

# Risk-Based Interest Rate Expectations

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# Risk-Based Interest Rate Expectations

## Abstract

I develop a method to extract interest rate expectations from option markets without specifying the dynamics of possibly non-stationary rate levels. I employ a no-arbitrage relationship to convert variance risk premium observable in the swaptions market to interest rate risk premium without modeling the process for interest rate levels. I find: (1) The unconditional interest rate risk premium averaged 44 basis points annually. Historical excess bond returns mostly reflected anticipated risk premia, not forecast errors. (2) Risk-based forecasts significantly outperform surveys and traditional term structure models, improving out of sample quarterly  $R^2$  by 28% and 10%. Improvements are particularly large during the zero rate period when stationary models forecast large negative risk premium. (3) FOMC announcement windows exhibit large, transitory declines in risk-neutral variance that are consistent with the net FOMC-window decline in rates being partially or fully driven by risk premium effects rather than information revelation. (4) The modern positive stock-yield correlation is driven by expectations rather than risk premium. The approach provides a simple, tractable alternative to traditional term structure modeling.

*Keywords:* term premium, term structure, interest rate risk, duration

*JEL Classification:* G12, G21

In January 2010 the yield curve sloped sharply upwards. A 1-year loan could be made at an interest rate of just 0.6%, but the same loan agreed to start in 1 year would pay 2%.<sup>1</sup> Were investors expecting inflation and interest rates to spike? Or was this just compensation for duration risk? Distinguishing between these possibilities and, more broadly, measuring interest rate expectations is a critical task for central banks and investors.

The finance profession has many tools to measure expected interest rates, from simple bond return regressions to full-fledged dynamic term structure models. What all of these approaches have in common is that they use a set of historical predictors to forecast interest rates or, equivalently, bond returns, next period.

This paper proposes instead to measure the interest rate risk premium from options markets, and use this risk premium to calculate expectations. I derive a no-arbitrage relationship that links the interest rate risk premium to interest rate variance and risk-neutral moments that are directly observable from options markets. I use this identity to create interest rate forecasts that do not make any assumptions about the dynamics or stationarity of the interest rate level, circumventing the thorny problems created by the secular decline of interest rates. The resulting forecasts significantly outperform traditional stationary term structure models out of sample. I use these forecasts to show that high historical bond returns have been driven by risk premium, not surprises, that bond risk premium is negatively correlated with stock returns, and that risk premium may account for much of the decline in long term rates around monetary policy announcements.

The key identity relates perceived risk must relate to perceived return. If investors expect high returns on bonds, they will take large interest rate exposures. A simple no-arbitrage relationship tells us that the interest rate risk premium  $RP_t$  perceived by an investor equals the investor's interest rate risk "exposure"  $\lambda_t$ , multiplied by the risk-neutral variance  $\sigma_t^{*2}$ :

$$RP_t = \lambda_t \times \sigma_t^{*2}$$

For a log-utility bond investor, the exposure equals portfolio duration; for CRRA utility

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<sup>1</sup>The 1-year LIBOR swap rate on January 1st was 0.6%, and the 1y-in-1y swap forward rate was 2%

with coefficient  $\gamma$ , it approximates duration times  $\gamma$ . Risk-neutral variance is directly observable from swaption prices.

To identify exposure, I use a parallel identity for squared interest rate changes. The risk premium on contracts that pay off squared changes in interest rates ( $RPSq_t$ ) approximately equals the level of interest rate exposure multiplied by the risk-neutral third moment (skewness) of interest rate risk exposure:

$$RPSq_t \approx \lambda_t \text{ThirdMoment}_t^*$$

Intuitively, when the level of interest rate risk exposure ( $\lambda$ ) is high, the equilibrium price of “insuring against” large changes in interest rates ( $RPSq$ ) should also be high. We can use the price of this insurance to learn about the level of risk exposure. This relationship will be stronger and the insurance more valuable when the distribution is right-skewed and large increases in rates are more common than large declines.

This identity allows us to back out the exposure, and hence interest rate expectations, based on a variance forecast rather than a level forecast. This offers two advantages: first, the variance, unlike the level, is clearly stationary (see Figure 1). Second, variance can be learned quickly from daily data, and hence variance-forecasting models have a strong track record of empirical success (e.g. [Bollerslev, 1986](#); [Corsi, 2009](#)).

The identity involves an additional unobservable that captures the covariance between squared interest rate changes and non-linear components of the investor’s payoffs. I argue that this term is likely to be small. Observable proxies using common SDF specifications are an order of magnitude too small to explain the variance risk premium, and do not line up with the time series of variance risk premium. For my main analysis, I therefore set this value to zero.

The resulting interest rate forecasts can be interpreted as the expected interest rate that must be perceived by an investor with log utility who chooses to take a linear exposure to interest rates. Up to a modest approximation error, this same interpretation can be applied

to an investor with power utility. These forecasts capture well known patterns in interest rate forecasting, for example, the association of the slope with the risk premium. However, they significantly outperform existing methods. At one-quarter horizons, they improve forecast accuracy by 28% relative to professional surveys and 10% relative to the popular [Adrian, Crump, and Moench \(2013\)](#) affine model (Table 4). The predictions generate economically large trading profits with Sharpe ratios of 0.73 vs. -0.17 for traditional term structure models.

Beyond forecasting performance, the variance-based approach provides new perspectives on three puzzles in fixed income markets. First, do the high returns on long term bonds over the past 25 years represent an anticipated risk premium, or a series of surprises? Forward rates have tended to point upwards, while rates stayed low, leading to large returns for bond investors. The causes of these returns are of central interest to finance research — they can potentially account for the entire equity risk premium in this period ([van Binsbergen, 2020](#)).

Standard approaches struggle to answer this question because of the secular decline in rates. Models that assume a stationary interest rate process always forecast rates to return to the historical average, leading to low or negative risk premium. If one instead allows the long-term “end point” to shift, then a wide range of different risk premia and expectations are compatible with a reasonable set of priors ([Farmer, Nakamura, and Steinsson, 2024](#)).

My method gathers information instead from the variance of interest rates, which, in contrast, exhibits mean reversion with no obvious long-term trend (see Figure 1). I find a highly significant variance risk premium (t-statistic 5), that implies an interest rate risk premium of 44 basis points (bp) per year, similar to the realized premium of 34 bp. The duration risk premium appears to have been expected risk compensation rather than forecasting errors.

Second, do investors learn their long-term interest rate expectations from the Fed? [Hillenbrand \(2025\)](#) documents that the entire secular decline in long-term rates occurred during three-day FOMC windows, interpreting this as markets learning about long-run fun-

damentals from the Fed. Standard term structure models are poorly suited to decompose these announcement effects into expectations versus risk premium. They either assume a constant long-run mean interest rate, ruling out learning about long run levels, or they update long-run mean estimates at low frequencies.

My variance-based decomposition offers an alternative explanation. Risk-neutral variance falls around FOMC meetings by amounts far exceeding mechanical effects from uncertainty resolution, and then reverses between meetings. This pattern is difficult to reconcile with permanent learning but is consistent with temporary risk premium compression. The implied term premium declines could potentially account for the entire FOMC-window rate puzzle. While I cannot definitively rule out concurrent shifts in physical variance expectations, the transitory nature and magnitude of these moves point toward monetary policy systematically compressing risk premia, rather than lowering long term expectations.

Third, why are stocks now positively correlated with yields? The stock-yield correlation flipped from negative to positive in the late 1990s. Is this driven by correlated risk premia (e.g., flight-to-safety shocks increasing stock risk premia while lowering bond risk premia (e.g. [Antolin-Diaz, 2025](#))) or correlated expectations about growth (e.g. [Campbell, Sunderam, and Viceira, 2017](#))? Standard models show positive risk premium-equity correlation, but this may reflect stationarity assumptions: growth news that raises long-run rate expectations could be misclassified as risk premium. Using the variance-based approach, I find interest rate risk premium correlates negatively with equity returns, opposite flight-to-safety predictions. The positive yield-stock correlation therefore appears to reflect correlated expectations rather than risk premium effects.

## Related Literature

This paper extends techniques from the literature on option-based expected returns ([Martin, 2017](#); [Kremens and Martin, 2019](#); [Chabi-Yo and Loudis, 2020](#); [Tetlock, McCoy, and Shah, 2024](#)) to apply to a broader range of asset classes. [Martin \(2017\)](#) takes a theory-driven benchmark of an investor with unit exposure to stock market returns. We cannot

take a similar approach for interest rates because it is not clear how large or even in what direction the market’s interest rate exposure should be. Moreover, we cannot take an approach that tries to link interest rates to equities, as in [Kremens and Martin \(2019\)](#) for FX, because there are no instruments that reveal the risk-neutral covariance of interest rates with equities.

I show that by instead taking the perspective of an investor with arbitrary linear exposures we can derive expectations for any asset class with options, including interest rates. This approach is closely related [Tetlock et al. \(2024\)](#), who calculate expected returns on equities using variance risk premium and risk-neutral third moment.

This paper also contributes to the extensive literature on interest rate forecasting, term premium, and dynamic term structure models. The conditional forecast methodology presents an alternative to traditional term structure models as a forecasting tool. But the results can also be interpreted as supporting the use of shifting end-point models that allow for non-stationary interest rates e.g., [Kozicki and Tinsley \(2001\)](#); [van Dijk, Koopman, van der Wel, and Wright \(2014\)](#); [Bauer and Rudebusch \(2020\)](#), and as supporting evidence for models with unspanned stochastic volatility, e.g., [Collin-Dufresne and Goldstein \(2002\)](#); [Collin-Dufresne, Goldstein, and Jones \(2009\)](#).

The empirical findings of this paper are also related to [Bauer and Chernov \(2024\)](#), who find that risk-neutral skewness predicts treasury yields, [Choi, Mueller, and Vedolin \(2017\)](#), who document the interest rate variance risk premium, and [Trolle and Schwartz \(2014\)](#), who measure risk-neutral yield moments from swaptions.

## Organization of the paper

Section [1](#) derives the key relationship between variance risk premia and interest rate risk premia. Section [2](#) describes the data and estimation of the model. Section [3](#) presents the the interest rate expectations and evaluates their forecasting performance. Section [4](#) applies the estimator to measure the contribution of risk premium vs expectations to the stock-bond correlation and the decline in rates around FOMC announcement windows. Section [5](#)

validates the key identifying assumption of the method: that the residual coskew is small. Section 6 concludes. An appendix contains extensions to other maturities, robustness tests, and technical details.

# 1 Measuring expectations from risk

If investors are highly exposed to interest rate risk, the equilibrium price to insure against variance in rates will be high. The price of that insurance can therefore inform us about their interest rate exposure and the expected interest rate.

This section formalizes this intuition by deriving an identity relating the interest rate risk premium to the variance risk premium and one unobservable quantity. I then discuss the key assumption needed to operationalize this relationship.

## 1.1 Defining interest rates and interest rate risk premium

This paper aims to measure expectations of next period’s interest rate:

$$E_t(y_{t+1})$$

The  $y_{t+1}$  could represent any rate or yield. When I implement the theory, I will use swap rates, focusing on the 10-year swap rate one quarter or one year in the future.

Consider a one-period linear interest rate forward. At time  $t + 1$  the buyer receives the level of the interest rate minus the pre-agreed forward price  $F_t$ . By definition, the price of this forward will be the risk-neutral expectation of the interest rate. The buyer’s payoff is equal to:

$$\Delta y_{t+1} = y_{t+1} - F_t = y_{t+1} - E_t^*(y_{t+1})$$

where the  $E_t^*$  operator represents risk-neutral expectations.

If we can measure the expected payoff on the interest forward, then we know the expected interest rates next period. I will define the “interest rate risk premium” as the expected



payoff from selling the linear interest rate forward. This is equal to the difference between the forward rate and the expected rate:

$$RP_t = E_t^*(y_{t+1}) - E_t(y_{t+1}) = -E_t(\Delta y_{t+1})$$

Under the expectations hypothesis, the interest rate risk premium is always zero, and forward rates tell us the expected rate in the future. In reality, of course, it turns out that forward rates are a very poor guide to the future path of rates, and so interest rate risk premia must be large and variable (Fama and Bliss, 1987; Campbell and Shiller, 1991). This paper attempts to measure those premia, and thus the expected interest rate.

I use linear interest rate forwards as the basic unit of my analysis, whereas most work in this field uses excess returns on zero-coupon bonds. While these two concepts are closely related, the forward framework allows us to work with any yield or interest rate, rather than just zero-coupon bonds. In practice we observe options for swaps or coupon bonds and not zero-coupon bonds, so this will prove convenient.

We do not observe linear forwards for most interest rates. However, their value can be constructed from the observable yield curve, with a minuscule convexity adjustment described in sections 2 and Appendix C.

## 1.2 Measuring interest rate risk premium without assumptions on the rate process

I will assume that there is no arbitrage and the fundamental theorem of asset pricing holds throughout this paper. In that case, the following result allows us to learn about expectations without specifying an interest rate process:

**Proposition 1.** *If no arbitrage holds, the expectation of any payoff  $X$  is given by:*

$$E_t^*(X) - E_t(X) = -\frac{1}{R_{f,t}} cov_t^* \left( \frac{1}{M_{t+1}}, X \right)$$

Where  $E^*$  represents risk-neutral expectations. From [Martin and Wagner \(2019\)](#); [Chabi-Yo and Loudis \(2020\)](#)<sup>2</sup>.

Applying this identity to the interest rate  $y_{t+1}$  yields:

$$RP_t = -\frac{1}{R_{f,t}} \text{cov}_t^* \left( \frac{1}{M_{t+1}}, y_{t+1} \right)$$

The risk premium is revealed by the risk-neutral covariance of interest rates with the SDF. This result resembles the more familiar asset pricing identity that relates expected returns with the *physical* covariance with the SDF. However, it has the advantage of working with directly observable quantities. Risk-neutral covariances are potentially observable from asset prices.

Hence if we can make some assumptions about the nature of the SDF (i.e., what constitutes a good or bad state) and find the price of an asset whose payoffs are linked to interest rates but also this SDF, then we can calculate interest rate expectations.

### 1.3 Projecting the SDF onto yields

The existing literature that uses proposition 1 to measure expectations has taken the perspective of an equity investor for whom  $\frac{1}{M_{t+1}}$  is a function of stock market returns ([Martin, 2017](#); [Martin and Wagner, 2019](#); [Kremens and Martin, 2019](#); [Chabi-Yo and Loudis, 2020](#); [Tetlock et al., 2024](#)). In this case, expected returns are given by the risk-neutral covariance with stock market returns.

This approach will not work for interest rates. We simply do not observe any liquid assets that reveal the risk-neutral covariance of interest rates with stocks [Martin \(2025\)](#).

Additionally, even if such an asset were available, we might question whether equity returns are a reasonable proxy for fixed-income investor wealth. The efforts to find joint

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<sup>2</sup>The proposition can be proven simply by calculating the price of an asset with payoff  $Z = \left( \frac{1}{M_{t+1}} - R_{f,t} \right) X$  using both SDF and risk-neutral pricing notation

risk factors across stocks and bonds have not always been successful.

I therefore propose a new approach that does not rely on equity returns. Instead, I will consider the SDF as a function of interest rates and residual terms.

