# TSA Tutorial

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## 1. Analysis of lynx Dataset

The lynx dataset contains annual lynx trappings from 1821 to 1934, known for its distinct cyclical pattern.

```
# Load libraries once for the entire document
library(forecast)
```

```
Registered S3 method overwritten by 'quantmod':
  method            from
  as.zoo.data.frame zoo
```

```
library(tseries)
library(imputeTS)
```

```
Attaching package: 'imputeTS'

The following object is masked from 'package:tseries':

    na.remove
```

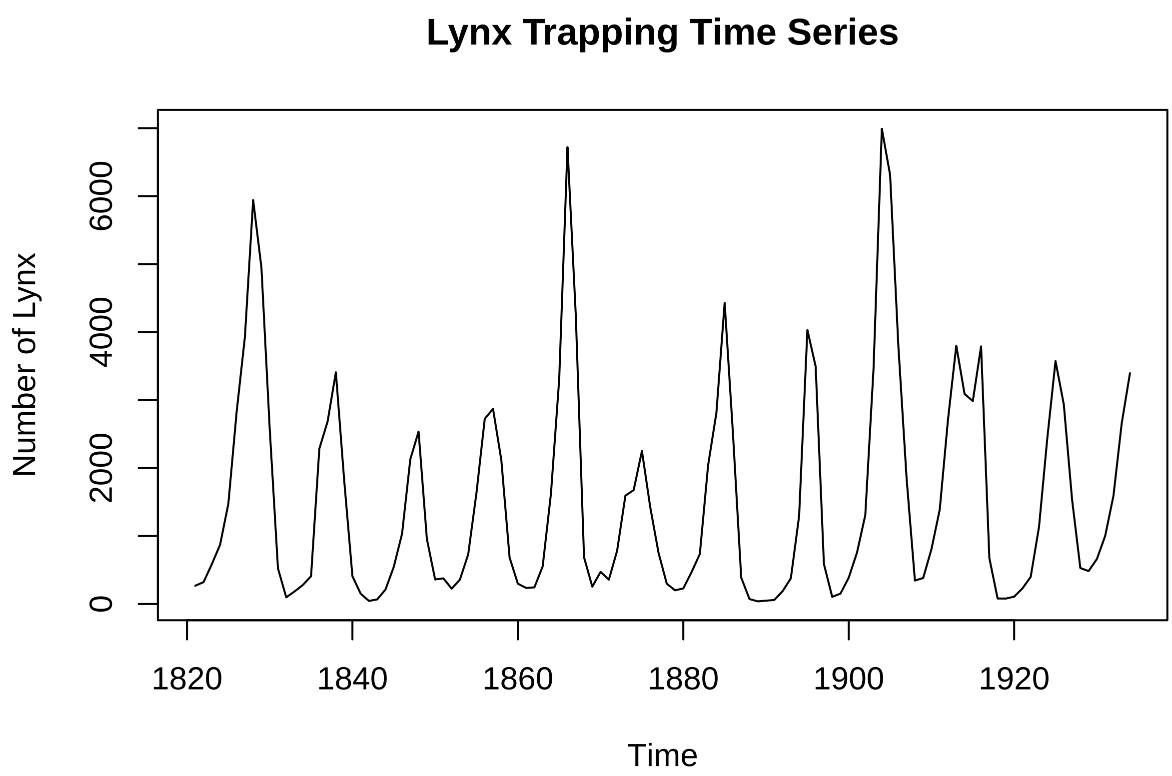
```
# Exploratory Data Analysis
# Check if the dataset is a time series object
data("lynx")
str(lynx)
```

```
 Time-Series [1:114] from 1821 to 1934: 269 321 585 871 1475 ...
```

```
print("Dataset: lynx - Annual lynx trappings (1821-1934), time series confirmed.")
```

```
[1] "Dataset: lynx - Annual lynx trappings (1821-1934), time series confirmed."
```

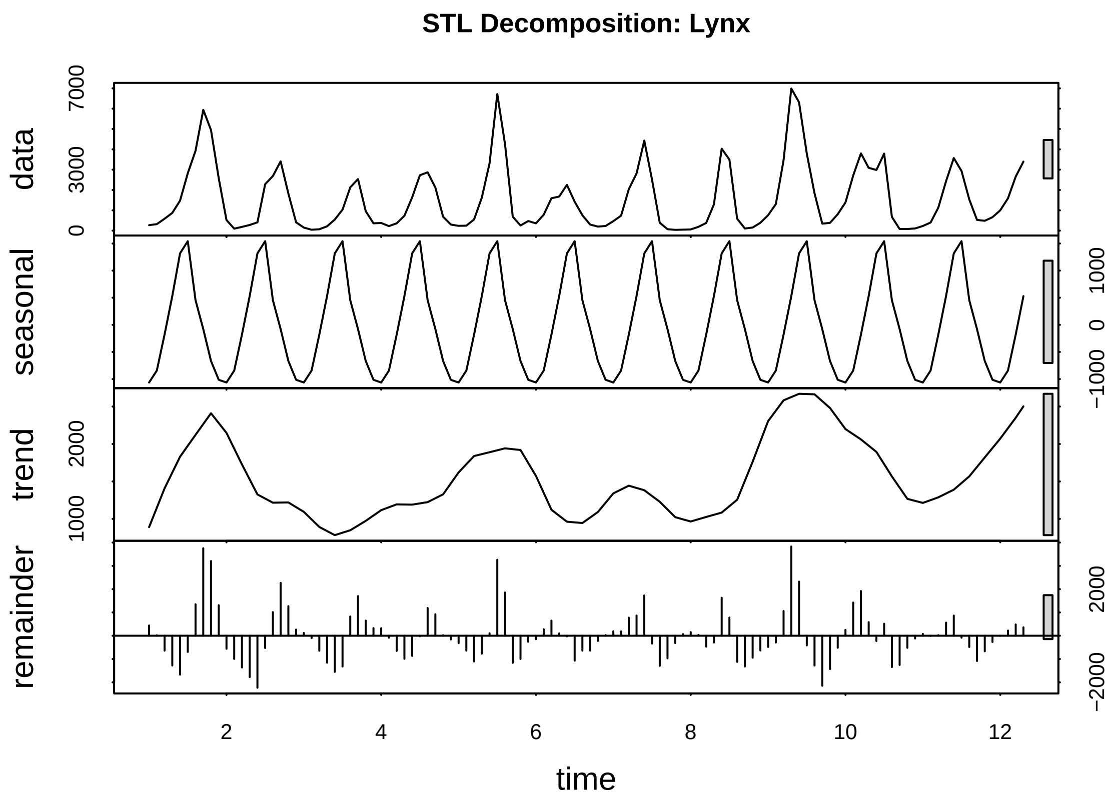
```
# Plot the time series
plot(lynx, main = "Lynx Trapping Time Series", ylab = "Number of Lynx")
```




```
summary(lynx)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
   39.0   348.2   771.0  1538.0  2566.8  6991.0 
```

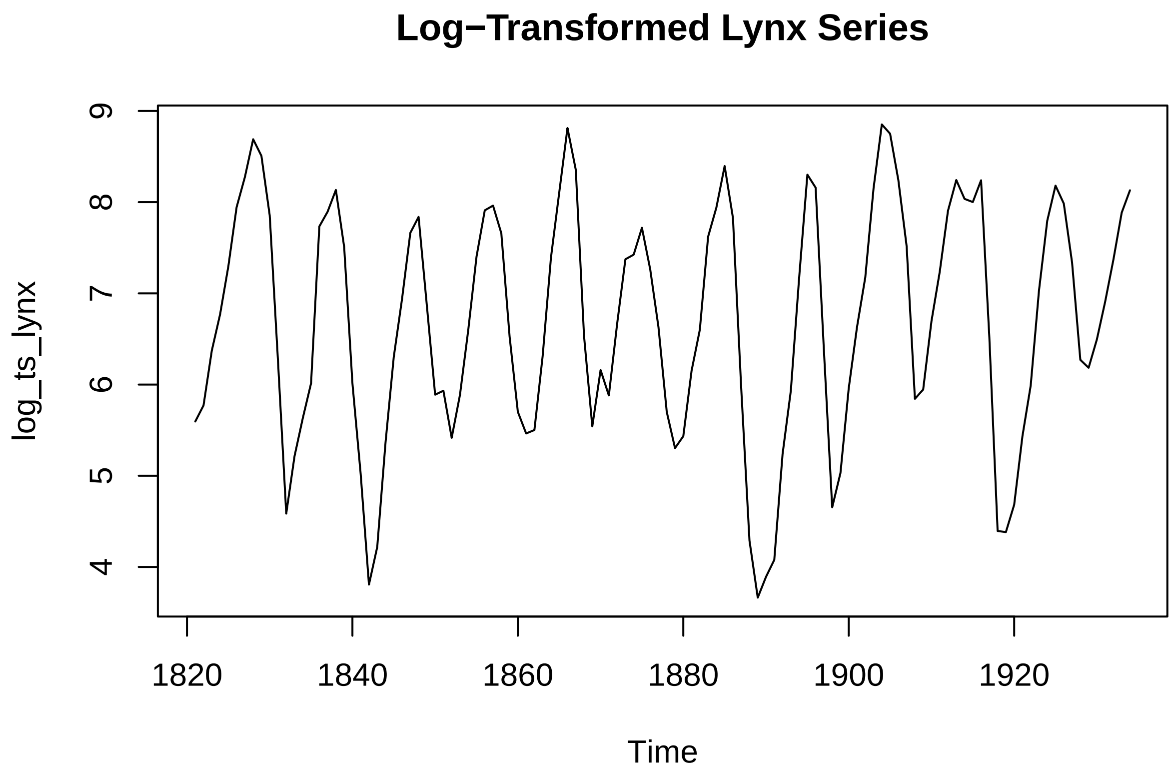
```
# Decomposition using STL for cyclical patterns
lynx_ts <- ts(lynx, frequency = 10) # 10 year cycle
decomp_lynx <- stl(lynx_ts, s.window = "periodic")
plot(decomp_lynx, main = "STL Decomposition: Lynx")
```



