Topic 4: Examining Numerical Data

Histograms

Histogram: a graphical method for analyzing the distribution of 1 numerical variable

IQ test scores for 60 randomly chosen fifth-grade students

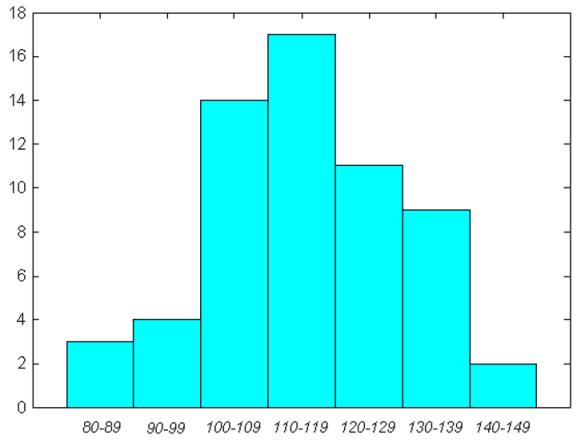
145	139	126	122	125	130	96	110	118	118
101	142	134	124	112	109	134	113	81	113
123	94	100	136	109	131	117	110	127	124
106	124	115	133	116	102	127	117	109	137
117	90	103	114	139	101	122	105	97	89
102	108	110	128	114	112	114	102	82	101

We can use a **histogram** to visually inspect the *distribution* of these IQ scores.

"Bins" = "classes" / "ranges"

Bin	Frequency
80 – 89	3
90 – 99	4
100 – 109	14
110 – 119	17
120 – 129	11
130 – 139	9
140 - 149	2

Frequency table: a sorting of data values into bins ("classes") of even width such that each value goes in exactly 1 bin

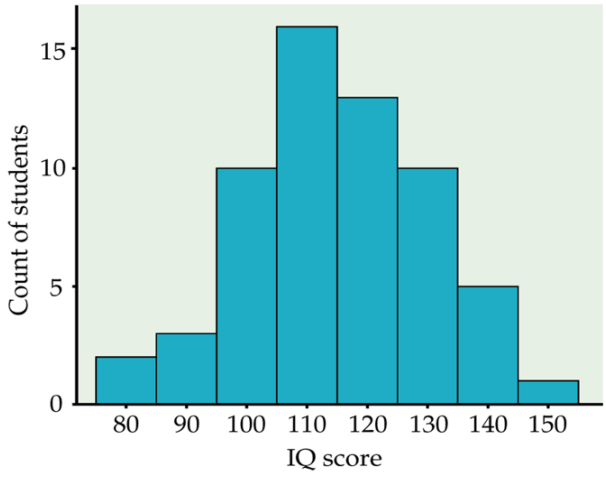


This is a histogram!

Bin etiquette: equal width/range, no gaps between their specified ranges, no overlap

Slightly different choice of bins:

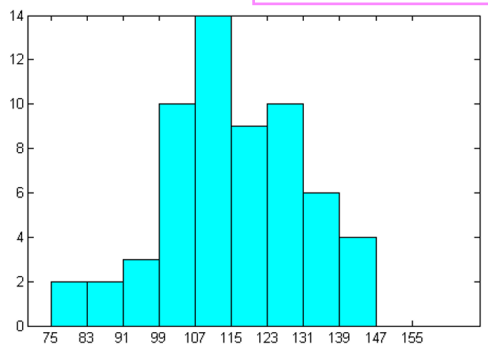
Bin	Frequency
75-84	2
85-94	3
95-104	10
105-114	16
115-124	13
125-134	10
135-144	5
145-154	1

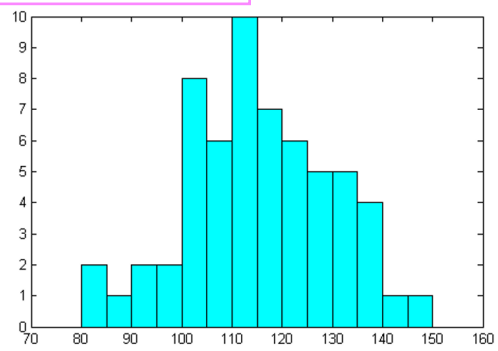


A single dataset has many valid bin construction options, as long as you obey the rules above!

