技術者リテラシー I (機械工学科) —— 第 4 回 2024/10/16 略解

問題 1.

(1) $f(x) = x^3$,

$$f(x+h) = (x+h)^3 = x^3 + 3hx^2 + 3h^2x + h^3.$$

(2) $$\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{(a+h) - a} = \frac{3ha^2 + 3h^2a + h^3}{h} = 3a^2 + 3ha + h^2.$$

(3) $$f'(a) = \lim_{h\to0}\frac{\Delta y}{\Delta x} = \lim_{h\to0}(3a^2 + 3ha + h^2) = 3a^2.$$

問題 2.

(1) $f'(x) = 6x^2 + 10x - 1.$

(2) $f'(x) = 5x^4 + 8x^3.$

(3) $f'(x) = \dfrac{8}{x^3}.$

(4) $f'(x) = \sin x + x\cos x.$

(5) $$\begin{aligned} f'(x) &= (3x^2-2)'(x^2+x+1) + (3x^2-2)(x^2+x+1)' \\ &= 6x(x^2+x+1) + (3x^2-2)(2x+1) \\ &= 12x^3 + 9x^2 + 2x - 2. \end{aligned}$$

(6) $$\begin{aligned} f'(x) &= (e^x)'\cos x + e^x(\cos x)' \\ &= e^x\cos x - e^x\sin x \quad (= e^x(\cos x - \sin x)). \end{aligned}$$

(7) $f'(x) = (x)'\log x + x(\log x)' = \log x + 1.$

(8) $$\begin{aligned} f'(x) &= \frac{(x^2)'(x+3) - x^2(x+3)'}{(x+3)^2} \\ &= \frac{2x(x+3) - x^2}{(x+3)^2} \\ &= \frac{x^2+6x}{(x+3)^2} \quad \left(= \frac{x(x+6)}{(x+3)^2}\right). \end{aligned}$$

(9) $$\begin{aligned} f'(x) &= \frac{(2x+1)'(x^2+1) - (2x+1)(x^2+1)'}{(x^2+1)^2} \\ &= \frac{2(x^2+1) - (2x+1)\cdot 2x}{(x^2+1)^2} \\ &= \frac{-2x^2 - 2x + 2}{(x^2+1)^2}. \end{aligned}$$

(10) $$\begin{aligned} f'(x) &= \frac{(\cos x)'\sin x - \cos x(\sin x)'}{\sin^2 x} \\ &= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} \\ &= -\frac{1}{\sin^2 x}. \end{aligned}$$

(11) $$\begin{aligned} f'(x) &= \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)'\cos x - \sin x(\cos x)'}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}. \end{aligned}$$

(12) $f'(x) = (2\log x)' = \dfrac{2}{x}.$

問題 3.

(1) $f'(x) = 3(2x^2+5)^2(2x^2+5)' = 12x(2x^2+5)^2.$

(2) $f'(x) = -\dfrac{3}{(x^2+1)^4}\cdot(x^2+1)' = -\dfrac{6x}{(x^2+1)^4}.$

(3) $$\begin{aligned} f'(x) &= -\sin\left(\frac{\pi}{4} - 2x\right)\cdot\left(\frac{\pi}{4} - 2x\right)' \\ &= 2\sin\left(\frac{\pi}{4} - 2x\right). \end{aligned}$$

(4) $f'(x) = 2\sin x\cdot(\sin x)' = 2\sin x\cos x \quad (= \sin 2x).$

(5) $f'(x) = \dfrac{1}{x^2+1}\cdot(x^2+1)' = \dfrac{2x}{x^2+1}.$

(6) $f'(x) = \dfrac{1}{\sin x}\cdot(\sin x)' = \dfrac{\cos x}{\sin x} \quad \left(= \dfrac{1}{\tan x}\right).$

(7) $f'(x) = e^{-x^2}\cdot(-x^2)' = -2xe^{-x^2}.$

(8) $f'(x) = e^{\sin x}(\sin x)' = e^{\sin x}\cos x.$

(9) $f'(x) = \dfrac{1}{4}(x^3+1)^{-\frac{3}{4}}\cdot(x^3+1)' = \dfrac{3}{4}x^2(x^3+1)^{-\frac{3}{4}}.$

(10) $f'(x) = n(ax+b)^{n-1}(ax+b)' = an(ax+b)^{n-1}.$

(11) $$\begin{aligned} f'(x) &= (e^{\log a^x})' = (e^{x\log a})' \\ &= e^{x\log a}\cdot(x\log a)' = a^x\log a. \end{aligned}$$

(12) $f'(x) = \left(\dfrac{\log x}{\log a}\right)' = \dfrac{1}{x\log a}.$

問題 4.

