Global vs. Local pMOR

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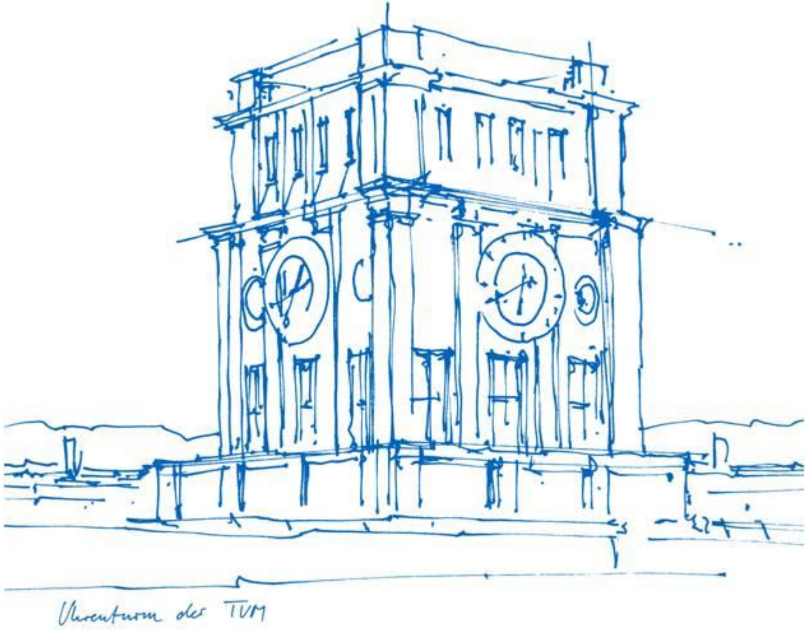


Motivation

- Complex and large numerical models **require substantial computational power** and large amount of memory

- **Model Order Reduction** techniques allow to reduce these requirements at the cost of an approximation of full system

- However, the reduced model becomes **invalid** when parameters of the original model such as **material properties, the geometry or boundary conditions are changed**

- Some applications like optimization, navigation, etc. require repeated model evaluations over a large **range** of such parameter values

Motivation

- A **naïve** approach would be to generate a reduced order model (ROM) **at each queried point** in the parameter space
- However, this is a costly task requiring large computations with full order system, thus generally **unfeasible**
- **Parametric Model Order Reduction (pMOR)** helps in overcoming these problems
- In pMOR, instead of generating ROMs at each point, a **sampling at different parameter combinations** is done to generate a database of reduced quantities
- Two main techniques – **Global pMOR and Local pMOR**

Global pMOR

- In global pMOR, the reduced bases (V_k) obtained at sampling points (p_k) are combined into **one global basis matrix** $(\boldsymbol{V})$

- The global basis matrix can then be reduced further using SVD to $\boldsymbol{V_r}$ with r most important bases

- Final reduced system: $M_r = V_r' M V_r$, $D_r = V_r' D V_r$, $K_r = V_r' K V_r$, $f_r = V_r' f$

- This is easy to implement but the parametric reduced model **grows with the number of samples**

- Also, it requires reduced bases to be on the **same mesh** as sample points

Local pMOR

- In local pMOR, **interpolation of the reduced matrices** is done to avoid full order systems

- However, a straightforward interpolation of the reduced matrices is **not meaningful** because **the reduced matrices relate to different reduced bases** $(\boldsymbol{V_k})$

- Therefore, a **generalized coordinate system** $(\boldsymbol{R})$ is computed first, followed by the transformation of reduced matrices to this system

Local pMOR

- $\boldsymbol{R} = \boldsymbol{U}(:,1\!:\!r)$ where $\boldsymbol{U}$ is obtained by SVD, $\boldsymbol{V} = \boldsymbol{U\Sigma Y}$

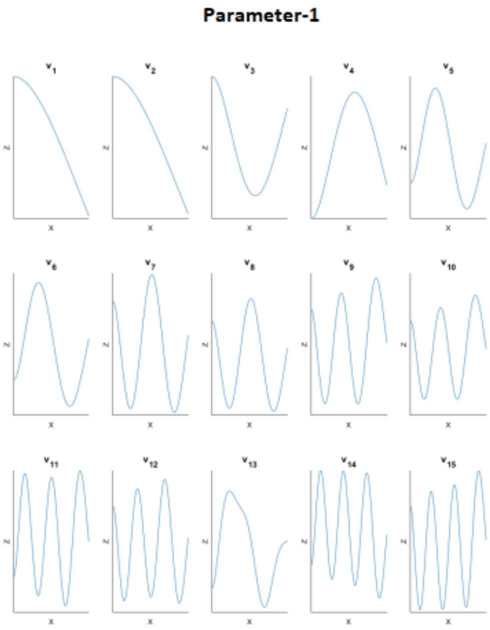
- Transformation matrix: $T_k = (R'V_k)^{-1}$

