技術者リテラシー I (機械工学科) —— 第 1 回 2023/9/27 略解

指数・対数

問題 1.

(1) 1.

(2) $\dfrac{1}{16}$.

(3) $\dfrac{1}{1024}$.

(4) $7^{5+(-4)} = 7$.

(5) $5^{8-5} = 5^3 = 125$.

(6) $2^{-3\times 2} = 2^{-6} = \dfrac{1}{64}$.

(7) $3^{-4} \times 5^{-2} = \dfrac{1}{2025}$.

別解: $(3^2 \times 5)^{-2} = 45^{-2} = \dfrac{1}{2025}$

問題 2.

(1) $\sqrt[3]{121}$.

(2) $\dfrac{1}{\sqrt[3]{5}}$.

(3) $2\sqrt[3]{3}$.

(4) $\sqrt[3]{5^4} = 5\sqrt[3]{5}$.

(5) $\sqrt[3]{18}\sqrt[3]{15} = \sqrt[3]{2\times 3^2}\sqrt[3]{3\times 5} = 3\sqrt[3]{10}$.

(6) $(2^5)^{\frac{1}{5}} = 2$.

(7) $\left(\dfrac{1}{3^4}\right)^{\frac{1}{4}} = \dfrac{1}{3}$.

(8) $\dfrac{\sqrt[4]{243}}{\sqrt[4]{3}} = \dfrac{3^{\frac{5}{4}}}{3^{\frac{1}{4}}} = 3^{\frac{5}{4}-\frac{1}{4}} = 3$.

ちなみに, $\dfrac{\sqrt[4]{273}}{\sqrt[4]{3}} = \sqrt[4]{91}$.

(9) $((2^3)^{\frac{1}{4}})^{\frac{1}{3}} = 2^{3\times\frac{1}{4}\times\frac{1}{3}} = 2^{\frac{1}{4}}$.

(10) $13^{\frac{1}{2}} \div 13^{\frac{5}{6}} \times 13^{\frac{1}{3}} = 13^{\frac{1}{2}-\frac{5}{6}+\frac{1}{3}} = 13^0 = 1$.

(11) $13^{\frac{2}{3}} \div 13^{\frac{1}{6}} \times 13^{\frac{1}{2}} = 13^{\frac{2}{3}-\frac{1}{6}+\frac{1}{2}} = 13$.

問題 3.

(1) 3.

(2) -3.

(3) 0.

(4) 5.

(5) -2.

(6) -2.

(7) $\dfrac{1}{4}$.

(8) $\dfrac{\log_3 9}{\log_3 \sqrt{3}} = \dfrac{2}{\frac{1}{2}} = 4$.

(9) $\log_{15}(3\times 5) = 1$.

(10) $\log_2 \dfrac{40}{5} = 3$.

(11) $\log_5 \dfrac{24\times 54}{6^3} = \log_5 6$.

(12) $\log_3 5 \times \dfrac{\log_3 27}{\log_3 5} = 3$.

三角関数の値

問題 4.

(1) $\dfrac{5}{6}\pi$.

(2) $\dfrac{5}{3}\pi$.

(3) $180°$.

(4) $330°$.

問題 5.

(1) $\sin\theta = \dfrac{\sqrt{3}}{2}$, $\cos\theta = \dfrac{1}{2}$, $\tan\theta = \sqrt{3}$.

(2) $\sin\theta = -\dfrac{1}{\sqrt{2}}$, $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\tan\theta = -1$.

(3) $\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$.

(4) $\sin\theta = 1$, $\cos\theta = 0$, $\tan\theta$ は定義されない.

方程式

問題 6.

(1) $x = \pm i$.

(2) $x = \pm\dfrac{\sqrt{6}}{4}$.

(3) 解の公式より $x = \dfrac{-2 \pm \sqrt{2}i}{3}$.

(4) $x^2 - 2\sqrt{5}x + 6 = 0$ の解は

解の公式より $x = \sqrt{5} \pm i$.

ちなみに, $x^2 - 2\sqrt{5} + 6 = 0$ の解は

$x = \pm(\sqrt{5} - 1)$.

(5) $x^4 + x^2 - 12 = (x^2 + 4)(x^2 - 3)$ より $x = \pm 2i, \pm\sqrt{3}$.

(6) $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ より $x = 2, -1 \pm \sqrt{3}i$.

(7) $x^3 - 4x^2 + 2x + 3 = (x - 3)(x^2 - x - 1)$ より $x = 3, \dfrac{1 \pm \sqrt{3}}{2}$.

(8) $x^4 + x^2 + 1 = (x^2 + 1)^2 - x^2$

$= (x^2 + x + 1)(x^2 - x + 1)$ より

$x = \dfrac{-1 \pm \sqrt{3}i}{2}, \dfrac{1 \pm \sqrt{3}i}{2}$.

問題 7.

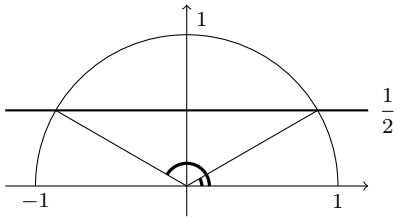
(1) $3^{3x} = 3^2$ より $3x = 2$, つまり $x = \dfrac{2}{3}$.

(2) $2^{2x} = 2^{x+1}$ より $2x = x + 1$, つまり $x = 1$.

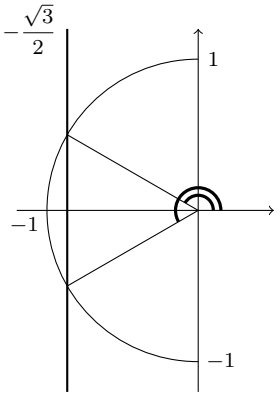
(3) $\log_3 x = 2 \iff x = 9$.

(4) $\log_{\frac{1}{4}} x = -\dfrac{3}{2} \iff x = \left(\dfrac{1}{4}\right)^{-\frac{3}{2}} = 8$.

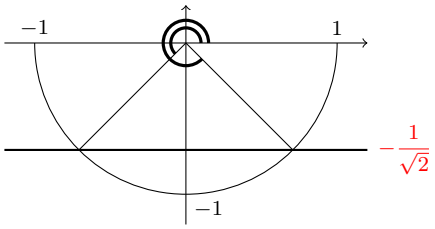
(5) $\sin\theta = \dfrac{1}{2}$ より, $\theta = \dfrac{\pi}{6}, \dfrac{5}{6}\pi$.



(6) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ より, $\theta = \dfrac{5}{6}\pi, \dfrac{7}{6}\pi$.



(7) $\sin\theta = -\dfrac{1}{\sqrt{2}}$ より, $\theta = \dfrac{5}{4}\pi, \dfrac{7}{4}\pi$.



(8) $\theta = \dfrac{\pi}{4}, \dfrac{5}{4}\pi$.