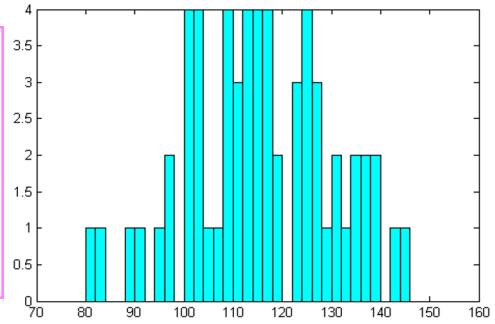
Other choices of bins:

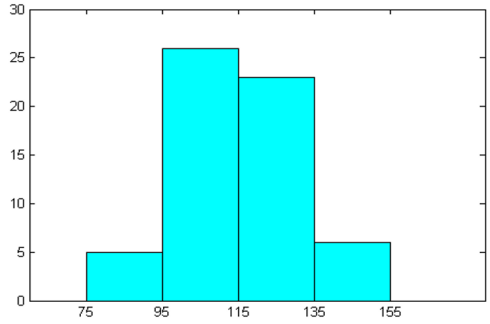
These top two both look like good bin choices with regards to width.





These bins are too narrow - the many gaps between bins and big jumps between bin heights obscure the distribution.





These bins are too wide - there's so few bins that we can't see the shape formed by the data very well.

Using relative frequencies instead:

Rel. freq. = freq. / n

Item Price	Frequency	Relative Frequency
$1 - $10	20	0.303
$11 - $20	21	0.318
$21 - $30	13	0.197
$31 - $40	8	0.121
$41 - $50	4	0.061

n = total number of data values

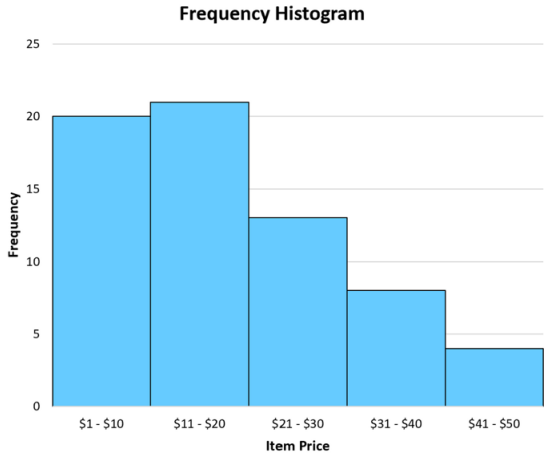
The relative frequencies will always all add up to 1.

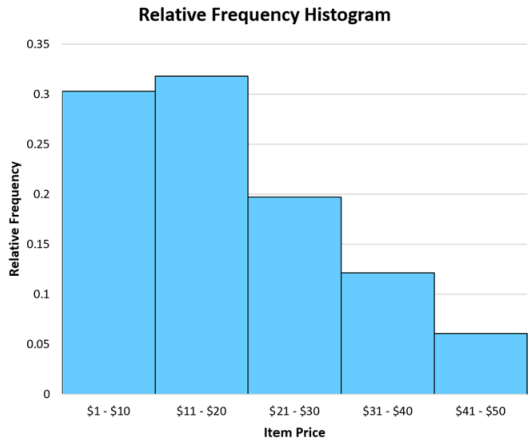
You can always find n by adding up all frequencies!

Relative frequency of "$41-$50":

freq / n = 4/66 = 0.061 (approx.)

n = 20+21+13+8+4 = 66





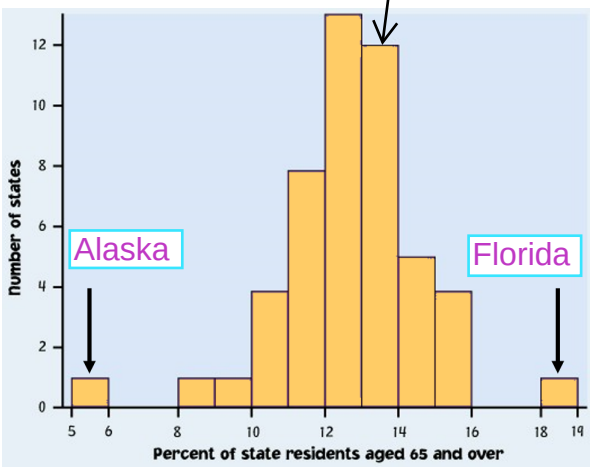
Histograms have the SAME SHAPE regardless of frequency vs relative frequency.

Using relative frequency allows you to COMPARE histograms of multiple datasets with DIFFERENT amounts of data (n values)

Using a histogram to identify **outliers**:

Outlier: a data value that is far away from the rest of the data

"12 states have 13%-14% of residents aged 65 and over"



Look for bars of height 1 with empty space between them and the rest of the (otherwise-well-structured) histogram

Describing Distributions

- Shape

  Peaks: unimodal (1), bimodal (2), multimodal (3+), or uniform(0)?

  Layout: symmetric or skew?

  WATCH OUT!
  "Right-skew" means the TAIL is on the RIGHT, and the peak is on the left.
  "Left-skew" means the TAIL is on the LEFT, and the peak is on the right.

