第 4 回リメディアル数学 (化学システム工学科) 2023/5/17 略解

問題 1.

(1) $y' = 5x^4 + 8x^3$.

(2) $y' = (3x^2-2)'(x^2+x+1) + (3x^2-2)(x^2+x+1)'$

$= 6x(x^2+x+1) + (3x^2-2)(2x+1)$

$= 12x^3 + 9x^2 + 2x - 2$.

(3) $y' = \dfrac{(x^2)'(x+3) - x^2(x+3)7}{(x+3)^2}$

$= \dfrac{2x(x+3) - x^2}{(x+3)^2}$

$= \dfrac{x^2+6x}{(x+3)^2} \quad \left(= \dfrac{x(x+6)}{(x+3)^2}\right)$.

(4) $y' = \dfrac{8}{x^3}$.

(5) $y' = \dfrac{(\cos x)' \sin x - \cos x (\sin x)'}{\sin^2 x}$

$= \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$

$= -\dfrac{1}{\sin^2 x}$.

(6) $y' = (e^x)' \sin x + e^x (\sin x)'$

$= e^x \sin x + e^x \cos x \quad (= e^x(\sin x + \cos x))$.

(7) $y' = (x)' \log x + x(\log x)' = \log x + 1$.

問題 2.

(1) $y' = 4(3x+1)^3(3x+1)' = 12(3x+1)^3$.

(2) $y' = 3(2x^2+5)^2(2x^2+5)' = 12x(2x^2+5)$.

(3) $y' = -\dfrac{3}{(x+1)^4} \cdot (x^2+1)' = -\dfrac{6x}{(x^2+1)^4}$.

(4) $y' = \sin\left(\dfrac{\pi}{4} - 2x\right) \cdot \left(\dfrac{\pi}{4} - 2x\right)' = 2\sin\left(\dfrac{\pi}{4} - 2x\right)$.

(5) $y' = 2\sin x (\sin)' = 2\sin x \cos x \quad (= \sin 2x)$.

(6) $y' = -\dfrac{1}{\tan^2 x} \cdot (\tan x)' = -\dfrac{1}{\tan^2 x} \cdot \dfrac{1}{\cos^2 x}$

$= -\dfrac{1}{\sin^2 x}$. [1]

(7) $y' = \dfrac{1}{x^2+1} \cdot (x^2+1)' = \dfrac{2x}{x^2+1}$.

[1] 問題 1(5) の別解になっている.

(8) $y' = \dfrac{1}{\sin x} \cdot (\sin x)' = \dfrac{\cos x}{\sin x} \quad \left(= \dfrac{1}{\tan x}\right)$.

(9) $y' = \dfrac{1}{\log 2} \cdot \dfrac{1}{x^2-4} \cdot (x^2-4)' = \dfrac{2x}{(x^2-4)\log 2}$.

(10) $y' = e^{-x^2} \cdot (-x^2)' = -2xe^{-x^2}$.

(11) $y' = e^{\sin x} (\sin x)' = e^{\sin x} \cos x$.

(12) $y' = \log 2 \cdot 2^{-2x} \cdot (-2x)' = -2^{-2x+1} \log 2$.

(13) $y' = \dfrac{1}{4}(x^3+1)^{-\frac{3}{4}} \cdot (x^3+1)' = \dfrac{3}{4}x^2(x^3+1)^{-\frac{3}{4}}$.

(14) $y' = n(ax+b)^{n-1}(ax+b)' = an(ax+b)^{n-1}$.

問題 3.

(1) $\log y = x \log x$ より, 両辺を x で微分すると

$$\frac{y'}{y} = (x)' \log x + x(\log x)' = \log x + 1.$$

よって, $y' = y(\log x + 1) = x^x(\log x + 1)$.

(2) $\log y = \log x^{\log x} = (\log x)^2$ より, 両辺を x で微分すると

$$\frac{y'}{y} = 2\log x \cdot (\log x)' = \frac{2\log x}{x}.$$

よって, $y' = \dfrac{2\log x}{x} y = \dfrac{2x^{\log x} \log x}{x}$.

