第1回リメディアル数学 (化学システム工学科) 2023/4/19 略解

指数・対数

問題 1.

(1) $3^0 = 1$.

(2) $3^{-3} = \dfrac{1}{27}$.

(3) $\left(\dfrac{1}{3}\right)^5 = \dfrac{1}{273}$.

(4) $2^6 \times 2^{-3} = 2^{6+(-3)} = 2^3 = 8$.

(5) $5^8 \div 5^5 = 5^{8-5} = 5^3 = 125$.

(6) $(2^{-2})^3 = 2^{-2\times 3} = 2^{-6} = \dfrac{1}{64}$.

(7) $(2 \times 3^4)^{-2} = 2^{-2} \times 3^{-8} = \dfrac{1}{26244}$.

問題 2.

(1) $11^{\frac{2}{3}} = \sqrt[3]{121}$.

(2) $5^{-\frac{1}{3}} = \dfrac{1}{\sqrt[3]{5}}$.

(3) $\sqrt[3]{24} = 2\sqrt[3]{3}$.

(4) $(\sqrt[3]{25})^2 = 5\sqrt[3]{5}$.

(5) $\sqrt[3]{18}\sqrt[3]{15} = \sqrt[3]{2\times 3^2}\sqrt[3]{3\times 5} = 3\sqrt[3]{10}$.

(6) $\sqrt[5]{32} = 2$.

(7) $\sqrt[4]{\dfrac{1}{81}} = \dfrac{1}{2}$.

(8) $\dfrac{\sqrt[4]{273}}{\sqrt[4]{3}} = \sqrt[4]{81} = 3$.

(9) $\sqrt[3]{\sqrt[4]{24}} = 24^{\frac{1}{12}}$.

(10) $13^{\frac{1}{2}} \div 13^{\frac{5}{6}} \times 13^{\frac{1}{3}} = 13^{\frac{1}{2}-\frac{5}{6}+\frac{1}{3}} = 13^0 = 1$.

(11) $\sqrt[3]{169} \div \sqrt[12]{169} \times \sqrt[4]{169} = 13^{\frac{2}{3}-\frac{1}{6}+\frac{1}{2}} = 13$.

問題 3.

(1) $\log_3 27 = 3$.

(2) $\log_5 \dfrac{1}{125} = \dfrac{1}{3}$.

(3) $\log_{691} 1 = 0$.

(4) $\log_{\frac{1}{2}} \dfrac{1}{32} = 5$.

(5) $\log_{10} 0.01 = -2$.

(6) $\log_{\frac{1}{4}} 2 = -\dfrac{1}{2}$.

(7) $\log_5 \sqrt[4]{5} = \dfrac{1}{4}$.

(8) $\log_{\sqrt{3}} 9 = \dfrac{\log_3 9}{\log_3 \sqrt{3}} = \dfrac{2}{\frac{1}{2}} = 4$.

(9) $\log_{15} 3 + \log_{15} 5 = \log_{15}(3\times 5) = 1$.

(10) $\log_2 40 - \log_2 5 = \log_2 \dfrac{40}{5} = 3$.

(11) $\log_5 24 - 3\log_5 6 + \log_5 54$
$= \log_5 \dfrac{24\times 54}{6^3} = \log_5 6$.

(12) $\log_3 5 \times \log_5 27 = \log_3 5 \times \dfrac{\log_3 27}{\log_3 5} = 3$.

三角関数の値

問題 4.

(1) $150^\circ = \dfrac{5}{6}\pi$.

(2) $300^\circ = \dfrac{5}{3}\pi$.

(3) $\pi = 180^\circ$.

(4) $\dfrac{11}{6}\pi = 330^\circ$.

問題 5.

(1) $\theta = \dfrac{\pi}{3}$

$\sin\theta = \dfrac{\sqrt{3}}{2}$, $\cos\theta = \dfrac{1}{2}$, $\tan\theta = \sqrt{3}$.

(2) $\theta = \dfrac{7}{4}\pi$

$\sin\theta = -\dfrac{1}{\sqrt{2}}$, $\cos\theta = \dfrac{1}{\sqrt{2}}$, $\tan\theta = -1$.

