

リメディアル数学 (化学システム工学科) ——— 第8回 2024/6/12 略解

問題 1. C を積分定数とする.

$$\begin{aligned}(1) \quad \int \left(\frac{1}{3}x^3\right)' \log x \, dx &= \frac{1}{3}x^3 \log x - \int \frac{1}{3}x^3 (\log x)' \, dx \\ &= \frac{1}{3}x^3 \log x - \frac{1}{3} \int x^2 \, dx \\ &= \frac{1}{3}x^3 \log x - \frac{1}{9}x^3 + C.\end{aligned}$$

$$\begin{aligned}(2) \quad \int x(-\cos x)' \, dx &= -x \cos x + \int (x)' \cos x \, dx \\ &= -x \cos x + \int \cos x \, dx \\ &= -x \cos x + \sin x + C.\end{aligned}$$

$$\begin{aligned}(3) \quad \int x \left(-\frac{1}{2} \cos 2x\right)' \, dx &= -\frac{1}{2}x \cos 2x + \frac{1}{2} \int (x)' \cos 2x \, dx \\ &= -\frac{1}{2}x \cos 2x + \frac{1}{2} \int \cos 2x \, dx \\ &= -\frac{1}{2}x \cos 2x + \frac{1}{4} \sin 2x + C.\end{aligned}$$

$$\begin{aligned}(4) \quad \int x^2 (\sin x)' \, dx &= x^2 \sin x - \int (x^2)' \sin x \, dx \\ &= x^2 \sin x - 2 \int x \sin x \, dx \\ &= x^2 \sin x + 2x \cos x - 2 \sin x + C \quad (\odot (2)).\end{aligned}$$

$$\begin{aligned}(5) \quad \int x(e^x)' \, dx &= xe^x - \int (x)' e^x \, dx \\ &= xe^x - \int e^x \, dx \\ &= xe^x - e^x + C.\end{aligned}$$

$$\begin{aligned}(6) \quad \int \left(\frac{1}{2}x^2\right)' \log x \, dx &= \frac{1}{2}x^2 \log x - \int \frac{1}{2}x^2 (\log x)' \, dx \\ &= \frac{1}{2}x^2 \log x - \frac{1}{2} \int x \, dx \\ &= \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C.\end{aligned}$$

問題 2.

$$\begin{aligned}(1) \quad \int_0^{\frac{\pi}{2}} x(\sin x)' \, dx &= \left[x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx \\ &= \frac{\pi}{2} - \left[-\cos x\right]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1.\end{aligned}$$

$$\begin{aligned}(2) \quad \int_0^1 x(-e^{-x})' \, dx &= \left[-xe^{-x}\right]_0^1 + \int_0^1 e^{-x} \, dx \\ &= -e^{-1} + \left[-e^{-x}\right]_0^1 = 1 - 2e^{-1}.\end{aligned}$$

$$\begin{aligned}(3) \quad \int_1^e \left(\frac{1}{2}x^2\right)' \log x \, dx &= \left[\frac{1}{2}x^2 \log x\right]_1^e - \frac{1}{2} \int_1^e x \, dx \\ &= \frac{1}{2}e^2 - \left[\frac{1}{4}x^2\right]_1^e = \frac{1}{4}(e^2 + 1).\end{aligned}$$

$$\begin{aligned}(4) \quad \int_1^e (x)' (\log x)^2 \, dx &= \left[x(\log x)^2\right]_1^e - \int_1^e x \cdot \frac{2 \log x}{x} \, dx \\ &= e - 2 \int_1^e \log x \, dx \\ &= e - 2 \left[x \log x - x\right]_1^e = e - 2.\end{aligned}$$

問題 3.

$$\begin{aligned}(1) \quad I &= \int_0^{\frac{\pi}{2}} (e^x)' \sin x \, dx \\ &= \left[e^x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} e^x \cos x \, dx = e^{\frac{\pi}{2}} - J.\end{aligned}$$

よって $I + J = e^{\frac{\pi}{2}}$. 一方で,

$$\begin{aligned}I &= \int_0^{\frac{\pi}{2}} e^x (-\cos x)' \, dx \\ &= \left[-e^x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} e^x \cos x \, dx = 1 + J.\end{aligned}$$

よって $I - J = 1$.

$$(2) \quad (1) \text{ より } I = \frac{1}{2}(e^{\frac{\pi}{2}} + 1), J = \frac{1}{2}(e^{\frac{\pi}{2}} - 1).$$

問題 4.

$$(1) \quad I_0 = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}, I_1 = \left[-\cos x\right]_0^{\frac{\pi}{2}} = 1.$$

(2) 部分積分を行うと

$$\begin{aligned}I_n &= \int_0^{\frac{\pi}{2}} (-\cos x)' \sin^{n-1} x \, dx \\ &= \left[-\cos x \sin^{n-1} x\right]_0^{\frac{\pi}{2}} + (n-1) \int_0^{\frac{\pi}{2}} \sin^{n-2} x \cos^2 x \, dx \\ &= (n-1) \int_0^{\frac{\pi}{2}} (\sin^{n-2} x - \sin^n x) \, dx \\ &= (n-1)(I_{n-2} - I_n).\end{aligned}$$

$$\text{よって, } I_n = \frac{n-1}{n} I_{n-2}.$$

(3) n の偶奇で場合分けする.