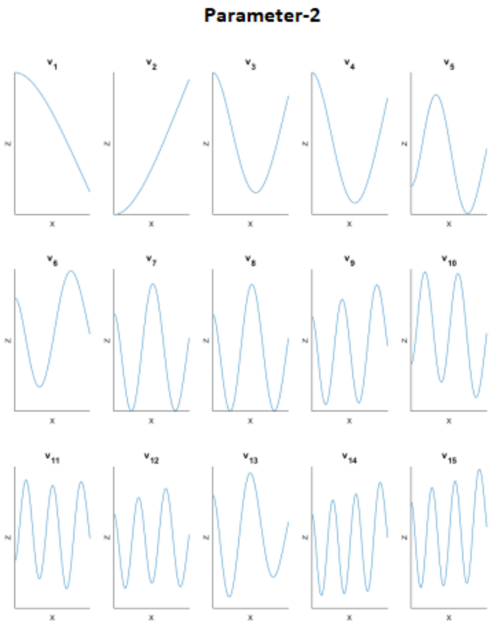
- $\widetilde{M_{r,k}} = T_k' M_{r,k} T_k,\ \ \widetilde{D_{r,k}} = T_k' D_{r,k} T_k,\ \ \widetilde{K_{r,k}} = T_k' K_{r,k} T_k,\ \ \widetilde{f_{r,k}} = T_k' f_{r,k}$

- The **evaluation efficiency is higher** than global pMOR since the parametric reduced model doesn't grow with the number of samples

- But the reduced bases still need to be generated on the **same mesh** as sample points

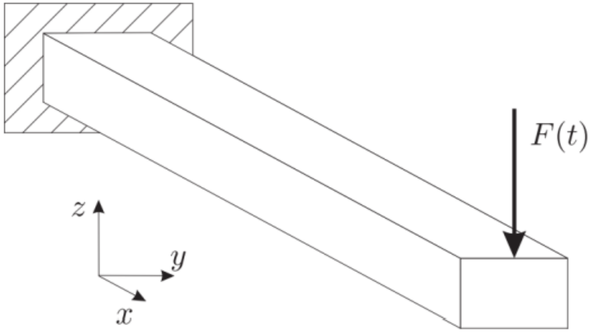
Local pMOR Interpolation





Methodology

- Generate a **FEM model** of the Timoshenko cantilever beam at different parameter samples (e.g. different beam lengths and thicknesses)
- Perform **MOR** at each parameter sample using SO-IRKA
- Collect **reduced model bases** together to obtain a parametric model
- Reduce the parametric model using either **global or local** pMOR technique
- Local pMOR interpolation done using **cubic splines**
- Solve the reduced system model at **required parameter value**
- Obtain the **frequency response** for a sweep of frequencies



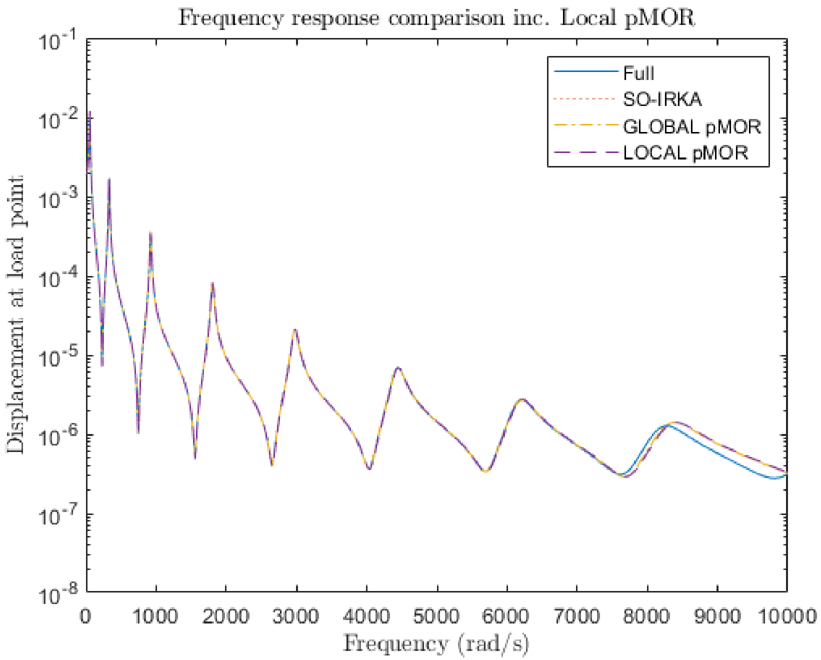
Second Order Iterative Rational Krylov Alogrithm (SO-IRKA)

