

第9回リメディアル数学 (化学システム工学科) 2023/6/21 略解

問題 1.

$$(1) \frac{2x^2 + 1}{x + 1} = 2x - 2 + \frac{3}{x + 1}.$$

$$(2) \frac{1}{x^2(x + 3)} = \frac{1}{9} \left(\frac{3}{x^2} - \frac{1}{x} + \frac{1}{x + 3} \right).$$

$$(3) \frac{9x^2}{(x - 1)^2(x + 2)} = \frac{5}{x - 1} + \frac{3}{(x - 1)^2} + \frac{4}{x + 2}.$$

$$(4) \frac{x^2 + x + 4}{x(x^2 + 4)} = \frac{1}{x} + \frac{1}{x^2 + 4}$$

問題 2.

$$(1) \frac{1}{x^2 - 1} = \frac{1}{2} \left(\frac{1}{x - 1} - \frac{1}{x + 1} \right) \text{ より,}$$

$$\begin{aligned} \int \frac{1}{x^2 - 1} dx &= \frac{1}{2} \int \left(\frac{1}{x - 1} - \frac{1}{x + 1} \right) dx \\ &= \frac{1}{2} (\log |x - 1| - \log |x + 1|) + C \\ &= \frac{1}{2} \log \left| \frac{x - 1}{x + 1} \right| + C. \end{aligned}$$

$$(2) \frac{x}{x^2 - 3x + 2} = \frac{2}{x - 2} - \frac{1}{x - 1} \text{ より,}$$

$$\begin{aligned} \int \frac{x}{x^2 - 3x + 2} dx &= \int \frac{x}{(x - 2)(x - 1)} dx \\ &= \int \left(\frac{2}{x - 2} - \frac{1}{x - 1} \right) dx \\ &= 2 \log |x - 2| - \log |x - 1| + C \\ &= \log \frac{(x - 2)^2}{|x - 1|} + C. \end{aligned}$$

$$(3) \frac{3x + 3}{x^3 - 1} = \frac{2}{x - 1} - \frac{2x + 1}{x^2 + x + 1} \text{ より,}$$

$$\begin{aligned} \int \frac{3x + 3}{x^3 - 1} dx &= \int \frac{3x + 3}{(x - 1)(x^2 + x + 1)} dx \\ &= \int \left(\frac{2}{x - 1} - \frac{2x + 1}{x^2 + x + 1} \right) dx \\ &= 2 \log |x - 1| - \log(x^2 + x + 1) + C \\ &= \log \frac{(x - 2)^2}{x^2 + x + 1} + C. \end{aligned}$$

問題 3.

$$(1) \int \sin^2 x dx = \frac{1}{2} \int (1 - \cos 2x) dx = \frac{1}{2} x - \frac{1}{4} \sin 2x + C.$$

$$(2) \int \sin x \cos x dx = \frac{1}{2} \int \sin 2x dx = -\frac{1}{2} \cos 2x + C.$$

$$(3) \sin 3x \cos x = \frac{1}{2} (\sin 4x + \sin 2x) \text{ より,}$$

$$\begin{aligned} \int \sin 3x \cos x dx &= \frac{1}{2} \int (\sin 4x + \sin 2x) dx \\ &= -\frac{1}{8} \cos 4x - \frac{1}{4} \cos 2x + C. \end{aligned}$$

$$(4) \cos 3x \cos 2x = \frac{1}{2} (\cos 5x + \cos x) \text{ より,}$$

$$\begin{aligned} \int \cos 3x \cos 2x dx &= \frac{1}{2} \int (\cos 5x + \cos x) dx \\ &= \frac{1}{10} \sin 5x + \frac{1}{2} \sin x + C. \end{aligned}$$

$$(5) t = \tan \frac{x}{2} \text{ とおくと, } \sin x = \frac{2t}{1 + t^2}, dx = \frac{2}{1 + t^2} dt \text{ より,}$$

$$\begin{aligned} \int \frac{1}{\sin x} dx &= \int \frac{1 + t^2}{2t} \cdot \frac{2}{1 + t^2} dt = \int \frac{1}{t} dt \\ &= \log |t| + C = \log \left| \tan \frac{x}{2} \right| + C. \end{aligned}$$

問題 4. 以下では, \textcircled{n} , \triangle と書いて, それぞれ行列の n 行目, n 列目を表すものとする.

