第 6 回 リメディアル数学 (化学システム工学科) 2023/5/31 略解

問題 1

(1) $\displaystyle\int x^5\ dx = \frac{1}{5+1}x^{5+1} = \frac{1}{6}x^6 + C.$

(2) $\displaystyle\int \frac{dx}{x^3} = \frac{1}{-3+1}x^{-3+1} + C = -\frac{1}{2x^2} + C.$

(3) $\displaystyle\int x^{\frac{1}{3}}\ dx = \frac{1}{\frac{1}{3}+1}x^{\frac{1}{3}+1} + C = \frac{3}{4}x^{\frac{4}{3}} + C.$

(4) $\displaystyle\int x^{-\frac{1}{3}}\ dx = \frac{1}{-\frac{1}{3}+1}x^{-\frac{1}{3}+1} + C = \frac{3}{2}x^{\frac{2}{3}} + C.$

(5) $\displaystyle\int x\sqrt{x}\ dx = \int x^{\frac{3}{2}}\ dx = \frac{1}{\frac{3}{2}+1}x^{\frac{3}{2}+1} + C = \frac{2}{5}x^{\frac{5}{2}} + C.$

(6) $\displaystyle\int \frac{dx}{\sqrt{x}} = \int x^{-\frac{1}{2}}\ dx = \frac{1}{-\frac{1}{2}+1}x^{-\frac{1}{2}+1} + C = 2\sqrt{x} + C.$

(7) $\displaystyle\int \frac{x^2-4x+1}{x^3}\ dx = \int \left(\frac{1}{x} - \frac{4}{x^2} + \frac{1}{x^3}\right)\ dx$
$= \log|x| + \frac{4}{x} - \frac{1}{2x^2} + C.$

(8) $\displaystyle\int \frac{(x^2-2)(x^2-3)}{x^4}\ dx = \int \left(1 - \frac{5}{x^2} + \frac{6}{x^4}\right)\ dx$
$= x + \frac{5}{x} - \frac{2}{x^3} + C.$

(9) $\displaystyle\int \frac{x+2}{\sqrt{x}}\ dx = \int \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right)\ dx$
$= \frac{2}{3}x\sqrt{x} + 4\sqrt{x} + C.$

(10) $\displaystyle\int \frac{(\sqrt{x}-1)^2}{x}\ dx = \int \left(1 - \frac{2}{\sqrt{x}} + \frac{1}{x}\right)\ dx$
$= x - 4\sqrt{x} + \log|x| + C.$

(11) $\displaystyle\int \frac{1-y-y^2}{y^2}\ dy = \int \left(\frac{1}{y^2} - \frac{1}{y} - 1\right)\ dy$
$= -\frac{1}{y} - \log|y| - y + C.$

(12) $\displaystyle\int \left(3t^2 - \frac{1}{t}\right)\ dt = t^3 - \log|t| + C.$

問題 2

(1) $\displaystyle\int (\cos x - 2\sin x)\ dx = \sin x + 2\cos x + C.$

(2) $\displaystyle\int \frac{2\cos^3 x - 1}{\cos x^2}\ dx = \int \left(2\cos x - \frac{1}{\cos^2 x}\right)\ dx$
$= 2\sin x - \tan x + C.$

(3) $\displaystyle\int \frac{1}{\sin^2 x - 1}\ dx = -\int \frac{1}{\cos^2 x}\ dx = -\tan x + C.$

(4) $\displaystyle\int (2 - \tan\theta)\cos\theta\ d\theta = \int (2\cos\theta - \sin\theta)\ d\theta$
$= 2\sin\theta + \cos\theta + C.$

(5) $\displaystyle\int 4^x\ dx = \frac{4^x}{\log 4} + C.$

(6) $\displaystyle\int (3^x - 2e^x)\ dx = \frac{3^x}{\log 3} - 2e^x + C.$

(7) $\displaystyle\int \tan^2 x\ dx = \int \frac{\sin^2 x}{\cos^2 x}\ dx = \int \frac{1-\cos^2 x}{\cos^2 x}\ dx$
$\displaystyle= \int \left(\frac{1}{\cos^2 x} - 1\right)\ dx = \tan x - x + C.$

(8) $\displaystyle\int \left(\frac{4}{\cos^2 x} + \frac{5}{\sin^2 x}\right)\ dx = 4\tan x - \frac{5}{\tan x} + C.$

(9) $\displaystyle\int \sin\frac{x}{2}\cos\frac{x}{2}\ dx = \frac{1}{2}\int \sin x\ dx = -\frac{1}{2}\cos x + C.$

(10) $\displaystyle\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2\ dx = \int (1 + \sin x)\ dx$
$= x - \cos x + C$

(11) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\ dx = \arcsin x + C.$

(12) $\displaystyle\int \frac{1}{1+x^2}\ dx = \arctan x + C.$

問題 3

$$\begin{cases} 3x + 4y = 1 \\ 2x + y = 4 \end{cases} \iff \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

$\begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = 3\cdot 1 - 4\cdot 2 = -5 \neq 0$ なので, $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ の逆行列は存在して,

$$\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}^{-1} = -\frac{1}{5}\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}$$

よって, $\displaystyle\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}.$