

技術者リテラシー I (機械工学科) ——— 第3回 2023/10/11 略解

問題 1.

$$(1) \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{10}{n}} = 1.$$

$$(2) \lim_{n \rightarrow \infty} n^3 \left(-4 + \frac{7}{n} \right) = -\infty.$$

$$(3) \lim_{n \rightarrow \infty} \frac{4 + \frac{3}{n} - \frac{4}{n^2}}{2 + \frac{3}{n^2}} = 2.$$

$$(4) \lim_{n \rightarrow \infty} \frac{n^2 + n + 1}{(1 - n)(1 + n + n^2)} = \lim_{n \rightarrow \infty} \frac{1}{1 - n} = 0.$$

$$(5) \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0.$$

$$(6) \lim_{n \rightarrow \infty} \frac{n + \sqrt{n^2 + 2n}}{-2n} = \lim_{n \rightarrow \infty} \frac{1 + \sqrt{1 + \frac{2}{n}}}{-2} = -1.$$

(7) 部分分数分解より

$$\begin{aligned} a_n &= \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) \\ &= \left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1} \right) \\ &= 1 - \frac{1}{n+1} \end{aligned}$$

$$\text{とわかるので, } \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n+1} \right) = 1.$$

問題 2.

$$(1) -1.$$

$$(2) \lim_{x \rightarrow +\infty} \frac{1 - \frac{1}{x} + \frac{1}{x^2}}{5x - 3} = 0.$$

$$(3) \lim_{x \rightarrow +\infty} \frac{2 + \frac{5}{x^2}}{5 + \frac{1}{x} + \frac{7}{x^2}} = \frac{2}{5}$$

$$(4) \lim_{x \rightarrow 3} \frac{(x-3)(x+2)}{x-3} = \lim_{x \rightarrow 3} (x+2) = 5.$$

$$(5) \lim_{x \rightarrow 1} \frac{(x-1)(2x-3)}{(x-1)(x-3)} = \lim_{x \rightarrow 1} \frac{2x-3}{x-3} = \frac{1}{2}.$$

$$(6) \lim_{x \rightarrow 0} \frac{-1}{3(x+3)} = -\frac{1}{9}.$$

$$\begin{aligned} (7) \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{1+x} + \sqrt{1-x})} \\ = \lim_{x \rightarrow 0} \frac{2}{\sqrt{1+x} + \sqrt{1-x}} = 1. \end{aligned}$$

$$(8) 0.$$

$$(9) +\infty.$$

$$(10) \lim_{x \rightarrow 0} 3 \cdot \frac{\sin 3x}{3x} = 3.$$

$$(11) \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2(1 + \cos x)} = \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \cdot \frac{1}{1 + \cos x} = \frac{1}{2}.$$

$$(12) 0 \leq \left| x^2 \sin \frac{1}{x} \right| \leq |x^2| \text{ より, はさみうちの原理から} \\ \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0.$$

問題 3.

$$\lim_{x \rightarrow 1-0} f(x) = \lim_{x \rightarrow 1-0} (2x^2 + 1) = 3,$$

$$\lim_{x \rightarrow 1+0} f(x) = \lim_{x \rightarrow 1+0} 3x^2 = 3,$$

$$f(1) = 3$$

より, $f(x)$ は $x = 1$ で連続.

問題 4. $\lim_{x \rightarrow -0} \frac{1}{x} = -\infty$, $\lim_{x \rightarrow +0} \frac{1}{x} = +\infty$ であることに注意すると,

$$\lim_{x \rightarrow -0} f(x) = \lim_{x \rightarrow -0} \frac{1}{1 + e^{1/x}} = 1$$

$$\lim_{x \rightarrow +0} f(x) = \lim_{x \rightarrow +0} \frac{1}{1 + e^{1/x}} = 0$$

より, $\lim_{x \rightarrow 0} f(x)$ は存在しない. よって $f(x)$ は $x = 0$ で連続でない.

$$\begin{aligned} \text{問題 5. } 3X = A + 2B &= \begin{pmatrix} 2 & -1 \\ 1 & -6 \end{pmatrix} + 2 \begin{pmatrix} 5 & -7 \\ 4 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 12 & -15 \\ 9 & 0 \end{pmatrix}. \end{aligned}$$

$$\text{よって } X = \frac{1}{3} \begin{pmatrix} 12 & -15 \\ 9 & 0 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 3 & 0 \end{pmatrix}.$$

問題 6.

(1) 定義できない.

$$(2) \text{定義できる. } AB = \begin{pmatrix} 27 & 44 & 3 & 6 \\ 6 & 24 & 6 & -12 \\ 29 & 36 & -1 & 17 \end{pmatrix}.$$

$$(3) \text{定義できる. } AB = \begin{pmatrix} 13 & 9 & 8 \\ 36 & 13 & 27 \end{pmatrix}.$$

$$(4) \text{定義できる. } AB = \begin{pmatrix} 10 & -15 \\ 8 & -12 \end{pmatrix}.$$

問題 7.

$$\begin{aligned} A^2 &= \begin{pmatrix} 2 \cdot 2 + 1 \cdot (-4) & 2 \cdot 1 + 1 \cdot (-2) \\ -4 \cdot 2 + (-2) \cdot (-4) & -4 \cdot 1 + (-2) \cdot (-2) \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} AB &= \begin{pmatrix} 2 \cdot 1 + 1 \cdot 0 & 2 \cdot (-1) + 1 \cdot 3 \\ -4 \cdot 1 + (-2) \cdot 0 & (-4) \cdot (-1) + (-2) \cdot 3 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 1 \\ -4 & -2 \end{pmatrix}. \end{aligned}$$

$$\begin{aligned} BA &= \begin{pmatrix} 1 \cdot 2 + (-1) \cdot (-4) & 1 \cdot 1 + (-1) \cdot (-2) \\ 0 \cdot (-2) + 3 \cdot (-4) & 0 \cdot 1 + 3 \cdot (-2) \end{pmatrix} \\ &= \begin{pmatrix} 6 & 3 \\ -12 & -6 \end{pmatrix}. \end{aligned}$$

問題 8. $AB = \begin{pmatrix} -3 & 6 & -6 \\ 0 & 5 & 1 \\ 8 & -8 & 18 \end{pmatrix}, BA = \begin{pmatrix} 1 & 6 & 4 \\ -4 & 15 & 5 \\ 4 & 2 & 4 \end{pmatrix}.$