(1) $f'(x) = \dfrac{1}{2}(4-x^2)^{-\frac{1}{2}}(4-x^2)' = -\dfrac{x}{\sqrt{4-x^2}}.$

(2) $$\begin{aligned} f'(x) &= -\frac{1}{(\tan x+1)^2}\cdot(\tan x+1)' \\ &= -\frac{1}{(\tan x+1)^2}\cdot\frac{1}{\cos^2 x} \\ &= -\frac{1}{(\sin x+\cos x)^2}. \end{aligned}$$

(3) $f'(x) = -\sin\dfrac{1}{x}\cdot\left(\dfrac{1}{x}\right)' = \dfrac{1}{x^2}\sin\dfrac{1}{x}.$

(4) $f'(x) = 4(e^x+1)^3(e^x+1)' - 2\cdot 2(e^x+1)(e^x+1)'$

$= 4e^x(e^x+1)^3 - 4e^x(e^x+1)$

$(= 4e^{2x}(e^x+1)(e^x+2))$.

(5) $f'(x) = \dfrac{1}{\frac{x-1}{x+1}}\left(\dfrac{x-1}{x+1}\right)'$

$= \dfrac{x+1}{x-1}\cdot\dfrac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$

$= \dfrac{x+1}{x-1}\cdot\dfrac{2}{(x+1)^2}$

$= \dfrac{2}{(x+1)(x-1)}$.

問題 5.

(1) $\log y = x\log x$ より, 両辺を x で微分すると

$$\frac{y'}{y} = (x)'\log x + x(\log x)' = \log x + 1.$$

よって, $y' = y(\log x+1) = x^x(\log x+1)$.

(2) $\log y = \dfrac{1}{2}\log(x^2+1) - \dfrac{1}{2}\log(x^2+2)$ より, 両辺を x で微分すると

$$\frac{y'}{y} = \frac{x}{x^2+1} - \frac{x}{x^2+2} = \frac{x}{(x^2+1)(x^2+2)}.$$

よって,

$$\begin{aligned} y' &= \frac{x}{(x^2+1)(x^2+2)}y \\ &= \frac{x}{\sqrt{(x^2+1)(x^2+2)}(x^2+2)}. \end{aligned}$$

問題 6.

(1) $f'(x) = \dfrac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\left(\dfrac{x}{a}\right)'$

$= \dfrac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\dfrac{1}{a} = \dfrac{1}{\sqrt{a^2-x^2}}$.

(2) $f'(x) = -\dfrac{1}{\sqrt{1-x}}\cdot(\sqrt{x})'$

$= -\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{2\sqrt{x}} = -\dfrac{1}{2\sqrt{x(x-1)}}$.

(3) $f'(x) = \dfrac{1}{a}\cdot\dfrac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\left(\dfrac{x}{a}\right)'$

$= \dfrac{1}{a^2+x^2}$.

(4) $f'(x) = (x)'\arctan x + x(\arctan x)'$

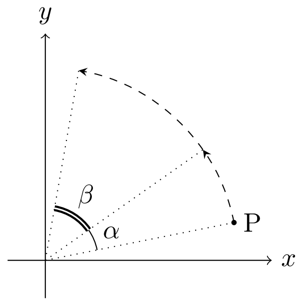
$= \arctan x + \dfrac{x}{1+x^2}$.

問題 7. 角度 $-\dfrac{\pi}{6}$ の回転行列は $\dfrac{1}{2}\begin{pmatrix}\sqrt{3} & 1 \\ -1 & \sqrt{3}\end{pmatrix}$ である.

よって, 求める点は $\dfrac{1}{2}\begin{pmatrix}\sqrt{3} & 1 \\ -1 & \sqrt{3}\end{pmatrix}\begin{pmatrix}2 \\ 3\end{pmatrix} = \begin{pmatrix}\frac{2\sqrt{3}+3}{2} \\ \frac{-2+3\sqrt{3}}{2}\end{pmatrix}$.

問題 8.

(1) 下図の通り. (角度は適当でよい.)



(2) $A = \begin{pmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{pmatrix}$, $B = \begin{pmatrix}\cos\beta & -\sin\beta \\ \sin\beta & \cos\beta\end{pmatrix}$ より,

$BA = \begin{pmatrix}\cos\beta & -\sin\beta \\ \sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{pmatrix}$

$= \begin{pmatrix}\cos\beta\cos\alpha - \sin\beta\sin\alpha & -(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \\ \sin\beta\cos\alpha + \cos\beta\sin\alpha & -\sin\beta\sin\alpha + \cos\beta\cos\alpha\end{pmatrix}$

$= \begin{pmatrix}\cos(\beta+\alpha) & -\sin(\beta+\alpha) \\ \sin(\beta+\alpha) & \cos(\beta+\alpha)\end{pmatrix}$.

この答えは BA が角度 $(\alpha+\beta)$ の回転行列である. したがって, 本問の計算は「α だけ回転した後, β だけ回転する作用」と「$(\alpha+\beta)$ だけ回転させる作用」が変わらないことを表している.