(i) n が偶数のとき, $n = 2k$ (k は自然数) と表せるので,

$$\begin{aligned} I_n &= \frac{n-1}{n} I_{n-2} = \frac{n-1}{n} \frac{n-3}{n-2} I_{n-4} \\ &= \dots \\ &= \frac{n-1}{n} \frac{n-3}{n-2} \dots \frac{5}{6} \cdot \frac{3}{4} I_0 \\ &= \frac{n-1}{n} \frac{n-3}{n-2} \dots \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{\pi}{2}. \end{aligned}$$

(ii) n が奇数のとき, $n = 2k-1$ (k は自然数) と表せるので,

$$\begin{aligned} I_n &= \frac{n-1}{n} I_{n-2} = \frac{n-1}{n} \frac{n-3}{n-2} I_{n-4} \\ &= \dots \\ &= \frac{n-1}{n} \frac{n-3}{n-2} \dots \frac{4}{5} \cdot \frac{2}{3} I_1 \\ &= \frac{n-1}{n} \frac{n-3}{n-2} \dots \frac{4}{5} \cdot \frac{2}{3}. \end{aligned}$$

問題 5. 以下, \textcircled{n} と書いて行列の n 行目を表すものとする.

$$(1) \left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\textcircled{1} \xrightarrow{\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\textcircled{3} + \textcircled{1} \xrightarrow{\times(-1)} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array} \right)$$

$$\textcircled{2} \leftrightarrow \textcircled{3} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 1 & 0 \end{array} \right)$$

$$\textcircled{3} + \textcircled{2} \times 1 \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right)$$

$$\textcircled{1} + \textcircled{3} \times 1 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{array} \right).$$

$$\text{よって, } A^{-1} = \left(\begin{array}{ccc} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array} \right).$$

$$(2) \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 2 & 4 & 4 & 0 & 1 & 0 \\ 1 & 3 & 2 & 0 & 0 & 1 \end{array} \right)$$

$$\textcircled{2} + \textcircled{1} \times (-2) \quad \textcircled{3} + \textcircled{1} \times (-1) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 2 & -2 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array} \right)$$

$$\textcircled{2} \leftrightarrow \textcircled{3} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array} \right)$$

$$\textcircled{1} + \textcircled{2} \times (-2) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 1 & 0 \end{array} \right)$$

$$\textcircled{3} \times \frac{1}{2} \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 3 & 0 & -2 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array} \right)$$

$$\textcircled{1} + \textcircled{3} \times 1 \quad \textcircled{2} + \textcircled{3} \times (-1) \rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & \frac{1}{2} & -2 \\ 0 & 1 & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & 1 & -1 & \frac{1}{2} & 0 \end{array} \right).$$

$$\text{よって, } A^{-1} = \left(\begin{array}{ccc} 2 & \frac{1}{2} & -2 \\ 0 & -\frac{1}{2} & 1 \\ -1 & \frac{1}{2} & 0 \end{array} \right).$$

$$(3) \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 2 & 2 & 0 & 1 & 0 & 0 \\ 2 & 3 & 4 & 4 & 0 & 0 & 1 & 0 \\ 2 & 3 & 4 & 5 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\textcircled{2} + \textcircled{1} \times (-1) \quad \textcircled{3} + \textcircled{1} \times (-2) \quad \textcircled{4} + \textcircled{1} \times (-2) \rightarrow \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 2 & -2 & 0 & 1 & 0 \\ 0 & 1 & 2 & 3 & -2 & 0 & 0 & 1 \end{array} \right)$$

$$\textcircled{1} + \textcircled{2} \times (-1) \quad \textcircled{3} + \textcircled{2} \times (-1) \quad \textcircled{4} + \textcircled{2} \times (-1) \rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -1 & 0 & 1 \end{array} \right)$$

$$\textcircled{2} + \textcircled{3} \times (-1) \quad \textcircled{4} + \textcircled{3} \times (-1) \rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array} \right)$$

$$\textcircled{3} + \textcircled{4} \times (-1) \rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 & -1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{array} \right).$$

$$\text{よって, } A^{-1} = \left(\begin{array}{cccc} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{array} \right).$$