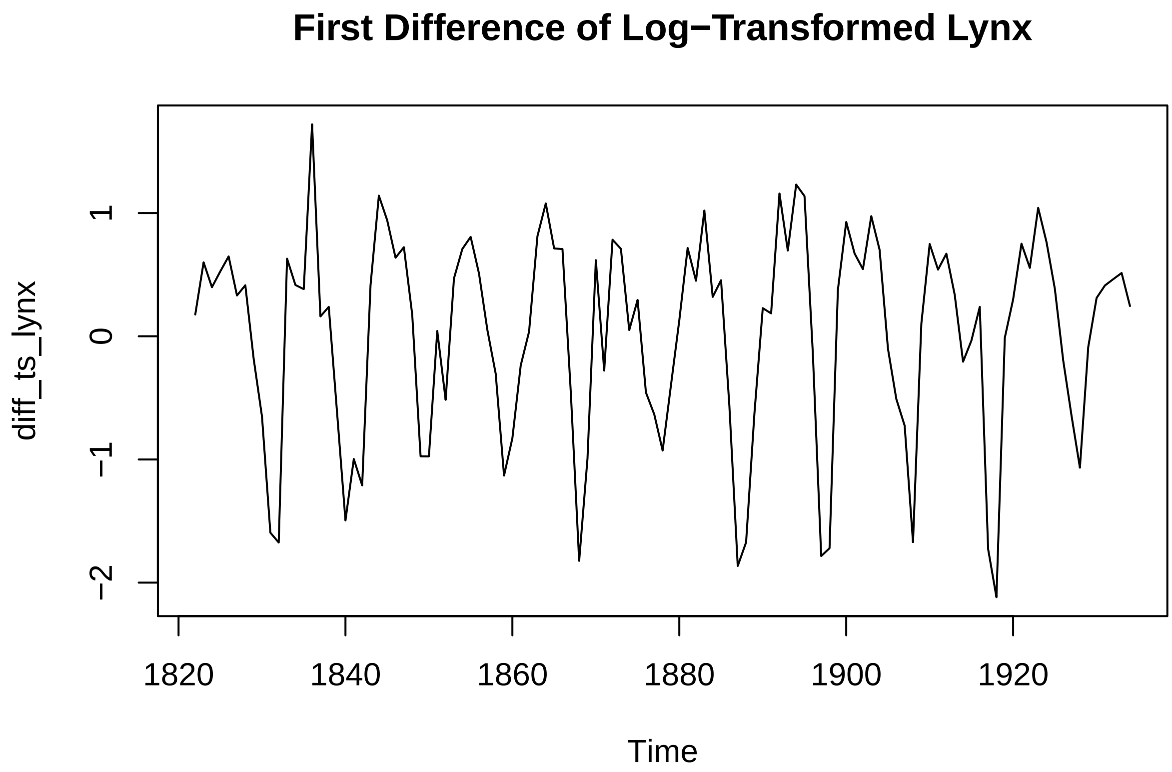

```r
# Data Preprocessing
# Check for missing values (none)
sum(is.na(lynx))
```

```
[1] 0
```

```r
# Apply log transformation to stabilize variance
log_ts_lynx <- log(lynx)
plot(log_ts_lynx, main = "Log-Transformed Lynx Series")
```




```r
# Apply first differencing to achieve stationarity
diff_ts_lynx <- diff(log_ts_lynx, differences = 1)
plot(diff_ts_lynx, main = "First Difference of Log-Transformed Lynx")
```




```r
# Stationarity Testing
# ADF Test for the original series
adf_original_lynx <- adf.test(lynx)
```

```
Warning in adf.test(lynx): p-value smaller than printed p-value
```

```r
print(adf_original_lynx)
```

```
    Augmented Dickey-Fuller Test

data:  lynx
Dickey-Fuller = -6.3068, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

```r
# KPSS Test for the original series
kpss_original_lynx <- kpss.test(lynx)
```

```
Warning in kpss.test(lynx): p-value greater than printed p-value
```

```r
print(kpss_original_lynx)
```

```
    KPSS Test for Level Stationarity

data:  lynx
KPSS Level = 0.070147, Truncation lag parameter = 4, p-value = 0.1
```

```
# ADF Test for the differenced series
adf_diff_lynx <- adf.test(diff_ts_lynx)
```

```
Warning in adf.test(diff_ts_lynx): p-value smaller than printed p-value
```

```
print(adf_diff_lynx)
```

```
    Augmented Dickey-Fuller Test

data:  diff_ts_lynx
Dickey-Fuller = -8.8658, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

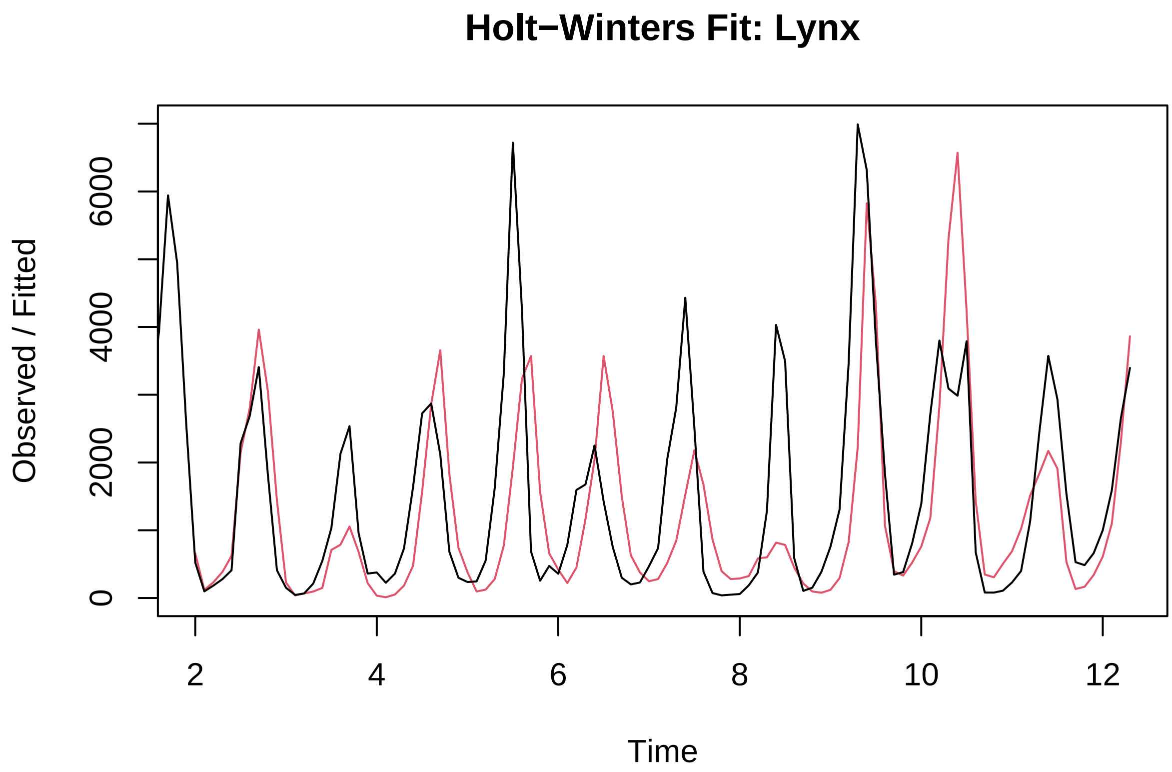
```
# KPSS Test for the differenced series
kpss_diff_lynx <- kpss.test(diff_ts_lynx)
```

```
Warning in kpss.test(diff_ts_lynx): p-value greater than printed p-value
```

```
print(kpss_diff_lynx)
```

```
    KPSS Test for Level Stationarity

data:  diff_ts_lynx
KPSS Level = 0.017293, Truncation lag parameter = 4, p-value = 0.1
```

```
# Modelling and Forecasting
# Fit Holt-Winters model on the original series
# Use a multiplicative model due to varying amplitude of the cycles
hw_model_lynx <- HoltWinters(lynx_ts, seasonal = "multiplicative")
plot(hw_model_lynx, main = "Holt-Winters Fit: Lynx")
```




```r
summary(hw_model_lynx)
```

```
             Length Class  Mode
fitted       416    mts    numeric
x            114    ts     numeric
alpha          1    -none- numeric
beta           1    -none- numeric
gamma          1    -none- numeric
coefficients  12    -none- numeric
seasonal       1    -none- character
SSE            1    -none- numeric
call           3    -none- call
```

```r
# Fit AR(1) and MA(1) models on the differenced series
ar_model_lynx <- arima(diff_ts_lynx, order = c(1, 0, 0))
summary(ar_model_lynx)
```

