

第3回リメディアル数学 (化学システム工学科) 2023/5/10 略解

問題 1.

$$(1) \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{10}{n}} = 1.$$

$$(2) \lim_{n \rightarrow \infty} n^3 \left(-4 + \frac{7}{n} \right) = -\infty.$$

$$(3) \lim_{n \rightarrow \infty} \frac{4 + \frac{3}{n} - \frac{4}{n^2}}{2 + \frac{3}{n^2}} = 2.$$

$$(4) \lim_{n \rightarrow \infty} \frac{n^2 + n + 1}{(1 - n)(1 + n + n^2)} = \lim_{n \rightarrow \infty} \frac{1}{1 - n} = 0.$$

$$(5) \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0.$$

$$(6) \lim_{n \rightarrow \infty} \frac{n + \sqrt{n^2 + 2n}}{-2n} = \lim_{n \rightarrow \infty} \frac{1 + \sqrt{1 + \frac{2}{n}}}{-2} = -1.$$

(7) 部分分数分解より

$$\begin{aligned} a_n &= \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) \\ &= \left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1} \right) \\ &= 1 - \frac{1}{n+1} \end{aligned}$$

$$\text{とわかるので, } \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = 1.$$

$$(8) \lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} (5n - 3)a_n \cdot \frac{1}{5n - 3} = 3 \cdot 0 = 0.$$

また, 線形性より

$$\begin{aligned} 3 &= \lim_{n \rightarrow \infty} (5n - 3)a_n \\ &= 5 \left(\lim_{n \rightarrow \infty} na_n \right) - 3 \left(\lim_{n \rightarrow \infty} a_n \right) \\ &= 5 \lim_{n \rightarrow \infty} na_n. \end{aligned}$$

$$\text{よって } \lim_{n \rightarrow \infty} na_n = \frac{3}{5}.$$

問題 2.

$$(1) -1.$$

$$(2) \lim_{x \rightarrow +\infty} \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{5x - 3} = 0.$$

$$(3) \lim_{x \rightarrow +\infty} \frac{2 + \frac{5}{x^2}}{5 + \frac{1}{x} + \frac{7}{x^2}} = \frac{2}{5}$$

$$(4) \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{x-3} = \lim_{x \rightarrow 3} (x+2) = 5.$$

$$(5) \lim_{x \rightarrow 1} \frac{(x-1)(2x-3)}{(x-1)(x-3)} = \lim_{x \rightarrow 1} \frac{2x-3}{x-3} = \frac{1}{2}.$$

$$(6) \lim_{x \rightarrow 0} \frac{-1}{3(x+3)} = -\frac{1}{9}.$$

$$\begin{aligned} (7) \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{1+x} + \sqrt{1-x})} \\ = \lim_{x \rightarrow 0} \frac{2}{\sqrt{1+x} + \sqrt{1-x}} = 1. \end{aligned}$$

$$(8) 0.$$

$$(9) +\infty.$$

$$(10) \lim_{x \rightarrow 0} 3 \cdot \frac{\sin 3x}{3x} = 3.$$

$$(11) \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \cdot \frac{1}{1 + \cos x} = \frac{1}{2}.$$

$$\begin{aligned} (12) \quad 0 \leq \left| x^2 \sin \frac{1}{x} \right| &\leq |x^2| \text{ より, はさみうちの原理から} \\ \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} &= 0. \end{aligned}$$

問題 3.

$$\lim_{x \rightarrow 1-0} f(x) = \lim_{x \rightarrow 1-0} (2x^2 + 1) = 3,$$

$$\lim_{x \rightarrow 1+0} f(x) = \lim_{x \rightarrow 1+0} 3x^2 = 3,$$

$$f(1) = 3$$

より, $f(x)$ は $x = 1$ で連続.

問題 4.

$$(1) f(x) = x^3,$$

$$f(x+h) = (x+h)^3 = x^3 + 3hx^2 + 3h^2x + h^3.$$

$$\begin{aligned} (2) \quad \frac{\Delta y}{\Delta x} &= \frac{f(a+h) - f(a)}{(a+h) - a} \\ &= \frac{3ha^2 + 3h^2a + h^3}{h} = 3a^2 + 3ha + h^2. \end{aligned}$$

$$(3) f'(a) = \lim_{h \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{h \rightarrow 0} (3a^2 + 3ha + h^2) = 3a^2.$$

問題 5.

$$(1) f'(x) = 6x^2 + 10x - 1.$$

$$(2) f'(x) = \sin x + x \cos x.$$

$$\begin{aligned}(3) \quad f'(x) &= (2^x)' \cos x + 2^x (\cos x)' \\ &= 2^x \log 2 \cos x - 2^x \sin x.\end{aligned}$$

$$(4) \quad f'(x) = (2 \log x)' = \frac{2}{x}.$$

$$\begin{aligned}(5) \quad (\tan x)' &= \frac{(\sin x)' \cos x - \sin x (\cos x)'}{\cos^2 x} \\ &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}.\end{aligned}$$