

問題 1.

- (1) $f(x) = x^3$,
 $f(x+h) = (x+h)^3 = x^3 + 3hx^2 + 3h^2x + h^3$.
- (2) $\frac{\Delta y}{\Delta x} = \frac{f(a+h) - f(a)}{(a+h) - a}$
 $= \frac{3ha^2 + 3h^2a + h^3}{h} = 3a^2 + 3ha + h^2$.
- (3) $f'(a) = \lim_{h \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{h \rightarrow 0} (3a^2 + 3ha + h^2) = 3a^2$.

問題 2.

- (1) $f'(x) = 6x^2 + 10x - 1$.
- (2) $y' = 5x^4 + 8x^3$.
- (3) $y' = \frac{8}{x^3}$.
- (4) $f'(x) = \sin x + x \cos x$.
- (5) $y' = (3x^2 - 2)'(x^2 + x + 1) + (3x^2 - 2)(x^2 + x + 1)'$
 $= 6x(x^2 + x + 1) + (3x^2 - 2)(2x + 1)$
 $= 12x^3 + 9x^2 + 2x - 2$.
- (6) $f'(x) = (e^x)' \cos x + e^x (\cos x)'$
 $= e^x \cos x - e^x \sin x \quad (= e^x (\cos x - \sin x))$.
- (7) $y' = (x)' \log x + x (\log x)' = \log x + 1$.
- (8) $y' = \frac{(x^2)'(x+3) - x^2(x+3)'}{(x+3)^2}$
 $= \frac{2x(x+3) - x^2}{(x+3)^2}$
 $= \frac{x^2 + 6x}{(x+3)^2} \quad \left(= \frac{x(x+6)}{(x+3)^2} \right)$.
- (9) $y' = \frac{(2x+1)'(x^2+1) - (2x+1)(x^2+1)'}{(x^2+1)^2}$
 $= \frac{2(x^2+1) - (2x+1) \cdot 2x}{(x^2+1)^2}$
 $= \frac{-2x^2 - 2x + 2}{(x^2+1)^2}$.
- (10) $y' = \frac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x}$
 $= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$
 $= -\frac{1}{\sin^2 x}$.

$$(11) \left(\frac{\sin x}{\cos x} \right)' = \frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.$$

$$(12) f'(x) = (2 \log x)' = \frac{2}{x}.$$

問題 3.

- (1) $y' = 3(2x^2 + 5)^2(2x^2 + 5)' = 12x(2x^2 + 5)$.
- (2) $y' = -\frac{3}{(x+1)^4} \cdot (x^2 + 1)' = -\frac{6x}{(x^2 + 1)^4}$.
- (3) $y' = \sin \left(\frac{\pi}{4} - 2x \right) \cdot \left(\frac{\pi}{4} - 2x \right)' = 2 \sin \left(\frac{\pi}{4} - 2x \right)$.
- (4) $y' = 2 \sin x (\sin x)' = 2 \sin x \cos x \quad (= \sin 2x)$.
- (5) $y' = \frac{1}{x^2 + 1} \cdot (x^2 + 1)' = \frac{2x}{x^2 + 1}$.
- (6) $y' = \frac{1}{\sin x} \cdot (\sin x)' = \frac{\cos x}{\sin x} \quad \left(= \frac{1}{\tan x} \right)$.
- (7) $y' = e^{-x^2} \cdot (-x^2)' = -2xe^{-x^2}$.
- (8) $y' = e^{\sin x} (\sin x)' = e^{\sin x} \cos x$.
- (9) $y' = \frac{1}{4} (x^3 + 1)^{-\frac{3}{4}} \cdot (x^3 + 1)' = \frac{3}{4} x^2 (x^3 + 1)^{-\frac{3}{4}}$.
- (10) $y' = n(ax + b)^{n-1} (ax + b)' = an(ax + b)^{n-1}$.
- (11) $f'(x) = (e^{\log a^x})' = (e^{x \log a})'$
 $= e^{x \log a} \cdot (x \log a)' = a^x \log a$.
- (12) $f'(x) = \left(\frac{\log x}{\log a} \right)' = \frac{1}{x \log a}$.

問題 4.

