

第1回リメディアル数学 (化学システム工学科) 2023/4/19 略解

指数・対数

問題 1.

$$(1) 3^0 = 1.$$

$$(2) 3^{-3} = \frac{1}{27}.$$

$$(3) \left(\frac{1}{3}\right)^5 = \frac{1}{273}.$$

$$(4) 2^6 \times 2^{-3} = 2^{6+(-3)} = 2^3 = 8.$$

$$(5) 5^8 \div 5^5 = 5^{8-5} = 5^3 = 125.$$

$$(6) (2^{-2})^3 = 2^{-2 \times 3} = 2^{-6} = \frac{1}{64}.$$

$$(7) (2 \times 3^4)^{-2} = 2^{-2} \times 3^{-8} = \frac{1}{26244}.$$

問題 2.

$$(1) 11^{\frac{2}{3}} = \sqrt[3]{121}.$$

$$(2) 5^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{5}}.$$

$$(3) \sqrt[3]{24} = 2\sqrt[3]{3}.$$

$$(4) (\sqrt[3]{25})^2 = 5\sqrt[3]{5}.$$

$$(5) \sqrt[3]{18}\sqrt[3]{15} = \sqrt[3]{2 \times 3^2}\sqrt[3]{3 \times 5} = 3\sqrt[3]{10}.$$

$$(6) \sqrt[5]{32} = 2.$$

$$(7) \sqrt[4]{\frac{1}{81}} = \frac{1}{2}.$$

$$(8) \frac{\sqrt[4]{273}}{\sqrt[4]{3}} = \sqrt[4]{81} = 3.$$

$$(9) \sqrt[3]{\sqrt[4]{24}} = 24^{\frac{1}{12}}.$$

$$(10) 13^{\frac{1}{2}} \div 13^{\frac{5}{6}} \times 13^{\frac{1}{3}} = 13^{\frac{1}{2} - \frac{5}{6} + \frac{1}{3}} = 13^0 = 1.$$

$$(11) \sqrt[3]{169} \div \sqrt[12]{169} \times \sqrt[4]{169} = 13^{\frac{2}{3} - \frac{1}{6} + \frac{1}{2}} = 13.$$

問題 3.

$$(1) \log_3 27 = 3.$$

$$(2) \log_5 \frac{1}{125} = \frac{1}{3}.$$

$$(3) \log_{691} 1 = 0.$$

$$(4) \log_{\frac{1}{2}} \frac{1}{32} = 5.$$

$$(5) \log_{10} 0.01 = -2.$$

$$(6) \log_{\frac{1}{4}} 2 = -\frac{1}{2}.$$

$$(7) \log_5 \sqrt[4]{5} = \frac{1}{4}.$$

$$(8) \log_{\sqrt{3}} 9 = \frac{\log_3 9}{\log_3 \sqrt{3}} = \frac{2}{\frac{1}{2}} = 4.$$

$$(9) \log_{15} 3 + \log_{15} 5 = \log_{15} (3 \times 5) = 1.$$

$$(10) \log_2 40 - \log_2 5 = \log_2 \frac{40}{5} = 3.$$

$$(11) \log_5 24 - 3 \log_5 6 + \log_5 54 \\ = \log_5 \frac{24 \times 54}{6^3} = \log_5 6.$$

$$(12) \log_3 5 \times \log_5 27 = \log_3 5 \times \frac{\log_3 27}{\log_3 5} = 3.$$

三角関数の値

問題 4.

$$(1) 150^\circ = \frac{5}{6}\pi.$$

$$(2) 300^\circ = \frac{5}{3}\pi.$$

$$(3) \pi = 180^\circ.$$

$$(4) \frac{11}{6}\pi = 330^\circ.$$

問題 5.

$$(1) \theta = \frac{\pi}{3}$$

$$\sin \theta = \frac{\sqrt{3}}{2}, \cos \theta = \frac{1}{2}, \tan \theta = \sqrt{3}.$$

$$(2) \theta = \frac{7}{4}\pi$$

$$\sin \theta = -\frac{1}{\sqrt{2}}, \cos \theta = \frac{1}{\sqrt{2}}, \tan \theta = -1.$$

$$(3) \theta = \pi$$

$$\sin \theta = 0, \cos \theta = -1, \tan \theta = 0.$$

方程式

問題 6.

