第 8 回 リメディアル数学（化学システム工学科）2023/6/14 略解

問題 1

(1) $\displaystyle\int x^2 \log x \; dx = \int \left(\frac{1}{3}x^3\right)' \log x \; dx$
$\displaystyle= \frac{1}{3}x^3 \log x - \int \frac{1}{3}x^3 (\log x)' \; dx$
$\displaystyle= \frac{1}{3}x^3 \log x - \frac{1}{3}\int x^2 \; dx$
$\displaystyle= \frac{1}{3}x^3 \log x - \frac{1}{9}x^3 + C.$

(2) $\displaystyle\int x \sin x \; dx = \int x(-\cos x)' \; dx$
$\displaystyle= -x\cos x + \int (x)' \cos x \; dx$
$\displaystyle= -x\cos x + \int \cos x \; dx$
$= -x\cos x + \sin x + C.$

(3) $\displaystyle\int x \sin 2x \; dx = \int x \left(-\frac{1}{2}\cos 2x\right)' \; dx$
$\displaystyle= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int (x)' \cos 2x \; dx$
$\displaystyle= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x \; dx$
$\displaystyle= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C.$

(4) $\displaystyle\int x^2 \cos x \; dx = \int x^2 (\sin x)' \; dx$
$\displaystyle= x^2 \sin x - \int (x^2)' \sin x \; dx$
$\displaystyle= x^2 \sin x - 2\int x \sin x \; dx$
$= x^2 \sin x + 2x\cos x - 2\sin x + C \quad (\because (2)).$

(5) $\displaystyle\int xe^x \; dx = \int x(e^x)' \; dx$
$\displaystyle= xe^x - \int (x)' e^x \; dx$
$\displaystyle= xe^x - \int e^x \; dx$
$= xe^x - e^x + C.$

(6) $\displaystyle\int \log x \; dx = \int (x)' \log x \; dx$
$\displaystyle= x\log x - \int x(\log x)' \; dx$
$\displaystyle= x\log x - \int dx$
$= x\log x - x + C.$

問題 2

(1) $\displaystyle\int_0^{\frac{\pi}{2}} x\cos x \; dx = \int_0^{\frac{\pi}{2}} x(\sin x)' \; dx$
$\displaystyle= [x\sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \; dx$
$\displaystyle= \frac{\pi}{2} - [-\cos x]_0^{\frac{\pi}{2}}$
$\displaystyle= \frac{\pi}{2} - 1$

(2) $\displaystyle\int_0^1 xe^{-x} \; dx = \int_0^1 x(-e^{-x})' \; dx$
$\displaystyle= \left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x} \; dx$
$= -e^{-1} + \left[-e^{-x}\right]_0^1$
$= 1 - 2e^{-1}$

(3) $\displaystyle\int_1^e x\log x \; dx = \int_1^e \left(\frac{1}{2}x^2\right)' \log x \; dx$
$\displaystyle= \left[\frac{1}{2}x^2 \log x\right]_1^e - \frac{1}{2}\int_1^e x \; dx$
$\displaystyle= \frac{1}{2}e^2 - \left[\frac{1}{4}x^2\right]_1^e$
$\displaystyle= \frac{1}{4}(e^2 + 1)$

(4) $\displaystyle\int_1^e (\log x)^2 \; dx = \int_1^e (x)'(\log x)^2 \; dx$
$\displaystyle= \left[x(\log x)^2\right]_1^e - \int_1^e x \cdot \frac{2\log x}{x} \; dx$
$\displaystyle= e - 2\int_1^e \log x \; dx$
$= e - 2\left[x\log x - x\right]_1^e \qquad (\because$ 問題 1 (5))
$= e - 2$

問題 3

$\displaystyle I = \int_0^{\frac{\pi}{2}} e^x \sin x \; dx = \int_0^{\frac{\pi}{2}} (e^x)' \sin x \; dx$
$\displaystyle= [e^x \sin x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x \; dx$
$= e^{\frac{\pi}{2}} - J$

よって $I + J = e^{\frac{\pi}{2}} \cdots$ ①.

一方で, $\displaystyle I = \int_0^{\frac{\pi}{2}} e^x \sin x \; dx = \int_0^{\frac{\pi}{2}} e^x(-\cos x)' \; dx$
$\displaystyle= [-e^x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x \cos x \; dx$
$= 1 + J$

よって $I - J = 1 \cdots$ ②.

①, ②より, $I = \dfrac{1}{2}(e^{\frac{\pi}{2}} + 1)$, $J = \dfrac{1}{2}(e^{\frac{\pi}{2}} - 1)$.

問題 4

以下では, ⓝと書いて, 行列の n 行目を表すものとする.

(1) $\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$

$\xrightarrow{①\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$

$\xrightarrow{③+①\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$

$\xrightarrow{②\leftrightarrow③} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \end{array}\right)$

$\xrightarrow{③+②\times1} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$

$\xrightarrow{①+③\times1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$

よって, $A^{-1} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

(2) $\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 4 & 4 & 0 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array}\right)$

$$\xrightarrow{\substack{\text{②}+\text{①}\times(-2) \\ \text{③}+\text{①}\times(-1)}} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\text{②}\leftrightarrow\text{③}} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{\text{①}+\text{②}\times(-2)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{\text{③}\times\frac{1}{2}} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array}\right)$$

$$\xrightarrow{\substack{\text{①}+\text{③}\times 1 \\ \text{②}+\text{③}\times(-1)}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & \frac{1}{2} & -2 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array}\right)$$

よって, $A^{-1} = \begin{pmatrix} 2 & \frac{1}{2} & -2 \\ 0 & -\frac{1}{2} & 1 \\ -1 & \frac{1}{2} & 0 \end{pmatrix}$.

(3) $\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 4 & 0 & 0 & 1 & 0 \\ 2 & 3 & 4 & 5 & 0 & 0 & 0 & 1 \end{array}\right)$

$$\xrightarrow{\substack{\text{②}+\text{①}\times(-1) \\ \text{③}+\text{①}\times(-2) \\ \text{④}+\text{①}\times(-2)}} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\substack{\text{①}+\text{②}\times(-1) \\ \text{③}+\text{②}\times(-1) \\ \text{④}+\text{②}\times(-1)}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{\substack{\text{②}+\text{③}\times(-1) \\ \text{④}+\text{③}\times(-1)}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{\text{③}+\text{④}\times(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

よって, $A^{-1} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$.