Without loss of generality, we can consider the projection of any SDF onto interest rates ( $\Delta y_{t+1}$ ) under the risk-neutral measure:

**Definition 1.** *Projection of the inverse SDF*

$$\frac{1}{M_{t+1}} = R_{f,t+1} (1 - \lambda_t \Delta y_{t+1} + \varepsilon_{t+1}) \quad (1)$$

where  $E_t^*(\varepsilon_{t+1}) = \text{cov}_t^*(\varepsilon_{t+1}, \Delta y_{t+1}) = 0$

This is simply a linear projection, and is a fully general statement for any model with no arbitrage. The intercept must be  $R_{f,t+1}$  because  $E_t^*\left(\frac{1}{M_{t+1}}\right) = R_{f,t+1}$  and  $E_t^*(\Delta y_{t+1}) = 0$ . The projection will be exact, and  $\varepsilon_{t+1} = 0$ , only in the case of a log investor with exactly linear exposures to the interest rate  $y_{t+1}$ , i.e., an investor who only holds the risk-free rate and interest rate forwards.

In general, I will refer to  $\lambda_t$  as the investor's "exposure" to interest rates, because it captures how much the investor suffers from a rise in interest rates.

For a CRRA investor with linear interest rate exposure and risk aversion coefficient  $\gamma$ :

$$\lambda_t \approx \gamma \times \text{duration}_t$$

For example, with a CRRA coefficient of 2 and a linear exposure of duration 20,  $\lambda \approx 40$ . In the special case of log utility where  $\gamma = 1$ , this approximation is exact and  $\lambda_t$  is the duration. More generally, there will be a small approximation error. Appendix B tests this approximation up to  $\gamma = 4$  and shows it is accurate within 4% for the average risk-neutral distribution, and 8% on the most extreme day.

## 1.4 Risk premium as a function of exposure

Applying Proposition 1 to measure interest rate expectations and plugging in the projection for  $\frac{1}{M_{t+1}}$  yields an expression for interest rate risk premium:

**Proposition 2.**

$$RP_t = \lambda_t var_t^*(\Delta y_{t+1}) \equiv \lambda_t \sigma_t^{*2} \quad (2)$$

In words, Equation (2) tells us the risk premium is equal to exposure times risk-neutral variance. Loosely speaking, this is because in equilibrium, the amount of risk agents take should tell us both the rewards to taking interest rate risk and the price to insure against large changes in interest rates (the risk-neutral variance).

Now we have an expression for the interest rate risk premium in terms of an observable variable. The risk-neutral variance represents the price of a contract that pays out squared interest rate changes and can be easily measured from option prices, using the methodology described in Section 2. It is the interest rate equivalent of the *SVIX*<sup>2</sup> equity index from [Martin \(2017\)](#), and is highly correlated with the well-known *VIX* “fear index” for equity markets or the *MOVE* index for bonds.

However, we still need to know our exposure term,  $\lambda_t$  before we know interest rate expectations. [Martin \(2017\)](#) derives a similar expression for equities and posits that the exposure should be near 1. But for interest rates, the sign and magnitude of  $\lambda$  is not obvious. Is the marginal investor long or short duration, and by how much?

## 1.5 Exposure as a function of variance

We want to use the value of  $\lambda_t$  to measure expected payoffs on interest rate forwards. But the same logic can be applied in reverse: if we can estimate any expected payoff, we can use it to learn about  $\lambda_t$ .

I will measure the expected payoff on a contract that pays out  $\Delta y_{t+1}^2$  and use this expected payoff to learn about  $\lambda_t$ . This approach resembles the strategy that [Tetlock et al.](#)

(2024) apply to equity markets.

Variance has three important characteristics that make it an attractive payoff to forecast:

1. Interest rate variance does not display the same long-term non-stationary trend as the level (see Figure 1)
2. Variance can be measured easily with high frequency data. For any function involving higher moments, this becomes more challenging due to volatility clustering.<sup>3</sup>
3. There is a long track record of empirical success with simple variance forecasting models (e.g., Bollerslev, 1986; Corsi, 2009)

Applying Proposition 1 to the payoff  $\Delta y_{t+1}^2$  yields:

**Proposition 3.**

$$\sigma_t^{*2} - E_t(\Delta y_{t+1}^2) \equiv RPSq_t = \lambda_t E_t^*(\Delta y_{t+1}^3) - cov_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1}) \quad (3)$$

This equation tells us the expected profit from selling variance contracts is equal to exposure scaled by the risk-neutral third moment minus an unobservable term that I will call the “residual coskew.”

The risk-neutral third moment can be measured from option data in exactly the same way as the risk-neutral variance. So if we can make an assumption about the size of the residual coskew term, then a forecast of the expected squared changes in rates ( $E_t(\Delta y_{t+1}^2)$ ) will yield a value for the exposure and interest rate risk premium.

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<sup>3</sup>Intuitively, the daily autocorrelation of changes in rates should be low. Otherwise there would be large potential trading profits. But the daily autocorrelation of squared changes in rates, or the last period’s change in rates with next period’s squared change may be very high. Variance is persistent. Hence we cannot find a good estimate for third or higher moments by simply dividing up the sample into small pieces and calculating the moments of the short observations.

## 1.6 The residual coskew assumption

$\varepsilon_{t+1}$  represents the portion of the inverse SDF that is uncorrelated with interest rates under the risk-neutral measure. The inverse SDF, loosely speaking, captures how “good” states are for the investor (1/ marginal utility). So the residual coskew term,  $cov_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1})$  represents the extent to which the investor tends to be better or worse off in states of the world with large interest rate changes, regardless of their sign.

This quantity is unobservable. We do not observe  $\varepsilon$ , and even if we did, we would not observe the option prices that would reveal the relevant risk-neutral moment.

Producing an estimate of the interest rate risk premium therefore requires us to take an assumption on the size of this covariance. The simplest assumption that is most consistent with the existing literature is simply that the value is zero. This assumption will approximately hold, for example, in all standard dynamic term structure models, because they assume that the log SDF is linear in interest rate factors, making the inverse SDF nearly linear over short horizons.

Setting the residual coskew to zero amounts to an assumption that the risk premium on variance contracts reflects directional interest rate risk by the investor ( $\lambda_t$ ) rather than a non-linear exposure to rates. In other words, the reason why the investor does not choose to sell variance contracts despite their positive payoff is because of his level of interest rate risk. This does not impose that other asset classes (e.g. equities) are not important to the investor, but only that the directional exposure of those assets to interest rate risk is more important than exposure to squared changes.

I will take this as my main benchmark in developing a measure of interest rate expectations. It turns out that despite the simplicity of this assumption, it produces forecasts that perform very well out of sample and lines up with the variables we expect to be associated with interest rate premium (e.g. the slope of the yield curve), as shown in Section 3.

In Section 5 I will measure the closest observable data and argue the assumption is empirically reasonable. I calculate the physical covariance of  $\varepsilon$  with squared changes in rates

under a few common specifications for the SDF and find they are an order of magnitude too small to explain the size variance risk premium. They also do not help to explain the time series of the variance risk premium compared to a simple no-residual-coskew model. I further demonstrate that if one assumes the investor has CRRA (power) utility with reasonable risk aversion parameters, the simple no-residual-coskew estimate will understate term premium by approximately 30%, but retain a  $> 99\%$  correlation with the true values.

## 1.7 Identifying the risk premium

Imposing the no-residual-coskew assumption, we can rewrite equation (3) in terms of more easily measurable items as:

$$\sigma_t^{*2} - \sigma_t^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \sigma_t^{*4} \lambda_t^2 \quad (4)$$

where I have here used the fact that the sum of squared changes is equal to the variance plus the expected change (i.e. risk premium) squared:

$$E_t(\Delta y_{t+1}^2) = \sigma_t^2 + RP_t^2 = \sigma_t^2 + \sigma_t^{*4} \lambda_t^2$$

I will call the left-hand side of (4) — the difference between the risk-neutral and physical variance — the “variance risk premium” (VRP), consistent with the existing literature (e.g., [Bollerslev, Tauchen, and Zhou, 2009](#); [Drechsler and Yaron, 2011](#)).

We now have an equation with only two items that cannot be directly read from asset prices: the physical variance  $\sigma_t^2$ , and the exposure  $\lambda_t$ . So an estimate of the physical variance will identify the value of  $\lambda_t$ .

The risk-neutral third moment ( $E_t^*(\Delta y_{t+1}^3)$ ) tells us how directional exposure translates into variance risk premium. If the distribution is highly right-skewed (i.e., the third moment is high), then the variance contract will mostly pay off in high-rate states of the world where the investor is poor and hence will be highly valuable. If, on the other hand, the risk-neutral third moment is very low, it will take a large amount of exposure to deliver

much variance risk premium. The identification of  $\lambda_t$  will therefore be stronger in periods when the risk-neutral third moment is high.

## 2 Data and estimation

Section 1 shows how exposure, and hence interest rate risk premium, can be identified by the physical variance, the risk-neutral variance, and the risk-neutral third moment of interest rates (Equation 4).

This section describes how I operationalize this relationship to calculate interest rate expectations. I first present the data sources that the paper employs. I then briefly describe how the risk-neutral variance and third moment are calculated, followed by the methodology for estimating physical variance. Finally I describe the functional form and estimation approach I use to solve for the exposure parameter.

### 2.1 Data

I measure realized variance from daily interest rate swap rates obtained from Refinitiv, and risk-neutral moments from swaptions quotes obtained from Bloomberg. The sample covers USD LIBOR swaps from December 2001 to June 2023, when LIBOR ceased publication. I track six tenors (1, 2, 5, 10, 20, and 30 years) at three forecast horizons (3 months, 1 year, and 5 years). I construct swap forwards from swap rates and treasury yields using the procedure described in Appendix C.

The main results of this paper focus on the 10-year swap rate, the most liquid tenor. The methodology can be readily applied to other tenors of swap rate with similar results, although a few additional complications are introduced by the zero lower bound for short tenors. Appendix A shows the results from forecasts of the 1-, 2-, 5-, 20-, and 30-year interest rate risk premium.

The results of this paper should be interpreted as risk premium on swap rates, rather than treasury rates. Arguably, this is the more relevant rate for many market participants



who are more likely to borrow at rates linked to swap rates or hedge using swaps.

The differences are unlikely to be large. The realized quarterly risk premium on 10-year swaps vs treasuries during my sample period differs by just 0.7 basis points. The spread between 10-year swaps and the equivalent (off the run) treasury yield averages just 12 basis points during the sample period and it explains only 3% of the variation of the 10-year yield. Since 2008, this spread has often been negative and is thought to be related to leverage constraints and capital requirements rather than bank credit risk ([Boyarchenko, Gupta, Steele, and Yen, 2018](#)).

I use swaptions to calculate risk-neutral interest rate moments. A swaption is a contract that gives its holder the right to enter an interest rate swap at a predetermined rate. Swaption markets represent the most liquid interest option market, with extensive participation of both buy-side and interdealer trading ([ISDA, 2014](#); [Barnes, 2024](#)). While swaptions trade over-the-counter rather than on exchanges, collateralization through initial and variation margin is standard practice and limits counterparty credit risk.

Before 2011 only at-the-money swaption quotes are available. Out-of-the-money quotes are available for 10-year treasuries from CME and for closely-related LIBOR caps and floors from Bloomberg. I therefore estimate pre-2011 risk-neutral variance and skewness using the parameters from a regression on at-the-money variance, the 10-year treasury risk-neutral skewness (calculated by [Bauer and Chernov, 2024](#)), and the risk-neutral skewness of caps and floors. These regressions have an  $R^2$  of 0.99 for variance and 0.7 for skewness. Data before 2007 is only available on a weekly basis.

Treasury options data from CME offers an alternative source of interest rate option data. While this data source has a longer history than swaptions, it covers fewer tenors, has lower trading volume, and trades as American options at a single, short and time-varying horizon of 1-3 months.

## 2.2 Constructing risk-neutral moments

I follow [Carr and Madan \(1998\)](#) and [Martin \(2017\)](#) to extract model-free measures of risk-neutral moments from option prices. The key insight is that a portfolio of options with strikes spanning all possible outcomes has a payoff equal to the squared change in rates vs the forward rate. The forward price of this portfolio therefore equals the risk-neutral variance. The third moment follows similarly. A portfolio of options weighted appropriately produces a payoff of cubed rate changes, and its forward price gives the risk-neutral third moment.

This result holds exactly when we can trade options on the underlying rate itself. However, swaptions are options on swap values rather than swap rates. The value of a swap depends on the price of an annuity, not just the swap rate. In [Appendix C](#), I show that we can nevertheless derive the risk neutral moments of the swap rate itself using a simple approximation. If we assume that changes in the 10-year annuity yield are the same as changes in the 10-year swap rate (historical correlation in quarterly or annual changes is  $> 99.5\%$ ), then we can calculate the risk neutral moments of the swap rate itself. [Appendix C](#) demonstrates this relationship and derives the relevant formulas.

[Table 1](#) presents summary statistics for these risk-neutral moments, and [Figure 9](#) in [Appendix D](#) plots the time series. Variance is high during and after the financial crisis, during the 2022 inflation, and, more surprisingly, in 2002-4. The high option-implied interest rate volatility of the early 2000s as noted at the time as puzzling by central banks ([Fornari, 2005](#); [ECB, 2005](#)).

## 2.3 Measuring physical variance

### 2.3.1 Realized variance

The simplest way to estimate physical variance is to simply use realized variance measured over the subsequent period. Replacing the physical variance in [Equation \(4\)](#) with the

realized variance from daily data plus an error term yields:

$$\sigma_t^{*2} - RV_{t \rightarrow t+1} = \lambda_t E_t^*(\Delta y_{t+1}^3) + \sigma^{*4} \lambda_t^2 + \eta_{t+1} \quad (5)$$

Now all terms in this equation are observable except for  $\lambda_t$  and the error term  $\eta_t$ . If the conditional variance is equal to the conditional expectation of realized variance, then this error term should be uncorrelated with the other terms in the equation.

I use this approach to construct a simple test of the average size of  $\lambda_t$  in Section 3.1. If we assume  $\lambda_t$  is constant then we can simply solve for the value of  $\lambda$  that minimizes the sum of squared errors  $\sum \eta_t^2$  with no further assumptions.

### 2.3.2 Conditional variance forecasts

To develop conditional measures of time varying exposure ( $\lambda_t$ ) for out-of-sample forecasting, we will want to use the latest information about variance available at time  $t$ . I therefore fit a simple variance forecasting model to produce estimates  $\hat{\sigma}_t^2$ .

I employ a simple linear regression estimator based on the heterogeneous autoregressive realized variance (HAR-RV) model of [Corsi \(2009\)](#), widely used in the variance risk premium literature (e.g., [Bollerslev et al., 2009](#); [Drechsler and Yaron, 2011](#)). I estimate the model by regressing quarterly or annual realized variance onto the variance realized over the preceding week, month, and quarter:

$$RV_{t \rightarrow t+63d} = \beta_0 + \beta_1 RV_{t-5d \rightarrow t} + \beta_2 RV_{t-21d \rightarrow t} + \beta_3 RV_{t-63d \rightarrow t} + \varepsilon_{t+H}$$

where  $RV_{t \rightarrow t+Hd}$  is the realized variance calculated from daily data from period  $t$  to  $t + H$  business days. I assume 21 days in a month and 63 in a quarter. I use weekly, monthly, and quarterly lags rather than the standard daily, weekly, and monthly from [Corsi \(2009\)](#) to better suit the longer forecast horizons in my application, and because I do not have intra-day swap rate data.