(1) $x = \pm i$.

(2) $x = \pm \frac{\sqrt{6}}{4}$.

(3) 解の公式より $x = \frac{-2 \pm \sqrt{2}i}{3}$.

(4) 解の公式より $x = \sqrt{5} \pm i$.

(5) $x^4 + x^2 - 12 = (x^2 + 4)(x^2 - 3)$ より $x = \pm 2i, \pm \sqrt{3}$.

(6) $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$ より $x = 2, -1 \pm \sqrt{3}i$.

(7) $x^3 - 4x^2 + 2x + 3 = (x - 3)(x^2 - x - 1)$ より
 $x = 3, \frac{1 \pm \sqrt{3}}{2}$.

(8) $x^4 + x^2 + 1 = (x^2 + 1)^2 - x^2$
 $= (x^2 + x + 1)(x^2 - x + 1)$ より
 $x = \frac{-1 \pm \sqrt{3}i}{2}, \frac{1 \pm \sqrt{3}i}{2}$.

問題 7.

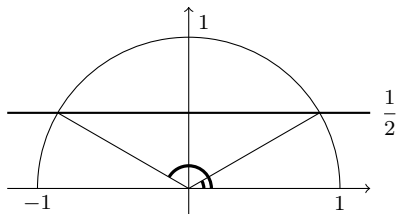
(1) $3^{3x} = 3^2$ より $3x = 2$, つまり $x = \frac{2}{3}$.

(2) $2^{2x} = 2^{x+1}$ より $2x = x + 1$, つまり $x = 1$.

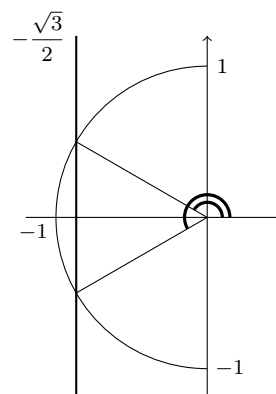
(3) $\log_3 x = 2 \iff x = 9$.

(4) $\log_{\frac{1}{4}} x = -\frac{3}{2} \iff x = \left(\frac{1}{4}\right)^{-\frac{3}{2}} = 8$.

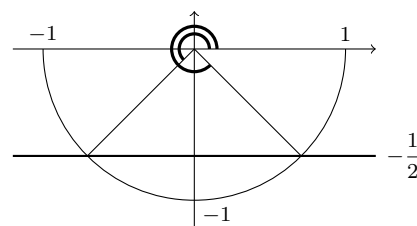
(5) $\sin \theta = \frac{1}{2}$ より, $\theta = \frac{\pi}{6}, \frac{5}{6}\pi$.



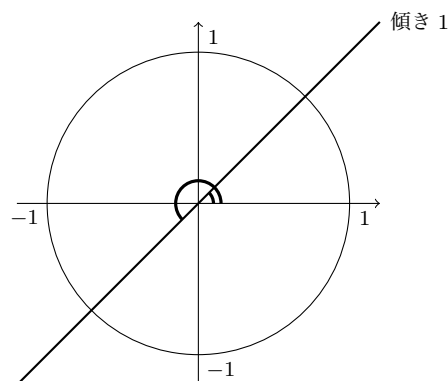
(6) $\cos \theta = -\frac{\sqrt{3}}{2}$ より, $\theta = \frac{5}{6}\pi, \frac{7}{6}\pi$.



(7) $\sin \theta = -\frac{1}{\sqrt{2}}$ より, $\theta = \frac{5}{4}\pi, \frac{3}{4}\pi$.



(8) $\theta = \frac{\pi}{4}, \frac{5}{4}\pi$.



問題 8. 求める時間を x 時間後とすると, $2^{2x} > 2 \times 10^5$. 両辺の対数を取ると, $2x \log_{10} 2 > 5 + \log_{10} 2$. よって

$$x > \frac{5 + \log_{10} 2}{2 \log_{10} 2} = \frac{5}{2 \log_{10} 2} + \frac{1}{2} = 8.80 \dots \approx 8.8.$$