# リメディアル数学 (化学システム工学科) —— 第9回 2024/6/19 略解

問題 1.

(1) $\dfrac{1}{x} + \dfrac{1}{x^2+4}$

(2) $\dfrac{5}{x-1} + \dfrac{3}{(x-1)^2} + \dfrac{4}{x+2}$.

(3) $1 - \dfrac{3x^2+3}{x^2(x+3)} = 1 - \dfrac{1}{3}\left(\dfrac{2}{3}\cdot\dfrac{1}{x^2} - \dfrac{2}{9}\cdot\dfrac{1}{x} + \dfrac{29}{9}\cdot\dfrac{1}{x+3}\right)$.

問題 2.

(1) $\dfrac{1}{x^2-1} = \dfrac{1}{2}\left(\dfrac{1}{x-1} - \dfrac{1}{x+1}\right)$ より,

$$\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\int\left(\frac{1}{x-1} - \frac{1}{x+1}\right)\,dx$$
$$= \frac{1}{2}\left(\log|x-1| - \log|x+1|\right) + C$$
$$= \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C.$$

(2) $\dfrac{x}{x^2-3x+2} = \dfrac{2}{x-2} - \dfrac{1}{x-1}$ より,

$$\int \frac{x}{x^2-3x+2}\,dx = \int \frac{x}{(x-2)(x-1)}\,dx$$
$$= \int\left(\frac{2}{x-2} - \frac{1}{x-1}\right)\,dx$$
$$= 2\log|x-2| - \log|x-1| + C$$
$$= \log\frac{(x-2)^2}{|x-1|} + C.$$

(3) $\dfrac{3x+3}{x^3-1} = \dfrac{2}{x-1} - \dfrac{2x+1}{x^2+x+1}$ より,

$$\int \frac{3x+3}{x^3-1}\,dx = \int \frac{3x+3}{(x-1)(x^2+x+1)}\,dx$$
$$= \int\left(\frac{2}{x-1} - \frac{2x+1}{x^2+x+1}\right)\,dx$$
$$= 2\log|x-1| - \log(x^2+x+1) + C$$
$$= \log\frac{(x-1)^2}{x^2+x+1} + C.$$

問題 3.

(1) $\displaystyle\int \sin^2 x\,dx = \frac{1}{2}\int(1-\cos 2x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C.$

(2) $\displaystyle\int \sin x\cos x\,dx = \frac{1}{2}\int \sin 2x\,dx = -\frac{1}{4}\cos 2x + C.$

(3) $\sin 3x\cos x = \dfrac{1}{2}(\sin 4x + \sin 2x)$ より,

$$\int \sin 3x\cos x\,dx = \frac{1}{2}\int(\sin 4x + \sin 2x)\,dx$$
$$= -\frac{1}{8}\cos 4x - \frac{1}{4}\cos 2x + C.$$

(4) $\cos 3x\cos 2x = \dfrac{1}{2}(\cos 5x + \cos x)$ より,

$$\int \cos 3x\cos 2x\,dx = \frac{1}{2}\int(\cos 5x + \cos x)\,dx$$
$$= \frac{1}{10}\sin 5x + \frac{1}{2}\sin x + C.$$

(5) $t = \tan\dfrac{x}{2}$ とおくと, $\sin x = \dfrac{2t}{1+t^2}$, $dx = \dfrac{2}{1+t^2}dt$ より,

$$\int \frac{1}{\sin x}\,dx = \int \frac{1+t^2}{2t}\cdot\frac{2}{1+t^2}dt = \int \frac{1}{t}\,dt$$
$$= \log|t| + C = \log\left|\tan\frac{x}{2}\right| + C.$$

問題 4. 以下では, ⓝ, △ と書いて, それぞれ行列の $n$ 行目, $n$ 列目を表すものとする.

(1) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \xrightarrow{②+①\times(-3)} \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \xrightarrow{②\times(-\frac{1}{2})} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$
$\xrightarrow{①+②\times(-2)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. よって, $\mathrm{rank}(A) = 2$.

(2) $A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \xrightarrow[③+①\times(-1)]{②+①\times 2} \begin{pmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix}$
$\xrightarrow[△+△\times(-1)]{△+△\times 2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix} \xrightarrow{②\times(-\frac{1}{3})} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{pmatrix}$
$\xrightarrow{③+②\times(-3)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{△+△\times 1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.
よって, $\mathrm{rank}(A) = 2$.

(3) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 5 & 10 & 15 \end{pmatrix} \xrightarrow[③+①\times(-5)]{②+①\times(-2)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow[△+△\times(-3)]{△+△\times(-2)}$
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. よって, $\mathrm{rank}(A) = 1$.

(4) $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \\ 3 & 7 & 11 \end{pmatrix} \xrightarrow[\text{③}+\text{①}\times(-3)]{\text{②}+\text{①}\times(-2)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 1 & 2 \end{pmatrix}$

$\xrightarrow{\text{②}\times(-1)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow[\text{③}+\text{②}\times(-1)]{\text{①}+\text{②}\times(-2)} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{pmatrix}$

$\xrightarrow{\text{③}\times(-1)} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow[\text{②}+\text{③}\times(-3)]{\text{①}+\text{③}\times 3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

よって, $\mathrm{rank}(A) = 3$.

(5) $A = \begin{pmatrix} 1 & a & 1 & 1 \\ a & 1 & a & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & 1 \end{pmatrix}$

$\xrightarrow[\substack{\text{④}+\text{①}\times(-1)}]{\substack{\text{②}+\text{①}\times(-a) \\ \text{③}+\text{①}\times(-1)}} \begin{pmatrix} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 0 & 1-a \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a & a-1 & 0 \end{pmatrix}$

$\xrightarrow[\substack{\text{④̲}+\text{①̲}\times(-1)}]{\substack{\text{②̲}+\text{①̲}\times(-a) \\ \text{③̲}+\text{①̲}\times(-1)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a^2 & 0 & 1-a \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a & a-1 & 0 \end{pmatrix}$

$\xrightarrow{\text{②}\leftrightarrow\text{④}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a^2 & 0 & 1-a \end{pmatrix}$

$\xrightarrow{\text{④}+\text{②}\times(-a-1)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix}$

$\xrightarrow{\text{③̲}+\text{②̲}\times 1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix}$

$\xrightarrow{\text{③̲}\leftrightarrow\text{④̲}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 1-a & 1-a^2 \end{pmatrix}$

$\xrightarrow{\text{④}+\text{③}\times 1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & 1-a^2 \end{pmatrix}$

$\xrightarrow[\text{④}\times(-1)]{\text{②}\times(-1)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a-1 & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & (a-1)(a+1) \end{pmatrix}.$

よって, $\mathrm{rank}(A) = \begin{cases} 1 & (a = 1), \\ 3 & (a = -1), \\ 4 & (a \neq \pm 1). \end{cases}$