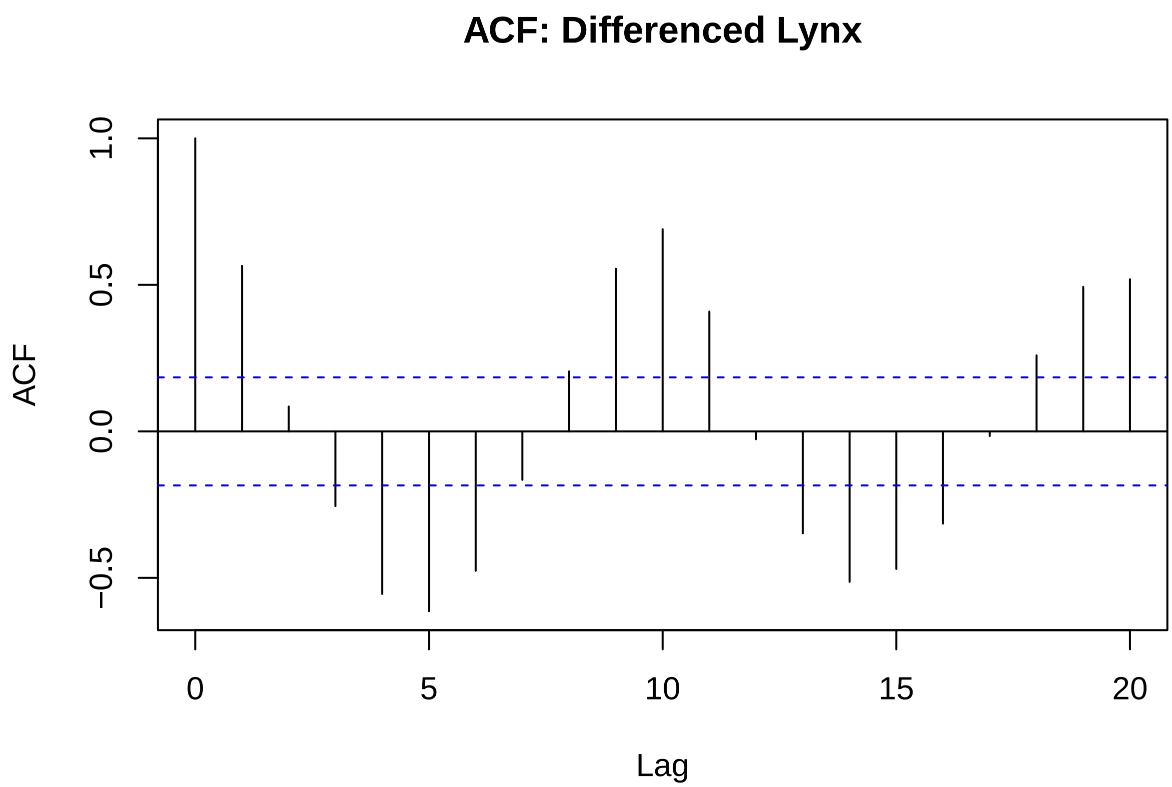
```
Call:
arima(x = diff_ts_lynx, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.5605     0.0266
```

```
s.e.  0.0770     0.1441

sigma^2 estimated as 0.4635:  log likelihood = -117.08,  aic = 240.16

Training set error measures:
                        ME      RMSE       MAE       MPE     MAPE      MASE
Training set -0.0009755728 0.6807822 0.5353477 -5.899207 211.5224 0.9166508
                  ACF1
Training set 0.1969268
```

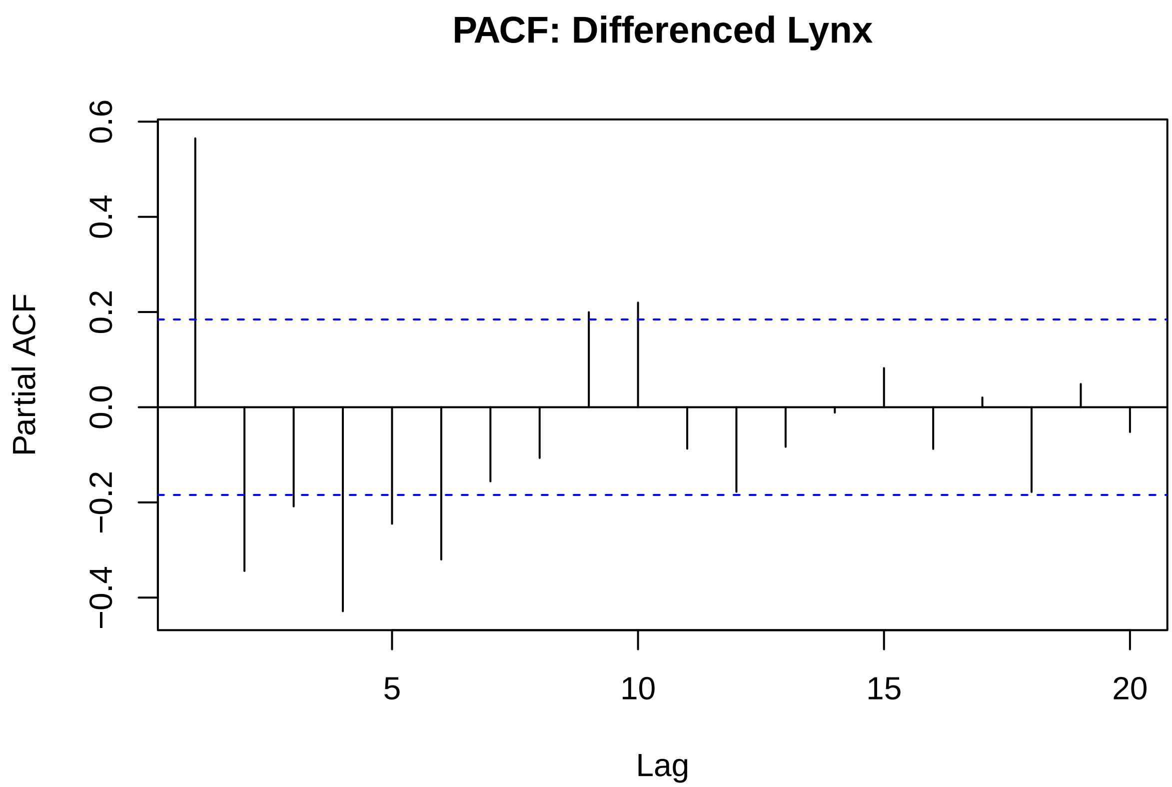
```r
ma_model_lynx <- arima(diff_ts_lynx, order = c(0, 0, 1))
summary(ma_model_lynx)
```

```
Call:
arima(x = diff_ts_lynx, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.6632     0.0217
s.e.  0.0689     0.1027

sigma^2 estimated as 0.4341:  log likelihood = -113.48,  aic = 232.95

Training set error measures:
                       ME      RMSE       MAE      MPE     MAPE      MASE
Training set 0.0002041239 0.6588284 0.5236113 35.71677 149.7891 0.8965551
                   ACF1
Training set 0.09711448
```

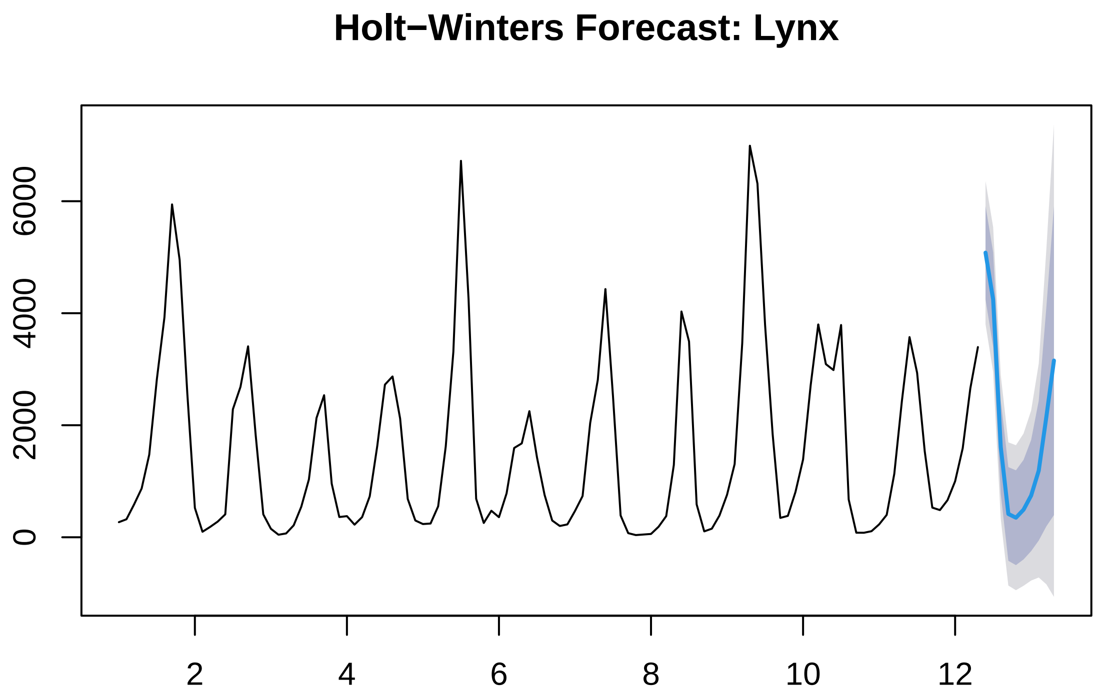
```r
# ACF/PACF plots to check for remaining patterns
acf(diff_ts_lynx, main = "ACF: Differenced Lynx")
```




```r
pacf(diff_ts_lynx, main = "PACF: Differenced Lynx")
```




```r
# Final Forecast
# Forecast using Holt-Winters
hw_forecast_lynx <- forecast(hw_model_lynx, h = 10)
plot(hw_forecast_lynx, main = "Holt-Winters Forecast: Lynx")
```




```
print(hw_forecast_lynx)
```

```
      Point Forecast       Lo 80    Hi 80      Lo 95    Hi 95
12.40      5081.6927 4246.97305 5916.412  3805.0991 6358.286
12.50      4243.9745 3398.72236 5089.227  2951.2729 5536.676
12.60      1613.0584  774.49396 2451.623   330.5847 2895.532
12.70       415.8762 -420.13052 1251.883  -862.6858 1694.438
12.80       348.9585 -497.37314 1195.290  -945.3941 1643.311
12.90       492.2393 -396.24164 1380.720  -866.5750 1851.054
13.00       745.0700 -247.37853 1737.518  -772.7490 2262.889
13.10      1190.0170  -58.08881 2438.123  -718.7961 3098.830
13.20      2148.4816  194.79919 4102.164  -839.4178 5136.381
13.30      3155.1187  396.76913 5913.468 -1063.4129 7373.650
```

## 2. Analysis of sunspots Dataset

The sunspots dataset contains monthly sunspot counts from 1749 to 1983, a classic example of cyclical data with a period of approximately 11 years.

