

# 技術者リテラシー I (機械工学科) ——— 第 11 回 2023/12/6 略解

問題 1.

$$(1) \lim_{n \rightarrow \infty} \frac{3 + \frac{2}{n} - \frac{3}{n^2}}{2 - \frac{4}{n} + \frac{1}{n^2}} = \frac{3}{2}.$$

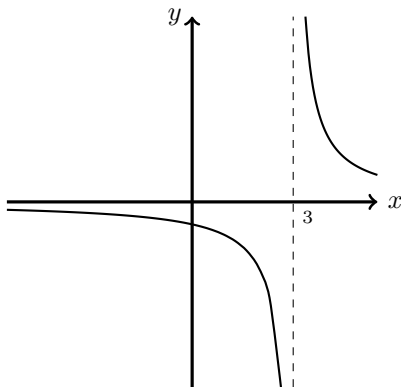
$$(2) \lim_{n \rightarrow \infty} \frac{\frac{5}{n} + \frac{2}{n^2}}{2 - \frac{5}{n} - \frac{3}{n^2}} = 0.$$

$$(3) \lim_{n \rightarrow \infty} \frac{(\sqrt{n+2} - \sqrt{n-1})(\sqrt{n+2} + \sqrt{n-1})}{\sqrt{n+2} + \sqrt{n-1}} \\ = \lim_{n \rightarrow \infty} \frac{3}{\sqrt{n+2} + \sqrt{n-1}} = 0.$$

$$(4) \lim_{n \rightarrow \infty} \frac{n + \sqrt{n^2 + 2n}}{(n - \sqrt{n^2 + 2n})(n + \sqrt{n^2 + 2n})} \\ = \lim_{n \rightarrow \infty} \frac{n + \sqrt{n^2 + 2n}}{-2n} \\ = \lim_{n \rightarrow \infty} \frac{1 + \sqrt{1 + \frac{2}{n}}}{-2} = -1.$$

$$(5) \lim_{x \rightarrow 3} \frac{(x-3)(x-2)}{x-3} = \lim_{x \rightarrow 3} (x-2) = 1.$$

$$(6) \text{ グラフより, } \lim_{x \rightarrow 3-0} \frac{2}{x-3} = -\infty.$$



$$(7) \lim_{x \rightarrow 0} 4 \cdot \frac{\sin 4x}{4x} = 4.$$

$$(8) \frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1} \text{ より} \\ \sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^n \left( \frac{1}{k} - \frac{1}{k+1} \right) = 1 - \frac{1}{n+1}. \\ \text{よって求める極限は 1.}$$

問題 2.

$$(1) f'(x) = 9x^2 - 4x + 1.$$

$$(2) f'(x) = -\frac{4}{x^3} + \frac{3}{x^2} - 3 + 4x.$$

$$(3) f'(x) = (e^x)' \sin x + e^x (\sin x)' = e^x \sin x + e^x \cos x.$$

$$(4) f'(x) = \frac{(\log x)' x^2 - \log x \cdot (x^2)'}{x^4} \\ = \frac{x - 2x \log x}{x^4} = \frac{1 - 2 \log x}{x^3}.$$

$$(5) f'(x) = 3(3x+2)^2(3x+2)' = 9(3x+2)^2.$$

$$(6) f'(x) = \cos \left( \frac{\pi}{5} - 2x \right) \cdot \left( \frac{\pi}{5} - 2x \right)' = -2 \cos \left( \frac{\pi}{5} - 2x \right).$$

$$(7) f'(x) = \frac{(4x^3 - 2x + 1)'}{4x^3 - 2x + 1} = \frac{12x^2 - 2}{4x^3 - 2x + 1}.$$

$$(8) \text{ 両辺の対数を取ると } \log f(x) = \log x^{\log x} = (\log x)^2.$$

$$\text{この両辺を微分して } \frac{f'(x)}{f(x)} = 2 \log x \cdot (\log x)' = \frac{2 \log x}{x}.$$

$$\text{よって, } f'(x) = \frac{2 \log x}{x} \cdot f(x) = \frac{2x^{\log x} \log x}{x}$$

問題 3. 以下, 積分定数を  $C$  とする.