- Center

  Mean - average
  Median - middle data value when sorted smallest to largest
  Mode - most frequent (can be local)

  Symmetric: mean ≈ median
  Right-skew: mean > median
  Left-skew: mean < median

- Spread

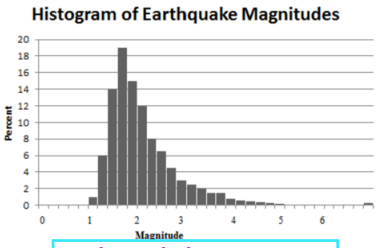
  Variance and/or standard deviation
  Inter-Quartile Range (IQR)

- Outliers

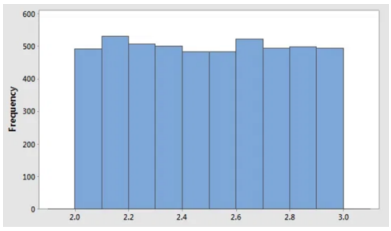
  Are there outliers?

2 LOCAL modes here!

Bimodal
Symmetric (roughly)
Mean ≈ median, around 49-50
1 minor outlier to left



Unimodal
Skew: right-skew
Mean > median
1 outlier far to right



Uniform
Symmetric
Mean ≈ median, around 2.5
No outliers

Example: A child's birthday party has 9 attendees of the following ages: 7, 1, 3, 4, 4, 6, 3, 5, 3

- Notation

Observations: label as x_1, x_2, x_3, ... x_n
n = total # of data points

Here: n=9, and $x_1 = 7$, $x_2 = 1$, etc.

Refer to a specific data point as "x_i" where "i" is an integer in the set {1, 2, ... n}

- Measures of center

Mean: average of all data points

Summation notation!
"The sum from i=1 to i=n of all values x_i"

Notation: $\bar{x}$

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\bar{x} = \frac{1}{9}(7+1+3+4+4+6+3+5+3)$$

$$= 4 \text{ years old}$$

Median: "middle data value", according to sorted data smallest to largest

1, 3, 3, 3, 4, 4, 5, 6, 7

Here: since there are 9 values, the median is equal to the 5th one when ordered smallest to largest

m = 4 years old

Mode: "most frequent data value"

Here: the value 3 occurs the most in the data (three times)

How does adding a 64-year old to the group change mean and median?

$$\bar{x} = \frac{1}{10}(7+1+3+4+4+6+3+5+3+64)$$

$$= 10 \text{ years old}$$

1, 3, 3, 3, 4, 4, 5, 6, 7, 64

For n even, have TWO middle values... Just average them

$$\text{Median} = \frac{4+4}{2} = 4 \text{ years old}$$

Effect of outliers on mean and median:

Mean is **sensitive** to outliers; outliers have a large effect on the mean.

Median is **robust** to outliers; outliers do NOT have a large effect on the median.

- Median as a percentile

Other concept for median: median is **the 50th percentile**

x^{th} percentile: the value greater than or equal to (≥) x% of the data values

Median is **the value ≥ 50%** of the data values (which also means it is less than 50% of the data values)

Same example: Birthday party attendees aged 7, 1, 3, 4, 4, 6, 3, 5, 3

- Measures of spread: variance and standard deviation

Variance and standard deviation: represent "how far" a typical (i.e. random) observation x_i is from the MEAN

WARNING!
Divide by **n-1** to get s^2, NOT by n.

Sample variance: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

"Average squared difference from mean"

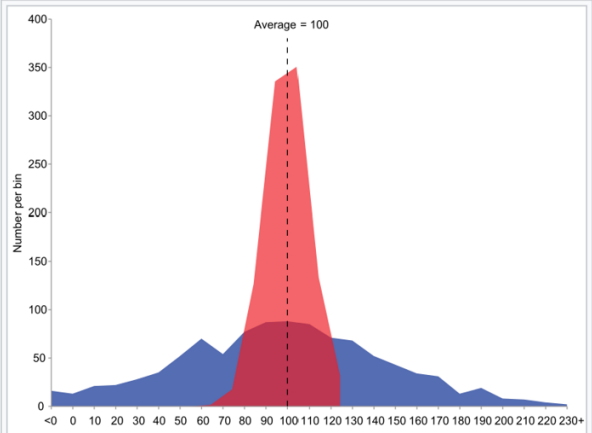
For birthday party (no Grandma):

$$= \frac{1}{9-1}\left((7-4)^2 + (1-4)^2 + 3\cdot(3-4)^2 + 2(4-4)^2 + (5-4)^2 + (6-4)^2\right)$$