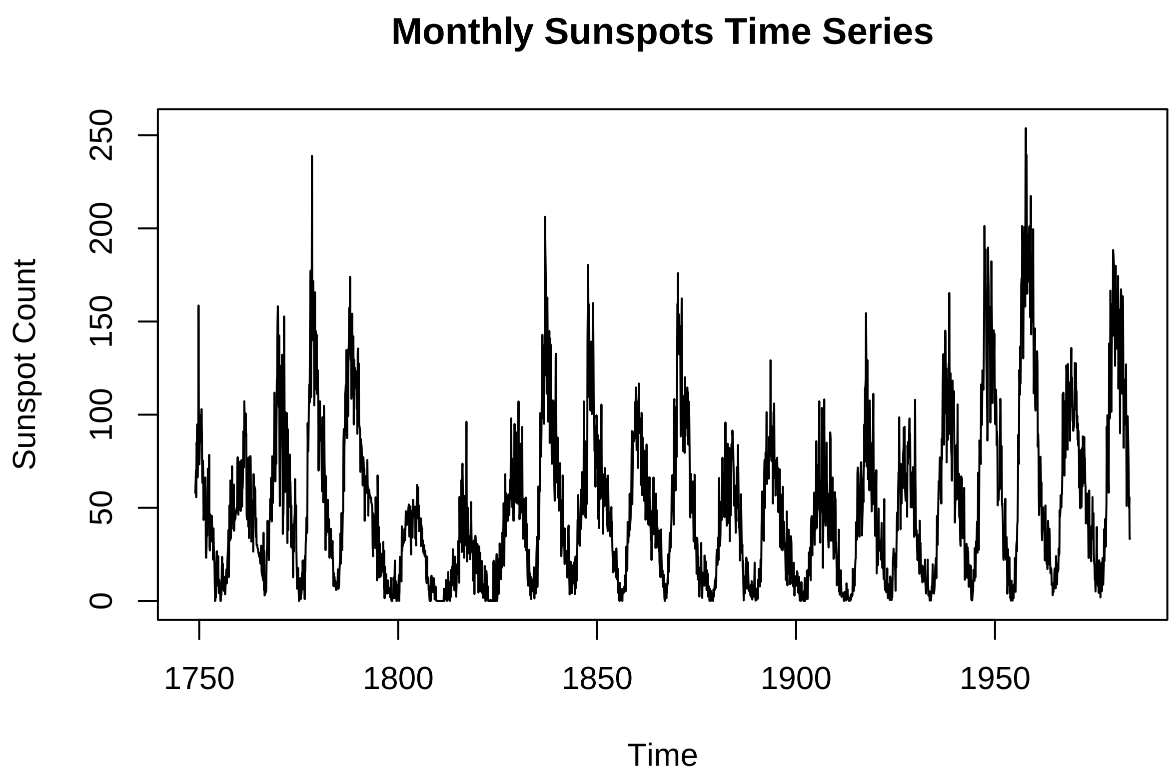
```
# Exploratory Data Analysis
data("sunspots")
str(sunspots)
```

```
 Time-Series [1:2820] from 1749 to 1984: 58 62.6 70 55.7 85 83.5 94.8 66.3 75.9 75.5 ...
```

```r
print("Dataset: sunspots - Monthly sunspots (1749-1983), time series confirmed.")
```

```
[1] "Dataset: sunspots - Monthly sunspots (1749-1983), time series confirmed."
```

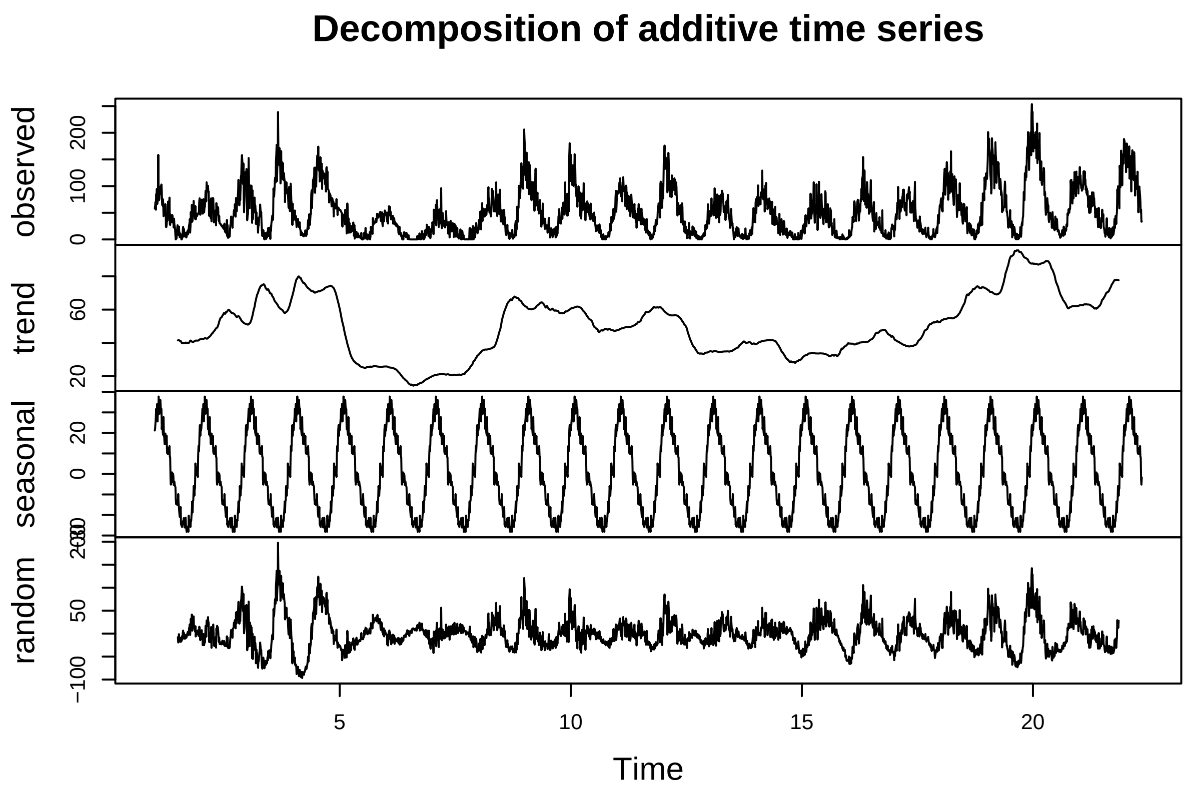
```r
plot(sunspots, main = "Monthly Sunspots Time Series", ylab = "Sunspot Count")
```




```r
summary(sunspots)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
   0.00   15.70   42.00   51.27   74.92  253.80
```

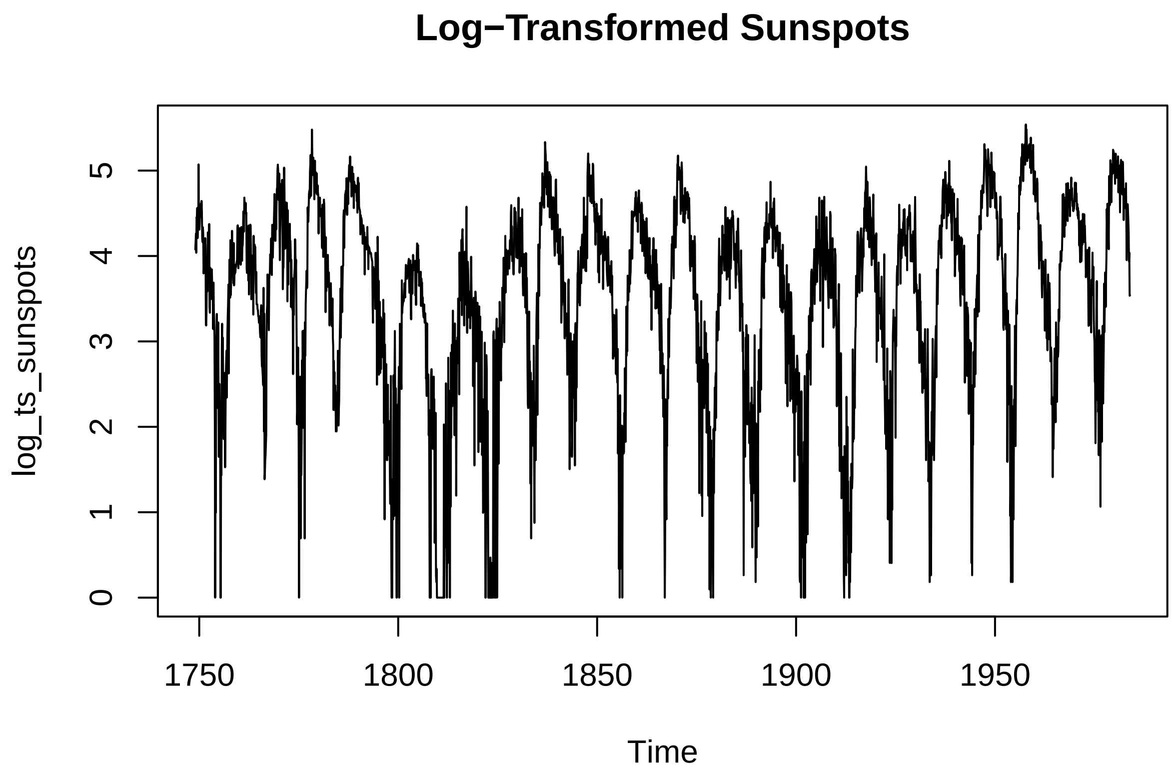
```r
# Decomposition (Solar cycle is ~11 years, which is 132 months)
sunspots_ts_adj <- ts(sunspots, frequency = 132)
decomp_sunspots <- decompose(sunspots_ts_adj, type = "additive")
plot(decomp_sunspots)
```