(3) $\log y = \dfrac{1}{2}\log(x^2+1) - \dfrac{1}{2}\log(x^2+2)$ より, 両辺を x で微分すると

$$\frac{y'}{y} = \frac{x}{x^2+1} - \frac{x}{x^2+2} = \frac{x}{(x^2+1)(x^2+2)}.$$

よって,

$$y' = \frac{x}{(x^2+1)(x^2+2)} y = \frac{x}{\sqrt{(x^2+1)(x^2+2)}(x^2+2)}.$$

問題 4.

(1) $y' = \dfrac{1}{2}(4-x^2)^{-\frac{1}{2}}(4-x^2)' = -\dfrac{x}{\sqrt{4-x^2}}$.

(2) $y' = -\dfrac{1}{(\tan x+1)^2} \cdot (\tan x+1)'$

$= -\dfrac{1}{(\tan x+1)^2} \cdot \dfrac{1}{\cos^2 x}$

$= -\dfrac{1}{(\sin x + \cos x)^2}$.

(3) $y' = -\sin\dfrac{1}{x}\cdot\left(\dfrac{1}{x}\right)' = \dfrac{1}{x^2}\sin\dfrac{1}{x}$.

(4) $y' = 4(e^x+1)^3(e^x+1)' - 2\cdot 2(e^x+1)(e^x+1)'$

$= 4e^x(e^x+1)^3 - 4e^x(e^x+1)$

$(= 4e^{2x}(e^x+1)(e^x+2))$.

(5) $y' = \dfrac{1}{\frac{x-1}{x+1}}\left(\dfrac{x-1}{x+1}\right)'$

$= \dfrac{x+1}{x-1}\cdot\dfrac{(x-1)'(x+1)-(x-1)(x+1)'}{(x+1)^2}$

$= \dfrac{x+1}{x-1}\cdot\dfrac{2}{(x+1)^2}$

$= \dfrac{2}{(x+1)(x-1)}$.

問題 5.

(1) $y' = \dfrac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\left(\dfrac{x}{a}\right)'$

$= \dfrac{1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\dfrac{1}{a} = \dfrac{1}{\sqrt{a^2-x^2}}$.

(2) $y' = -\dfrac{1}{\sqrt{1-x}}\cdot(\sqrt{x})'$

$= -\dfrac{1}{\sqrt{1-x}}\cdot\dfrac{1}{\sqrt{x}} = -\dfrac{1}{\sqrt{x(x-1)}}$.

(3) $y' = \dfrac{1}{a}\cdot\dfrac{1}{1+\left(\frac{x}{a}\right)^2}\cdot\left(\dfrac{x}{a}\right)'$

$= \dfrac{1}{a^2+x^2}$.

(4) $y' = (x)'\arctan x + x(\arctan x)'$

$= \arctan x + \dfrac{x}{1+x^2}$.

問題 6. $3X = A + 2B = \begin{pmatrix}2 & -1\\ 1 & -6\end{pmatrix} + 2\begin{pmatrix}5 & -7\\ 4 & 3\end{pmatrix}$

$= \begin{pmatrix}12 & -15\\ 9 & 0\end{pmatrix}$.

よって $X = \begin{pmatrix}4 & -5\\ 3 & 0\end{pmatrix}$.

問題 7.

$A^2 = \begin{pmatrix}2\cdot 2+1\cdot(-4) & 2\cdot 1+1\cdot(-2)\\ -4\cdot 2+(-2)\cdot(-4) & -4\cdot 1+(-2)\cdot(-2)\end{pmatrix}$

$= \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$.

$AB = \begin{pmatrix}2\cdot 1+1\cdot 0 & 2\cdot(-1)+1\cdot 3\\ -4\cdot 1+(-2)\cdot 0 & (-4)\cdot(-1)+(-2)\cdot 3\end{pmatrix}$

$= \begin{pmatrix}2 & 1\\ -4 & -2\end{pmatrix}$.

$BA = \begin{pmatrix}1\cdot 2+(-1)\cdot(-4) & 1\cdot 1+(-1)\cdot(-2)\\ 0\cdot(-2)+3\cdot(-4) & 0\cdot 1+3\cdot(-2)\end{pmatrix}$

$= \begin{pmatrix}6 & 3\\ -12 & -6\end{pmatrix}$.