)
20Y	-2.6% (1.1%)	-2.5% (0.7%)	-1.4% (0.6%)
30Y	-2.9% (1.0%)	-2.9% (0.8%)	-1.2% (0.6%)

Table 10: Risk-neutral variance changes during FOMC windows. The table reports the average proportional decline in risk-neutral variance during 3-day windows around FOMC meetings for different swap rate tenors (rows) and swaption maturities (columns). Changes are calculated from close on day  $t - 1$  to close on day  $t + 1$  where day  $t$  is the FOMC announcement. Sample period is 2007–2023. Standard errors in parentheses are clustered by FOMC announcement date.

# Appendices

## A Forecasting other interest rate tenors

Table 11 below reports the relative improvement in forecasting performance by applying this same methodology to the 1 year, 2 year, 5 year, 20 year, and 30 year swap yield. As a simple approximation I assume the value of  $\lambda_t$  for all tenors is the same as the values calculated in the main body of this paper.

Recalculating different values of  $\lambda_t$  for each tenor based on their respective variance risk premia can be done. However, there are some challenges in estimating physical variance for shorter tenors because the rates are at the zero lower bound for much of the sample and more subject to jumps around policy announcements.

Comparison model	Tenor	Monthly	Quarterly	Annual
Expectations Hypothesis	1	0.052	0.112	0.154
	2	0.063	0.119	0.185
	5	0.068	0.102	0.229
	10	0.065	0.081	0.230
	20	0.066	0.042	0.191
	30	0.072	0.022	0.174
<a href="#">Adrian et al. (2013)</a>	1	0.126	0.228	0.153
	2	0.077	0.141	0.170
	5	0.060	0.096	0.162
	10	0.070	0.095	0.184

Table 11: Multi-tenor out of sample forecast performance. The table reports the improvement in  $R^2$  from risk-based interest rate expectation measure versus the expectations hypothesis and the [Adrian et al. \(2013\)](#) DTSM. Improvement in  $R^2$  is defined as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the risk-based model and  $\nu_t$  is the forecast error from the alternative. 2002-2023, monthly data.

## B Constructing risk neutral interest rate moments from swaptions

This section describes how I approximate the risk neutral moments of swap yields from swaption data. Swaptions are not precisely options on the interest rate. I therefore need to employ a simple approximation. I assume that changes in the annuity yield are the same as changes in the swap yield. This approximation is unlikely to lead to substantive errors — changes in the 10 year annuity yield and swap yield are >99% correlated at the quarterly or annual horizon.

### Swaps and swaption prices

By a standard result, we can write the value of a swap agreed at fixed rate  $K$  with tenor  $T$  as  $(y - K)A$  where  $A$  is the price of the  $T$ -period annuity, and  $y$  is the swap rate (i.e. the rate that sets the value of the swap to 0).

The payoff of a swaption agreed at rate  $K$  is therefore

- Pay-fixed:  $\text{Max}\{(y_{t+j} - K) A_{t+j}, 0\}$
- Receive-fixed:  $\text{Max}\{(K - y_{t+j}) A_{t+j}, 0\}$

And the forward price of the swaption can be written:

- Pay-fixed:  $C(k) = E_t^*(\text{Max}\{(y_{t+j} - K) A_{t+j}, 0\})$
- Receive-fixed:  $P(k) = E_t(\text{Max}\{(K - y_{t+j}) A_{t+j}, 0\})$

If we change measure to the  $T$ -tenor “annuity measure” we can write these swaption prices as options directly on the swap rate:

- Pay-fixed:  $C(k) = E_t^A(\text{Max}\{(y_{t+j} - K), 0\})$
- Receive-fixed:  $P(k) = E_t^A(\text{Max}\{(K - y_{t+j}), 0\})$

## Applying Breeden & Litzenberger

Applying [Breeden and Litzenberger \(1978\)](#), we can write the expectation of any function of the swap rate under this annuity measure as:

$$E^A(g(y)) = \left( g(y^f) + \int_{-\infty}^{y^f} g''(y) P(k) dk + \int_{y^f}^{\infty} g''(y) C(k) dk \right)$$

Where  $y^f$  is the swap forward rate.