- (1) $\log y = x \log x$ より, 両辺を x で微分すると

$$\frac{y'}{y} = (x)' \log x + x (\log x)' = \log x + 1.$$

$$\text{よって, } y' = y(\log x + 1) = x^x (\log x + 1).$$

- (2) $\log y = \log x^{\log x} = (\log x)^2$ より, 両辺を x で微分すると

$$\frac{y'}{y} = 2 \log x \cdot (\log x)' = \frac{2 \log x}{x}.$$

$$\text{よって, } y' = \frac{2 \log x}{x} y = \frac{2x^{\log x} \log x}{x}.$$

(3) $\log y = \frac{1}{2} \log(x^2 + 1) - \frac{1}{2} \log(x^2 + 2)$ より, 両辺を x で微分すると

$$\frac{y'}{y} = \frac{x}{x^2 + 1} - \frac{x}{x^2 + 2} = \frac{x}{(x^2 + 1)(x^2 + 2)}.$$

よって,

$$\begin{aligned} y' &= \frac{x}{(x^2 + 1)(x^2 + 2)} y \\ &= \frac{x}{\sqrt{(x^2 + 1)(x^2 + 2)(x^2 + 2)}}. \end{aligned}$$

問題 5.

$$(1) y' = \frac{1}{2} (4 - x^2)^{-\frac{1}{2}} (4 - x^2)' = -\frac{x}{\sqrt{4 - x^2}}.$$

$$\begin{aligned} (2) y' &= -\frac{1}{(\tan x + 1)^2} \cdot (\tan x + 1)' \\ &= -\frac{1}{(\tan x + 1)^2} \cdot \frac{1}{\cos^2 x} \\ &= -\frac{1}{(\sin x + \cos x)^2}. \end{aligned}$$

$$(3) y' = -\sin \frac{1}{x} \cdot \left(\frac{1}{x}\right)' = \frac{1}{x^2} \sin \frac{1}{x}.$$

$$\begin{aligned} (4) y' &= 4(e^x + 1)^3 (e^x + 1)' - 2 \cdot 2(e^x + 1)(e^x + 1)' \\ &= 4e^x (e^x + 1)^3 - 4e^x (e^x + 1) \\ &= (4e^{2x} (e^x + 1)(e^x + 2)). \end{aligned}$$

$$\begin{aligned} (5) y' &= \frac{1}{\frac{x-1}{x+1}} \left(\frac{x-1}{x+1}\right)' \\ &= \frac{x+1}{x-1} \cdot \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2} \\ &= \frac{x+1}{x-1} \cdot \frac{2}{(x+1)^2} \\ &= \frac{2}{(x+1)(x-1)}. \end{aligned}$$

問題 6.

$$\begin{aligned} (1) y' &= \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \left(\frac{x}{a}\right)' \\ &= \frac{1}{\sqrt{1 - \left(\frac{x}{a}\right)^2}} \cdot \frac{1}{a} = \frac{1}{\sqrt{a^2 - x^2}}. \end{aligned}$$

$$\begin{aligned} (2) y' &= -\frac{1}{\sqrt{1-x}} \cdot (\sqrt{x})' \\ &= -\frac{1}{\sqrt{1-x}} \cdot \frac{1}{\sqrt{x}} = -\frac{1}{\sqrt{x(x-1)}}. \end{aligned}$$

$$\begin{aligned} (3) y' &= \frac{1}{a} \cdot \frac{1}{1 + \left(\frac{x}{a}\right)^2} \cdot \left(\frac{x}{a}\right)' \\ &= \frac{1}{a^2 + x^2}. \end{aligned}$$

$$\begin{aligned} (4) y' &= (x)' \arctan x + x(\arctan x)' \\ &= \arctan x + \frac{x}{1 + x^2}. \end{aligned}$$

問題 7. $F = \frac{1}{2} \begin{pmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{pmatrix}$ である.

$$(1) \text{ 求める点は } F \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{2\sqrt{3}+3}{2} \\ \frac{-2+3\sqrt{3}}{2} \end{pmatrix}.$$

$$(2) \text{ (i) 求める点は } F \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix},$$

$$F \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}+4}{2} \\ \frac{-1+4\sqrt{3}}{2} \end{pmatrix}$$

(ii) 求める直線の方程式を $y = ax + b$ とおくと,
(i) より連立方程式

$$\begin{cases} \frac{\sqrt{3}}{2} = \frac{1}{2}a + b \\ \frac{-1+4\sqrt{3}}{2} = \frac{\sqrt{3}+4}{2}a + b \end{cases}$$

を得る. これを解くと $a = \frac{-6+5\sqrt{3}}{3}$, $b = \frac{3-\sqrt{3}}{3}$. よって求める直線の方程式は $y = \frac{-6+5\sqrt{3}}{3}x + \frac{3-\sqrt{3}}{3}$.