Estimated conditional variance is then given by the regression coefficients multiplied by the latest realized variance:

$$\hat{\sigma}_t^2 = \hat{\beta}_0 + \hat{\beta}_1 RV_{t-5d \rightarrow t} + \hat{\beta}_2 RV_{t-21d \rightarrow t} + \hat{\beta}_3 RV_{t-63d \rightarrow t} \quad (6)$$

The model performs remarkably well out-of-sample, achieving  $R^2$  values exceeding 20% even at quarterly horizons, as shown in Table 2. This predictability confirms that it is easier to learn about variance than means, one of the key advantages of the variance-based forecasting approach.

Results are robust to including interest rate levels or allowing variance to depend on rate levels through a constant elasticity of variance specification.

## 2.4 Parameterizing exposure

The variance risk premium identity in Equation (4) and the conditional variance forecasts from Section 2.3.2 allow us to solve for the exposure,  $\lambda_t$ , in each period. However, in periods where the risk neutral third moment is low, the exposure will be poorly identified and highly sensitive to noise in the variance and skew estimates, as discussed in Section 1.7. To estimate exposure for the whole time series, I therefore assume that  $\lambda_t$  is a smooth function of state variables. This allows us to use the periods where the third moment is high to identify the plausible exposure when it is low.

In the Appendix E, I consider the alternative approach that estimates  $\lambda_t$  period-by-period without functional form assumptions. The results closely resemble the main results, indicating the choice of functional form is not critical.

I parameterize  $\lambda_t$  as a linear function of the first three principal components of the yield curve and the two risk-neutral moments used to construct my estimates (variance and skewness):

$$\lambda_t = \lambda_0 + \lambda_1 f_{1,t} + \lambda_2 f_{2,t} + \lambda_3 f_{3,t} + \lambda_4 \sigma_t^{*2} + \lambda_5 skew_t^* \quad (7)$$

where  $f_1$ ,  $f_2$ , and  $f_3$  are the first three principal components of yields, which capture the level, slope, and curvature of the yield curve.

This can be interpreted as an assumption that investors choose their level of exposure depending on the shape of the yield curve and risk-neutral distribution of future yields, or that the shape of the yield curve responds to investor choices and exposure.

The first three principle components explain  $> 99\%$  of variation in the shape of the yield curve and are empirically well known to be related to interest rate risk premium. It has been standard practice in term structure modeling since [Duffee \(2002\)](#) to assume that the price of risk is a function of yield curve factors.

I include the risk-neutral moments for the sake of completeness because they are used in construction of the estimator and because empirical work has suggested they contain additional information on interest rate risk premium that is not included in the yield curve ([Joslin and Konchitchki, 2018](#); [Bauer and Chernov, 2024](#)). Ultimately I find these are not significant explainers of the variance risk premium (shown in Table 6 in Section 3), but I allow my estimator to determine this rather than assuming this relationship a priori.

This paper’s methodology is flexible, and more state variables could easily be added in the future to improve prediction or test theoretical predictions.

## 2.5 Estimation

Substituting the variance forecast (Equation 6) and the exposure parameterization (Equation 7) into (4) yields:

$$\sigma_t^{*2} - \hat{\sigma}_t^2 = \lambda_t E_t^*(\Delta y_{t+1}^3) + \lambda_t^2 \sigma_t^{*4} + \eta_t \quad (8)$$

where:

$$\lambda_t = \lambda_0 + \lambda_1 f_{1,t} + \lambda_2 f_{2,t} + \lambda_3 f_{3,t} + \lambda_4 \sigma_t^{*2} + \lambda_5 skew_t^*$$

The risk neutral moments ( $\sigma_t^{*2}$  and  $E_t^*(\Delta y_{t+1}^3)$ ) can be measured from options data.

The estimated physical variance  $\hat{\sigma}_t^2$  is measured from a variance forecasting regression. The error term  $\eta_t$  captures three sources of error: (i) forecast errors in physical variance, (ii) time variation in  $\lambda_t$  orthogonal to the principal components, and (iii) the residual coskew term.

I estimate the parameters  $\{\lambda_0, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$  using nonlinear least squares. To address heteroskedasticity and reduce the influence of a few high-variance states, I weight observations by the inverse of risk-neutral variance. This is equivalent to dividing equation (2.5) through by  $\sigma_t^*$  and improves the stability of estimates, reducing the excess kurtosis of the left hand side variable from 4.2 to 0.1.

Where relevant, indicative standard errors are calculated using GMM with Newey-West corrections, using lags at least equal to the forecast horizon. Some caution is warranted in interpreting these standard errors as the appropriate autocorrelation structure for  $\eta_t$  depends on the relative importance of its three components. If variance forecasting errors are large and persistent, for example, the true error autocorrelation may be high. The main conclusions on the accuracy of these forecasts relies on out of sample prediction rather than in sample standard errors.

### 3 Risk-based interest rate expectation results

This section reports the results and forecasting performance of the risk-based interest rate expectation measure. I first show that the average interest rate risk premium was large. I then show that conditional forecasts of interest rates capture well-known relationships between the yield curve slope and risk premium, but outperform existing measures out of sample. I close by briefly discussing what the forecasts tell us about investors' exposure to rates.

### 3.1 A simple variance-based test of interest rate risk premium

Equation (2) relates risk premium to exposure and tells us that if there are no interest rate risk premia and the expectations hypothesis holds, then we must have  $\lambda = 0$ . Equation (4) gives us a powerful way to test this hypothesis in terms of variance risk premium. If we set  $\lambda_t$  to a constant level, and replace the physical variance with the realized variance from daily data plus an error term, we have:

$$\sigma_t^{*2} - RV_{t \rightarrow t+1} = \lambda E_t^*(\Delta y_{t+1}^3) + \sigma^{*4} \lambda_t^2 + \eta_{t+1} \quad (9)$$

We can simply solve for the value of  $\lambda$  that minimizes the sum of squared errors  $\sum \eta_t^2$ . If we can reject  $\lambda = 0$ , then we reject the expectations hypothesis and show there was a risk premium.

I find  $\hat{\lambda} = 0.41$ , with a standard error of 0.06, using monthly observations and GMM Newey West standard errors with 3 lags. This amounts to a very strong rejection of the expectations hypothesis with a t-statistic of over 5. The statistical significance is high enough that a simple difference in priors could not account for the apparent risk premium, as suggested by [Farmer et al. \(2024\)](#) for the level-based forecasts.

Multiplying this exposure by the risk neutral variance yields an average interest rate risk premium of 11 basis points (bp), or 44 bp annualized. This is similar to the 34 bp realized risk premium. Markets seem to have been demanding compensation for bearing interest rate risk, rather than making systematic mistakes.

Since I denominate  $\Delta y_t$  in percentage points, this exposure estimate corresponds to a duration of 42 years for a log-utility fixed-income investor (with a confidence interval from 26 to 58). If we instead consider an investor with higher risk aversion, the implied duration would be lower. For example, in appendix F I show that with a CRRA risk aversion coefficient of 4, duration would be approximately 15. This high sensitivity to interest rate changes could be consistent with models where highly leveraged intermediaries are

the marginal investors in fixed-income markets. For example, [Kekre, Lenel, and Mainardi \(2024\)](#) suggest durations of 10 to 30 years for fixed-income arbitrageurs.

### 3.2 Forecasting performance

Applying the conditional interest rate forecasts and exposure parameterization described in [Section 2](#) lets us create a time-varying conditional measure of exposure and interest rate expectations. In this subsection I demonstrate that this conditional measure successfully predicts changes in interest rates out of sample.

[Table 3](#) reports regressions of realized rate changes on estimated risk premia at various horizons:

$$\Delta y_{t,t+h} = \alpha - \beta \times \widehat{\text{RP}}_{t,h} + \varepsilon_{t+h} \quad (10)$$

where  $\widehat{\text{RP}}_{t,h}$  denotes the predicted risk premium from time  $t$  to  $t + h$ .

The predictions are highly significant at monthly (t-statistic 2.9), quarterly (t-statistic 2.9), and annual (t-statistic 2.4) horizons. More importantly, I cannot reject  $\beta = 1$  or  $\alpha = 0$  at any horizon. This suggests the measure captures risk premia accurately in levels, not just direction. The variance-based approach, using no information from the yield curve itself, recovers the compensation investors demand for bearing duration risk.

The  $R^2$  values are high even at short horizons (4% monthly, and 6% quarterly), where term structure models have traditionally struggled. The short-term performance allows for high trading profits as I will demonstrate in [Section 3.2.1](#).

This predictive power is greater than that of professional forecasters or traditional term structure models. [Table 4](#) reports out-of-sample relative  $R^2$  statistics of the variance-based model vs these other models, calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the error when the variance-based model is used to forecast changes in the yield and  $\nu_t$  is the error when the alternative model is used. This represents the improvement in forecast accuracy from using the variance-based measure versus alternatives.



I compare my forecasts of the 10-year yield to the expectations hypothesis, the survey of professional forecasters (SPF), the two most commonly used dynamic term structure models (DTSMs) (Adrian et al., 2013; Kim and Wright, 2005), the Bauer and Rudebusch (2020) non-stationary “observed shifting endpoint” model and a simple forecast of no change in rates (“random walk”). The SPF and Bauer and Rudebusch (2020) data are produced quarterly, and Bauer and Rudebusch (2020) forecasts are only produced up to 2018. Details on the construction of the alternative benchmarks are included in the appendix.

The variance-based interest rate expectation measure delivers more accurate forecasts than all alternative benchmarks at all horizons. Improvements are highly statistically significant vs the expectations hypothesis and the stationary DTSMs at the 1 month and 1-year horizons. The improvements versus the Bauer and Rudebusch (2020) model are weaker (9% improvement in annual  $R^2$ ), and with just 18 years of data, the difference is not statistically significant.

The power of the variance-based forecasts comes from the use of theory to derive the level of interest rate risk premium. If instead we were to take our predictor and regress it on changes in interest rates in each period, we would get a substantially worse out-of-sample performance, as the coefficient loadings swing around based on past performance. The last row of Table 4 compares the out-of-sample  $R^2$  from the risk based measure to a regression-based forecast constructed in this fashion. Using theory to derive the levels of the forecasts delivers a large (although only borderline significant) improvement in  $R^2$ .

The random walk forecast (i.e., the current rate) remains surprisingly competitive, beating all benchmarks except for my variance-based forecast. This reminds us that assuming no mean reversion was, ex post, a successful strategy during this period. However, ex-ante it had no obvious justification.

### 3.2.1 Economic significance

To demonstrate that the  $R^2$  from the predictive regressions in Section 3.2 are large, Figure 7 shows cumulative returns from trading strategies based on different interest rate risk

premium estimates.

The basic strategy takes positions in interest rate forwards proportional to expected interest rate risk premia: buy duration when risk premia are high, sell when they are low. Panel (b) shows returns using simple monthly position sizes equal to the expected risk premium in percentage points. Panel (a) scales positions by the inverse of risk-neutral variance, investing more aggressively when uncertainty is low, mimicking the behavior of the implied log investor.

The variance-based strategy generates substantial cumulative returns over the sample period, with a Sharpe ratio of 0.75. Using variance-weighted positions, a dollar invested in 2002 grows to over \$100 by 2023, compared to <\$5 using SPF forecasts or ACM. Much of the outperformance comes from 2013–2021, when stationary models consistently predicted rising rates that did not arrive.

The large cumulative returns on the variance-weighted strategy underscores that the log investor benchmark may be more aggressive than the typical institutional investor. If we instead consider an investor with CRRA coefficient 2 with the same  $\lambda_t$ , he will take positions approximately one half as large and have a cumulative return of around 10 instead of  $> 100$ . This is roughly in line with the cumulative return of stock indices during this period.

These results should be interpreted cautiously. The strategies exhibit high volatility, and transaction costs, though likely small in liquid swap markets, would reduce returns. Nevertheless, the magnitude of outperformance suggests that the statistical improvements in forecasting are economically meaningful.

### 3.3 Characterizing the conditional interest rate forecasts

The time series of interest rate risk premium is shown in Figure 3. We see two large peaks in 2002–4 and 2009–11. Risk premium is then low 2012–2022 except for a brief spike during the “taper tantrum” in 2013. To contextualize the size of these interest rate risk premia,

Figure 4 compares expectations hypothesis versus risk-based forecasts over time. The dark line plots the time series of the 10-year swap rate, while at each quarter the faint gray and red lines plot the forward rates and the expected rate (using this paper’s estimates) one year ahead. Periods that were characterized by steep upwards sloping yield curves were associated with large risk premium, such that expectations were almost always for interest rates to remain flat over the following year.

This time series shows many similarities to well known term premium models. Table 5 compares the forecast to various alternative benchmarks including the Survey of Professional Forecasters (SPF) and the widely used Adrian et al. (2013) (ACM) affine term structure model, and Figure 5 plots the ACM series risk premium versus my estimates. The SPF and ACM forecasts both show  $> 50\%$  correlation with the risk based estimate at the 1y horizon, despite the different underlying methods and sources of information.

The key difference lies in the behavior at low rates. The stationary assumption built into standard DTSMs meant that they always forecast rates to return to their long-term averages eventually. So when current and forward rates were low, as in 2016–2020, the ACM model forecast a very large negative risk premium. At their lowest, these forecasts implied investors were expecting to lose  $-1.5\%$  on 10-year bonds in a single quarter.<sup>4</sup>

In contrast, my estimates make no assumption about stationarity of interest rate levels, and end up forecasting almost no interest rate risk premium in this period. These forecasts are more in line with the suggestive evidence from surveys of institutional investors who mostly do not seem to report negative expected excess returns on bonds (Dahlquist and Ibert, 2024; Coutts, Gonçalves, and Loudis, 2023).

### 3.4 Variation in investor exposure to rates

Besides a risk premium estimate, the variance based approach delivers an estimate of  $\lambda_t$ : the exposure to interest rate increases of the log investor who chooses a linear interest rate exposure. Figure 6 shows the evolution of the estimated (in-sample)  $\lambda_t$  over time. The

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<sup>4</sup>Assuming duration of 8, and 10y-in-1q interest rate risk premium of  $-0.2$

series shows a gradual decline from 0.8 to approximately 0 in 2020, followed by an increase to 0.35 in 2023.

The gradual pace of changes suggests that the simple example of the log investor with constant exposure (discussed in Section 1) works fairly well over short horizons. The  $\lambda$  parameter is relatively stable over the course of a month, with changes in risk-neutral variance explaining 77% of the variance in 1 month changes in 1 quarter risk premium.<sup>5</sup>

However, over the long run, the drop is substantial. To provide suggestive evidence on the causes, I repeat the conditional risk premium estimation exercise (i.e., equation 2.5) allowing  $\lambda_t$  to be a function of economically-motivated variables instead of yield curve principal components. Table 6 reports coefficients from these alternative specifications. I find two significant relationships in column 5.

