リメディアル数学 (化学システム工学科) —— 第 11 回 2024/7/3 略解

問題 1.

(1) $\displaystyle\lim_{n\to\infty}\frac{3+\frac{2}{n}-\frac{3}{n^2}}{2-\frac{4}{n}+\frac{1}{n^2}}=\frac{3}{2}.$

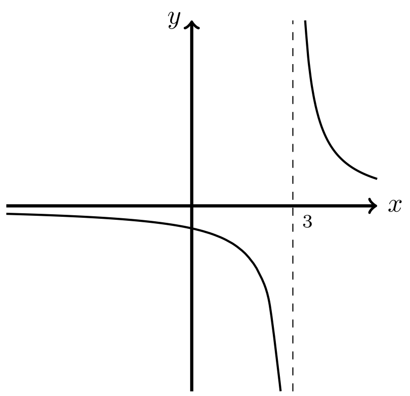
(2) $\displaystyle\lim_{n\to\infty}\frac{\frac{5}{n}+\frac{2}{n^2}}{2-\frac{5}{n}-\frac{3}{n^2}}=0.$

(3) $$\lim_{n\to\infty}\frac{(\sqrt{n+2}-\sqrt{n-1})(\sqrt{n+2}+\sqrt{n-1})}{\sqrt{n+2}+\sqrt{n-1}}$$
$$=\lim_{n\to\infty}\frac{3}{\sqrt{n+2}+\sqrt{n-1}}=0.$$

(4) $$\lim_{n\to\infty}\frac{n+\sqrt{n^2+2n}}{(n-\sqrt{n^2+2n})(n+\sqrt{n^2+2n})}$$
$$=\lim_{n\to\infty}\frac{n+\sqrt{n^2+2n}}{-2n}$$
$$=\lim_{n\to\infty}\frac{1+\sqrt{1+\frac{2}{n}}}{-2}=-1.$$

(5) $\displaystyle\lim_{x\to3}\frac{(x-3)(x-2)}{x-3}=\lim_{x\to3}(x-2)=1.$

(6) グラフより, $\displaystyle\lim_{x\to3-0}\frac{2}{x-3}=-\infty.$



(7) $\displaystyle\lim_{x\to0}4\cdot\frac{\sin 4x}{4x}=4.$

(8) $\displaystyle\frac{1}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}$ より
$$\sum_{k=1}^{n}\frac{1}{k(k+1)}=\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+1}\right)=1-\frac{1}{n+1}.$$
よって求める極限は 1.

問題 2.

(1) $f'(x)=9x^2-4x+1.$

(2) $\displaystyle f'(x)=-\frac{4}{x^3}+\frac{3}{x^2}-3+4x.$

(3) $f'(x)=(e^x)'\sin x+e^x(\sin x)'=e^x\sin x+e^x\cos x.$

(4) $$f'(x)=\frac{(\log x)'x^2-\log x\cdot(x^2)'}{x^4}$$
$$=\frac{x-2x\log x}{x^4}=\frac{1-2\log x}{x^3}.$$

(5) $f'(x)=5(3x+2)^4(3x+2)'=15(3x+2)^4.$

(6) $\displaystyle f'(x)=\cos\left(\frac{\pi}{5}-2x\right)\cdot\left(\frac{\pi}{5}-2x\right)'=-2\cos\left(\frac{\pi}{5}-2x\right).$

(7) $\displaystyle f'(x)=\frac{(4x^3-2x+1)'}{4x^3-2x+1}=\frac{12x^2-2}{4x^3-2x+1}.$

(8) 両辺の対数を取ると $\log f(x)=\log x^{\log x}=(\log x)^2.$

この両辺を微分して $\displaystyle\frac{f'(x)}{f(x)}=2\log x\cdot(\log x)'=\frac{2\log x}{x}.$

よって, $\displaystyle f'(x)=\frac{2\log x}{x}\cdot f(x)=\frac{2x^{\log x}\log x}{x}$

問題 3. 以下, 積分定数を C とする.