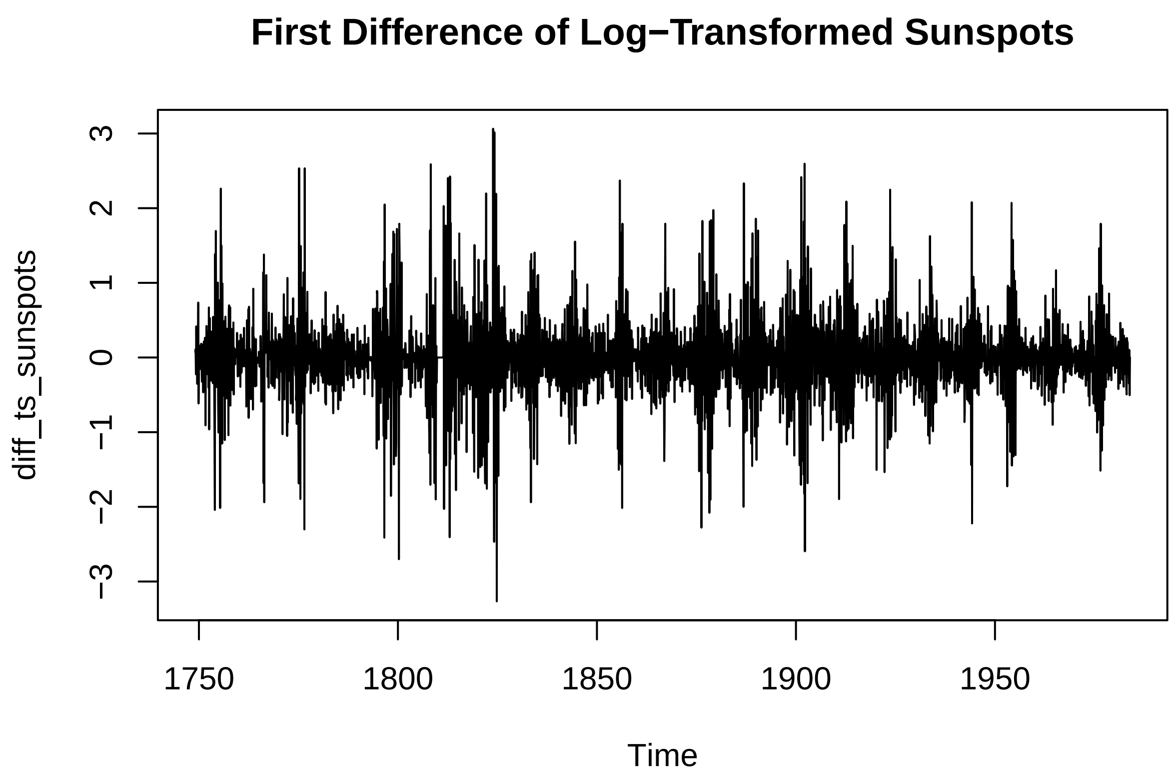

```
# Data Preprocessing
# Check for missing values (none)
sum(is.na(sunspots))
```

```
[1] 0
```

```
# Log transformation to handle zeroes and stabilize variance
log_ts_sunspots <- log(sunspots + 1) # Add 1 to avoid log(0)
plot(log_ts_sunspots, main = "Log-Transformed Sunspots")
```




```
# Apply first differencing
diff_ts_sunspots <- diff(log_ts_sunspots, differences = 1)
plot(diff_ts_sunspots, main = "First Difference of Log-Transformed Sunspots")
```




```r
# Stationarity Testing
# Original series tests
adf_original_sunspots <- adf.test(sunspots)
```

```
Warning in adf.test(sunspots): p-value smaller than printed p-value
```

```r
print(adf_original_sunspots)
```

```

    Augmented Dickey-Fuller Test

data:  sunspots
Dickey-Fuller = -6.494, Lag order = 14, p-value = 0.01
alternative hypothesis: stationary
```

```r
kpss_original_sunspots <- kpss.test(sunspots)
```

```
Warning in kpss.test(sunspots): p-value smaller than printed p-value
```

```r
print(kpss_original_sunspots)
```

```

    KPSS Test for Level Stationarity
```

```
data:  sunspots
KPSS Level = 1.1672, Truncation lag parameter = 9, p-value = 0.01
```

```
# Differenced series tests
adf_diff_sunspots <- adf.test(diff_ts_sunspots)
```

```
Warning in adf.test(diff_ts_sunspots): p-value smaller than printed p-value
```

```
print(adf_diff_sunspots)
```

```

    Augmented Dickey-Fuller Test

data:  diff_ts_sunspots
Dickey-Fuller = -12.875, Lag order = 14, p-value = 0.01
alternative hypothesis: stationary
```

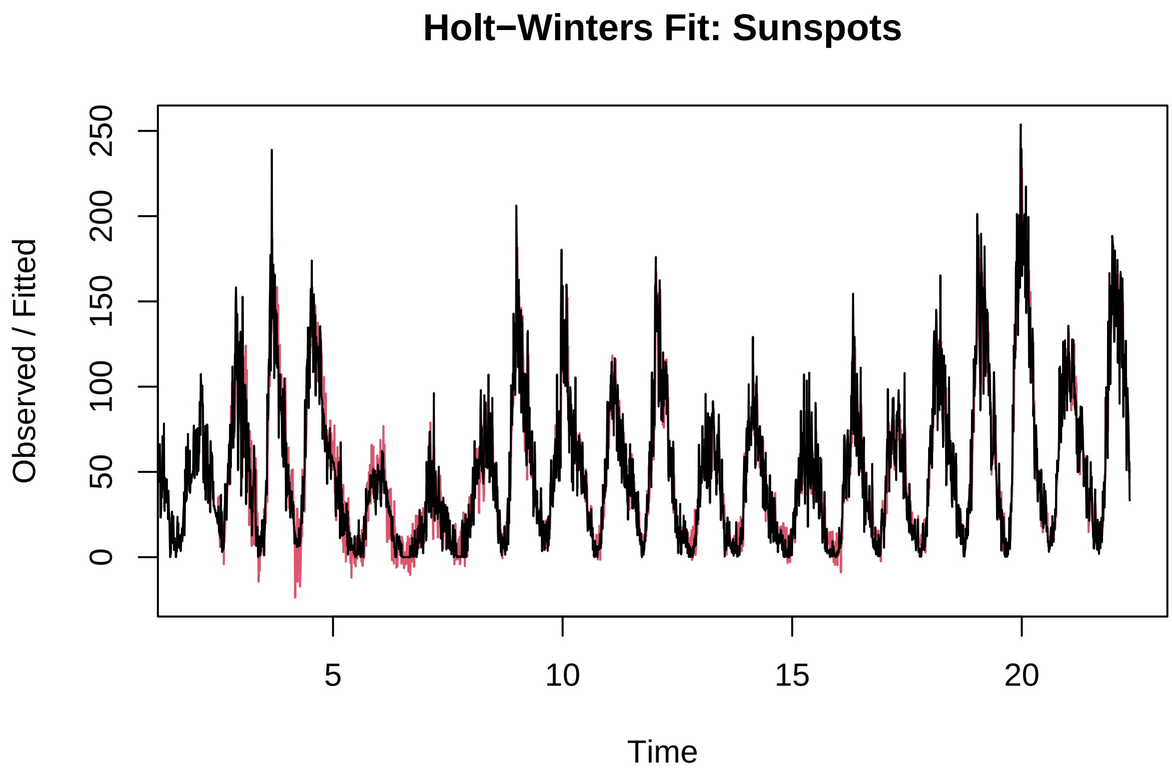
```
kpss_diff_sunspots <- kpss.test(diff_ts_sunspots)
```

```
Warning in kpss.test(diff_ts_sunspots): p-value greater than printed p-value
```

```
print(kpss_diff_sunspots)
```

```

    KPSS Test for Level Stationarity

data:  diff_ts_sunspots
KPSS Level = 0.0074249, Truncation lag parameter = 9, p-value = 0.1
```

```
# Modelling and Forecasting
# Fit Holt-Winters model
hw_model_sunspots <- HoltWinters(sunspots_ts_adj, seasonal = "additive")
plot(hw_model_sunspots, main = "Holt-Winters Fit: Sunspots")
```




```r
summary(hw_model_sunspots)
```

```
             Length Class  Mode
fitted       10752  mts    numeric
x             2820  ts     numeric
alpha            1  -none- numeric
beta             1  -none- numeric
gamma            1  -none- numeric
coefficients   134  -none- numeric
seasonal         1  -none- character
SSE              1  -none- numeric
call             3  -none- call
```

```r
# Fit AR(1) and MA(1) models on the differenced series
ar_model_sunspots <- arima(diff_ts_sunspots, order = c(1, 0, 0))
summary(ar_model_sunspots)
```

