Topic 7: Linear Regression

Practice Test Questions: Correlation

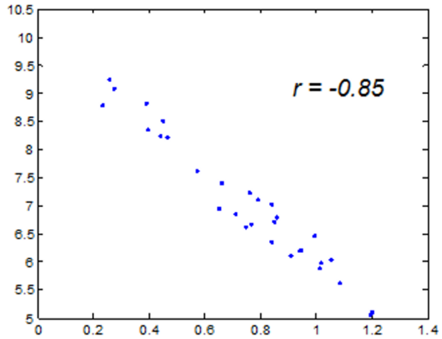
i	x_i	y_i	$(x_i - \bar{x})/s_x$	$(y_i - \bar{y})/s_y$	product
1	0	5	–1.1	+1.17	–1.28
2	2	2	***T***	–0.83	+0.25
3	3	2	+0.1	–0.83	–0.08
4	6	4	+1.3	+0.65	+0.65
	$\bar{x} = 2.75$	$\bar{y} = 3.25$			Sum = –0.46
	$s_x = 2.5$	$s_y = 1.5$			***r*** =

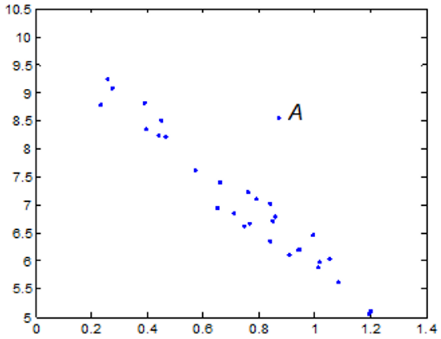
1. What is ***T*** ?
   - A +0.3
   - B –0.3
   - C –0.75
   - D 0.75
   - E None of the above.

2. What is ***r*** ?
   - A –0.46
   - B –0.115
   - C –0.153
   - D 0
   - E None of the above.

The two scatterplots show the same data, except that the point at *A* has been added to the dataset at the bottom. What can you say about the correlation coefficient of the dataset with *A* added?

- A It is greater than –0.85.
- B It is less than –0.85.
- C It equals –0.85.
- D It is impossible to tell without computing the correlation.





Least Squares Regression

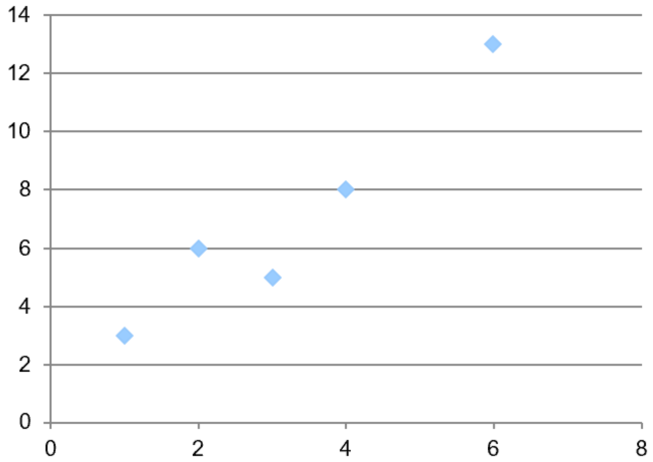
A **regression line** describes ...

Unlike with correlation, it matters

General equation of a line:

Small example:

X	Y
1	3
2	6
3	5
4	8
6	13



How to construct a line to fit these data?

Notation:

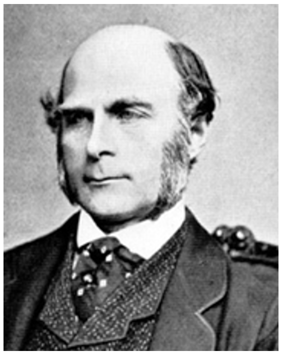
Definition: A **residual**

Mathematically, a least-squares regression line minimizes the sum

What values of b_0 and b_1 minimize this quantity?

Why the term “regression”?