However, we want the moments under the risk neutral measure, not the annuity measure. So I will assume that changes in the annuity yield vs its forward yield are always identical to changes in the swap yield vs its forward yield. The annuity price then becomes a function of the swap yield:

$$A = \sum_{t=1}^{4T} \frac{1}{(1+y)^{t/4}} = A(y)$$

Now we can find the risk neutral expectation of any arbitrary function  $f(y)$  by letting:

$$g(y) = \frac{E^*(A)}{A(y)} f(y)$$

The annuity measure is defined such that for any X:

$$E_t^A \left( \frac{E^*(A)}{A(y)} X \right) = E^*(X)$$

And hence we have

$$E^A(g(y)) = E^*(f(y))$$

## Calculating the moments

For this paper I need to estimate  $E^*(f(y))$  for  $f(y) = y$ ,  $f(y) = (y - E_t^*(y))^2$ , and

$$f(y) = (y - E_t^*(y))^3.$$

In each case, I need to first find the second derivative of the function:

$$g(y) = \frac{E^*(A)}{A(y)} f(y)$$

I then use my data on the forward prices of payer and receiver swaptions to calculate, using the standard:

$$E^*(f(y)) = \left( g(y^f) + \int_{-\infty}^{y^f} g''(y) P(k) dk + \int_{y^f}^{\infty} g''(y) C(k) dk \right)$$

For yields between the observed swaption strikes, I calculate the price using the Bachelier (normal) option pricing formula, as is standard in fixed income markets.

## C Risk neutral moment time series

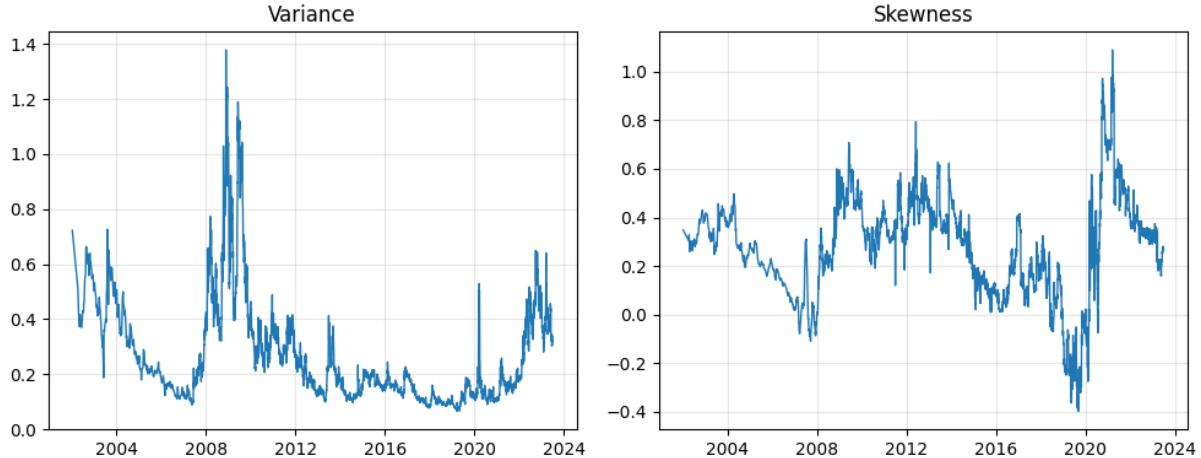


Figure 8: Quarterly risk-neutral variance (left) and skewness (right) of 10-year swap rates, 2002–2023. Variance and skewness are extracted from swaption prices, using the model free approach approach of Carr and Madan (1998). Variance is expressed in percentage points squared.

## D Term premium with non-parametric form for $\lambda_t$

Figure 9 compares term premium estimated from the parametric specification for  $\lambda_t$ , described in section 4, with term premium estimated by solving for  $\lambda$  separately each period, without a functional form. Since two roots are possible for lambda, I always choose the root with the minimum magnitude of the term premium. Confidence intervals are provided for each period for the non-parametric version. Where no value of  $\lambda_t$  was consistent with the measured variance risk premium, I take the closest value. If this implies a value for the variance risk premium that is outside of its 95% confidence interval of the variance-prediction regression, I plot this observation with a dotted line.