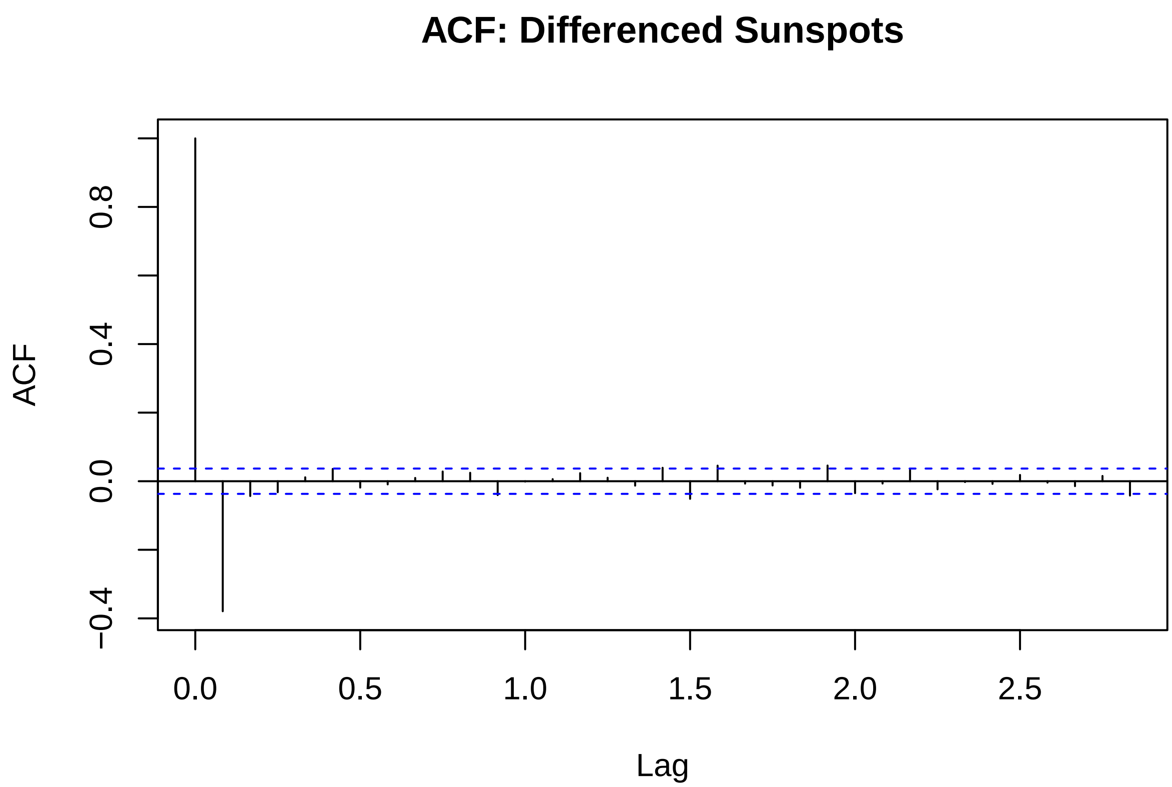
```
Call:
arima(x = diff_ts_sunspots, order = c(1, 0, 0))

Coefficients:
          ar1  intercept
      -0.3791    -0.0002
```

```
s.e.   0.0174     0.0071

sigma^2 estimated as 0.2696:  log likelihood = -2152.32,  aic = 4310.64

Training set error measures:
                       ME      RMSE       MAE MPE MAPE      MASE        ACF1
Training set 3.372672e-06 0.5192026 0.3500949 NaN  Inf 0.5705126 -0.08291571
```

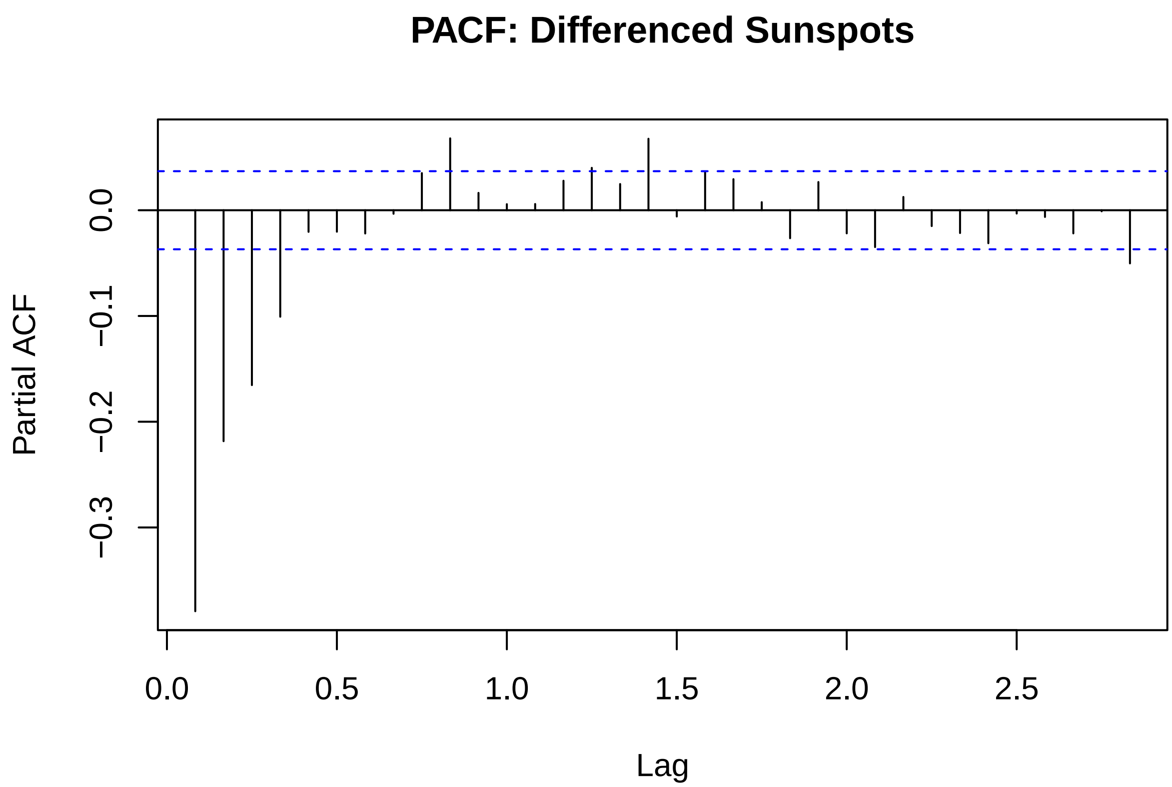
```r
ma_model_sunspots <- arima(diff_ts_sunspots, order = c(0, 0, 1))
summary(ma_model_sunspots)
```

```
Call:
arima(x = diff_ts_sunspots, order = c(0, 0, 1))

Coefficients:
          ma1  intercept
      -0.5517    -0.0002
s.e.   0.0163     0.0042

sigma^2 estimated as 0.2487:  log likelihood = -2038.65,  aic = 4083.3

Training set error measures:
                       ME      RMSE       MAE MPE MAPE      MASE       ACF1
Training set 3.797023e-05 0.4986648 0.3422004 NaN  Inf 0.5576478 0.03456532
```

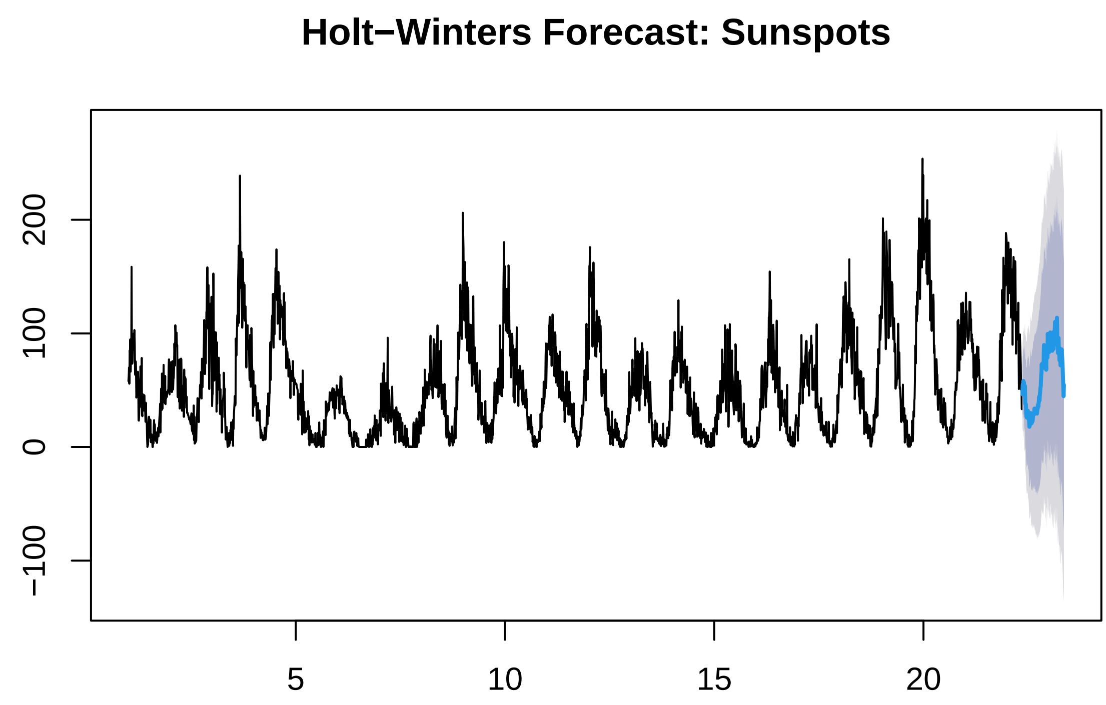
```r
# ACF/PACF plots
acf(diff_ts_sunspots, main = "ACF: Differenced Sunspots")
```




```r
pacf(diff_ts_sunspots, main = "PACF: Differenced Sunspots")
```




```r
# Final Forecast
# Forecast using Holt-Winters
hw_forecast_sunspots <- forecast(hw_model_sunspots, h = 132) # 11 years
plot(hw_forecast_sunspots, main = "Holt-Winters Forecast: Sunspots")
```




```r
print(hw_forecast_sunspots)
```

```
         Point Forecast        Lo 80    Hi 80      Lo 95     Hi 95
22.36364       46.40565  24.0709667 68.74033  12.247701  80.56359
22.37121       54.70815  30.1593657 79.25693  17.164027  92.25226
22.37879       54.30686  27.7277796 80.88593  13.657667  94.95604
22.38636       58.16247  29.6975438 86.62739  14.629123 101.69581
22.39394       55.64495  25.4115794 85.87832   9.407001 101.88290
22.40152       56.78671  24.8827674 88.69064   7.993843 105.57957
22.40909       46.82933  13.3380482 80.32061  -4.391165  98.04983
22.41667       50.16853  15.1618031 85.17525  -3.369636 103.70669
22.42424       53.71939  17.2601627 90.17862  -2.040187 109.47897
22.43182       38.49650   0.6404557 76.35254 -19.399323  96.39232
22.43939       37.81370  -1.3894171 77.01682 -22.142295  97.76970
22.44697       37.55118  -2.9542416 78.05660 -24.396517  99.49888
22.45455       29.33043 -12.4367057 71.09757 -34.546894  93.20776
22.46212       27.06775 -15.9240901 70.05960 -38.682598  92.81811
22.46970       25.85804 -18.3245769 70.04065 -41.713440  93.42951
22.47727       31.17518 -14.1669449 76.51730 -38.169614 100.51997
22.48485       29.94000 -16.5327046 76.41271 -41.133871 101.01388
22.49242       31.91456 -15.6618746 79.49100 -40.847319 104.67644
22.50000       29.10853 -19.5466038 77.76366 -45.303076 103.52014
22.50758       30.54653 -19.1639016 80.25696 -45.479013 106.57207
22.51515       28.89909 -21.8446940 79.64287 -48.706829 106.50500
22.52273       23.19495 -28.5615524 74.95146 -55.959792 102.34970
```