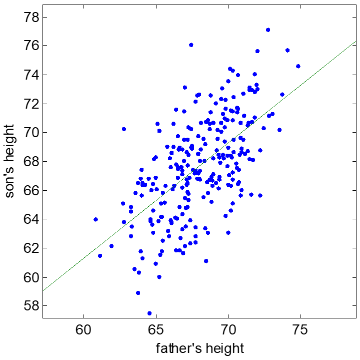
The term was coined by Francis Galton (1822 - 1911) in a study on the heights of fathers and sons. He found that fathers taller than average would more likely have shorter sons, and fathers shorter than average would more likely have taller sons. Galton called this “regression towards the mean height.” (His 1886 paper was titled “Regression towards mediocrity in hereditary stature.”)



Galton’s data looked something like this:

For fathers taller than average, the least-squares line predicts a shorter height for their sons, and vice-versa. That is, the slope b_1 of the line is less than 1.

The name “regression line” stuck, even though it’s only a “regression” when the units are the same on both axes and slope is less than 1.



Practice: Observations for explanatory variable X and response variable Y have

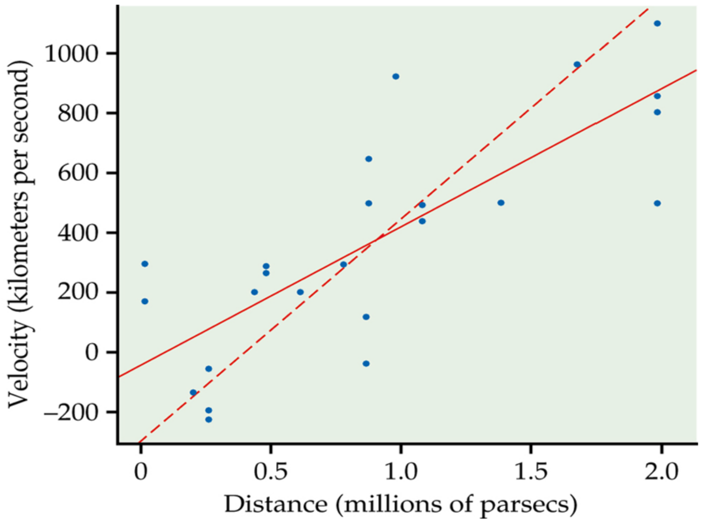
$$\bar{x} = 7, \quad \bar{y} = 5, \quad s_x = 4, \quad s_y = 2, \quad r = 0.85$$

Find the equation of a least-squares regression line that we could use to predict an observation of Y given an observation of X.

Question: Will switching which variable is explanatory and which is the response change the least-squares line?

Example: Data on galaxies

X: galaxy's distance from Earth
Y: velocity at which galaxy is moving away from Earth



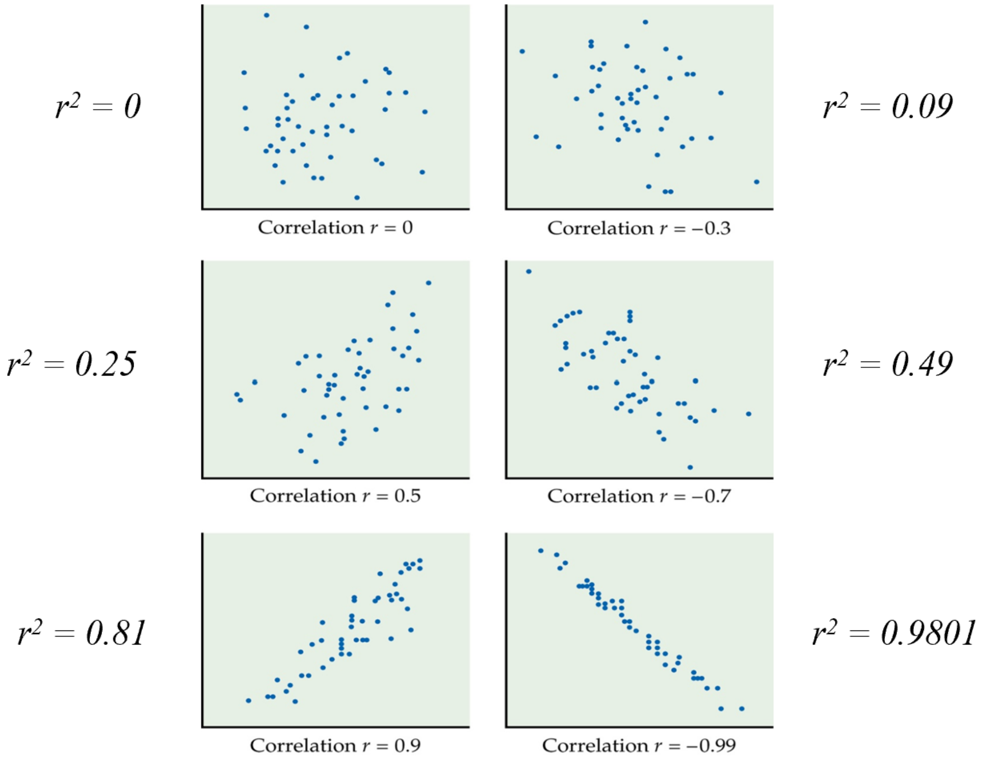
Solid line: prediction of velocity for a given distance.