- Useful for MOR of single-input single-output (SISO) second order systems

1. Starts with an **initial set** of expansion points (assumed to be complex)
2. Compute **initial projection matrices** V_r, W_r (two sided case)
3. **Reduce** matrices: $M_r = W_r^* M V_r$, $D_r = W_r^* D V_r$, $K_r = W_r^* K V_r$
4. Solve the quadratic **eigenvalue** problem: $\lambda^2 M_r + \lambda D_r + K_r = 0$
5. Choose $\boldsymbol{r}$ eigenvalues and **update** expansion points $S_0 \leftarrow -\lambda_r$
6. Repeat **steps 2-5** while $\frac{|\lambda_i^{m+1} - \lambda_i^m|}{|\lambda_i^m|} > tol, i = 1 \dots r$ or limit of iterations
7. Return reduced bases matrix $\boldsymbol{V_r}$ **and** $\boldsymbol{W_r}$ on convergence

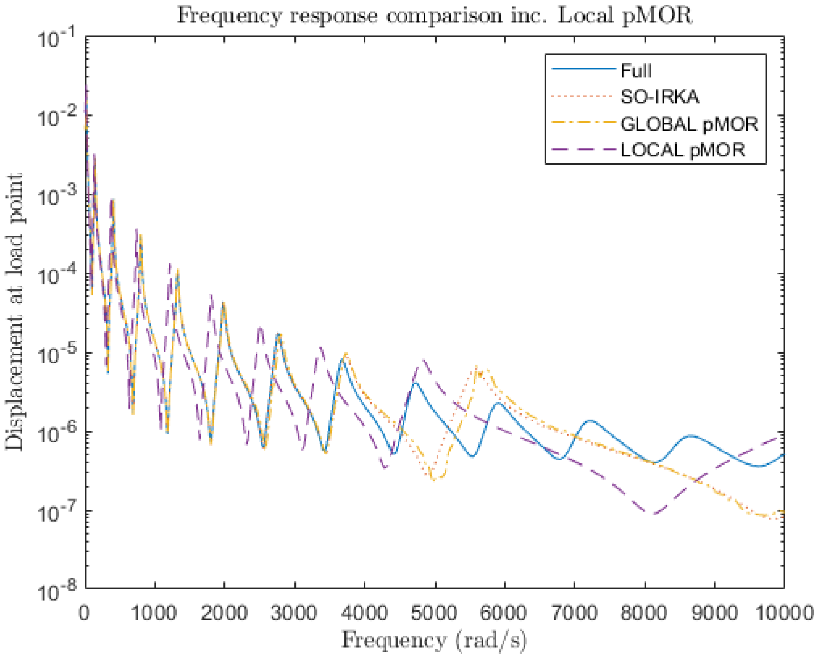
Results

- Parameter – **Beam length**
- Parameter range, P = [0.214 0.3 0.4 0.5 0.6 0.75 0.9 1.25]
- Models evaluated at **L = 1m** and **100 elements**
- All models give approximately the same results
- Accuracy of reduced-order models decreases at high frequencies



Extrapolation Results

- Models evaluated at **L = 1.5m** and **100 elements**
- Accuracy of all reduced-order models is worse than the interpolation case before
- Accuracy of reduced-order models decreases even further at high frequencies (>4000 rad/s)
- Accuracy of **local pMOR is the worst** since it is based on interpolation and thus, is unable to extrapolate the results