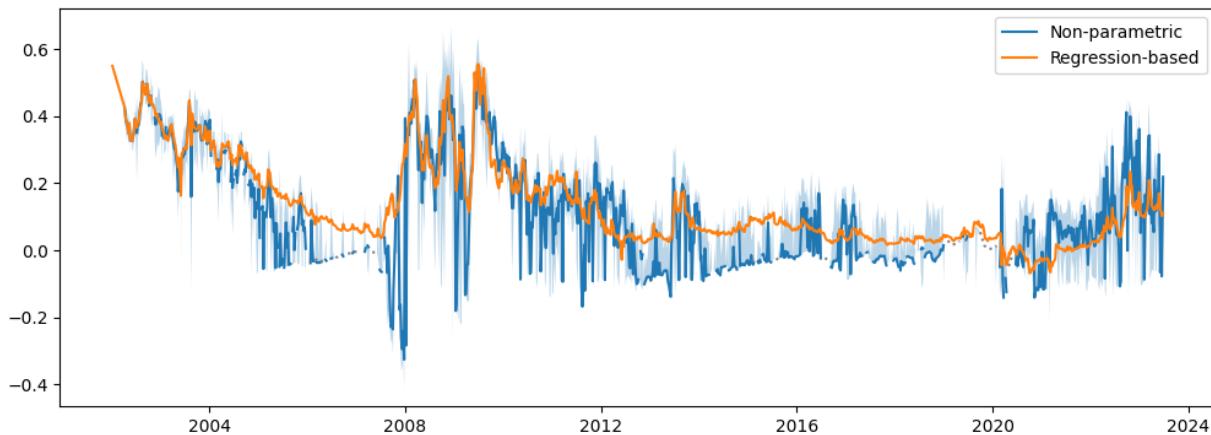


Figure 9: The left hand plot shows term premium estimated from the parametric specification for  $\lambda_t$ . The right hand plot shows term premium estimated by solving for  $\lambda$  separately each period. Quarterly forecasts in ppt, 2002-2023.

## E Additional content on the residual coskew

### E.1 Risk aversion

Higher risk aversion might seem like a natural explanation for the variance risk premium. After all, more risk-averse investors should demand higher compensation for bearing variance risk. But this intuition proves exactly backward—higher risk aversion alone actually

makes the residual coskew positive, not negative, and therefore cannot explain the observed positive variance risk premium.

Consider an investor with constant relative risk aversion  $\gamma$  and linear exposure  $D$  to interest rates. Her inverse stochastic discount factor takes the form:

$$\frac{1}{M_{t+1}} = (a - D\Delta y_{t+1})^\gamma$$

Projecting this onto  $\Delta y_{t+1}$  and  $(\Delta y_{t+1}^2 - \sigma_t^{*2})$  under the risk-neutral measure yields:

$$\frac{1}{M_{t+1}} = R_{f,t+1} - \lambda_1 \Delta y_{t+1} + \lambda_2 (\Delta y_{t+1}^2 - \sigma_t^{*2}) + \eta_{t+1}$$

where  $\lambda_1 = \gamma a^{\gamma-1} D$  and  $\lambda_2 = \frac{\gamma(\gamma-1)a^{\gamma-2}D^2}{2}$ .

The residual coskew term becomes:

$$E_t^*(\Delta y_{t+1}^2 \varepsilon_{t+1}) = \lambda_2 \text{var}_t^*(\Delta y_{t+1}^2) = \frac{\gamma(\gamma-1)a^{\gamma-2}D^2}{2} \text{var}_t^*(\Delta y_{t+1}^2)$$

This expression is always positive when  $\gamma > 1$ . Higher risk aversion therefore generates a smaller variance risk premium for any given level of term premium, not a larger one.