First,  $\lambda_t$  increases 1:1 with the sensitivity of equity markets to interest rate changes. The coefficient on equity beta (the sensitivity of stock returns to yield changes) is 0.97, significant at the 1% level. This relationship is consistent with a log investor fully invested in equities.<sup>6</sup>

Second,  $\lambda_t$  appears to rise following increases in aggregate bond supply, although the statistical significance is weak. A 1% increase in aggregate bond duration (scaled by GDP) raises  $\lambda$  by 0.004, or approximately 1%. While the effect is only marginally significant, it aligns with models where slow-moving capital forces specialized arbitrageurs to absorb supply shocks, increasing their exposure and the compensation they demand. So the long-term decline in exposure could be related to the long-term growth of arbitrageur capital relative to the aggregate bond duration.

In general, economically-motivated variables do a poor job explaining the exposure variable  $\lambda$  relative to the simple principal components, with none adding more than 2 ppt

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<sup>5</sup>Variance of 4-week changes in log risk-neutral variance is 5.25, and of changes in log risk premium is 6.75

<sup>6</sup>It is also consistent with an arbitrageur whose equity value is correlated with the overall market (e.g. a bank). For example, the rolling sensitivity to interest rate changes of stock returns of the primary dealers from the He, Kelly, and Manela (2017) closely resembles that of the overall market.

of  $R^2$  over the constant-only model. The underlying causes of the reduction in sensitivity of investors to interest rates remains an open question.

## 4 Applications

This section considers two applications that traditional interest rate forecasting methods are poorly suited to answer. I consider the average decline in long term rates around FOMC meetings and the positive stock-yield correlation of 2000–2021 and ask whether they are driven by interest rate expectations or interest rate risk premium. Both of these questions require measuring short-term changes in term premium. I therefore start by developing a simple measure of daily or weekly changes in term premium that loosens the exposure-parameterization assumptions from Section 2.

### 4.1 Measuring short-term changes in interest rate risk premium

To measure short-term changes in conditional interest rate risk premium, we could simply take the daily changes of the risk premium estimates from Section 3. However, we might be concerned that the functional form we have imposed for  $\lambda_t$  does not accurately capture small changes in the relationship between interest rate risk premium and the yield curve, particularly around shocks such as FOMC announcements.

To find an alternative measure with looser functional form assumptions, we can combine equations (2) and (4) to state interest rate risk premium in terms of variance risk premium, and then differentiate the result for small changes, to find:

$$dRP_t \approx \frac{d(\sigma_t^{*2} - \sigma_t^2)}{\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}} + 2RP_t} \quad (11)$$

Change in interest rate risk premium is revealed by change in variance risk premium, scaled by skewness and risk premium.<sup>7</sup>

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<sup>7</sup>I omit a term involving changes in skewness,  $-E_t(\Delta y_{t+1})d\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}}$ , which is empirically negligible.

This equation naturally suggests an estimator for short-term changes in risk premium. Simply plug in the fitted values for interest rate risk premium ( $\widehat{TP}_t$ ) and physical variance ( $\Delta\hat{\sigma}_t^2$ ) from Section 3:

$$\widehat{\Delta RP}_t \approx \frac{\Delta\sigma_t^{*2} - \Delta\hat{\sigma}_t^2}{\frac{E_t^*(\Delta y_{t+1}^3)}{\sigma_t^{*2}} + 2\widehat{RP}_t}$$

This estimator no longer depends on short-term changes in exposure exactly aligning with changes in the interest rate principal components used to parameterize  $\lambda_t$ . Of course the functional form assumptions are still embedded in the risk premium estimates ( $\widehat{RP}_t$ ), under the assumption that these do a decent job of describing the average level of exposure, even if they don't capture short-term changes well.

It is not possible in general to measure short-term changes in physical variance ( $\Delta\hat{\sigma}_t^2$ ) around shocks with any accuracy. I therefore only consider long term averages and covariances of this change in term premium measure, rather than considering specific days. In the case of FOMC announcement window results I will treat these results as indicative and check other indicators of the average change in physical variance.

## 4.2 Stock yield correlation

The correlation between stocks and long-term yields famously switched from negative to positive in the late 90s. A substantial literature seeks to explain the causes of the modern positive correlation.

By some explanations, the source is risk premium: when stocks go up, bond risk premium and yields go up. For example, [Antolin-Diaz \(2025\)](#) proposes flight to safety shocks move institutional investors out of stocks into long-term bonds.<sup>8</sup>

By others, the covariance is expectations driven: news that leads to low stock returns lowers expected rates. For example, [Campbell et al. \(2017\)](#) propose inflation shocks now

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<sup>8</sup>The convenience yield covariance from [Acharya and Laarits \(2023\)](#) and the “hedging premium” shocks from [Cieslak and Pang \(2021\)](#) should also involve positive correlation of stock returns with bond risk premia.

correlate with stock returns, or [Gormsen and Lazarus \(2025\)](#) argue long-term cash-flow news can push stocks and yields in the same direction.

This paper lets us test between these explanations. Does the 1-year interest rate risk premium correlate positively or negatively with stock returns? And is this effect large enough to explain the stock-yield correlation?

I show that the correlation is negative. Risk premium shocks cannot explain the positive stock-yield correlation 2002-2022. Table 10 shows the correlation of estimated changes in 10y-in-1y risk premium with stocks. I show results for multiple observation horizons (daily, weekly, monthly) and methods for calculating the interest rate risk premium (using simple changes in the estimated term premium in Column 1, or the short-term change measure developed in Section 4.1 in Column 2). All show a negative correlation. Risk-neutral variance tends to increase on days with bad stock returns more than the regression-estimated physical variance, suggesting risk premium rises.

In contrast, a traditional term structure model finds the opposite. Column 3 shows the correlation with changes in the [Kim and Wright \(2005\)](#) term structure model, which are all positive. The model loads heavily on the level of interest rates due to the stationary structure, and hence interprets a portion of all increases in the level of interest rates to be due to risk premium, yielding a positive term-premium-yield correlation.

Figure 8 plots the time series of correlations of 10y-in-1y forward rates, risk premium, and expected rates. Interestingly, the return of negative stock-yield correlation in 2022-2023 seems to be driven entirely by correlation of risk premium shocks with equities, not rate expectations. While I cannot observe data before 2002, this does suggest the possibility that the earlier negative stock-bond correlation was at least partially driven by very large negative correlation of risk premium with equity returns.

### 4.3 Interest rate risk premium during FOMC announcement windows

[Hillenbrand \(2025\)](#) documents an intriguing fact: the entire secular decline in long-term rates from the 1990s to 2020 occurred during three-day windows around FOMC meeting. This has been interpreted as evidence that markets primarily learn about long-run interest rate levels from the Federal Reserve. I provide suggestive evidence that, instead, this FOMC-window decline may reflect falling risk premia.

Standard dynamic term structure models are not very useful to decompose announcement effects. They either inherently assume constant long-term expectations (e.g. [Adrian et al., 2013](#)) or update long-run expectations at lower frequencies than daily (e.g. [Bauer and Rudebusch, 2020](#)).

However, this paper gives us a tool to learn about interest rate risk premium from daily observations of option prices. If the decline in interest rates comes from risk premium, we should expect to see a sharp drop in risk-neutral variance and variance risk premium that reverses after the announcements. Whereas if the decline is a result of learning about long-term expectations from the fed, we should see no increase in variance risk premium.

In this section I first document that there is a large decline in risk-neutral variance around FOMC meetings that appears to be consistent with a change in risk premium. I then show that the size of the declines is large enough to be potentially consistent with full FOMC-window effect being driven by interest rate risk premium rather than expectations. The fact that the FOMC-window declines match the magnitude of overall declines in rates could simply represent a coincidence.

#### 4.3.1 The FOMC-window variance decline

Table 9 shows that risk-neutral variance of interest rates declines across tenors and maturities during FOMC windows. The proportional declines are similar whether we look at short or long tenors, 3-month or 1-year horizons. Even five-year variance—capturing



uncertainty about rates far in the future—shows substantial FOMC-window declines.

This decline is much larger than the net change in physical or risk-neutral variance during the sample period. For example, over the full sample, five-year risk-neutral variance for the 10-year rate actually increased from 3.0 to 4.2 percentage points squared. Yet during FOMC windows alone, it declined by a cumulative 12.2 percentage points squared. For variance to end up higher despite falling dramatically during FOMC meetings requires an offsetting increase of 13.4 percentage points squared between meetings.

The evidence for the idea that the fed primarily learns about the long term from the fed was the fact that the size FOMC-window and aggregate declines in long-term forward rates coincide. This is not the case for the declines in long-term risk-neutral variance. If there is information in the FOMC announcement windows about variance, it must be systematically countered by other information received outside the FOMC windows.

It seems likely that instead, these FOMC-window declines in risk-neutral variance are a result of declines in risk premia that reverse outside the windows. This would be consistent with demand-based mechanisms by which declines in short-term rates push down the risk premium on long-term rates. For example, reaching for yield (e.g. [Hanson and Stein, 2015](#)), improvements in intermediary capital (e.g. [Kekre et al., 2024](#)), or MBS or bank deposit duration hedging effects (e.g. [Hanson, 2014](#); [Rogers, 2024](#)).

In Appendix [G](#) I show two further pieces of evidence to support the idea that these declines come from variance risk premium rather than physical variance. First, they are too large to be explained by simple “mechanical effects” from the high-variance FOMC window dropping out of the forecast period. Second, using the information from these changes does not improve on the regression-based forecast of interest rate variance.

#### **4.3.2 Measuring the implied risk premium decline**

To understand how much interest rate risk premium decline could be implied by the declines in variance risk premium, I will find the result if we assume there is no change in the physical

variance during the estimation windows, i.e.,  $\Delta\sigma_t^2 = 0$ . This is roughly what is implied by the regression-based variance estimates from Section 3.

From 2007–2018 (the end of the sample period from [Hillenbrand, 2025](#)), the average FOMC-window decline in the 10y-in-1y forward rate is 2.8 bp per meeting. The estimation procedure above yields a decline in 10y-in-1y risk premium of **3.4 bp**, with a standard error of **1.2 bp**, using daily data with standard errors clustered by meeting.

In other words, the entire FOMC-window decline in long-term rates could be attributable to declines in risk premia that are expected to be realized over the course of a year, with no change in long-term expectations at all.

In contrast, a traditional term structure model that does not use the information from the risk-neutral variance will find a much smaller decline. For example, the [Kim and Wright \(2005\)](#) stationary DTSM provided by the Federal Reserve Board shows only a 0.4 bp decline in the 10-year term premium from 2007–2018.

Of course to the extent that physical variance declined during these windows as well, the decline in risk premium could be smaller. It is likely that the true decline involved some combination of risk premium and expectations.

## 5 Validating the residual coskew assumption

The main theoretical result of this paper is that the interest rate risk premium is a function of the variance risk premium and an unobservable term, the “residual coskew,” that captures the correlation of squared rate changes with the inverse SDF (i.e., the investor’s marginal utility).

In Section 3 I show that if we assume this residual coskew term is 0, the implied interest rate risk premium is a strong out-of-sample predictor of interest rates.

In this section I argue that this is a reasonable assumption based on the data that we can observe. I show first that for common models of the SDF, the equivalent *physical*

moment ( $cov_t(\Delta y_{t+1}^2, \varepsilon_{t+1})$ ) is an order of magnitude smaller than what would be needed to explain the observed variance risk premium. Where we can measure equity and interest rate risk premia, they seem too small to make up this difference between the physical and risk neutral covariance.

Second, I show that in the time series the physical residual coskewness and other related measures do not help explain the variance risk premium versus term-premium-only model.

Finally, I show that allowing for higher risk aversion levels cannot alone explain the variance risk premium. If we allow for CRRA utility with risk aversion coefficients up to 4, then my exposure and interest rate risk premium estimates understate the true values by approximately 30%, but retain a 99% correlation.

In Appendix F, I further show that the levels of bond market convexity observed in the aggregate market are far too small to explain the variance risk premium.

## 5.1 Physical residual coskew under different SDF models

The average realized quarterly variance risk premium from Section 3.1 is 0.04 percentage points squared. To fully explain this through the residual coskew term alone would require  $cov_t^*(\Delta y_{t+1}^2, \varepsilon_{t+1}) = -0.04$ .

Measuring the residual coskew would require observing a set of options connecting interest rates with the parts of the SDF that are uncorrelated with interest rates. Since these options do not exist in reality, the best we can do is measure the physical covariance under various common models of the SDF.

In Table 7 I calculate the residual coskew term under the SDF implied by the log-equity-investor model of Martin (2017), a linear Fama French three factor model, and a typical dynamic term structure model. In all three cases, the covariance is at least an order of magnitude smaller than the variance risk premium.

Of course the risk-neutral moment may be larger or different than the physical moment,

but the risk premium we can observe on stocks and squared interest rate changes does not appear large enough to make up the difference. From 2007-2023, the risk-neutral standard deviation  $\Delta y_{t+1}^2$  is approximately  $1.3\times$  larger than the physical standard deviation.<sup>9</sup> Tetlock et al. (2024) estimate the risk-neutral standard deviation of  $R_m$  is about  $1.3\times$  the physical standard deviation. So if the physical and risk-neutral correlations were the same, we would only expect risk-neutral residual coskew to be roughly  $2\times$  as large as physical coskew, not the  $10\times$  needed to explain the variance risk premium.

To estimate the covariance figures I first regress daily observations of the SDF onto daily changes in the 10-year rate and collect the residuals:  $\varepsilon_t$ . I then calculate quarterly covariance of epsilon with squared interest rate changes, using this daily data<sup>10</sup>

For the Fama French three factor model I use the price of risk coefficients that match the observed average returns on the three factors. For the dynamic term structure model SDF, I calibrate the terms by fitting the 5 factor model of Adrian et al. (2013).

## 5.2 Time series evidence

Equation (4) tells us that the variance risk premium is a function of exposure multiplied by the risk-neutral third moment as well as the residual coskew. If the residual coskew plays an important role, then we should find that it helps explain the time series of variance risk premium. If it doesn't, then we should find the third moment alone explains the time series well.

In Section 3, I showed that the third moment explains a large share of the time series variance in estimated variance risk premium, even with a simplistic constant exposure assumption (see column 1 of Table 6).

Do measures of residual coskew improve upon this fit? To answer this question I run

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<sup>9</sup>Variance calculated from moments of daily data, allowing for skewness and volatility clustering but assuming conditional mean of daily rate changes is 0.

<sup>10</sup>Since there are 63 business days in the quarter, this is calculated as  $63cov(\varepsilon_t, \Delta y_t^2) + \sum_{j=1}^{62} (63 - j) (cov(\varepsilon_{t-j}, \Delta y_t^2) + cov(\varepsilon_t, \Delta y_{t-j}^2))$

simple regressions of VRP on the risk-neutral third moment alone, as well as the risk-neutral third moment plus variables that might plausibly be associated with residual coskew. This is a simplified version of the nonlinear estimation from Section 3, where we ignore the  $RP^2$  term and assume  $\lambda_t$  is constant.

