

リメディアル数学 (化学システム工学科) ——— 第7回 2024/6/5 略解

問題 1.

$$(1) \quad t = \sqrt{2x-1} \text{ とおくと, } x = \frac{t^2+1}{2}, \frac{dx}{dt} = t. \text{ よって,}$$

$$\begin{aligned} \int x\sqrt{2x-1} \, dx &= \int \frac{t^2+1}{2} \cdot t \cdot t \, dt \\ &= \frac{1}{2} \int (t^4 + t^2) \, dt \\ &= \frac{1}{2} \left(\frac{1}{5}t^5 + \frac{1}{3}t^3 \right) + C \\ &= \frac{1}{30}t^3(3t^2+5) + C \\ &= \frac{1}{15}(3x+1)(2x-1)\sqrt{2x-1} + C. \end{aligned}$$

$$(2) \quad t = \sin x \text{ とおくと, } \frac{dt}{dx} = \cos x. \text{ よって,}$$

$$\int \sin^3 x \cos x \, dx = \int t^3 \, dt = \frac{1}{4}t^4 + C = \frac{1}{4}\sin^4 x + C.$$

$$(3) \quad t = \log x \text{ とおくと, } \frac{dt}{dx} = \frac{1}{x}. \text{ よって,}$$

$$\int \frac{\log x}{x} \, dx = \int t \, dt = \frac{1}{2}t^2 + C = \frac{1}{2}(\log x)^2 + C.$$

$$(4) \quad t = x^2 \text{ とおくと, } \frac{dt}{dx} = 2x. \text{ よって,}$$

$$\int xe^{x^2} \, dx = \frac{1}{2} \int e^t \, dt = \frac{1}{2}e^t + C = \frac{1}{2}e^{x^2} + C.$$

問題 2.

$$(1) \quad \int (3x+1)^4 \, dx = \frac{1}{15}(3x+1)^5 + C.$$

$$(2) \quad \int (4x-3)^{-3} \, dx = -\frac{1}{8}(4x-3)^{-2} + C.$$

$$(3) \quad \int \frac{1}{\sqrt{1-2x}} \, dx = -\sqrt{1-2x} + C.$$

$$(4) \quad \int \sin 2x \, dx = -\frac{1}{2}\cos 2x + C.$$

$$(5) \quad \int e^{3x-1} \, dx = \frac{1}{3}e^{3x-1} + C.$$

問題 3.

$$(1) \quad \int \frac{2x}{x^2-3} \, dx = \int \frac{(x^2-3)'}{x^2-3} \, dx = \log|x^2-3| + C.$$

$$\begin{aligned} (2) \quad \int \frac{2x+1}{x^2+x-1} \, dx &= \int \frac{(x^2+x-1)'}{x^2+x-1} \, dx \\ &= \log|x^2+x-1| + C. \end{aligned}$$

$$(3) \quad \int \frac{e^x}{e^x+1} \, dx = \int \frac{(e^x+1)'}{e^x+1} \, dx = \log(e^x+1) + C.$$

$$\begin{aligned} (4) \quad \int \tan x \, dx &= \int \frac{\sin x}{\cos x} \, dx = -\int \frac{(\cos x)'}{\cos x} \, dx \\ &= -\log|\cos x| + C. \end{aligned}$$

$$\begin{aligned} (5) \quad \int \frac{1}{\tan x} \, dx &= \int \frac{\cos x}{\sin x} \, dx = \int \frac{(\sin x)'}{\sin x} \, dx \\ &= \log|\sin x| + C. \end{aligned}$$

問題 4.

$$(1) \quad t = 2-x \text{ とおくと, } x = 2-t, \frac{dx}{dt} = -1. \text{ また積分範囲}$$

は右図の通り. よって,

$$\begin{array}{c|cc} x & 1 & \longrightarrow & 2 \\ t & 1 & \longrightarrow & 0 \end{array}$$

$$\begin{aligned} \int_1^2 x(2-x)^4 \, dx &= \int_1^0 (2-t)t^4 \cdot (-1) \, dt \\ &= \int_0^1 (2t^4 - t^5) \, dt \\ &= \left[\frac{2}{5}t^5 - \frac{1}{6}t^6 \right]_0^1 = \frac{7}{30}. \end{aligned}$$

$$(2) \quad t = 1-x \text{ とおくと, } x = 1-t, \frac{dx}{dt} = -1. \text{ また, 積分範囲}$$