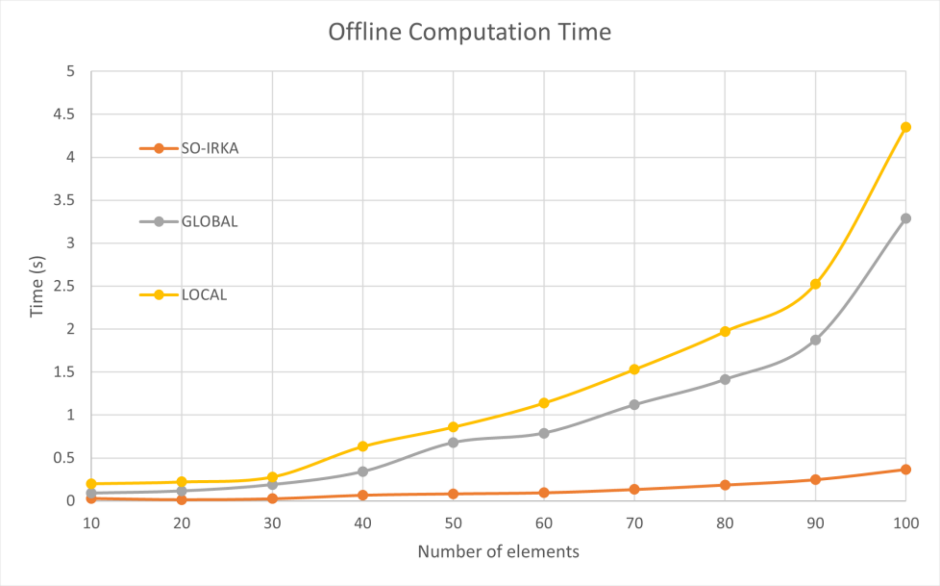


Performance Study – **Offline** Computation Time

Time taken to assemble the parametric model and reduce it.

- **Higher for local pMOR** since it requires further steps like transformation and interpolation as compared to global pMOR
- After pMOR, obtained reduced model size:

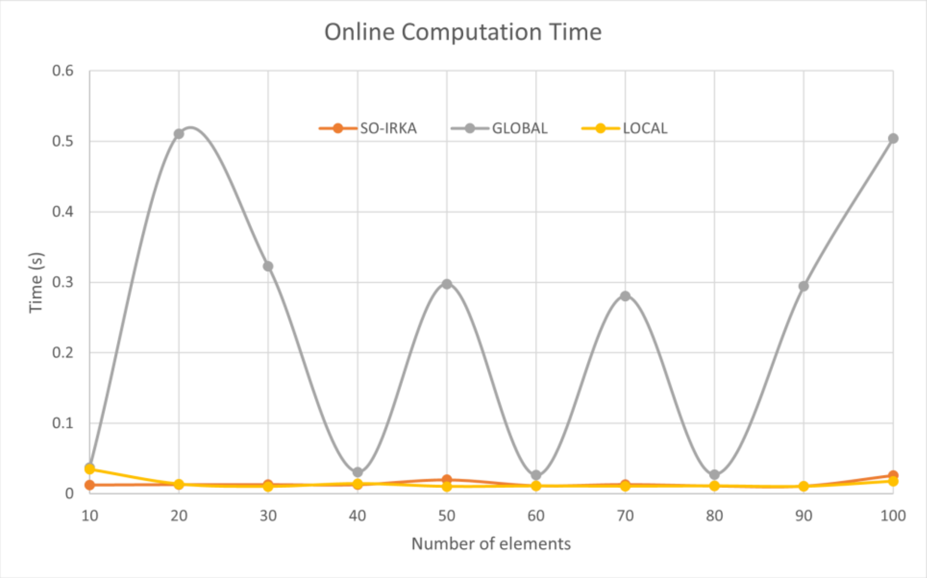
$$n_{Initial} = 300$$
$$r_{Global} = 24$$
$$r_{Local} = 10$$



Performance Study – **Online** Computation Time

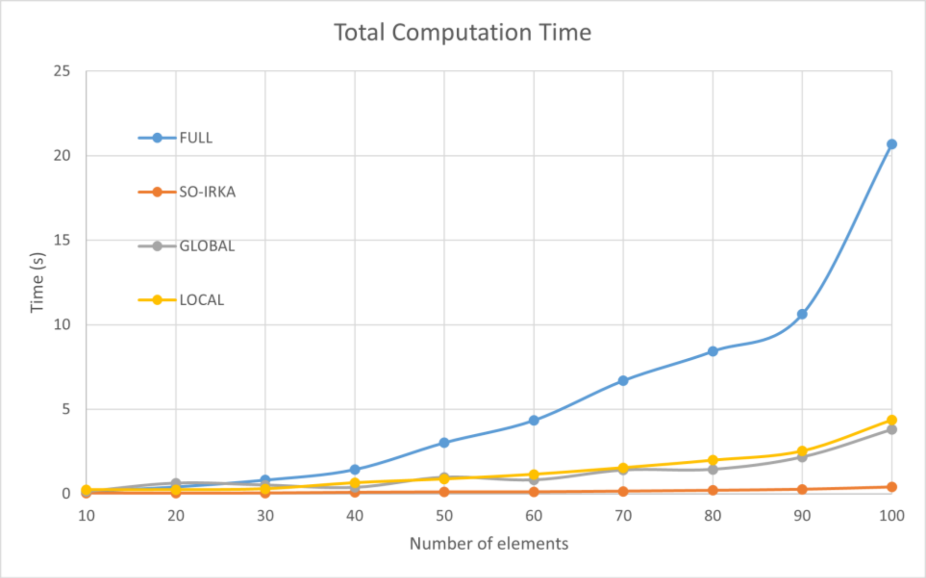
Time to solve and compute the frequency response of the reduced system.