Dashed line: prediction of distance for a given velocity.

Connection between Correlation and Regression

The value r^2 measures ...

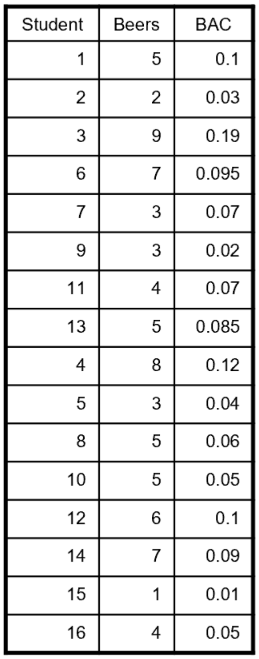
Rule of thumb:



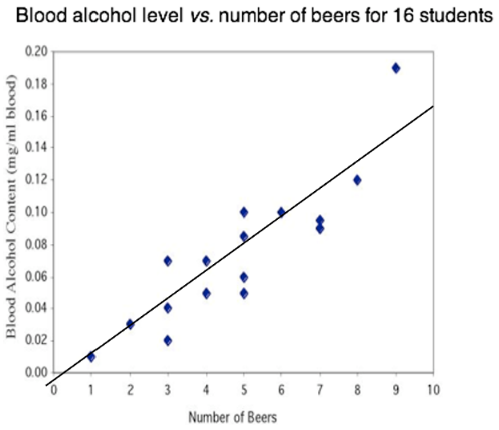
$r = 0.5 \implies$

$r = -0.7 \implies$

Limitations of using regression to make predictions



Student	Beers	BAC
1	5	0.1
2	2	0.03
3	9	0.19
6	7	0.095
7	3	0.07
9	3	0.02
11	4	0.07
13	5	0.085
4	8	0.12
5	3	0.04
8	5	0.06
10	5	0.05
12	6	0.1
14	7	0.09
15	1	0.01
16	4	0.05



Regression equation:

$\hat{y} = -0.0127 + 0.018x$

Extrapolation vs. interpolation:

Practice test question: Researchers measured the two variables below for a large number of subjects.

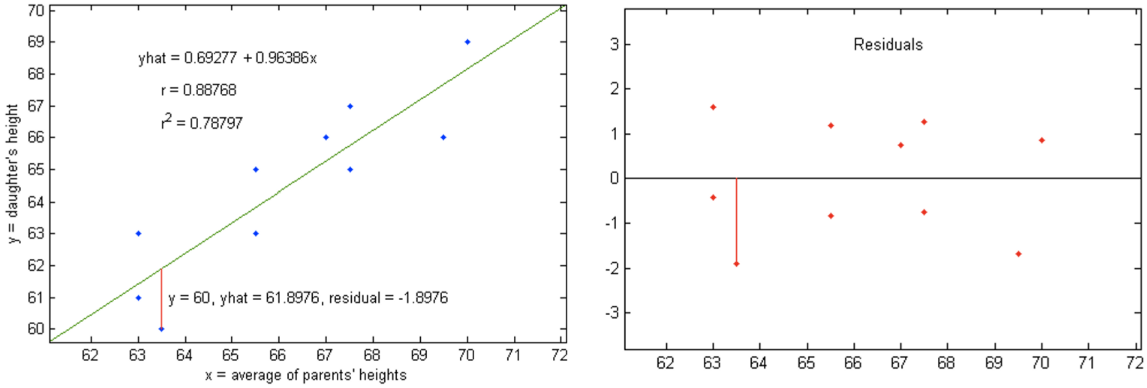
X = sodium intake in mg per day, and
Y = systolic blood pressure