(1) $\displaystyle\int\left(1-\frac{1}{x^2}\right)\,dx=x+\frac{1}{x}+C.$

(2) $\displaystyle\frac{3}{2}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+C.$

(3) $\displaystyle\left[\frac{1}{3}x^3-x^{-1}\right]_1^2=\frac{1}{3}(8-1)-\left(\frac{1}{2}-1\right)=\frac{17}{6}.$

(4) $\log|x^3+x^2-3x+9|+C.$ ($t=x^3+x^2-3x+9$ による置換積分.)

(5) $\displaystyle-\frac{1}{4}\cos^4 x+C.$ ($t=\cos x$ による置換積分.)

(6) $\displaystyle\int_0^{\pi/2}(1-\sin^2 x)^2\cos x\,dx$ と変形できるので, $t=\sin x$ で置換積分を行う. 積分範囲は右下図の通りになるので,

$$\int_0^1(1-t^2)^2\,dt$$

x	0	$\longrightarrow$	$\frac{\pi}{2}$
t	0	$\longrightarrow$	1

$$=\int_0^1(1-2t^2+t^4)\,dt=\left[t-\frac{2}{3}t^3+\frac{1}{5}t^5\right]_0^1$$
$$=1-\frac{2}{3}+\frac{1}{5}=\frac{8}{15}.$$

(7) $\displaystyle xe^x-\int(x)'e^x\,dx=xe^x-e^x+C=(x-1)e^x+C.$

(8) 求める積分を I とおくと,

$$I = -e^{-x}\cos x - \int (-e^{-x}(\cos x)')\ dx$$
$$= -e^{-x}\cos x - \int e^{-x}\sin x\ dx$$
$$= -e^{-x}\cos x - \left(-e^{-x}\sin x - \int (-e^{-x}(\sin x)')\ dx\right)$$
$$= -e^{-x}\cos x + e^{-x}\sin x - \int e^{-x}\cos x\ dx$$
$$= e^{-x}(\sin x - \cos x) - I.$$

よって, $I = \dfrac{e^{-x}}{2}(\sin x - \cos x) + C.$

(9) $\left[\dfrac{1}{2}x^2\log x\right]_1^3 - \displaystyle\int_1^3 \frac{1}{2}x^2(\log x)'\ dx$

$$= \frac{9}{2}\log 3 - \frac{1}{2}\int_1^3 x\ dx$$
$$= \frac{9}{2}\log 3 - \frac{1}{2}\left[\frac{1}{2}x^2\right]_1^3$$
$$= \frac{9}{2}\log 3 - \frac{1}{2}\left(\frac{9}{2} - \frac{1}{2}\right) = \frac{9}{2}\log 3 - 2.$$

(10) $\dfrac{1}{x^2-4x+3} = \dfrac{1}{(x-1)(x-3)} = \dfrac{1}{2}\left(\dfrac{1}{x-3} - \dfrac{1}{x-1}\right)$

より, 求める積分は

$$\frac{1}{2}\int \left(\frac{1}{x-3} - \frac{1}{x-1}\right)\ dx$$
$$= \frac{1}{2}(\log|x-3| - \log|x-1|) + C$$
$$= \frac{1}{2}\log\left|\frac{x-3}{x-1}\right| + C.$$

(11) $\dfrac{x-4}{x^2-2x-3} = \dfrac{x-4}{(x+1)(x-3)} = \dfrac{5}{4}\dfrac{1}{x+1} - \dfrac{1}{4}\dfrac{1}{x-3}$

より, 求める積分は

$$\frac{5}{4}\int \frac{1}{x+1}\ dx - \frac{1}{4}\int \frac{1}{x-3}\ dx$$
$$= \frac{5}{4}\log|x+1| - \frac{1}{4}\log|x-3| + C.$$

(12) $\dfrac{x^2}{x^2-1} = 1 + \dfrac{1}{(x-1)(x+1)} = 1 + \dfrac{1}{2}\left(\dfrac{1}{x-1} - \dfrac{1}{x+1}\right)$

より, 求める積分は

$$\int \left\{1 + \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)\right\}\ dx$$
$$= x + \frac{1}{2}\left(\log|x-1| - \log|x+1|\right) + C$$
$$= x + \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C.$$