技術者リテラシー I (機械工学科) —— 第 8 回 2024/11/13 略解

問題 1. C を積分定数とする.

(1) $\displaystyle\int \left(\frac{1}{3}x^3\right)' \log x\ dx = \frac{1}{3}x^3 \log x - \int \frac{1}{3}x^3 (\log x)'\ dx$

$\displaystyle= \frac{1}{3}x^3 \log x - \frac{1}{3}\int x^2\ dx$

$\displaystyle= \frac{1}{3}x^3 \log x - \frac{1}{9}x^3 + C.$

(2) $\displaystyle\int x(-\cos x)'\ dx = -x\cos x + \int (x)' \cos x\ dx$

$\displaystyle= -x\cos x + \int \cos x\ dx$

$= -x\cos x + \sin x + C.$

(3) $\displaystyle\int x\left(-\frac{1}{2}\cos 2x\right)' dx = -\frac{1}{2}x\cos 2x + \frac{1}{2}\int (x)' \cos 2x\ dx$

$\displaystyle= -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x\ dx$

$\displaystyle= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C.$

(4) $\displaystyle\int x^2(\sin x)'\ dx = x^2\sin x - \int (x^2)'\sin x\ dx$

$\displaystyle= x^2 \sin x - 2\int x\sin x\ dx$

$= x^2\sin x + 2x\cos x - 2\sin x + C \quad (\because (2)).$

(5) $\displaystyle\int x(e^x)'\ dx = xe^x - \int (x)' e^x\ dx$

$\displaystyle= xe^x - \int e^x\ dx$

$= xe^x - e^x + C.$

(6) $\displaystyle\int \left(\frac{1}{2}x^2\right)' \log x\ dx = \frac{1}{2}x^2 \log x - \int \frac{1}{2}x^2 (\log x)'\ dx$

$\displaystyle= \frac{1}{2}x^2\log x - \frac{1}{2}\int x\ dx$

$\displaystyle= \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + C.$

問題 2.

(1) $\displaystyle\int_0^{\frac{\pi}{2}} x(\sin x)'\ dx = \Big[x\sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\ dx$

$\displaystyle= \frac{\pi}{2} - \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1.$

(2) $\displaystyle\int_0^1 x(-e^{-x})'\ dx = \Big[-xe^{-x}\Big]_0^1 + \int_0^1 e^{-x}\ dx$

$= -e^{-1} + \left[-e^{-x}\right]_0^1 = 1 - 2e^{-1}.$

(3) $\displaystyle\int_1^e \left(\frac{1}{2}x^2\right)' \log x\ dx = \left[\frac{1}{2}x^2\log x\right]_1^e - \frac{1}{2}\int_1^e x\ dx$

$\displaystyle= \frac{1}{2}e^2 - \left[\frac{1}{4}x^2\right]_1^e = \frac{1}{4}(e^2+1).$

(4) $\displaystyle\int_1^e (x)'(\log x)^2\ dx = \Big[x(\log x)^2\Big]_1^e - \int_1^e x\cdot\frac{2\log x}{x}\ dx$

$\displaystyle= e - 2\int_1^e \log x\ dx$

$= e - 2\Big[x\log x - x\Big]_1^e = e - 2.$

問題 3.

(1) $\displaystyle I = \int_0^{\frac{\pi}{2}} (e^x)' \sin x\ dx$

$\displaystyle= \Big[e^x \sin x\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x\ dx = e^{\frac{\pi}{2}} - J.$

よって $I + J = e^{\frac{\pi}{2}}$. 一方で,

$\displaystyle I = \int_0^{\frac{\pi}{2}} e^x(-\cos x)'\ dx$

$\displaystyle= \Big[-e^x\cos x\Big]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x \cos x\ dx = 1 + J.$

よって $I - J = 1$.

(2) (1) より $I = \dfrac{1}{2}(e^{\frac{\pi}{2}}+1)$, $J = \dfrac{1}{2}(e^{\frac{\pi}{2}}-1)$.

問題 4.

(1) $I_0 = \left[x\right]_0^{\frac{\pi}{2}} = \dfrac{\pi}{2}$, $I_1 = \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = 1.$

(2) 部分積分を行うと

$\displaystyle I_n = \int_0^{\frac{\pi}{2}} (-\cos x)' \sin^{n-1} x\ dx$

$\displaystyle= \Big[-\cos x \sin^{n-1} x\Big]_0^{\frac{\pi}{2}} + (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x\ dx$

$\displaystyle= (n-1)\int_0^{\frac{\pi}{2}} (\sin^{n-2} x - \sin^n x)\ dx$

$= (n-1)(I_{n-2} - I_n).$

よって, $I_n = \dfrac{n-1}{n} I_{n-2}.$

(3) n の偶奇で場合分けする.

(i) n が偶数のとき, $n=2k$ (k は自然数) と表せるので,

$$\begin{aligned}
I_n &= \frac{n-1}{n}I_{n-2} = \frac{n-1}{n}\frac{n-3}{n-2}I_{n-4} \\
&= \cdots \\
&= \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{5}{6}\cdot\frac{3}{4}I_0 \\
&= \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{5}{6}\cdot\frac{3}{4}\cdot\frac{\pi}{2}.
\end{aligned}$$

(ii) n が奇数のとき, $n=2k-1$ (k は自然数) と表せるので,

$$\begin{aligned}
I_n &= \frac{n-1}{n}I_{n-2} = \frac{n-1}{n}\frac{n-3}{n-2}I_{n-4} \\
&= \cdots \\
&= \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{4}{5}\cdot\frac{2}{3}I_1 \\
&= \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{4}{5}\cdot\frac{2}{3}.
\end{aligned}$$

問題 5. 以下, ⓝと書いて行列の n 行目を表すものとする.

(1)
$$\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{①\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{③+①\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{②\leftrightarrow③} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{③+②\times1} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right)$$

$$\xrightarrow{①+③\times1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right).$$

よって, $A^{-1} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

(2)
$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 4 & 4 & 0 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[③+①\times(-1)]{②+①\times(-2)} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{②\leftrightarrow③} \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{①+②\times(-2)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array}\right)$$

$$\xrightarrow{③\times\frac{1}{2}} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array}\right)$$

$$\xrightarrow[②+③\times(-1)]{①+③\times1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & \frac{1}{2} & -2 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array}\right).$$

よって, $A^{-1} = \begin{pmatrix} 2 & \frac{1}{2} & -2 \\ 0 & -\frac{1}{2} & 1 \\ -1 & \frac{1}{2} & 0 \end{pmatrix}$.

(3)
$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 4 & 0 & 0 & 1 & 0 \\ 2 & 3 & 4 & 5 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{③+①\times(-2) \\ ④+①\times(-2)}]{②+①\times(-1)} \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{③+②\times(-1) \\ ④+②\times(-1)}]{①+②\times(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[④+③\times(-1)]{②+③\times(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{③+④\times(-1)} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array}\right).$$

よって, $A^{-1} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$.