As variables that might be correlated with conditional coskew, I include rolling 1-year estimates of the physical residual coskew under the log equity investor ( $M_t = \frac{1}{R_m}$ ) model<sup>11</sup>, risk-neutral equity volatility (*SVIX*), aggregate bond market convexity (from the Bloomberg “Agg” index), and the He et al. (2017) intermediary capital ratio.

The results are reported in Table 8. In all cases the third moment is highly significant with a t-statistic of over 7, while the alternative explanatory variables are at best marginally significant. Most of the coefficients also have the wrong sign compared to what we would expect for a residual-covariance based explanation. For example, we would expect periods of negative residual coskew to be associated with high variance risk premium, but we find the opposite (although insignificant). Low intermediary capital ratios are also associated with lower variance risk premium, surprisingly.

In short, measures of conditional residual coskewness do not improve on the simple zero-residual-coskew constant  $\lambda$  model at explaining the time series of variance risk premium.

### 5.3 Risk aversion

Another possible source of residual coskew is the convexity of the utility function itself, rather than the non-linearity of the payoffs. However, high risk aversion alone cannot explain the positive variance risk premium. In fact, the simple zero-residual-coskew estimate will tend to underestimate the term premium if the investor is more risk averse than a log investor

In Appendix F I repeat the constant- $\lambda$  estimation of term premium from Section 3.1 allowing for CRRA coefficients ( $\gamma$ ) of up to 4. I find that the zero-residual-coskew estimate

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<sup>11</sup>I do not include the Fama French 3 factor estimate because the two time series of coskewness is 99% correlated

would underestimate true risk premium by approximately 30%, but retain an  $> 99\%$  correlation with the true estimates

Intuitively, two investors can generate the same interest rate risk premium through different combinations of risk aversion and leverage. A less risk-averse investor needs high leverage (large  $D$ , small  $\gamma$ ), while a more risk-averse investor needs less leverage (small  $D$ , large  $\gamma$ ). But the highly levered investor fears far-away states more—a 5 percentage point rate move could wipe her out, while merely denting the unleveraged investor’s wealth. The levered investor therefore pays more for variance protection despite having the same directional exposure.

## 6 Conclusion

This paper introduces a new approach to measuring interest rate expectations and term premia. By shifting focus from the non-stationary level of interest rates to their stationary variance, we can extract expectations from options data and test hypotheses about interest rate dynamics in a new way.

The core theoretical result links the interest rate risk premium perceived by an investor to the variance risk premium perceived by the same investor and an unobserved “residual coskew” quantity related to the non-linearity of the the his marginal utility with respect to interest rates. If we assume as a benchmark, that this residual coskew term is 0, an assumption that does not appear unreasonable based on the data, we can derive the exact perceived interest rate risk premium from the variance risk premium.

The empirical findings are fourfold. First, the historical interest rate risk premium is large. The strong average bond returns over the past 25 years appear to be mostly due to risk premium rather than forecasting errors.

Second, I construct a conditional measure of the interest rate risk premium that outperforms standard dynamic term structure models and survey-based measures in forecasting excess returns at short horizons, with a trading strategy based on the measure generating

large economic profits.

Third, I find that the secular decline in long-term rates concentrated in FOMC announcement windows may be better explained by a compression of risk premia than by learning about the long-run level of rates. Risk-neutral variance falls by a large amount during these windows, a pattern more consistent with temporary changes in risk appetite or exposure than with information updates.

Finally, I show that the positive stock-yield correlation of the 2000s onward is not driven by bond risk premium. This suggests explanations based on inflation or growth expectations may have more promise than explanations driven by investor flight to quality.

## References

- Acharya, Viral V., and Toomas Laarits, 2023, When do treasuries earn the convenience yield? — a hedging perspective, *NBER Working Paper* 31863.
- Adrian, Tobias, Richard K Crump, and Emanuel Moench, 2013, Pricing the term structure with linear regressions, *Journal of Financial Economics* 110, 110–138.
- Antolin-Diaz, Juan, 2025, How did government bonds become safe?, *MIT Sloan Working Paper* 7314.
- Barnes, Chris, 2024, Swaption volumes by strike, *Clarus FT* Q2.
- Bauer, Michael D., and Mikhail Chernov, 2024, Interest rate skewness and biased beliefs, *Journal of Finance* 79, 173–217.
- Bauer, Michael D., and Glenn D. Rudebusch, 2020, Interest rates under falling stars, *American Economic Review* 110, 1316–1354.
- Bollerslev, Tim, 1986, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics* 31, 307–327.
- Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected stock returns and variance risk premia, *Review of Financial Studies* 22, 4463–4492.
- Boyarchenko, Nina, Pooja Gupta, Nick Steele, and Jacqueline Yen, 2018, Negative swap spreads, *Federal Reserve Bank of New York Economic Policy Review* 24.
- Breeden, Douglas T., and Robert H. Litzenberger, 1978, Prices of state-contingent claims implicit in option prices, *The Journal of Business* 51, 621–651.
- Campbell, John Y., and Robert J. Shiller, 1991, Yield spreads and interest rate movements: A bird’s eye view, *The Review of Economic Studies* 58, 495–514.



- Campbell, John Y., Adi Sunderam, and Luis M. Viceira, 2017, Inflation bets or deflation hedges? the changing risks of nominal bonds, *Critical Finance Review* 6, 263–301.
- Carr, Peter, and Dilip Madan, 1998, Towards a theory of volatility trading, in Robert A. Jarrow, ed., *Volatility: New Estimation Techniques for Pricing Derivatives*, chapter 29, 417–427 (Risk Books, London).
- Chabi-Yo, Fousseni, and Johnathan Loudis, 2020, The conditional expected market return, *Journal of Financial Economics* 137, 752–786.
- Choi, Hoyong, Philippe Mueller, and Andrea Vedolin, 2017, Bond variance risk premiums, *Review of Finance* 21, 987–1022.
- Cieslak, Anna, and Hao Pang, 2021, Common shocks in stocks and bonds, *Journal of Financial Economics* 142, 880–904.
- Collin-Dufresne, Pierre, and Robert S. Goldstein, 2002, Do bonds span the fixed income markets? theory and evidence for unspanned stochastic volatility, *The Journal of Finance* 57, 1685–1730.
- Collin-Dufresne, Pierre, Robert S. Goldstein, and Christopher S. Jones, 2009, Can interest rate volatility be extracted from the cross section of bond yields?, *Journal of Financial Economics* 94, 47–66.
- Corsi, Fulvio, 2009, A simple approximate long-memory model of realized volatility, *Journal of Financial Econometrics* 7, 174–196.
- Couts, Spencer J., Andrei S. Gonçalves, and Johnathan Loudis, 2023, The subjective risk and return expectations of institutional investors Working paper.
- Dahlquist, Magnus, and Markus Ibert, 2024, Equity return expectations and portfolios: Evidence from large asset managers, *The Review of Financial Studies* 37, 1887–1928.
- Drechsler, Itamar, and Amir Yaron, 2011, What’s vol got to do with it, *Review of Financial Studies* 24, 1–45.

- Duffee, Gregory R., 2002, Term premia and interest rate forecasts in affine models, *Journal of Finance* 57, 405–443.
- ECB, 2005, Box 2: The term structure of implied volatilities, *ECB Monthly Bulletin* December.
- Fama, Eugene F., and Robert R. Bliss, 1987, The information in long-maturity forward rates, *The American Economic Review* 77, 680–692.
- Farmer, Leland E., Emi Nakamura, and Jón Steinsson, 2024, Learning about the long run, *Journal of Political Economy* 132, 3334–3377.
- Fornari, Fabio, 2005, The rise and fall of us dollar interest rate volatility: evidence from swaps, *BIS Quarterly Review* September.
- Gormsen, Niels Joachim, and Eben Lazarus, 2025, Interest rates and equity valuations, *Working paper* .
- Hanson, Samuel G., 2014, Mortgage convexity, *Journal of Financial Economics* 113, 270–299.
- Hanson, Samuel G., and Jeremy C. Stein, 2015, Monetary policy and long-term real rates, *Journal of Financial Economics* 115, 429–448.
- He, Zhiguo, Bryan Kelly, and Asaf Manela, 2017, Intermediary asset pricing: New evidence from many asset classes, *Journal of Financial Economics* 126, 1–35.
- Hillenbrand, Sebastian, 2025, The fed and the secular decline in interest rates, *Review of Financial Studies* 38, 981–1013.
- ISDA, 2014, Size and use of the non-cleared derivatives market, *An ISDA study* April.
- Joslin, Scott, and Yaniv Konchitchki, 2018, Interest rate volatility, the yield curve, and the macroeconomy, *Journal of Financial Economics* 128, 344–362.

- Kekre, Rohan, Moritz Lenel, and Federico Mainardi, 2024, Monetary policy, segmentation, and the term structure NBER Working Paper No. 32324.
- Kim, Don H, and Jonathan H Wright, 2005, An arbitrage-free three-factor term structure model and the recent behavior of long-term yields and distant-horizon forward rates, Technical Report 2005-33, Board of Governors of the Federal Reserve System.
- Kozicki, Sharon, and P A Tinsley, 2001, Shifting endpoints in the term structure of interest rates, *Journal of Monetary Economics* 47, 613–652.
- Kremens, Lukas, and Ian Martin, 2019, The quanto theory of exchange rates, *American Economic Review* 109, 810–843.
- Martin, Ian, 2017, What is the expected return on the market?, *The Quarterly Journal of Economics* 132, 367–433.
- Martin, Ian, and Christian Wagner, 2019, What is the expected return on a stock?, *Journal of Finance* 74, 1887–1929.
- Martin, Ian W. R., 2025, Information in derivatives markets: Forecasting prices with prices, *Annual Review of Financial Economics* .
- Rogers, Robert, 2024, Bye bye beta: deposit duration with fixed rates, *Working paper* .
- Tetlock, Paul C., Jack McCoy, and Neel Shah, 2024, The implied equity premium Working paper.
- Trolle, Anders B., and Eduardo S. Schwartz, 2014, The swaption cube, *Review of Financial Studies* 27, 2307–2353.
- van Binsbergen, Jules, 2020, Duration-based stock valuation, CEPR Discussion Papers 14904, C.E.P.R. Discussion Papers.
- van Dijk, Dick JC, Siem Jan Koopman, Michel van der Wel, and Jonathan H Wright, 2014,

Forecasting interest rates with shifting endpoints, *Journal of Applied Econometrics* 29, 693–712.

## Figures

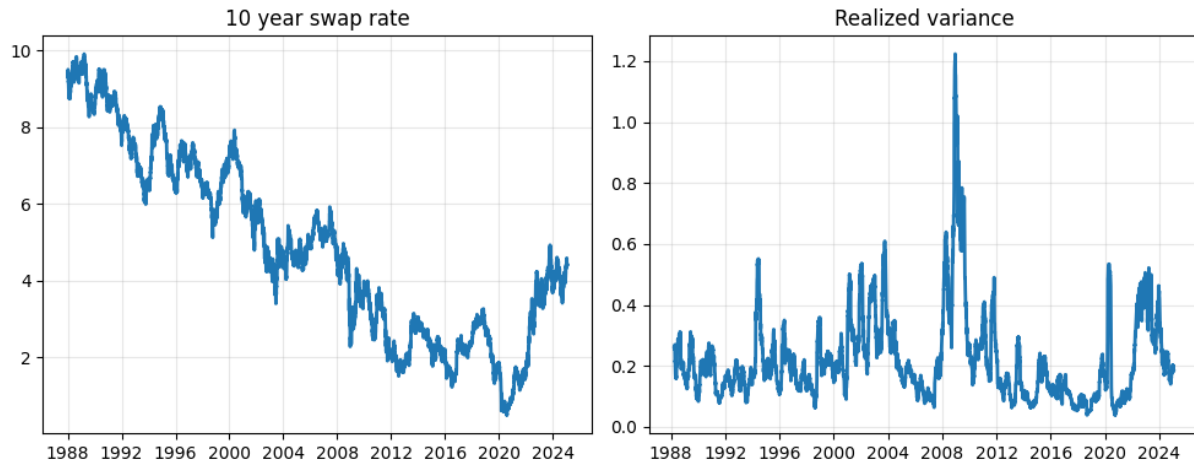


Figure 1: 10-year swap rate (left hand side) and quarterly realized variance on 10-year swap rate (right hand side), 1988-2025. The time series of interest rate levels shows strong signs of non-stationarity (e.g. Dickey Fuller test does not reject unit root), whereas the time series of variance does not (e.g. Dickey-Fuller test rejects unit root with p value  $< 0.0001$ ). Swap rates are taken from Refinitiv, and realized variance is calculated from daily data.

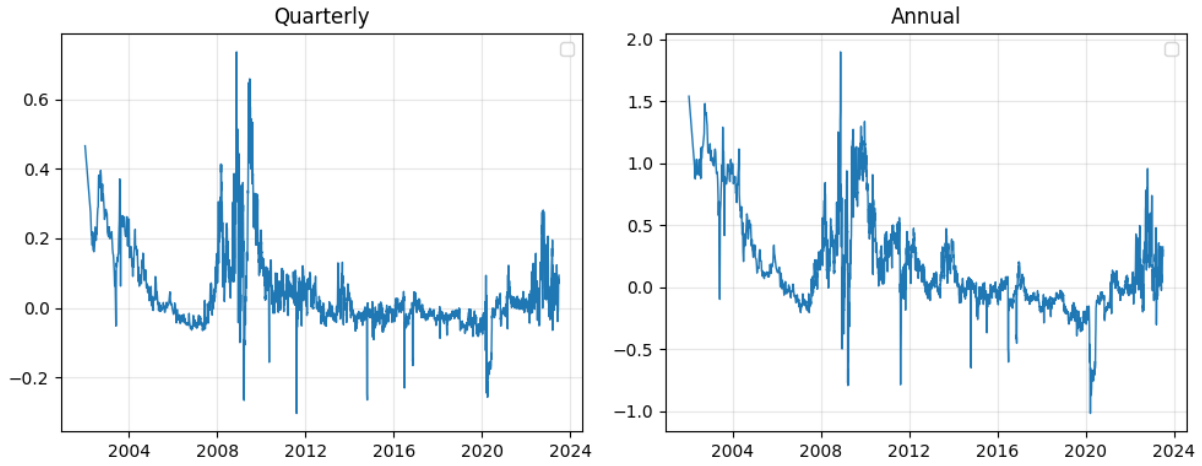


Figure 2: Conditional variance risk premium for 10-year swap rate at quarterly and annual horizons. These figures plot the difference between risk-neutral variance and forecasted physical variance, 2002–June 2023. Positive values indicate that investors are willing to pay a premium for protection against large changes in interest rates. Methodology is described in Section 3, and units are percentage point squared.