The least-squares regression line for predicting Y given X is

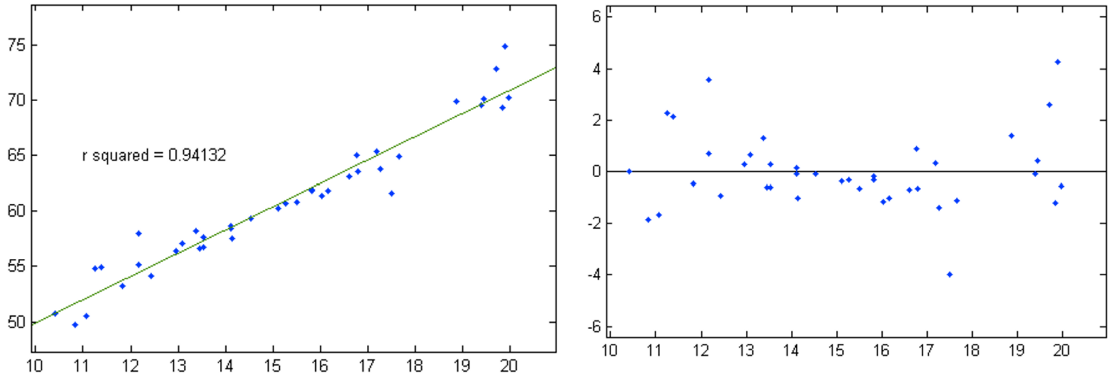
$$\hat{y} = -15.4 + 2.3x$$

What is the predicted systolic blood pressure for someone whose sodium intake is 60 mg per day?