Intuitively, two investors can generate the same term premium through different combinations of risk aversion and leverage. A less risk-averse investor needs high leverage (large  $D$ , small  $\gamma$ ), while a more risk-averse investor needs less leverage (small  $D$ , large  $\gamma$ ). But the highly levered investor fears far-away states more—a 5 percentage point rate move could wipe her out, while merely denting the un-leveraged investor's wealth. The levered investor therefore pays more for variance protection despite having the same directional exposure.

## E.2 Bond market convexity

Bonds with embedded options, particularly mortgage-backed securities, can exhibit negative convexity. When rates fall, prepayments accelerate and duration shrinks; when rates rise, prepayments slow and duration extends. Could this convexity explain the variance risk premium?

For a log investor holding a portfolio with duration  $D$  and convexity  $C$ , the inverse SDF becomes:

$$\frac{1}{M_{t+1}} = R_{f,t+1} - D\Delta y_{t+1} + \frac{1}{2}C(\Delta y_{t+1})^2$$

The residual covariance term equals:

$$E_t^*(\Delta y_{t+1}^2 \varepsilon_{t+1}) = \frac{1}{2}C \times \text{var}_t^*(\Delta y_{t+1}^2)$$

Using the relationship  $\text{var}_t^*(\Delta y_{t+1}^2) = (\text{kurt}_t^*(\Delta y_{t+1}) - 1)\sigma_t^{*4}$ , and noting that risk-neutral kurtosis averages 4.5 from 2011-2023 while quarterly risk-neutral variance squared averages 0.1, the convexity needed to generate the observed variance risk premium would be:

$$C = 2 \frac{-E(\text{VRP})}{E((\text{kurt}^*(\Delta y) - 1)\sigma^{*4})} \approx 2 \frac{-0.04}{3.5 \times 0.1} = -0.22$$

To put this in perspective, the Bloomberg Barclays Aggregate Bond Index recorded its most negative convexity at -0.005 in 2004 and most positive convexity at 0.006. A log investor would therefore need negative convexity equivalent to a 44-times levered position in the aggregate bond portfolio at its moment of peak negative convexity. This is implausible for a reasonable marginal investor.

## E.3 Robustness to relaxing the assumption

To further validate my results, I re-estimate the key findings from section 3 and ?? while explicitly accounting for potential nonlinearities. I consider three modifications:

1. Constant relative risk aversion of  $\gamma = 2$
2. Bond convexity / duration proportional to that of the Bloomberg Barclays Aggregate Index
3. Full equity exposure using historical correlations

The unconditional tests in Table 12 show that accounting for equity correlation reduces term premium estimates by about 15% but does not change any of the key rejections.

Table 13 shows that the conditional term premium estimates from section ?? maintain approximately 97% correlation with the baseline measure under all three modifications.

The only substantial effect comes from allowing  $\gamma = 2$ , which increases the variability of term premium estimates. This makes intuitive sense: higher risk aversion amplifies the conversion from variance risk premium to term premium. From this perspective, my baseline estimates assuming log utility could be viewed as conservative lower bounds on the true term premium.

The robustness of these results suggests that the residual coskew term is indeed small enough to ignore for practical purposes. While I cannot rule out all possible sources of nonlinearity, the magnitudes required to explain the observed variance risk premium appear economically implausible.

Test: Exp hypothesis	Test: Random walk	Constant lambda coefficient
0.89	0.33	-0.39
(0.36)	(0.33)	(0.09)

Table 12: Robustness of unconditional tests to equity correlation. The table repeats the unconditional tests from Table 2 after adjusting for the average correlation between equity returns and squared yield changes. Adjusted variance risk premium subtracts the estimated contribution from equity exposure assuming a log investor fully invested in equities. Test statistics and constant lambda are recalculated using the adjusted VRP. Monthly data from 2002–2023. Standard errors calculated using Newey-West with 3 lags.