$$(1) \int \left( 1 - \frac{1}{x^2} \right) dx = x + \frac{1}{x} + C.$$

$$(2) \frac{3}{2} x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + C.$$

$$(3) \left[ \frac{1}{3} x^3 - x^{-1} \right]_1^2 = \frac{1}{3} (8 - 1) - \left( \frac{1}{2} - 1 \right) = \frac{17}{6}.$$

$$(4) \log |x^3 + x^2 - 3x + 9| + C. \quad (t = x^3 + x^2 - 3x + 9 \text{ による置換積分.})$$

$$(5) -\frac{1}{4} \cos^4 x + C. \quad (t = \cos x \text{ による置換積分.})$$

$$(6) \int_0^{\pi/2} (1 - \sin^2 x)^2 \cos x \, dx \text{ と変形できるので, } t = \sin x \\ \text{で置換積分を行う. 積分範囲は右下図の通りになるので,}$$

$$\int_0^1 (1 - t^2)^2 dt \quad \begin{array}{c|cc} x & 0 & \rightarrow \frac{\pi}{2} \\ t & 0 & \rightarrow 1 \end{array}$$

$$= \int_0^1 (1 - 2t^2 + t^4) dt = \left[ t - \frac{2}{3} t^3 + \frac{1}{5} t^5 \right]_0^1$$

$$= 1 - \frac{2}{3} + \frac{1}{5} = \frac{8}{15}.$$

$$(7) x e^x - \int (x)' e^x dx = x e^x - e^x + C = (x - 1) e^x + C.$$

(8) 求める積分を  $I$  とおくと,

$$\begin{aligned}
 I &= -e^{-x} \cos x - \int (-e^{-x}(\cos x)') dx \\
 &= -e^{-x} \cos x - \int e^{-x} \sin x dx \\
 &= -e^{-x} \cos x - \left( -e^{-x} \sin x - \int (-e^{-x}(\sin x)') dx \right) \\
 &= -e^{-x} \cos x + e^{-x} \sin x - \int e^{-x} \cos x dx \\
 &= e^{-x}(\sin x - \cos x) - I. \\
 \text{よって, } I &= \frac{e^{-x}}{2}(\sin x - \cos x) + C.
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad & \left[ \frac{1}{2} x^2 \log x \right]_1^3 - \int_1^3 \frac{1}{2} x^2 (\log x)' dx \\
 &= \frac{9}{2} \log 3 - \frac{1}{2} \int_1^3 x dx \\
 &= \frac{9}{2} \log 3 - \frac{1}{2} \left[ \frac{1}{2} x^2 \right]_1^3 \\
 &= \frac{9}{2} \log 3 - \frac{1}{2} \left( \frac{9}{2} - \frac{1}{2} \right) = \frac{9}{2} \log 3 - 2.
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad & \frac{1}{x^2 - 4x + 3} = \frac{1}{(x-1)(x-3)} = \frac{1}{2} \left( \frac{1}{x-3} - \frac{1}{x-1} \right) \\
 & \text{より, 求める積分は} \\
 & \frac{1}{2} \int \left( \frac{1}{x-3} - \frac{1}{x-1} \right) dx \\
 &= \frac{1}{2} (\log |x-3| - \log |x-1|) + C \\
 &= \frac{1}{2} \log \left| \frac{x-3}{x-1} \right| + C.
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad & \frac{x-4}{x^2-2x-3} = \frac{x-4}{(x+1)(x-3)} = \frac{5}{4} \frac{1}{x+1} - \frac{1}{4} \frac{1}{x-3} \\
 & \text{より, 求める積分は} \\
 & \frac{5}{4} \int \frac{1}{x+1} dx - \frac{1}{4} \int \frac{1}{x-3} dx \\
 &= \frac{5}{4} \log |x+1| - \frac{1}{4} \log |x-3| + C.
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad & \frac{x^2}{x^2-1} = 1 + \frac{1}{(x-1)(x+1)} = 1 + \frac{1}{2} \left( \frac{1}{x-1} - \frac{1}{x+1} \right) \\
 & \text{より, 求める積分は} \\
 & \int \left\{ 1 + \frac{1}{2} \left( \frac{1}{x-1} - \frac{1}{x+1} \right) \right\} dx \\
 &= x + \frac{1}{2} (\log |x-1| - \log |x+1|) + C \\
 &= x + \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + C.
 \end{aligned}$$