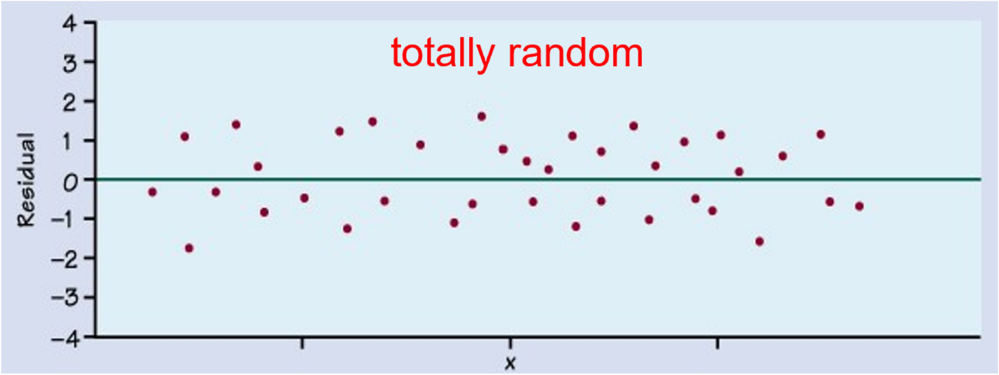
Residual Plots



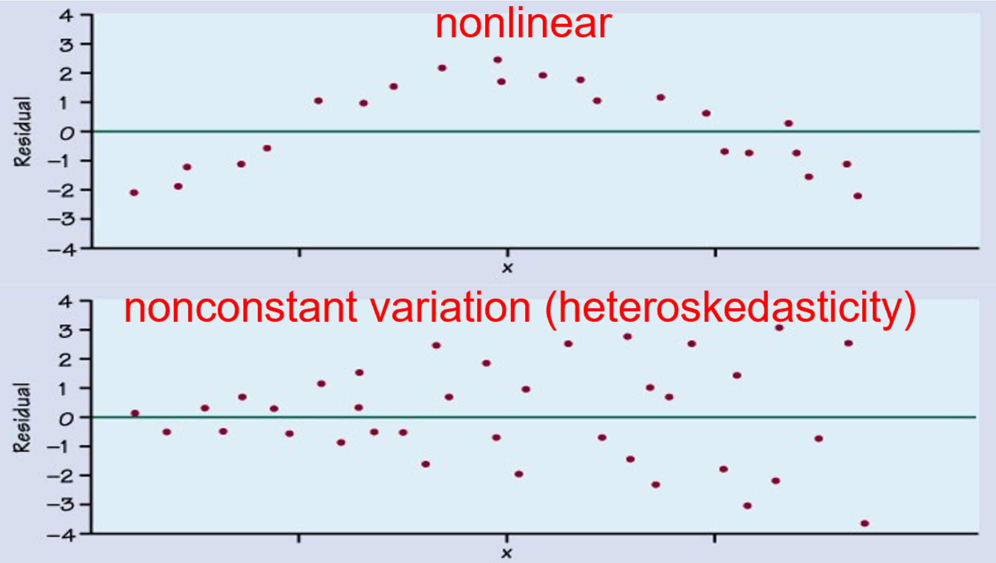
Why would we want to plot residuals by themselves?



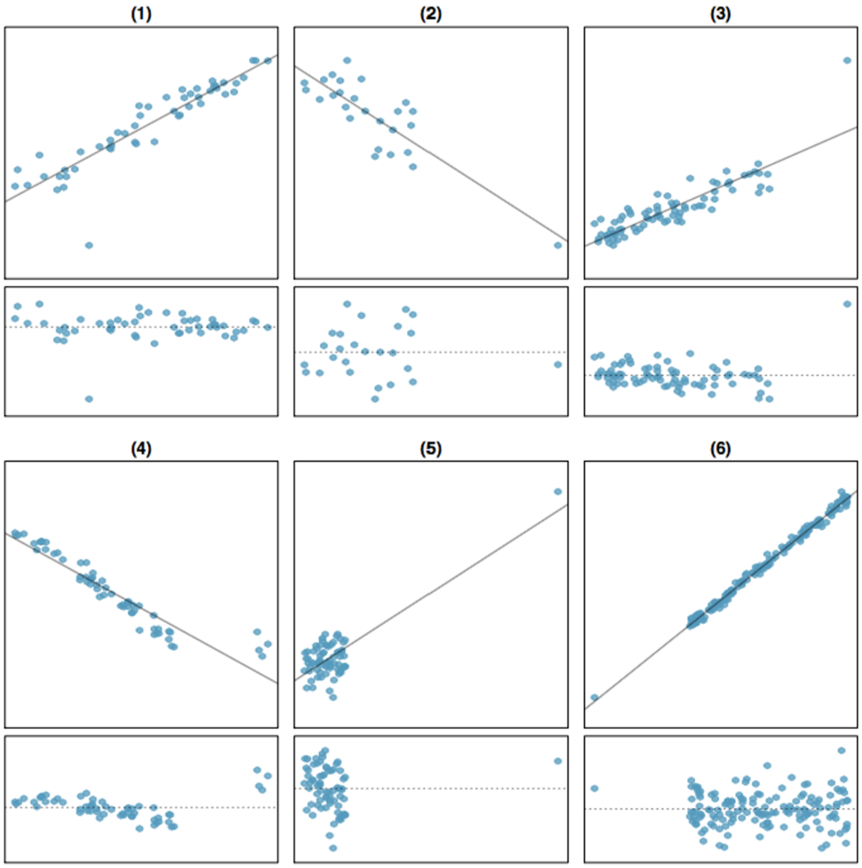
Ideal residual plot:



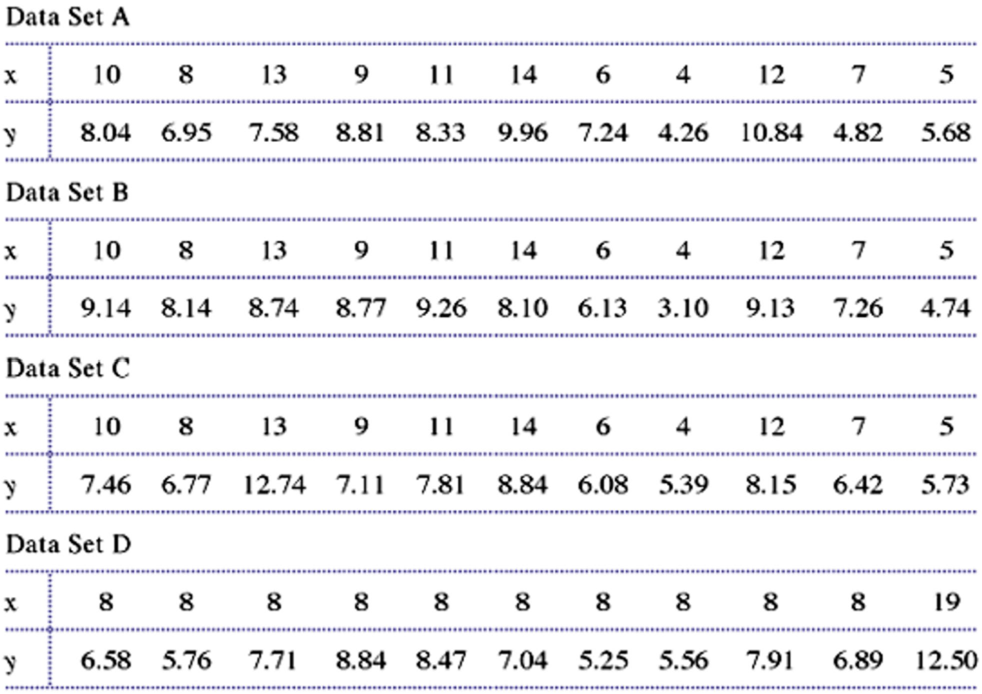
Not-so-ideal residual plots:



Influence of outliers:



Importance of Plotting Data



Data Set A

x	10	8	13	9	11	14	6	4	12	7	5
y	8.04	6.95	7.58	8.81	8.33	9.96	7.24	4.26	10.84	4.82	5.68

Data Set B

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.26	8.10	6.13	3.10	9.13	7.26	4.74

Data Set C

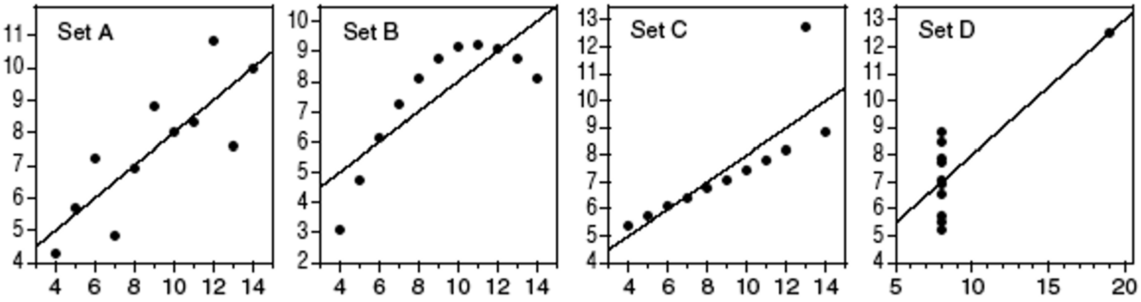
x	10	8	13	9	11	14	6	4	12	7	5
y	7.46	6.77	12.74	7.11	7.81	8.84	6.08	5.39	8.15	6.42	5.73

Data Set D

x	8	8	8	8	8	8	8	8	8	8	19
y	6.58	5.76	7.71	8.84	8.47	7.04	5.25	5.56	7.91	6.89	12.50

Source: Frank J. Anscombe, "Graphs in statistical analysis," The American Statistician, 27 (1973), pp. 17–21.

All have correlation $r = .816$, regression equation $\hat{y} = 3 + 0.5x$


Set A
Set B
Set C
Set D