(3) $\theta = \pi$

$\sin\theta = 0$, $\cos\theta = 1$, $\tan\theta = 0$.

方程式

問題 6.

(1) $x = \pm i$.

(2) $x = \pm\dfrac{\sqrt{6}}{4}$.

(3) 解の公式より $x = \dfrac{-2 \pm \sqrt{2}i}{3}$.

(4) 解の公式より $x = \sqrt{5} \pm i$.

(5) $x^4 + x^2 - 12 = (x^2+4)(x^2-3)$ より $x = \pm 2i, \pm\sqrt{3}$.

(6) $x^3 - 8 = (x-2)(x^2+2x+4)$ より $x = 2, -1 \pm \sqrt{3}i$.

(7) $x^3 - 4x^2 + 2x + 3 = (x-3)(x^2-x-1)$ より $x = 3, \dfrac{1 \pm \sqrt{3}}{2}$.

(8) $x^4 + x^2 + 1 = (x^2+1)^2 - x^2$
$= (x^2+x+1)(x^2-x+1)$ より
$x = \dfrac{-1 \pm \sqrt{3}i}{2}, \ \dfrac{1 \pm \sqrt{3}i}{2}$.

問題 7.

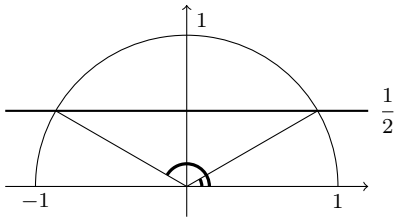
(1) $3^{3x} = 3^2$ より $3x = 2$, つまり $x = \dfrac{2}{3}$.

(2) $2^{2x} = 2^{x+1}$ より $2x = x+1$, つまり $x = 1$.

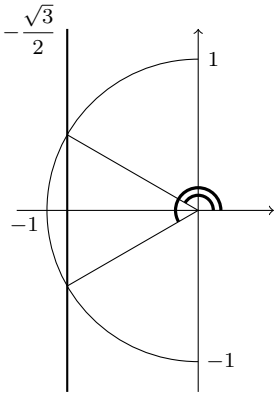
(3) $\log_3 x = 2 \iff x = 9$.

(4) $\log_{\frac{1}{4}} x = -\dfrac{3}{2} \iff x = \left(\dfrac{1}{4}\right)^{-\frac{3}{2}} = 8$.

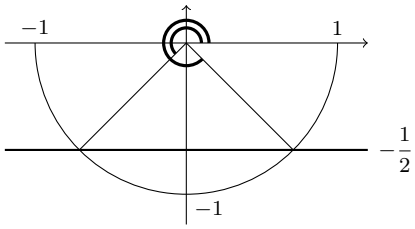
(5) $\sin\theta = \dfrac{1}{2}$ より, $\theta = \dfrac{\pi}{6}, \ \dfrac{5}{6}\pi$.



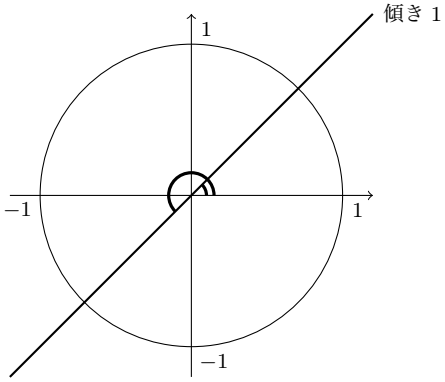
(6) $\cos\theta = -\dfrac{\sqrt{3}}{2}$ より, $\theta = \dfrac{5}{6}\pi, \ \dfrac{7}{6}\pi$.



(7) $\sin\theta = -\dfrac{1}{\sqrt{2}}$ より, $\theta = \dfrac{5}{4}\pi, \ \dfrac{3}{4}\pi$.



(8) $\theta = \dfrac{\pi}{4}, \ \dfrac{5}{4}\pi$.



問題 8. 求める時間を x 時間後とすると, $2^{2x} > 2\times 10^5$. 両辺の対数を取ると, $2x\log_{10}2 > 5 + \log_{10}2$. よって $x > \dfrac{5+\log_{10}2}{2\log_{10}2} = 8.80\cdots \approx 8.8$.