は右図の通り. よって,

$$\begin{array}{c|cc} x & 0 & \longrightarrow & 1 \\ t & 1 & \longrightarrow & 0 \end{array}$$

$$\begin{aligned} \int_0^1 x(1-x)^5 \, dx &= \int_1^0 (1-t)t^5 \cdot (-1) \, dt \\ &= \int_0^1 (t^5 - t^6) \, dt \\ &= \left[\frac{1}{6}t^6 - \frac{1}{7}t^7 \right]_0^1 = \frac{1}{42} \end{aligned}$$

$$(3) \quad t = x-1 \text{ とおくと, } x = t+1, \frac{dx}{dt} = 1. \text{ また積分範囲は}$$

右図の通り. よって,

$$\begin{array}{c|cc} x & 2 & \longrightarrow & 5 \\ t & 1 & \longrightarrow & 4 \end{array}$$

$$\begin{aligned} \int_2^5 x\sqrt{x-1} \, dx &= \int_1^4 (t+1)\sqrt{t} \, dt \\ &= \int_1^4 (t^{\frac{3}{2}} + t^{\frac{1}{2}}) \, dt \\ &= \left[\frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}} \right]_1^4 = \frac{256}{15} \end{aligned}$$

問題 5. $t = x + \sqrt{x^2 + A}$ とおくと,

$$\begin{aligned}\frac{dt}{dx} &= 1 + \frac{x}{\sqrt{x^2 + A}} = \frac{t}{\sqrt{x^2 + A}}. \\ \therefore \frac{1}{t} \frac{dt}{dx} &= \frac{1}{\sqrt{x^2 + A}}.\end{aligned}$$

よって,

$$\begin{aligned}\int \frac{1}{\sqrt{x^2 + A}} dx &= \int \frac{1}{t} dt = \log |t| + C \\ &= \log (x + \sqrt{x^2 + A}) + C.\end{aligned}$$

問題 6.

(1) $t = a + b - x$ で置換すると, $x = a + b - t$, $dx = -dt$ より

$$\int_a^b f(x) dx = \int_b^a f(a + b - t)(-1) dt = \int_a^b f(a + b - x) dx.$$

(2) 求める定積分を I とすると, (1) より

$$I = \int_{-1}^1 \frac{(-x)^2}{1 + e^{-x}} dx = \int_{-1}^1 \frac{e^x x^2}{1 + e^x} dx.$$

したがって,

$$\begin{aligned}2I &= \int_{-1}^1 \frac{x^2}{1 + e^x} dx + \int_{-1}^1 \frac{e^x x^2}{1 + e^x} dx \\ &= \int_{-1}^1 \left(\frac{x^2}{1 + e^x} + \frac{e^x x^2}{1 + e^x} \right) dx \\ &= \int_{-1}^1 x^2 dx \\ &= \left[\frac{1}{3} x^3 \right]_{-1}^1 = \frac{2}{3},\end{aligned}$$

つまり $I = \frac{1}{3}$.

問題 7. 以下では, 行列の n 行目を \textcircled{n} と表すこととする.

(1) 考える拡大係数行列は $\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 2 & 1 & 3 \end{array} \right)$ である. これに対して, 行基本変形を行うと

$$\begin{aligned}\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 2 & 1 & 3 \end{array} \right) &\xrightarrow{\textcircled{2} + \textcircled{1} \times (-2)} \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 5 & -5 \end{array} \right) \\ &\xrightarrow{\textcircled{2} \times \frac{1}{5}} \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -1 \end{array} \right) \xrightarrow{\textcircled{1} + \textcircled{2} \times 2} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \end{array} \right).\end{aligned}$$

よって, 求める解は $(x, y) = (2, -1)$.