| | | | | | |
|---|---|---|---|---|---|
| 22.53030 | 17.79374 | -34.9560548 | 70.54353 | -62.880109 | 98.46759 |
| 22.53788 | 18.84939 | -34.8753290 | 72.57411 | -63.315477 | 101.01426 |
| 22.54545 | 22.18253 | -32.4997387 | 76.86479 | -61.446781 | 105.81183 |
| 22.55303 | 25.19787 | -30.4254563 | 80.82120 | -59.870668 | 110.26641 |
| 22.56061 | 29.88564 | -26.6630914 | 86.43438 | -56.598183 | 116.36947 |
| 22.56818 | 23.55247 | -33.9067725 | 81.01170 | -64.323856 | 111.42879 |
| 22.57576 | 21.44128 | -36.9142587 | 79.79682 | -67.805814 | 110.68837 |
| 22.58333 | 20.87892 | -38.3593565 | 80.11720 | -69.718206 | 111.47605 |
| 22.59091 | 22.47765 | -37.6304005 | 82.58571 | -69.449682 | 114.40499 |
| 22.59848 | 26.63052 | -34.3349075 | 87.59594 | -66.608053 | 119.86908 |
| 22.60606 | 22.68551 | -39.1253924 | 84.49641 | -71.846107 | 117.21712 |
| 22.61364 | 25.28008 | -37.3648856 | 87.92505 | -70.527129 | 121.08729 |
| 22.62121 | 29.14548 | -34.3225971 | 92.61356 | -67.920567 | 126.21153 |
| 22.62879 | 28.70145 | -35.5791934 | 92.98210 | -69.607312 | 127.01022 |
| 22.63636 | 28.48923 | -36.5938361 | 93.57231 | -71.046733 | 128.02520 |
| 22.64394 | 29.40567 | -36.4700498 | 95.28139 | -71.342551 | 130.15390 |
| 22.65152 | 33.84516 | -32.8137917 | 100.50411 | -68.100908 | 135.79122 |
| 22.65909 | 30.03165 | -37.4014251 | 97.46473 | -73.098341 | 133.16165 |
| 22.66667 | 31.58734 | -36.6110824 | 99.78576 | -72.713146 | 135.88783 |
| 22.67424 | 31.19863 | -37.7566459 | 100.15390 | -74.259362 | 136.65661 |
| 22.68182 | 29.74557 | -39.9583336 | 99.44947 | -76.857351 | 136.34849 |
| 22.68939 | 31.24089 | -39.2036848 | 101.68547 | -76.494793 | 138.97658 |
| 22.69697 | 30.36640 | -40.8111461 | 101.54395 | -78.490265 | 139.22307 |
| 22.70455 | 33.82264 | -38.0804059 | 105.72569 | -76.143580 | 143.78886 |
| 22.71212 | 29.37514 | -43.2461547 | 101.99644 | -81.689547 | 140.43983 |
| 22.71970 | 32.90401 | -40.4285077 | 106.23652 | -79.248395 | 145.05641 |
| 22.72727 | 33.69710 | -40.3397924 | 107.73400 | -79.532558 | 146.92677 |
| 22.73485 | 37.11113 | -37.6235135 | 111.84577 | -77.185644 | 151.40790 |
| 22.74242 | 35.27712 | -40.1488179 | 110.70305 | -80.076896 | 150.63113 |
| 22.75000 | 38.13103 | -37.9799200 | 114.24197 | -78.270621 | 154.53267 |
| 22.75758 | 41.04253 | -35.7473216 | 117.83238 | -76.397412 | 158.48247 |
| 22.76515 | 44.24404 | -33.2187623 | 121.70684 | -74.225092 | 162.71317 |
| 22.77273 | 40.91069 | -37.2192660 | 119.04065 | -78.578767 | 160.40015 |
| 22.78030 | 45.62613 | -33.1653316 | 124.41760 | -74.875013 | 166.12728 |
| 22.78788 | 48.70142 | -30.7460482 | 128.14888 | -72.802995 | 170.20583 |
| 22.79545 | 52.24121 | -27.8568805 | 132.33930 | -70.258249 | 174.74067 |
| 22.80303 | 53.99754 | -26.7459341 | 134.74102 | -69.488948 | 177.48403 |
| 22.81061 | 61.77406 | -19.6096810 | 143.15780 | -62.691632 | 186.23976 |
| 22.81818 | 65.44815 | -16.5708568 | 147.46717 | -59.989099 | 190.88541 |
| 22.82576 | 72.88565 | -9.7637516 | 155.53504 | -53.515700 | 199.28699 |
| 22.83333 | 69.26475 | -14.0102612 | 152.53976 | -58.093390 | 196.62289 |
| 22.84091 | 69.35904 | -14.5369251 | 153.25500 | -58.948764 | 197.66684 |
| 22.84848 | 73.42049 | -11.0918577 | 157.93284 | -55.829992 | 202.67097 |
| 22.85606 | 70.37733 | -14.7469430 | 155.50160 | -59.809010 | 200.56367 |
| 22.86364 | 72.67318 | -13.0586449 | 158.40501 | -58.442333 | 203.78870 |
| 22.87121 | 75.75749 | -10.5776142 | 162.09260 | -56.280660 | 207.79565 |
| 22.87879 | 89.75322 | 2.8190132 | 176.68742 | -43.201174 | 222.70761 |
| 22.88636 | 86.42616 | -1.1030406 | 173.95535 | -47.438198 | 220.29051 |
| 22.89394 | 82.82810 | -5.2920749 | 170.94827 | -51.940077 | 217.59627 |
| 22.90152 | 85.80860 | -2.8986122 | 174.51581 | -49.857374 | 221.47458 |
| 22.90909 | 87.01266 | -2.2777287 | 176.30306 | -49.545208 | 223.57054 |
| 22.91667 | 76.14976 | -13.7200247 | 166.01955 | -61.294217 | 213.59375 |
| 22.92424 | 75.28586 | -15.1596115 | 165.73134 | -63.038553 | 213.61028 |
| 22.93182 | 67.98651 | -23.0310101 | 159.00402 | -71.212773 | 207.18579 |

