技術者リテラシー I (機械工学科) —— 第 6 回 2023/11/1 略解

問題 1.

(1) $\dfrac{1}{5+1}x^{5+1} = \dfrac{1}{6}x^6 + C.$

(2) $\dfrac{1}{-3+1}x^{-3+1} + C = -\dfrac{1}{2x^2} + C.$

(3) $\displaystyle\int x^{\frac{3}{2}}\,dx = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + C = \frac{2}{5}x^{\frac{5}{2}} + C.$

(4) $\displaystyle\int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{1}{x^3}\right)\,dx = \log|x| + \frac{4}{x} - \frac{1}{2x^2} + C.$

(5) $\displaystyle\int \left(1 - \frac{5}{x^2} + \frac{6}{x^4}\right)\,dx = x + \frac{5}{x} - \frac{2}{x^3} + C.$

(6) $\dfrac{1}{3+1}x^{3+1} - 2e^x + C = \dfrac{1}{4}x^4 - 2e^x + C.$

(7) $t^3 - \log|t| + C.$

問題 2.

(1) $\sin x + 2\cos x + C.$

(2) $4\tan x - \dfrac{5}{\tan x} + C.$

(3) $\displaystyle\int \left(2\cos x - \frac{1}{\cos^2 x}\right)\,dx$
$= 2\sin x - \tan x + C.$

(4) $\displaystyle\int (2\cos\theta - \sin\theta)\,d\theta = 2\sin\theta + \cos\theta + C.$

(5) $\displaystyle\int \frac{\sin^2 x}{\cos^2 x}\,dx = \int \frac{1-\cos^2 x}{\cos^2 x}\,dx$
$\displaystyle = \int \left(\frac{1}{\cos^2 x} - 1\right)\,dx = \tan x - x + C.$

(6) $\displaystyle\frac{1}{2}\int \sin x\,dx = -\frac{1}{2}\cos x + C.$

(7) $\displaystyle\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2\,dx = \int (1+\sin x)\,dx$
$= x - \cos x + C$

問題 3.

(1) $\left[\dfrac{3}{4}x^{\frac{4}{3}}\right]_{-8}^{27} = \dfrac{3}{4}\left(27^{\frac{4}{3}} - (-8)^{\frac{4}{3}}\right) = \dfrac{195}{4}.$

(2) $\left[2x^{\frac{1}{2}}\right]_{2}^{4} = 2\left(4^{\frac{1}{2}} - 2^{\frac{1}{2}}\right) = 4 - 2\sqrt{2}.$

(3) $\displaystyle\int_1^2 \left(\frac{1}{y^2} - \frac{1}{y} - 1\right)\,dy = \left[-\frac{1}{y} - \log|y| - y\right]_1^e$
$= \left(-\dfrac{1}{e} - \log|e| - e\right) - (-1-1) = -\dfrac{1}{e} - e + 1.$

(4) $\displaystyle\int_1^6 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)\,dx = \left[\frac{2}{3}x\sqrt{x} + 4\sqrt{x}\right]_1^6$
$= \left(4\sqrt{6} + 4\sqrt{6}\right) - \left(\dfrac{2}{3} + 4\right) = 8\sqrt{6} - \dfrac{14}{3}.$

(5) $\displaystyle -\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x}\,dx = -\Big[\tan x\Big]_{-\frac{\pi}{4}}^{\frac{\pi}{3}} = -\left(\sqrt{3}+1\right).$

(6) $\Big[\arcsin x\Big]_{-\frac{\pi}{6}}^{\frac{\pi}{2}} = \left(1 - \left(-\dfrac{1}{2}\right)\right) = \dfrac{3}{2}.$

(7) $\Big[\arctan x\Big]_0^{\frac{\pi}{4}} = 1 - 0 = 1.$

問題 4.

(1) 対応する行列は $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ である. $\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = 3\cdot 1 - 4\cdot 2 = -5 \neq 0$ なので,

$$\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}^{-1} = -\frac{1}{5}\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}.$$

よって, $\begin{pmatrix} x \\ y \end{pmatrix} = -\dfrac{1}{5}\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$

(2) 対応する行列は $\begin{pmatrix} 4 & 1 \\ 8 & -7 \end{pmatrix}$ である. $\begin{vmatrix} 4 & 1 \\ 8 & -7 \end{vmatrix} = 4\cdot(-7) - 1\cdot 8 = -36 \neq 0$ なので,

$$\begin{pmatrix} 4 & 1 \\ 8 & -7 \end{pmatrix}^{-1} = -\frac{1}{36}\begin{pmatrix} -7 & -1 \\ -8 & 4 \end{pmatrix}.$$

よって, $\begin{pmatrix} x \\ y \end{pmatrix} = -\dfrac{1}{36}\begin{pmatrix} -7 & -1 \\ -8 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

(3) 対応する行列は $\begin{pmatrix} 3 & -2 \\ 5 & -3 \end{pmatrix}$ である. $\begin{vmatrix} 3 & -2 \\ 5 & -3 \end{vmatrix} = 3\cdot(-3) - (-2)\cdot 5 = 1 \neq 0$ なので,

$$\begin{pmatrix} 3 & -2 \\ 5 & -3 \end{pmatrix}^{-1} = \begin{pmatrix} -3 & 2 \\ -5 & 3 \end{pmatrix}.$$

よって, $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 & 2 \\ -5 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$