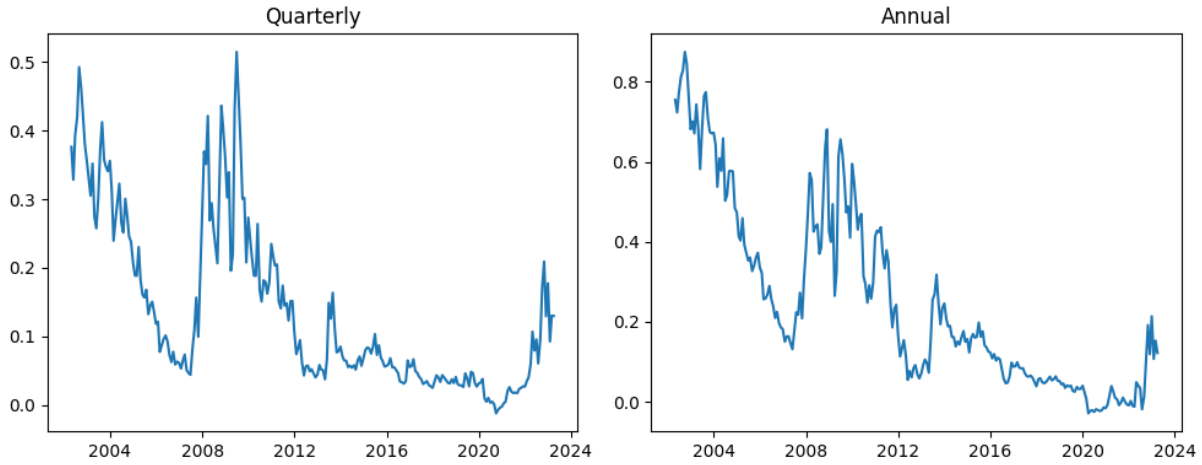


Figure 3: Variance-based interest rate risk premium estimates for 10-year swap rates. The figure displays quarterly (left hand side) and annual (right hand side) estimates of the difference between the 10y forward swap rate and the expected 10-year swap rate. Estimates are calculated using the methodology described in Section 3 and expressed in percentage points.



Figure 4: Expectations hypothesis vs variance-based forecasts of the 10-year swap rate. The black line plots the 10-year swap rate from 2002 to 2023. The light gray lines each quarter plot the 1q and 1y forward rate (i.e., the forecasts of the swap rate if there is no risk premium). The red lines plot the 1q and 1y expected interest rates calculated using the methodology from this paper. Units are percentage points

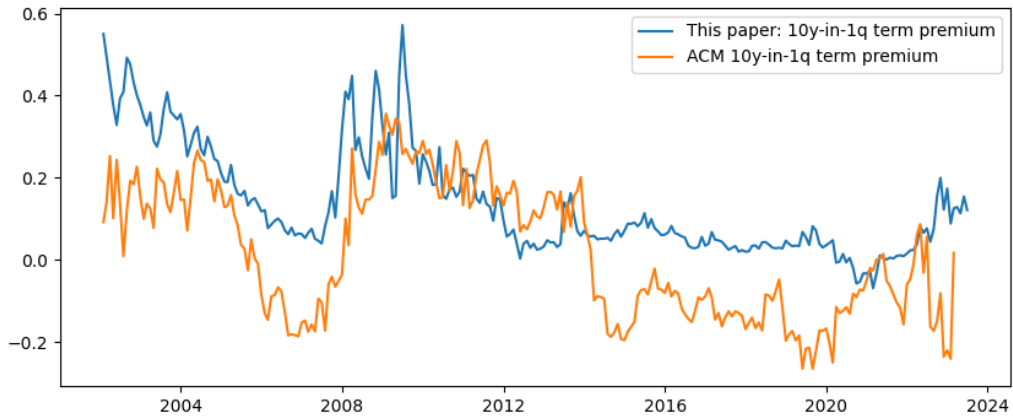


Figure 5: Out-of-sample 10y-in-1q interest rate risk premium estimates from this paper versus the [Adrian et al. \(2013\)](#) dynamic term structure model. These figures represent the difference between 10y-in-1q forward rates, and the 10y rate 1 quarter from now. Term premia are expressed in percentage points. Methodology is described in Section 3

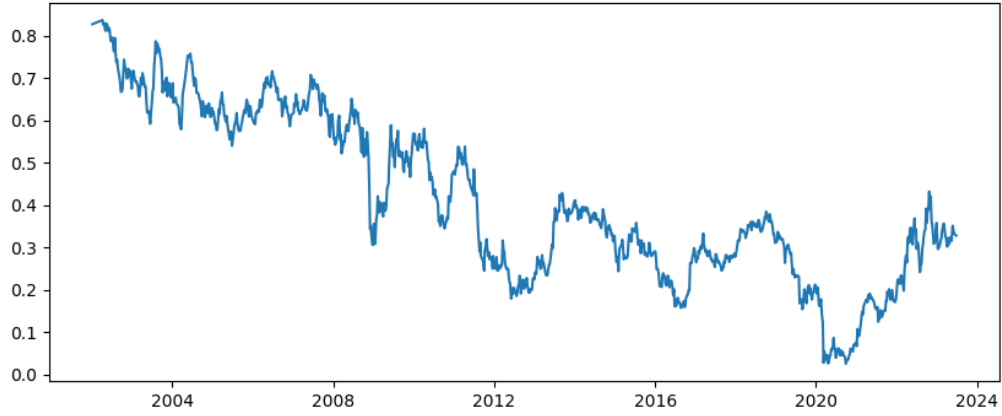


Figure 6: The investor's exposure to interest rate risk. The figure plots the estimated exposure parameter  $\lambda_t$  (blue line), which captures the sensitivity to interest rate increases of the log investor with linear interest rate exposures. The exposure parameter is estimated on the whole sample 2002-2023 using the methodology described in Section 3. Interest rate changes are measured in percentage points, so an exposure of 0.5 is consistent with a portfolio duration of 50 for the log investor, 25 for a CRRA investor with risk aversion coefficient 2, or 12.5 for an investor with CRRA coefficient 4.



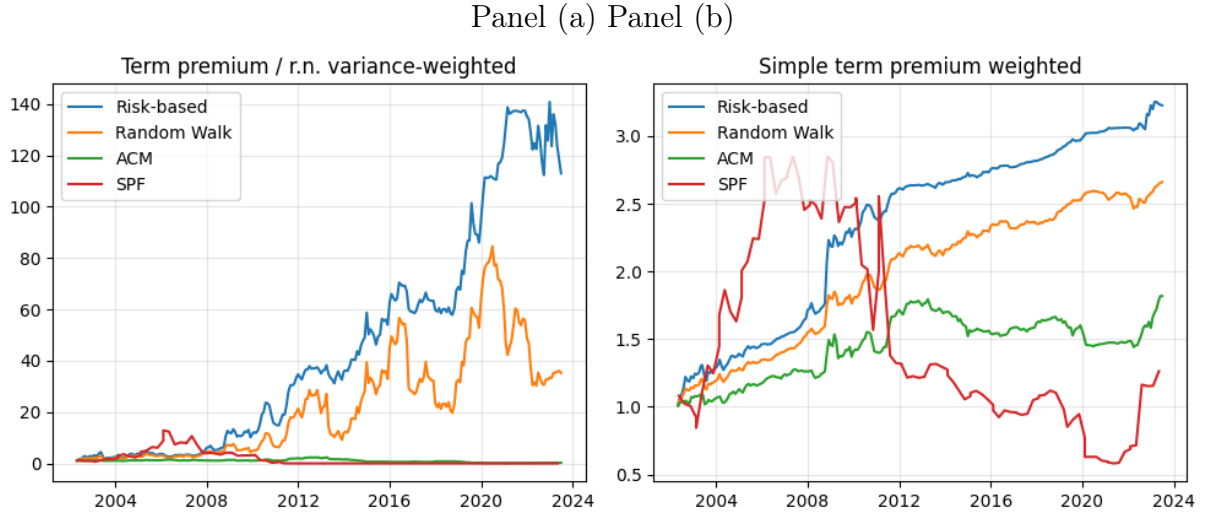


Figure 7: Cumulative returns from interest rate trading strategies. The figure shows cumulative dollar returns from strategies that take positions in 1-month interest rate forwards proportional to risk premia estimated from various models. Initial investment is normalized to \$1 in April 2002. Panel (b) uses position sizes equal to the expected monthly risk premium. Panel (a) scales positions by the inverse of risk-neutral variance, investing more aggressively when uncertainty is low, consistent with the behavior of the implied log marginal investor. Strategies are implemented using the variance-based measure, Survey of Professional Forecasters expectations, the [Adrian et al. \(2013\)](#) affine model, and a random walk assumption. Returns are calculated before transaction costs. The survey of professional forecasters strategy uses quarterly forwards instead of monthly due to the quarterly frequency of the surveys.

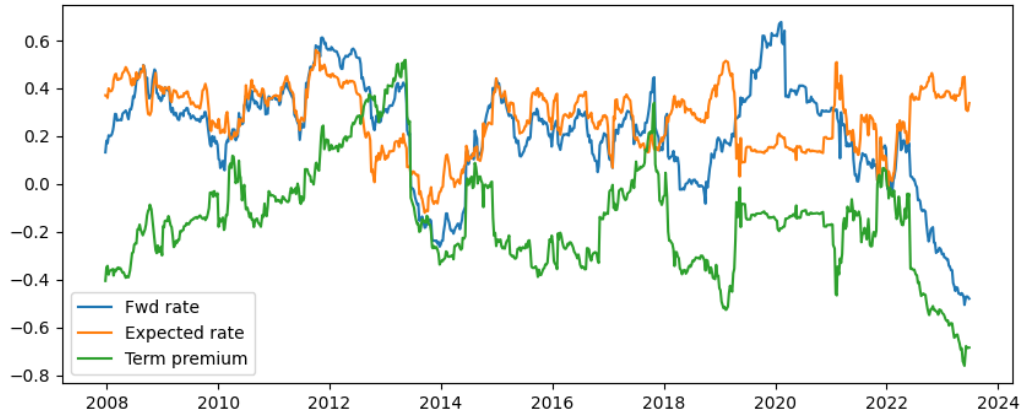


Figure 8: Rolling annual correlation of 1-week changes in 10y-in-1y forward rates, expected rates and interest rate risk premium with 1-week stock returns. Changes in risk premium are 1-week differences in the values calculated in Section 3. Market returns are the CRSP value-weighted return. 2002-2023.

## Tables

		Quarterly		Annual	
		Mean	Std Dev	Mean	Std Dev
Data	10 year rate ( $y_t$ )	3.18	1.36	3.18	1.36
	Change in 10y rate ( $\Delta y_{t+1}$ )	-0.09	0.48	-0.33	0.94
	R.n variance ( $\sigma_t^{*2}$ )	0.27	0.19	1.07	0.57
	R.n skewness ( $skew_t^*$ )	0.27	0.21	0.36	0.20
	R.n third moment ( $E_t(\Delta y_{t+1}^3)$ )	0.06	0.09	0.52	0.56
	Realized variance ( $RV_t$ )	0.23	0.18	0.92	0.59
Outputs	Conditional variance	0.22	0.10	0.89	0.29
	Variance risk prem.	0.05	0.11	0.18	0.39
	Term premium	0.13	0.13	0.25	0.23

Table 1: Summary statistics. The table reports summary means and standard deviations for the key model inputs and outputs. All statistics are quoted for January 2002 - June 2023 based on weekly data. Interest rates are reported in  $ppt$ , variances in  $ppt^2$ , and the third moment in  $ppt^3$ . Risk-neutral moments are extracted from swaption prices using the model-free approach of Carr and Madan (1998).

	EH	RW
Monthly	0.22	0.29
Quarterly	0.30	0.17
Yearly	0.26	0.33

Table 2: Variance forecasting performance vs alternative benchmarks. The table reports out-of-sample improvement in  $R^2$  values for forecasts of realized variance of 10-year swap rates using the HAR-RV model described in Section 3 compared to two benchmarks: risk-neutral variance from swaptions and lagged realized variance (random walk). Relative  $R^2$  is calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the regression-based forecaster and  $\nu_t$  is the forecast error from the alternative. The regression model regresses realized variance on lagged weekly, monthly, and quarterly realized variance. Forecasts are calculated using an expanding window starting in 1988. Sample period is 2002–2023.

	1m	1q	1y
Expected change in rates	1.8341*** (0.64)	0.9702*** (0.34)	1.4803** (0.61)
Intercept	0.0109 (0.02)	0.0362 (0.06)	0.0413 (0.25)
R2	0.0414	0.0602	0.1439
Observations	1064.0	1064.0	244.0
Frequency	Weekly	Weekly	Monthly
NW lags	5	13	12

Table 3: Interest rate risk premium forecasting regressions. The table reports results from regressions of realized changes in 10-year swap rates on predicted term premia:  $\Delta y_{t,t+h} = \alpha - \beta \times \widehat{RP}_{t,h} + \varepsilon_{t+h}$ , where  $\widehat{RP}_{t,h}$  is the variance-based interest rate risk premium estimate. Under the null hypothesis that the measure correctly captures term premia,  $\alpha = 0$  and  $\beta = 1$ . Sample period is March 2002– June 2023. Standard errors calculated using Newey-West with lags equal to the forecast horizon.

		Monthly	Quarterly	Annual
Expectations Hypothesis	R2	0.065	0.081	0.230
	p-value	0.014	0.160	0.010
Survey of Prof Forecasters	R2		0.275	0.271
	p-value		0.005	0.015
<a href="#">Adrian et al. (2013)</a>	R2	0.070	0.095	0.184
	p-value	0.003	0.077	0.203
<a href="#">Kim and Wright (2005)</a>	R2	0.100	0.150	0.497
	p-value	0.006	0.066	0.005
<a href="#">Bauer and Rudebusch (2020)</a>	R2		0.045	0.087
	p-value		0.406	0.311
Random Walk	R2	0.020	0.022	0.043
	p-value	0.073	0.347	0.287
Regression-rescaled	R2	0.049	0.136	0.216
	p-value	0.096	0.100	0.091

Table 4: Improvement in interest rate forecast performance vs alternative benchmarks. The table reports the improvement in  $R^2$  from using the variance-based interest rate risk premium measure versus alternative models. Relative  $R^2$  is calculated as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the variance-based model and  $\nu_t$  is the forecast error from the alternative. [Adrian et al. \(2013\)](#) estimates calculate the 10-year par-yield forecasts with rolling out-of-sample windows. [Kim and Wright \(2005\)](#) uses the 1-year instantaneous term premium provided by the Federal Reserve Board. The Board provides a "newer" and "older" calibration. I use the older calibration up to 2019 to ensure estimates are out of sample. [Bauer and Rudebusch \(2020\)](#) uses the 8y spot yield forecasts from the [Bauer and Rudebusch \(2020\)](#) observed-shifting-endpoints model up to 2018, calculated using the authors replication code. "Regression-rescaled" denotes forecasts made from expanding window regressions of my variance-based expectation measure on changes interest rates, instead of using the theoretically-motivated magnitude. P-values are from one-sided Diebold-Mariano tests for an improvement in forecast accuracy. To increase power without using long overlaps, I use weekly frequencies for the monthly forecast R2, and monthly frequencies for the quarterly and annual forecast. Monthly frequencies are not available for the SPF or [Bauer and Rudebusch \(2020\)](#). Sample period is March 2002–June 2023.