$$= \frac{1}{8}\cdot 26 = 3.25 \text{ years}^2$$

Sample standard deviation: $s = \sqrt{\text{sample variance}} = \sqrt{s^2}$

For birthday party (no Grandma): $= \sqrt{3.25} \approx 1.8$ years

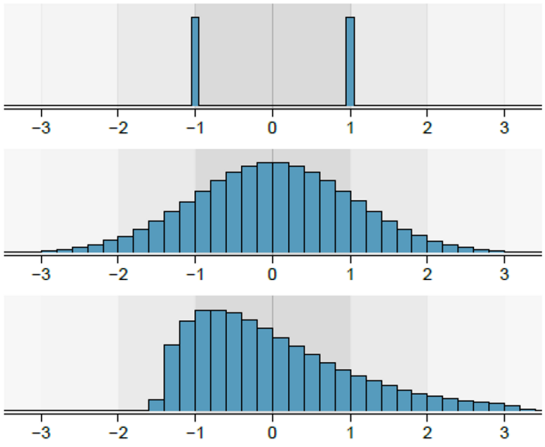


HIGHER variance / standard deviation means data is MORE spread out, tends to be FURTHER AWAY from the mean

Above: two datasets with the SAME mean=100 and SAME amount of data

Red data: SMALL variance, data concentrates very close to mean=100

Blue data: BIG variance, data spreads out very far from mean=100



WARNING!
Mean + var / std dev still leaves out A LOT of info about data shape! You should always examine your data graphically, too!

Above: three datasets with the SAME mean=0 and SAME std dev = 1

Top: all data is 1 or -1 (bimodal, very concentrated)

Middle: symmetric around mean=0

Bottom: heavy right-skew

Same example: Birthday party attendees aged 7, 1, 3, 4, 4, 6, 3, 5, 3

- Another measure of spread: IQR

IQR = range of "middle 50%" of data

Q1: "first quartile"
25th percentile
value ≥ 25% of data
median of "bottom half" of data

Q3: "third quartile"
75th percentile
value ≥ 75% of data
median of "top half" of data

IQR = Q3 - Q1

WARNING!
When n is odd, there are TWO WAYS to identify the "halves" of the data!
The halves MUST have equal numbers of data points for this calculation!

Inclusive: include median in both halves

1, 3, 3, 3, 4, 4, 5, 6, 7

Q1 = 3 (middle of lower)
Q3 = 5 (middle of upper)
IQR = 5 - 3 = 2 years

Exclusive: exclude median from both halves

1, 3, 3, 3, 4, 4, 5, 6, 7

Q1 = 3 (avg of middle two of lower)
Q3 = 5.5 (avg of middle two of upper)
IQR = 5.5 - 3 = 2.5 years

- IQR criterion for outliers:

According to IQR, a datapoint is an **outlier** if it is more than 1.5 x IQR BELOW Q1 or ABOVE Q3

Rephrase:
A point x_i is an outlier, IF...
x_i < Q1 - 1.5 IQR
OR...
x_i > Q3 + 1.5 IQR

1, 3, 3, 3, 4, 4, 5, 6, 7, 64

Q1 = 3 (middle of lower)
Q3 = 6 (middle of upper)
IQR = 6 - 3 = 3 years

The outliers are...
anything < 3 - 1.5(3) = -1.5 years old
anything > 6 + 1.5(3) = 10.5 years old
64 > 10.5, so 64 is an outlier!

- 5-number summary and box plot:

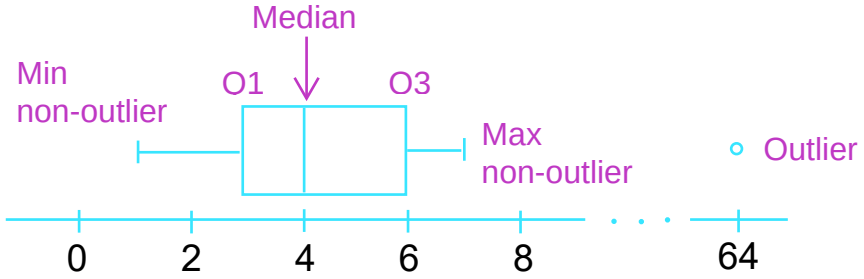
5-number summary of numerical data:

Minimum, Q1, Median, Q3, Maximum

This summary should include outliers!

1, 3, 3, 3, 4, 4, 5, 6, 7, 64

5-number summary: 1, 3, 4, 6, 64



We can graphically summarize all this information in a **boxplot**.

Boxplot instructions:

Box is from Q1 to Q3
Put median line in box
Whiskers extend out to min/max values that are NOT OUTLIERS
Indicate outliers with dots beyond whiskers

Data analysis in Excel: some easy commands to try on sheet `unc2017.xlsx` (posted on Canvas)!

Mean: **AVERAGE**(data)

Variance: **VAR.S**(data)

Standard deviation: **STDEV.S**(data)

Minimum: **MIN**(data)

Quartile 1: **QUARTILE.INC**(data,1)

Median: **MEDIAN**(data)

Quartile 3: **QUARTILE.INC**(data,3)

Maximum: **MAX**(data)