- **Higher for global pMOR** since the size of reduced system is more as compared to local pMOR
- Number of elements doesn't affect the size of reduced system and thus, it doesn't affect online times as well
- **Variations in global pMOR** online time are probably due to processor usage by other processes



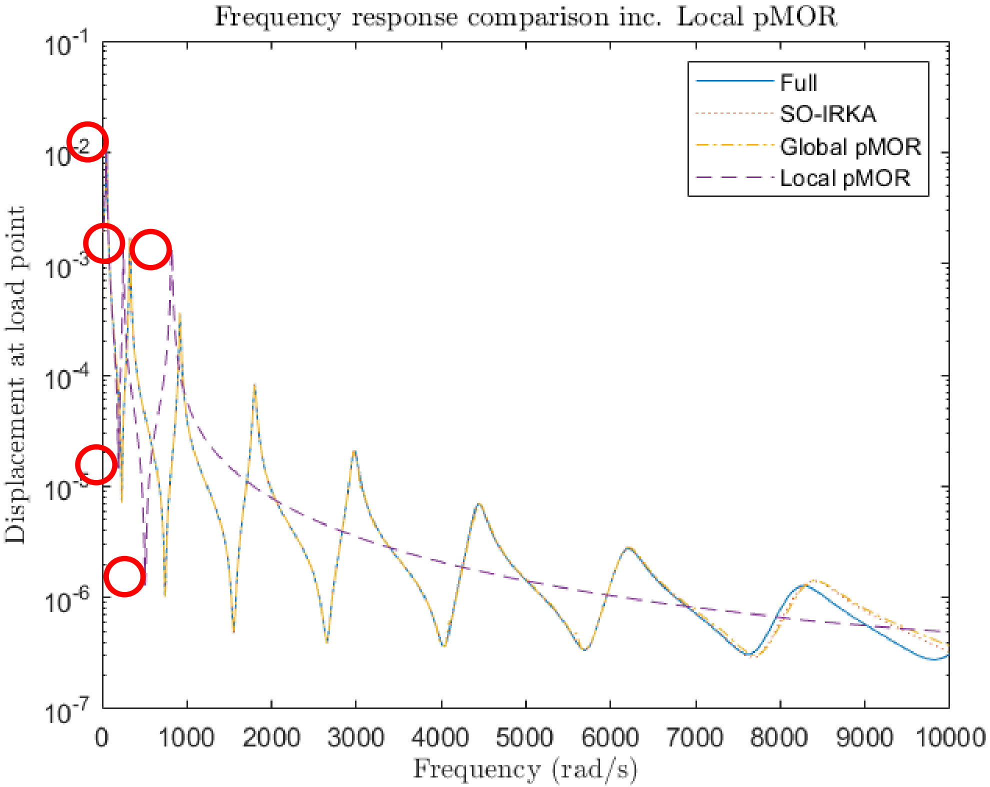
Performance Study – **Total** Computation Time

- For **small-sized systems**, solving the full order system takes less time as compared to pMOR techniques due to **overhead** involved in setting up the MOR
- But as the number of elements increase, both **local and global pMOR** take considerable **less time as** compared to the full order system
- Local pMOR is taking more time than global pMOR because only **one online computation** performed.
- SO-IRKA takes least because it is performing MOR for just a single parameter (**not easily scalable**)



Reduced base and stiffness

- H = 0.01 m to reduce stiffness
- Many modes (peaks) represented
- Global pMOR accurate because r based on SVD
- Local pMOR with r = 5 can only **represent 5 modes**



Conclusions

- Both pMOR techniques have been **successfully implemented**

- pMOR techniques are quite effective and result in considerable **resource-saving** specially if the **parameter range** is chosen appropriately

- **Local pMOR** is the best MOR technique for applications like optimization, navigation, etc. that require **repeated model evaluations** in online phase

- This is true specially **if extrapolation isn’t involved**

- **FUTURE WORK:** Different **interpolation** methods, **sampling** of parameter space.

Thank You