	Mean	Quarterly Correlation	Mean	Yearly Correlation
Realized	0.08		0.34	
Risk-based estimate	0.13		0.26	
Expectations Hypothesis	0.00		0.00	
Survey of Prof Forecasters	-0.07	0.38	-0.12	0.60
<a href="#">Adrian et al. (2013)</a>	0.02	0.65	0.06	0.74
<a href="#">Kim and Wright (2005)</a>			-0.36	0.38
<a href="#">Bauer and Rudebusch (2020)</a>	-0.06	-0.12	0.08	0.10
Random Walk	0.07	0.63	0.29	0.74

Table 5: Comparison of interest rate risk premium estimates. The table reports average term premia from different models for 10-year swap rates, and their correlation with the variance-based estimates from this paper. The left two columns show the results for quarterly forecasts, and the right two columns for annual forecasts. Realized shows the ex post change in rates vs forward-implied rates. variance-based uses the variance risk premium method developed in this paper. Other models are as described in Figure 4. Monthly data from 2002–2023. All values in percentage points per period.

	(1) Constant	(2) PCs only	(3) Main	(4) $\Delta$ Agg duration	(5) Equity $\beta$	(6): 3 & 4
Intercept	0.51 (0.04)	-0.06 (0.07)	-0.02 (0.11)	0.50 (0.04)	0.61 (0.05)	0.62 (0.04)
PC 1		0.04 (0.01)	0.04 (0.01)			
PC 2		-0.06 (0.01)	-0.06 (0.02)			
PC 3		0.04 (0.03)	0.04 (0.03)			
Var			0.01 (0.09)			
Skew			-0.07 (0.25)			
$\Delta$ Dur				0.22 (0.26)		0.42 (0.24)
Equity $\beta$					-0.80 (0.28)	-0.97 (0.31)
R2	0.54	0.65	0.65	0.54	0.56	0.57

Table 6: Determinants of marginal investor exposure. The table reports coefficients from nonlinear least squares regressions where the exposure parameter  $\lambda_t$  is specified as a linear function of economic variables.  $\lambda_t$  captures the marginal investor's sensitivity to interest rate increases. PC1, PC2, and PC3 are the first three principal components of the yield curve. Skew is the risk-neutral quarterly skewness of the 10-year rate.  $\Delta$ Dur is log changes in aggregate bond duration (from the Bloomberg Agg index) to GDP. Equity beta is sensitivity of S&P 500 returns to 10-year rate changes (calculated using physical 1-year correlation and risk-neutral volatilities). Quarterly data from 2002–2023. Standard errors in parentheses calculated using GMM with Newey-West corrections with four lags.

Model	Residual Physical Covariance
Log equity investor	−0.004
Fama-French 3 factor	−0.001
Dynamic term structure model	−0.001

Table 7: Estimations of average physical quarterly residual coskew under different models of the SDF. Residual coskew is the covariance of squared changes in yields with the residual portion of the inverse SDF left after regressing on changes in yields. See Section 5 for details.



	<i>Dependent variable: VRP</i>					
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	-0.010 (0.010)	-0.008 (0.010)	0.044 (0.032)	-0.008 (0.009)	-0.069** (0.031)	-0.018 (0.060)
3rd moment	0.893*** (0.094)	1.013*** (0.148)	0.968*** (0.098)	0.862*** (0.095)	0.941*** (0.089)	0.966*** (0.117)
Resid coskew		3.360 (3.161)				0.728 (2.465)
Equity vol			-0.306* (0.179)			-0.159 (0.220)
Bond convexity				-5.859* (3.016)		-4.388 (3.280)
Int. capital					0.840** (0.402)	0.538 (0.477)
Observations	81	81	81	81	81	81
$R^2$	0.677	0.689	0.700	0.697	0.709	0.726
Adjusted $R^2$	0.673	0.681	0.692	0.689	0.701	0.708
Resid Std. Error	0.067	0.066	0.065	0.065	0.064	0.063
F Statistic	91.077***	43.311***	49.473***	47.405***	56.905***	29.394***

Note:

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Table 8: Measures associated with residual coskew do not seem to explain the variance risk premium well. This table shows the results from regressing estimated conditional variance risk premium onto the risk-neutral third moment alone (column 1) or the risk-neutral third moment and other possible explanatory variables (columns 2-6). VRP being fully explained by the third moment is consistent with the mechanism in this paper by which VRP is a result of directional interest rate risk. The other variables could potentially be associated with non-linear interest rate risk. The residual coskew estimate uses the log-equity investor model, calculated on a rolling 1y basis (see Section 5 for more detail). risk-neutral equity vol uses the  $VIX$  index rescaled by a regression onto the  $SVIX$  index from [Martin \(2017\)](#). Bond convexity is the convexity of the Bloomberg Barclays Aggregate Index. Intermediary capital is from [He et al. \(2017\)](#). Data is quarterly and standard errors in parentheses calculated using Newey-West with 4 lags. Sample period is 2002–2023.

	Quarter	Year	5 Year
1Y	-2.5% (3.4%)	-4.1% (1.3%)	-1.6% (0.6%)
2Y	-4.3% (1.7%)	-3.4% (1.3%)	-1.8% (0.5%)
5Y	-3.2% (1.3%)	-2.8% (0.8%)	-1.5% (0.6%)
10Y	-2.6% (1.0%)	-2.3% (0.7%)	-1.4% (0.6%)
20Y	-2.6% (1.1%)	-2.5% (0.7%)	-1.4% (0.6%)
30Y	-2.9% (1.0%)	-2.9% (0.8%)	-1.2% (0.6%)

Table 9: Risk-neutral variance changes during FOMC windows. The table reports the average proportional decline in risk-neutral variance during 3-day windows around FOMC meetings for different swap rate tenors (rows) and swaption maturities (columns). Changes are calculated from close on day  $t - 1$  to close on day  $t + 1$  where day  $t$  is the FOMC announcement. Sample period is 2007–2023. Standard errors in parentheses are clustered by FOMC announcement date.

	Risk based (1)	Risk based (2)	<a href="#">Kim and Wright (2005)</a>
Daily	-0.011 (0.0087)	-0.088 (0.032)	0.29 (0.025)
Weekly	-0.058 (0.019)	-0.052 (0.05)	0.2 (0.037)
Monthly	-0.092 (0.034)	-0.22 (0.075)	0.18 (0.052)

Table 10: Correlation of different measures of changes in interest rate risk premium with market returns. The first column calculates simple changes in 10y-in-1y risk premium as calculated in Section 3. The second column weakens the functional form assumption for lambda by calculating changes in 10y-in-1y risk premium using differential approximation from Section 4.3. The third column uses the 1y instantaneous out-of-sample term premium from [Kim and Wright \(2005\)](#). Horizon for changes and market returns are given by the rows. Market returns are the CRSP value-weighted return. Sample period starts in 2002 for weekly or monthly horizons or 2007 for the daily horizon. Column (2) uses data ending in 2018 due to issues with some days zero denominators in the differential approximation from 2019 to 2020. Standard errors in parenthesis with Newey West standard errors.

# Appendices

## A Forecasting other interest rate tenors

Table 11 below reports the relative improvement in forecasting performance by applying this same methodology to the 1-year, 2-year, 5-year, 20-year, and 30-year swap yield. As a simple approximation I assume the value of  $\lambda_t$  for all tenors is the same as the values calculated in the main body of this paper.

Recalculating different values of  $\lambda_t$  for each tenor based on their respective variance risk premia can be done. However, there are some challenges in estimating physical variance for shorter tenors because the rates are at the zero lower bound for much of the sample and more subject to jumps around policy announcements.

Comparison model	Tenor	Monthly	Quarterly	Annual
Expectations Hypothesis	1	0.052	0.112	0.154
	2	0.063	0.119	0.185
	5	0.068	0.102	0.229
	10	0.065	0.081	0.230
	20	0.066	0.042	0.191
	30	0.072	0.022	0.174
<a href="#">Adrian et al. (2013)</a>	1	0.126	0.228	0.153
	2	0.077	0.141	0.170
	5	0.060	0.096	0.162
	10	0.070	0.095	0.184

Table 11: Multi-tenor out-of-sample forecast performance. The table reports the improvement in  $R^2$  from variance-based interest rate expectation measure versus the expectations hypothesis and the [Adrian et al. \(2013\)](#) DTSM. Improvement in  $R^2$  is defined as  $1 - \sum \varepsilon_t^2 / \sum \nu_t^2$ , where  $\varepsilon_t$  is the forecast error from the variance-based model and  $\nu_t$  is the forecast error from the alternative. 2002-2023, monthly data.

## B Duration and exposure for a CRRA fixed-income investor

CRRA utility with coefficient  $\gamma$  implies:

$$\frac{1}{M} = (R_f(1 - D\Delta y))^\gamma$$

Where I will ignoring the time subscripts throughout in this subsection

D is here the “duration” of the CRRA investor’s portfolio. Assume that  $\gamma$  is a positive integer. Since  $\lambda$  is the projection coefficient of the inverse SDF on yields, by binomial expansion we have:

$$\lambda = -\frac{1}{R_f} \frac{\text{cov}^*(\frac{1}{M}, \Delta y)}{\sigma^{*2}} = \sum_{k=1}^{\gamma} \binom{\gamma}{k} R_f^{k-1} D^k \frac{\mu_{k+1}^*}{\sigma^{*2}}$$

Where  $\mu_k^*$  is the kth risk-neutral central moment of  $y$ .

The first term in this expansion is  $\gamma R_f^\gamma D \approx \gamma D$ . To demonstrate that the other terms are small, Table 12 calculates the duration implied by a  $\lambda$  of 0.4 under different values of  $\gamma$  using the time series average levels of the risk-free rate and the first four quarterly risk-neutral moments after 2011. Here I am using interest rates in percentage points terms, so a lambda of 0.4 is equivalent to a duration of 40 if  $\gamma = 1$ . The approximation  $D \approx \frac{\lambda}{\gamma}$  is never off by more than 4%.

Repeating the exercise for every day from 2011 – 2023 gives a highest absolute approximation error of 8% for  $\gamma = 4$ .

CRRA Coefficient	Implied Duration	Approximation error	Note
1	0.400	0.0%	
2	0.196	2.0%	
3	0.129	3.1%	
4	0.096	3.7%	Assuming 5th moment is 0

Table 12: Accuracy of the Duration  $= \frac{\lambda}{\gamma}$  approximation. Each row provides the duration that would deliver  $\lambda = 0.4$  for a CRRA investor with various levels of risk aversion and with linear exposure to the interest rate  $y_t$ . The first 4 quarterly risk-neutral moments are taken as their time series average 2011-2023, and the 5th moment is assumed to be 0. Approximation error shows the percentage difference between the calculated duration and the simple approximation  $D \approx \frac{\lambda}{\gamma}$

## C Constructing risk-neutral interest rate moments from swaptions

This section describes how I approximate the risk-neutral moments of swap yields from swaption data. Swaptions are not precisely options on the interest rate. I therefore need to employ a simple approximation. I assume that changes in the annuity yield are the same as changes in the swap yield. This approximation is unlikely to lead to substantive errors — changes in the 10-year annuity yield and swap yield are >99% correlated at the quarterly or annual horizon.

### Swaps and swaption prices

By a standard result, we can write the value of a swap agreed at fixed rate  $K$  with tenor  $T$  as  $(y - K)A$  where  $A$  is the price of the  $T$ -period annuity, and  $y$  is the swap rate (i.e., the rate that sets the value of the swap to 0).

The payoff of a swaption agreed at rate  $K$  is therefore

- Pay-fixed:  $\text{Max} \{(y_{t+j} - K) A_{t+j}, 0\}$
- Receive-fixed:  $\text{Max} \{(K - y_{t+j}) A_{t+j}, 0\}$

And the forward price of the swaption can be written:

- Pay-fixed:  $C(k) = E_t^*(\text{Max}\{(y_{t+j} - K) A_{t+j}, 0\})$
- Receive-fixed:  $P(k) = E_t(\text{Max}\{(K - y_{t+j}) A_{t+j}, 0\})$

If we change measure to the T-tenor “annuity measure” we can write these swaption prices as options directly on the swap rate:

- Pay-fixed:  $C(k) = E_t^A(\text{Max}\{(y_{t+j} - K), 0\})$
- Receive-fixed:  $P(k) = E_t^A(\text{Max}\{(K - y_{t+j}), 0\})$

### Applying Breeden & Litzenberger

Applying [Breeden and Litzenberger \(1978\)](#), we can write the expectation of any function of the swap rate under this annuity measure as:

$$E^A(g(y)) = \left( g(y^f) + \int_{-\infty}^{y^f} g''(y) P(k) dk + \int_{y^f}^{\infty} g''(y) C(k) dk \right)$$

Where  $y^f$  is the swap forward rate.

However, we want the moments under the risk-neutral measure, not the annuity measure. So I will assume that changes in the annuity yield vs its forward yield are always identical to changes in the swap yield vs its forward yield. The annuity price then becomes a function of the swap yield:

$$A = \sum_{t=1}^{4T} \frac{1}{(1+y)^{t/4}} = A(y)$$

Now we can find the risk-neutral expectation of any arbitrary function  $f(y)$  by letting:

$$g(y) = \frac{E^*(A)}{A(y)} f(y)$$

The annuity measure is defined such that for any  $X$ :

$$E_t^A \left( \frac{E^*(A)}{A(y)} X \right) = E^*(X)$$

And hence we have

$$E^A(g(y)) = E^*(f(y))$$

### Calculating the moments

For this paper I need to estimate  $E^*(f(y))$  for  $f(y) = y$ ,  $f(y) = (y - E_t^*(y))^2$ , and  $f(y) = (y - E_t^*(y))^3$ .

In each case, I need to first find the second derivative of the function:

$$g(y) = \frac{E^*(A)}{A(y)} f(y)$$

I then use my data on the forward prices of payer and receiver swaptions to calculate, using the standard:

$$E^*(f(y)) = \left( g(y^f) + \int_{-\infty}^{y^f} g''(y) P(k) dk + \int_{y^f}^{\infty} g''(y) C(k) dk \right)$$

For yields between the observed swaption strikes, I calculate the price using the Bachelier (normal) option pricing formula, as is standard in fixed-income markets.

## D Risk-neutral moment time series

Figure 9 plots the time series of risk neutral variance and skewness 2002-2023.

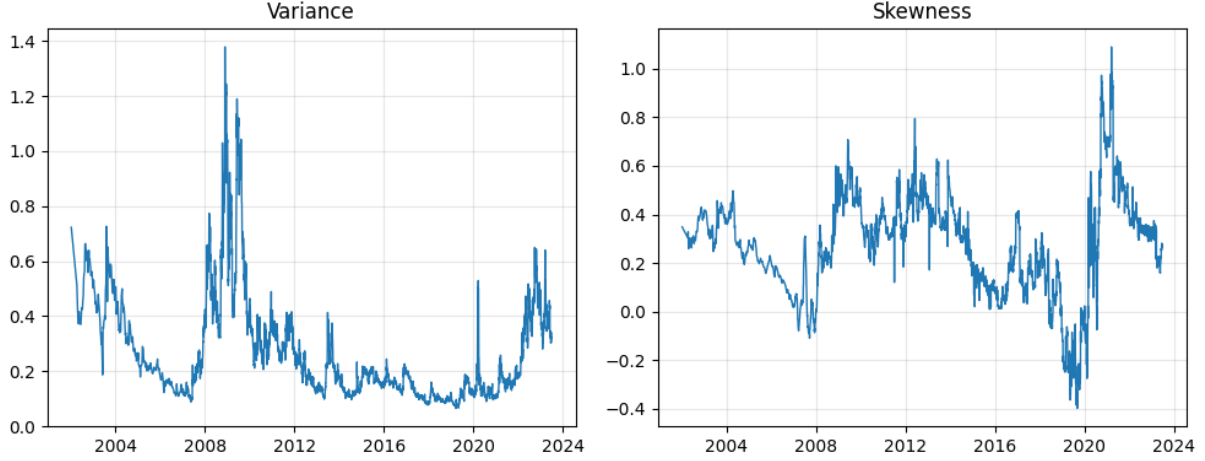


Figure 9: Quarterly risk-neutral variance (left) and skewness (right) of 10-year swap rates, 2002–2023. Variance and skewness are extracted from swaption prices, using the model free approach approach of Carr and Madan (1998). Variance is expressed in percentage points squared.