(2) 考える拡大係数行列は $\left(\begin{array}{cc|c} 1 & -2 & 4 \\ -2 & 4 & 7 \end{array} \right)$ である. これに対して, 行基本変形を行うと

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ -2 & 4 & 7 \end{array} \right) \xrightarrow{\textcircled{2} + \textcircled{1} \times 2} \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 0 & 15 \end{array} \right).$$

よって, 求める解は存在しない.

(3) 考える拡大係数行列は $\left(\begin{array}{cc|c} 1 & -2 & 4 \\ -3 & 6 & -12 \end{array} \right)$ である. これに対して, 行基本変形を行うと

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ -3 & 6 & -12 \end{array} \right) \xrightarrow{\textcircled{2} + \textcircled{1} \times 3} \left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 0 & 0 \end{array} \right).$$

よって, 求める解は $(x, y) = (2t + 4, t)$ (t は任意の実数).

問題 8.

(1) 考える拡大係数行列は $\left(\begin{array}{ccc|c} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -7 \\ -3 & 1 & 4 & 8 \end{array} \right)$ である.

これに対して, 行基本変形を行うと

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & -2 \\ 1 & 2 & -2 & -7 \\ -3 & 1 & 4 & 8 \end{array} \right) \xrightarrow{\textcircled{1} + \textcircled{2} \times (-2), \textcircled{3} + \textcircled{2} \times 3} \left(\begin{array}{ccc|c} 0 & -3 & 3 & 12 \\ 1 & 2 & -2 & -7 \\ 0 & 7 & -2 & -13 \end{array} \right)$$

$$\xrightarrow{\textcircled{1} \leftrightarrow \textcircled{2}} \left(\begin{array}{ccc|c} 1 & 2 & -2 & -7 \\ 0 & -3 & 3 & 12 \\ 0 & 7 & -2 & -13 \end{array} \right) \xrightarrow{\textcircled{2} \times (-\frac{1}{3})} \left(\begin{array}{ccc|c} 1 & 2 & -2 & -7 \\ 0 & 1 & -1 & -4 \\ 0 & 7 & -2 & -13 \end{array} \right)$$

$$\xrightarrow{\textcircled{1} + \textcircled{2} \times (-2), \textcircled{3} + \textcircled{2} \times (-7)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 5 & 15 \end{array} \right) \xrightarrow{\textcircled{3} \times \frac{1}{5}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 3 \end{array} \right)$$

$$\xrightarrow{\textcircled{2} + \textcircled{3} \times 1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right).$$

よって, 求める解は $(x, y, z) = (1, -1, 3)$.

(2) 考える拡大係数行列は $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 3 & 5 & 3 \\ 3 & 4 & 7 & 5 \end{array} \right)$ である. これ

に対して, 行基本変形を行うと

$$\begin{aligned}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 2 & 3 & 5 & 3 \\ 3 & 4 & 7 & 5 \end{array} \right) &\xrightarrow{\textcircled{2} + \textcircled{1} \times (-2), \textcircled{3} + \textcircled{1} \times (-3)} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{array} \right) \\ &\xrightarrow{\textcircled{1} + \textcircled{2} \times (-1), \textcircled{3} + \textcircled{2} \times (-1)} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right)\end{aligned}$$

よって, 求める解は存在しない.

(3) 考える拡大係数行列は $\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & 2 & 4 & 4 \\ 2 & -1 & -2 & 3 \end{array} \right)$ である. こ

れに対して, 行基本変形を行うと

$$\begin{aligned}\left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & 2 & 4 & 4 \\ 2 & -1 & -2 & 3 \end{array} \right) &\xrightarrow{\textcircled{2} + \textcircled{1} \times (-1), \textcircled{3} + \textcircled{1} \times (-2)} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & -3 & -6 & -3 \end{array} \right) \\ &\xrightarrow{\textcircled{1} + \textcircled{2} \times (-1), \textcircled{3} + \textcircled{2} \times 3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)\end{aligned}$$

よって, 求める解は $(x, y, z) = (2, -2t + 1, t)$ (t は任意の実数).