	Main	Equity conv	Bond conv	Risk aversion
Mean	-0.09	-0.08	-0.09	-0.12
Std Dev	0.15	0.15	0.16	0.26
Corr	1.00	0.98	0.97	0.97

Table 13: Robustness of conditional term premium estimates. The table reports summary statistics for quarterly term premium estimates under alternative assumptions about the stochastic discount factor. Main specification assumes log utility with linear exposure to rates. Equity conv allows for equity market exposure based on historical correlations. Bond conv includes convexity proportional to the aggregate bond market. Risk aversion assumes constant relative risk aversion of  $\gamma = 2$ . Mean and standard deviation in percentage points per quarter. Correlation reports correlation with the main specification. Sample period is 2002–2023.

## F Testing for information effects in risk neutral variance

While the magnitudes suggest term premium changes, I conduct three tests to evaluate whether information effects could explain the patterns.

### F.1 Mechanical variance effects

FOMC meetings are high-variance events. Once a meeting passes, forward-looking variance mechanically declines by removing this event from the forecast window. Could this explain the observed patterns?

The mechanical effect is far too small. Three-day FOMC-window variance for daily 10-year rate changes is 37% higher than non-FOMC periods. Passing through an FOMC window should therefore reduce:

- Quarterly variance by  $0.37 \times \frac{3}{61} = 1.8\%$
- Annual variance by  $0.37 \times \frac{3}{250} = 0.4\%$
- Five-year variance by less than 0.1%

I observe declines of 2.9% for annual variance—more than seven times the mechanical

effect.

Moreover, the cross-sectional pattern contradicts the mechanical story. FOMC-day variance is twice as high for 1-year rates as for 10-year rates, yet the proportional variance declines are similar across tenors. The risk-neutral variance of rates five years forward falls substantially, which mechanical effects cannot explain.

## F.2 Size of changes

For forward rates, the FOMC-window decline equals the total decline, consistent with gradual learning. But for variance, the cumulative FOMC effect dwarfs the total change.

For five-year risk-neutral variance on 10-year rates:

- Cumulative FOMC-window decline: -12.2 percentage points squared
- Net change 2007-2023: +1.2 percentage points squared
- Implied between-meeting increase: +13.4 percentage points squared

If markets were learning about future volatility, they would need to systematically “over-learn” during FOMC meetings that volatility will be low, then receive opposite information between every meeting. This asymmetric updating pattern seems implausible for rational learning but fits naturally with temporary risk premium compression.

## F.3 Forecasting ability

If FOMC meetings reveal information about future volatility, then FOMC-window changes in risk-neutral variance should predict realized variance. I test:

$$\sigma_{RV,t}^2 - \hat{\sigma}_{t-1}^2 = \beta \Delta \sigma_t^{*,\text{FOMC}} + \varepsilon_t \quad (12)$$

where  $\sigma_{RV,t}^2$  is subsequently realized variance and  $\hat{\sigma}_{t-1}^2$  is the pre-FOMC forecast from Section 5.

Table 14 shows the results. The  $R^2$  is below 0.01 for both quarterly and annual horizons. The coefficient is insignificantly different from zero and, for annual horizons, significantly different from one. FOMC-window variance changes contain essentially no information about future realized variance.

	Quarterly	Yearly
const	0.013 (0.021)	0.070 (0.124)
$\Delta\sigma^{*2}$	0.232 (0.617)	0.239 (0.311)
R-squared	0.006	0.003
R-squared Adj.	-0.002	-0.004
N	137	137

Table 14: Information content of FOMC variance changes. The table reports regressions of realized variance forecast errors on FOMC-window changes in risk-neutral variance:  $\sigma_{RV,t}^2 - \hat{\sigma}_{t-1}^2 = \alpha + \beta\Delta\sigma_{FOMC,t}^{*2} + \varepsilon_t$ . If FOMC variance changes reflect information about future volatility,  $\beta$  should equal one.  $\sigma_{RV,t}^2$  is subsequently realized variance,  $\hat{\sigma}_{t-1}^2$  is the pre-FOMC HAR-RV forecast, and  $\Delta\sigma_{FOMC,t}^{*2}$  is the change in risk-neutral variance during the FOMC window. Standard errors calculated using Newey-West with 2 lags (quarterly) and 8 lags (annual). Sample includes 137 FOMC meetings from 2007–2023.