## E Interest rate risk premium with non-parametric form for $\lambda_t$

Figure 10 compares interest rate risk premium estimated from the parametric specification for  $\lambda_t$ , described in Section 3, with interest rate risk premium estimated by solving for  $\lambda$  separately each period, without a functional form. Since two roots are possible for lambda, I always choose the root with the minimum magnitude of the interest rate risk premium. Confidence intervals are provided for each period for the non-parametric version. Where no value of  $\lambda_t$  was consistent with the measured variance risk premium, I take the closest value. If this implies a value for the variance risk premium that is outside of its 95% confidence interval of the variance-prediction regression, I plot this observation with a dotted line.



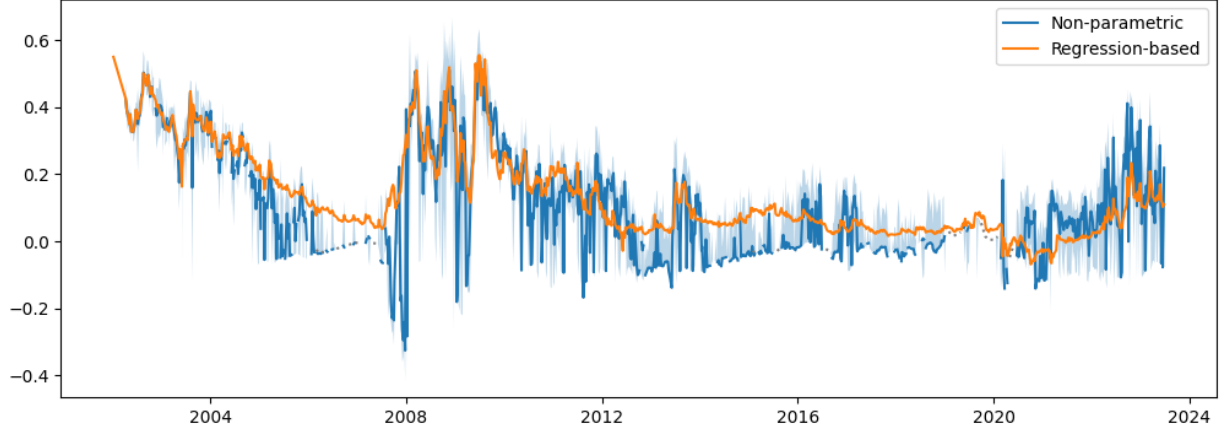


Figure 10: The left hand plot shows interest rate risk premium estimated from the parametric specification for  $\lambda_t$ . The right hand plot shows interest rate risk premium estimated by solving for  $\lambda$  separately each period. Quarterly forecasts in ppt, 2002-2023.

## F Additional content on the residual coskew

### F.1 Risk aversion

If the investor has CRRA utility, then we can write the inverse SDF as:

$$\frac{1}{M} = (R_f(1 - D\Delta y))^\gamma$$

where I will ignoring the time subscripts throughout in this subsection. D is here the “duration” of the CRRA investor’s portfolio.

With a positive integer risk aversion coefficient ( $\gamma$ ), applying the binomial expansion formula we can write the residual coskew as:

$$cov^*(\varepsilon, \Delta y^2) = cov^*\left(\frac{1}{M} - \lambda\Delta y, \Delta y^2\right) = \sum_{k=1}^{\gamma} \binom{\gamma}{k} R_f^{k-1} D^k \frac{\mu_{k+1}^*}{\sigma^{*2}}$$

And the residual covariance as:

To see how large an effect this term has on the estimated term premium, I repeating the constant- $\lambda$  estimation of term premium from Section 3.1 for a CRRA investor. I estimate the constant duration that best explains the observed variance risk premium, allowing for the residual coskew above. I use the same non-linear least squares method as in the main body.

The table below shows the resulting duration and term premium estimates for  $\gamma = 1, 2, 3$ , and 4.  $\gamma = 1$  represents the log-investor case with no residual coskew. The term premium for higher levels of risk aversion is higher, but 99% correlated with the zero-residual-coskew estimate. The zero-residual-coskew (i.e.  $\gamma = 1$ ) estimate underestimates true values by roughly 30% for higher risk aversion parameters. For the  $\gamma = 4$  calculation I have assumed the 5th moment is 0 (as in a normal distribution).

CRRA coefficient	Est. duration	Est. risk prem	Corr w/ log case RP
1	0.413	0.444	1.000
2	0.274	0.607	1.000
3	0.183	0.625	0.999
4	0.134	0.621	0.998

Table 13: Estimated duration and term premium for a linear-interest-rate-exposure investor with CRRA utility with different levels of risk aversion. The first column denotes the risk aversion coefficient. The second column shows the corresponding estimate of the constant duration that best explains the observed variance risk premium. The third column gives the annualized risk premium corresponding to that duration. The fourth column shows the correlation of the risk premium with the zero-residual-coskew (i.e.  $\gamma = 1$ ) estimate.

Intuitively, two investors can generate the same interest rate risk premium through different combinations of risk aversion and leverage. A less risk-averse investor needs high leverage (large  $D$ , small  $\gamma$ ), while a more risk-averse investor needs less leverage (small  $D$ , large  $\gamma$ ). But the highly levered investor fears far-away states more—a 5 percentage point rate move could wipe her out, while merely denting the un-leveraged investor’s wealth. The levered investor therefore pays more for variance protection despite having the same directional exposure.

A brief proof is provided below

*Proof.* We need to show that  $cov_t^*(\varepsilon_{t+1}, (\Delta y_{t+1})^2) > 0$  where  $\varepsilon_{t+1} = \frac{1}{M_{t+1}} - \lambda_t \Delta y_{t+1}$ , with  $\frac{1}{M_{t+1}} = (R_{f,t+1} + D\Delta y_{t+1})^\gamma$  for  $\gamma > 1$  and  $E_t^*[\Delta y_{t+1}] = 0$ .

First, expand  $\frac{1}{M_{t+1}}$  around  $\Delta y_{t+1} = 0$ :

$$\begin{aligned} \frac{1}{M_{t+1}} &= R_{f,t+1} + \gamma R_{f,t+1}^{\gamma-1} D \Delta y_{t+1} + \frac{\gamma(\gamma-1)}{2} R_{f,t+1}^{\gamma-2} (D \Delta y_{t+1})^2 + \\ &\quad \frac{\gamma(\gamma-1)(\gamma-2)}{6} R_{f,t+1}^{\gamma-2} (D \Delta y_{t+1})^3 + O((\Delta y_{t+1})^4) \end{aligned}$$

Since  $E_t^*[\Delta y_{t+1}] = 0$  and  $\lambda_t = \frac{cov_t^*\left(\frac{1}{M_{t+1}}, \Delta y_{t+1}\right)}{\sigma_t^{*2}}$ , we have:

$$\lambda_t = \gamma R_{f,t+1}^{\gamma-1} D + \frac{\gamma(\gamma-1)}{2} R_{f,t+1}^{\gamma-2} D^2 \frac{E_t^*[\Delta y_{t+1}^3]}{\sigma_t^{*2}} + O(E_t^*[\Delta y_{t+1}^2])$$

The covariance of interest is:

$$cov_t^*(\varepsilon_{t+1}, \Delta y_{t+1}^2) = cov_t^*\left(\frac{1}{M_{t+1}}, \Delta y_{t+1}^2\right) - \lambda_t E_t^*[\Delta y_{t+1}^3]$$

Computing the first term using the Taylor expansion:

$$cov_t^*\left(\frac{1}{M_{t+1}}, \Delta y_{t+1}^2\right) = \gamma R_{f,t+1}^{\gamma-1} D E_t^*[\Delta y_{t+1}^3] + \frac{\gamma(\gamma-1)}{2} R_{f,t+1}^{\gamma-2} D^2 var_t^*[\Delta y_{t+1}^2] + O(E_t^*[\Delta y_{t+1}^4])$$

Substituting the expression for  $\lambda_t$  and simplifying, the leading-order term becomes:

$$cov_t^*(\varepsilon_{t+1}, (\Delta y_{t+1})^2) = \frac{\gamma(\gamma-1)}{2} R_{f,t+1}^{\gamma-2} D^2 \left( var_t^*[\Delta y_{t+1}^2] - \frac{(E_t^*[\Delta y_{t+1}^3])^2}{\sigma_t^{*2}} \right) + O(E_t^*[\Delta y_{t+1}^4]) \quad (12)$$

By the Cauchy-Schwarz inequality:

$$(E_t^*[\Delta y_{t+1}^3])^2 \leq \sigma_t^{*2} \cdot E_t^*[\Delta y_{t+1}^4] \quad (13)$$

Therefore,  $var_t^*[\Delta y_{t+1}^2] - \frac{(E_t^*[\Delta y_{t+1}^3])^2}{\sigma_t^{*2}} > 0$  unless  $\Delta y_{t+1}$  is degenerate. Since  $\gamma > 1$  implies  $\gamma(\gamma - 1) > 0$ , we conclude that  $cov_t^*(\varepsilon_{t+1}, \Delta y_{t+1}^2) > 0$ .  $\square$

## F.2 Bond market convexity

Bonds with embedded options, particularly mortgage-backed securities, can exhibit negative convexity. When rates fall, prepayments accelerate and duration shrinks; when rates rise, prepayments slow and duration extends. Could this convexity explain the variance risk premium?

For a log investor holding a portfolio with duration  $D$  and convexity  $C$ , the inverse SDF becomes:

$$\frac{1}{M_{t+1}} = R_{f,t+1} - D\Delta y_{t+1} + \frac{1}{2}C(\Delta y_{t+1})^2$$

The residual covariance term equals:

$$E_t^*(\Delta y_{t+1}^2 \varepsilon_{t+1}) = \frac{1}{2}C \times \text{var}_t^*(\Delta y_{t+1}^2)$$

Using the relationship  $\text{var}_t^*(\Delta y_{t+1}^2) = (\text{kurt}_t^*(\Delta y_{t+1}) - 1)\sigma_t^{*4}$ , and noting that risk-neutral kurtosis averages 4.5 from 2011-2023 while quarterly risk-neutral variance squared averages 0.1, the convexity needed to generate the observed variance risk premium would be:

$$C = 2 \frac{-E(\text{VRP})}{E((\text{kurt}^*(\Delta y) - 1)\sigma^{*4})} \approx 2 \frac{-0.04}{3.5 \times 0.1} = -0.22$$

To put this in perspective, the Bloomberg Barclays Aggregate Bond Index recorded its most negative convexity at -0.005 in 2004 and most positive convexity at 0.006. A log

investor would therefore need negative convexity equivalent to a 44-times levered position in the aggregate bond portfolio at its moment of peak negative convexity. This is implausible for a reasonable marginal investor.

## G Testing for information effects in risk-neutral variance

While the magnitudes suggest risk premium changes, I conduct three tests to evaluate whether information effects could explain the patterns.

### G.1 Mechanical variance effects

FOMC meetings are high-variance events. Once a meeting passes, forward-looking variance mechanically declines by removing this event from the forecast window. Could this explain the observed patterns?

The mechanical effect is far too small. Three-day FOMC-window variance for daily 10-year rate changes is 37% higher than non-FOMC periods. Passing through an FOMC window should therefore reduce:

- Quarterly variance by  $0.37 \times \frac{3}{61} = 1.8\%$
- Annual variance by  $0.37 \times \frac{3}{250} = 0.4\%$
- Five-year variance by less than 0.1%

I observe declines of 2.9% for annual variance—more than seven times the mechanical effect.

Moreover, the cross-sectional pattern contradicts the mechanical story. FOMC-day variance is twice as high for 1-year rates as for 10-year rates, yet the proportional variance declines are similar across tenors. The risk-neutral variance of rates five years forward falls substantially, which mechanical effects cannot explain.

## G.2 Size of changes

For forward rates, the FOMC-window decline equals the total decline, consistent with gradual learning. But for variance, the cumulative FOMC effect dwarfs the total change.

For five-year risk-neutral variance on 10-year rates:

- Cumulative FOMC-window decline: -12.2 percentage points squared
- Net change 2007-2023: +1.2 percentage points squared
- Implied between-meeting increase: +13.4 percentage points squared

If markets were learning about future volatility, they would need to systematically “over-learn” during FOMC meetings that volatility will be low, then receive opposite information between every meeting. This asymmetric updating pattern seems implausible for rational learning but fits naturally with temporary risk premium compression.

## G.3 Forecasting ability

If FOMC meetings reveal information about future volatility, then FOMC-window changes in risk-neutral variance should predict realized variance. I test:

$$\sigma_{RV,t}^2 - \hat{\sigma}_{t-1}^2 = \beta \Delta \sigma_t^{*2, \text{FOMC}} + \varepsilon_t \quad (14)$$

where  $\sigma_{RV,t}^2$  is subsequently realized variance and  $\hat{\sigma}_{t-1}^2$  is the pre-FOMC forecast from Section 3.

Table 14 shows the results. The  $R^2$  is below 0.01 for both quarterly and annual horizons. The coefficient is insignificantly different from zero and, for annual horizons, significantly different from one. FOMC-window variance changes contain essentially no information about future realized variance.

	Quarterly	Yearly
const	0.013 (0.021)	0.070 (0.124)
$\Delta\sigma^{*2}$	0.232 (0.617)	0.239 (0.311)
R-squared	0.006	0.003
R-squared Adj.	-0.002	-0.004
N	137	137

Table 14: Information content of FOMC variance changes. The table reports regressions of realized variance forecast errors on FOMC-window changes in risk-neutral variance:  $\sigma_{RV,t}^2 - \hat{\sigma}_{t-1}^2 = \alpha + \beta \Delta\sigma_{FOMC,t}^{*2} + \varepsilon_t$ . If FOMC variance changes reflect information about future volatility,  $\beta$  should equal one.  $\sigma_{RV,t}^2$  is subsequently realized variance,  $\hat{\sigma}_{t-1}^2$  is the pre-FOMC HAR-RV forecast, and  $\Delta\sigma_{FOMC,t}^{*2}$  is the change in risk-neutral variance during the FOMC window. Standard errors calculated using Newey-West with 2 lags (quarterly) and 8 lags (annual). Sample includes 137 FOMC meetings from 2007–2023.