$$(1) A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \xrightarrow{\textcircled{2} + \textcircled{1} \times (-3)} \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} \xrightarrow{\textcircled{2} \times (-\frac{1}{2})} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \xrightarrow{\textcircled{1} + \textcircled{2} \times (-2)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \text{ よって, rank}(A) = 2.$$

$$(2) A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix} \xrightarrow{\textcircled{2} + \textcircled{1} \times 2, \textcircled{3} + \textcircled{1} \times (-1)} \begin{pmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix} \xrightarrow{\triangle + \triangle \times 2, \triangle + \triangle \times (-1)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix} \xrightarrow{\textcircled{2} \times (-\frac{1}{3})} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 3 & -3 \end{pmatrix} \xrightarrow{\textcircled{3} + \textcircled{2} \times (-3)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\triangle + \triangle \times 1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

よって, $\text{rank}(A) = 2$.

$$(3) A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 5 & 10 & 15 \end{pmatrix} \xrightarrow{\textcircled{2} + \textcircled{1} \times (-2), \textcircled{3} + \textcircled{1} \times (-5)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\triangle + \triangle \times (-2), \triangle + \triangle \times (-3)} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \text{ よって, rank}(A) = 1.$$

$$\begin{aligned}
(4) \quad A &= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \\ 3 & 7 & 11 \end{pmatrix} \xrightarrow{\substack{\textcircled{2}+\textcircled{1}\times(-2) \\ \textcircled{3}+\textcircled{1}\times(-3)}} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 1 & 2 \end{pmatrix} \\
&\xrightarrow{\textcircled{2}\times(-1)} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 3 \\ 0 & 1 & 2 \end{pmatrix} \xrightarrow{\substack{\textcircled{1}+\textcircled{2}\times(-2) \\ \textcircled{3}+\textcircled{2}\times(-1)}} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & -1 \end{pmatrix} \\
&\xrightarrow{\textcircled{3}\times(-1)} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{\textcircled{1}+\textcircled{3}\times 3 \\ \textcircled{2}+\textcircled{3}\times(-3)}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.
\end{aligned}$$

よって, $\text{rank}(A) = 3$.

$$(5) \quad A = \begin{pmatrix} 1 & a & 1 & 1 \\ a & 1 & a & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & a & 1 & 1 \\ a & 1 & a & 1 \\ 1 & a & 1 & a \\ 1 & 1 & a & 1 \end{pmatrix} \xrightarrow{\substack{\textcircled{2}+\textcircled{1}\times(-a) \\ \textcircled{3}+\textcircled{1}\times(-1) \\ \textcircled{4}+\textcircled{1}\times(-1)}} \begin{pmatrix} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 0 & 1-a \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a & a-1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & a & 1 & 1 \\ 0 & 1-a^2 & 0 & 1-a \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a & a-1 & 0 \end{pmatrix} \xrightarrow{\substack{\textcircled{2}+\textcircled{4}\times(-a) \\ \textcircled{3}+\textcircled{4}\times(-1) \\ \textcircled{4}+\textcircled{4}\times(-1)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a^2 & 0 & 1-a \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a & a-1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a^2 & 0 & 1-a \end{pmatrix} \xrightarrow{\textcircled{2}\leftrightarrow\textcircled{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 1-a^2 & 0 & 1-a \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix} \xrightarrow{\textcircled{4}+\textcircled{2}\times(-a-1)} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & a-1 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix} \xrightarrow{\textcircled{3}+\textcircled{2}\times 1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & 0 & a-1 \\ 0 & 0 & 1-a^2 & 1-a \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 1-a & 1-a^2 \end{pmatrix} \xrightarrow{\textcircled{3}\leftrightarrow\textcircled{4}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 1-a & 1-a^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & 1-a^2 \end{pmatrix} \xrightarrow{\textcircled{4}+\textcircled{3}\times 1} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1-a & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & 1-a^2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a-1 & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & (a-1)(a+1) \end{pmatrix} \xrightarrow{\substack{\textcircled{2}\times(-1) \\ \textcircled{4}\times(-1)}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & a-1 & 0 & 0 \\ 0 & 0 & a-1 & 0 \\ 0 & 0 & 0 & (a-1)(a+1) \end{pmatrix}.$$

$$\text{よって, } \text{rank}(A) = \begin{cases} 1 & (a=1), \\ 3 & (a=-1), \\ 4 & (a \neq \pm 1). \end{cases}$$