| | | | | | |
|---|---|---|---|---|---|
| 22.93939 | 85.31535 | -6.2706356 | 176.90134 | -54.753328 | 225.38403 |
| 22.94697 | 88.48932 | -3.6616348 | 180.64027 | -52.443401 | 229.42203 |
| 22.95455 | 85.42423 | -7.2882451 | 178.13670 | -56.367263 | 227.21572 |
| 22.96212 | 78.70624 | -14.5643729 | 171.97685 | -63.938852 | 221.35133 |
| 22.96970 | 99.57523 | 5.7497932 | 193.40066 | -43.918390 | 243.06884 |
| 22.97727 | 98.04207 | 3.6650828 | 192.41906 | -46.295078 | 242.37922 |
| 22.98485 | 87.12092 | -7.8044225 | 182.04627 | -58.054864 | 232.29671 |
| 22.99242 | 91.42867 | -4.0418801 | 186.89922 | -54.580936 | 237.43827 |
| 23.00000 | 83.38603 | -12.6266323 | 179.39868 | -63.452663 | 230.22471 |
| 23.00758 | 85.68757 | -10.8641555 | 182.23929 | -61.975550 | 233.35068 |
| 23.01515 | 89.16018 | -7.9276114 | 186.24798 | -59.322785 | 237.64315 |
| 23.02273 | 97.27965 | -0.3412719 | 194.90057 | -52.018666 | 246.57797 |
| 23.03030 | 100.87891 | 2.7277527 | 199.03006 | -49.230330 | 250.98814 |
| 23.03788 | 90.60323 | -8.0753052 | 189.28177 | -60.312567 | 241.51903 |
| 23.04545 | 87.67679 | -11.5263317 | 186.87990 | -64.041290 | 239.39486 |
| 23.05303 | 97.35265 | -2.3722892 | 197.07759 | -55.163482 | 249.86878 |
| 23.06061 | 93.73604 | -6.5080005 | 193.98008 | -59.573990 | 247.04607 |
| 23.06818 | 94.91579 | -5.8446799 | 195.67626 | -59.184051 | 249.01563 |
| 23.07576 | 84.65878 | -16.6154843 | 185.93305 | -70.226843 | 239.54441 |
| 23.08333 | 98.17277 | -3.6127011 | 199.95824 | -57.494674 | 253.84021 |
| 23.09091 | 87.90648 | -14.3876411 | 190.20059 | -68.538876 | 244.35183 |
| 23.09848 | 86.63050 | -16.1697486 | 189.43075 | -70.588913 | 243.84991 |
| 23.10606 | 86.27304 | -17.0308636 | 189.57694 | -71.716645 | 244.26272 |
| 23.11364 | 101.42770 | -2.3774090 | 205.23281 | -57.328514 | 260.18391 |
| 23.12121 | 99.76289 | -4.5410173 | 204.06680 | -59.756171 | 259.28195 |
| 23.12879 | 92.98284 | -11.8174891 | 197.78318 | -67.295434 | 253.26112 |
| 23.13636 | 102.65831 | -2.6361124 | 207.95272 | -58.375610 | 263.69222 |
| 23.14394 | 110.14802 | 4.3618278 | 215.93422 | -51.638001 | 271.93405 |
| 23.15152 | 96.71939 | -9.5563114 | 202.99508 | -65.815267 | 259.25404 |
| 23.15909 | 95.04843 | -11.7145267 | 201.81138 | -68.231421 | 258.32828 |
| 23.16667 | 90.14487 | -17.1031305 | 197.39287 | -73.876791 | 254.16653 |
| 23.17424 | 101.08222 | -6.6486389 | 208.81308 | -63.677910 | 265.84235 |
| 23.18182 | 98.24167 | -9.9698924 | 206.45323 | -67.253634 | 263.73698 |
| 23.18939 | 113.80126 | 5.1111175 | 222.49140 | -52.425969 | 280.02849 |
| 23.19697 | 97.15456 | -12.0120656 | 206.32118 | -69.801386 | 264.11050 |
| 23.20455 | 96.21424 | -13.4267988 | 205.85527 | -71.467257 | 263.89573 |
| 23.21212 | 82.78394 | -27.3294621 | 192.89734 | -85.619976 | 251.18785 |
| 23.21970 | 87.03687 | -23.5468814 | 197.62062 | -82.086383 | 256.16012 |
| 23.22727 | 96.03704 | -15.0150684 | 207.08915 | -73.802503 | 265.87658 |
| 23.23485 | 84.82337 | -26.6951239 | 196.34187 | -85.729450 | 255.37620 |
| 23.24242 | 84.71784 | -27.2651029 | 196.70079 | -86.545293 | 255.98098 |
| 23.25000 | 76.35935 | -36.0861240 | 188.80482 | -95.611162 | 248.32986 |
| 23.25758 | 86.87895 | -26.0271604 | 199.78506 | -85.796044 | 259.55394 |
| 23.26515 | 84.72719 | -28.6376772 | 198.09206 | -88.649415 | 258.10380 |
| 23.27273 | 71.87527 | -41.9465119 | 185.69706 | -102.200125 | 245.95067 |
| 23.28030 | 72.17790 | -42.0989735 | 186.45477 | -102.593495 | 246.94929 |
| 23.28788 | 80.27297 | -34.4571853 | 195.00312 | -95.191659 | 255.73759 |
| 23.29545 | 83.45643 | -31.7252207 | 198.63808 | -92.698703 | 259.61156 |
| 23.30303 | 85.90203 | -29.7293566 | 201.53341 | -90.940914 | 262.74497 |
| 23.31061 | 80.89949 | -35.1798833 | 196.97887 | -96.628594 | 258.42758 |
| 23.31818 | 75.78031 | -40.7453357 | 192.30596 | -102.430288 | 253.99091 |
| 23.32576 | 69.35153 | -47.6186822 | 186.32175 | -109.538974 | 248.24204 |
| 23.33333 | 61.01667 | -56.3964307 | 178.42977 | -118.551171 | 240.58451 |
| 23.34091 | 51.42838 | -66.4259410 | 169.28270 | -128.814249 | 231.67101 |

```
23.34848       44.72717 -73.5667227 163.02106 -136.187727 225.64207
23.35606       54.57811 -64.1537329 173.30995 -127.006573 236.16279
```

## 3. Analysis of USAccDeaths Dataset

The USAccDeaths dataset contains monthly totals of accidental deaths in the USA from 1973 to 1978. It exhibits a clear seasonal pattern.

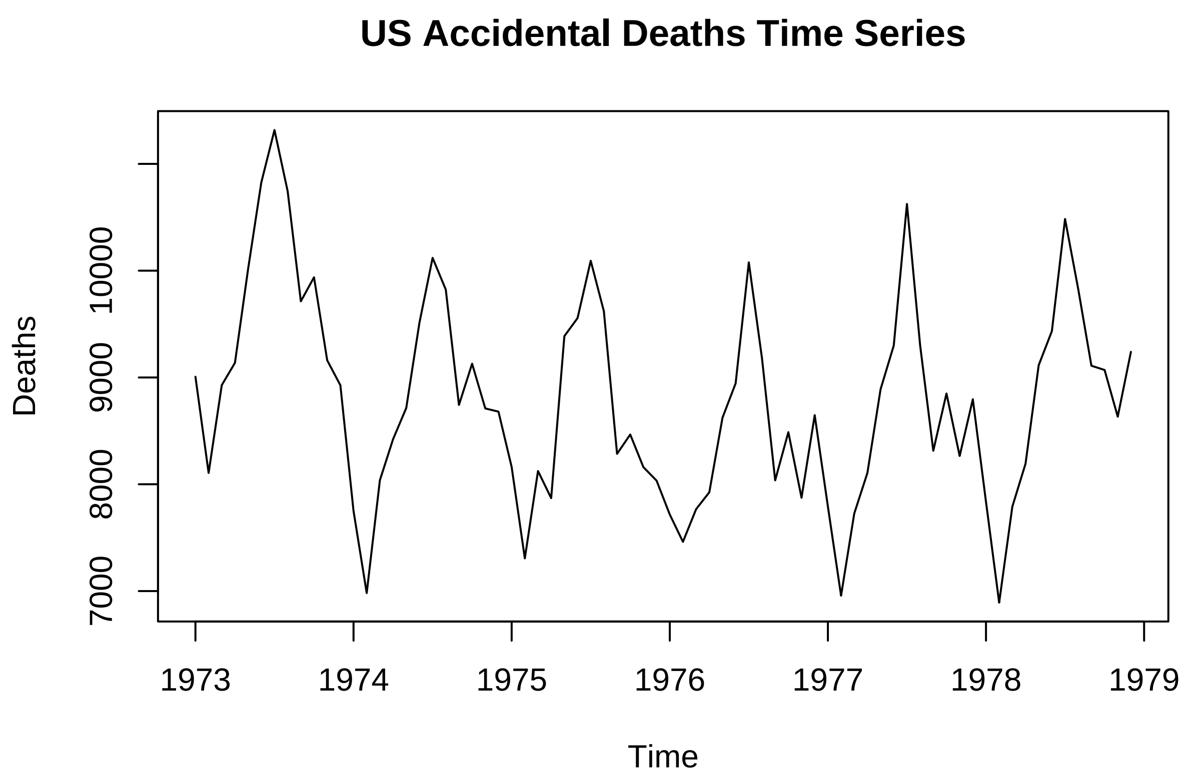
```r
# Exploratory Data Analysisdata("USAccDeaths")
str(USAccDeaths)
```

```
 Time-Series [1:72] from 1973 to 1979: 9007 8106 8928 9137 10017 ...
```

```r
print("Dataset: USAccDeaths - Monthly accidental deaths (1973-1978), time series confirmed.")
```

```
[1] "Dataset: USAccDeaths - Monthly accidental deaths (1973-1978), time series confirmed."
```

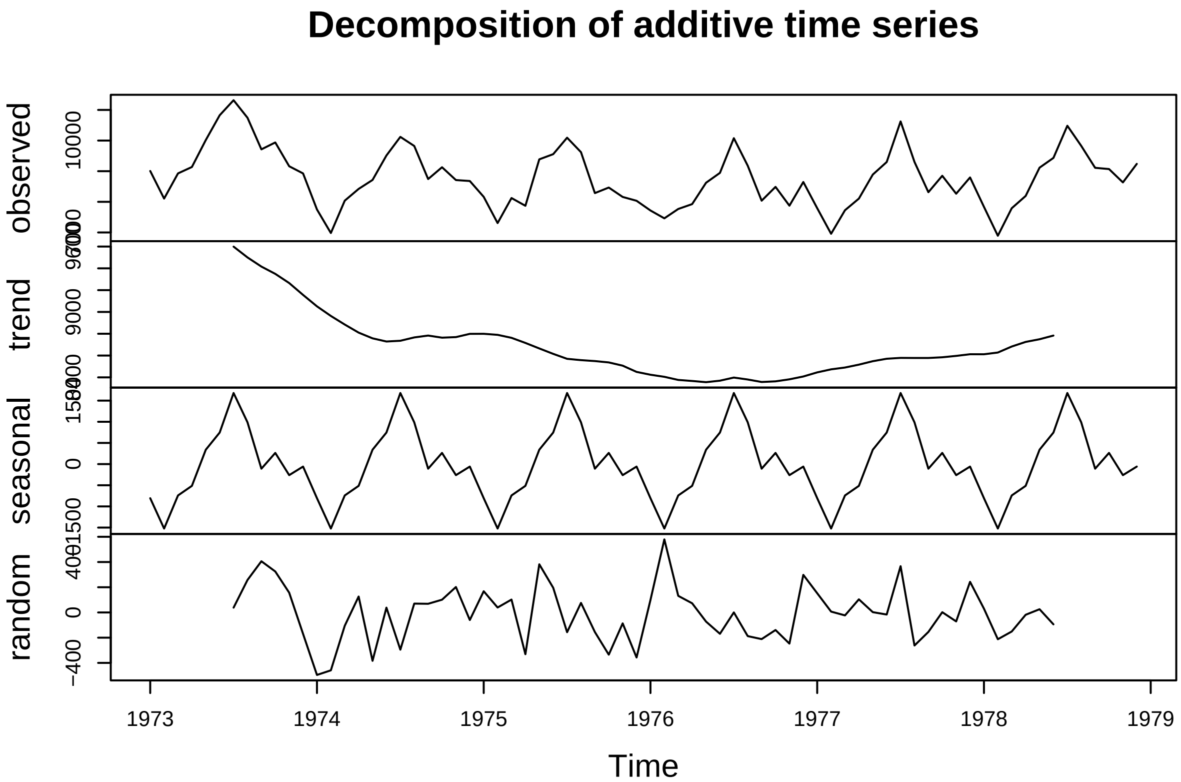
```r
plot(USAccDeaths, main = "US Accidental Deaths Time Series", ylab = "Deaths")
```




```r
summary(USAccDeaths)
```

```
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
   6892    8089    8728    8789    9323   11317 
```

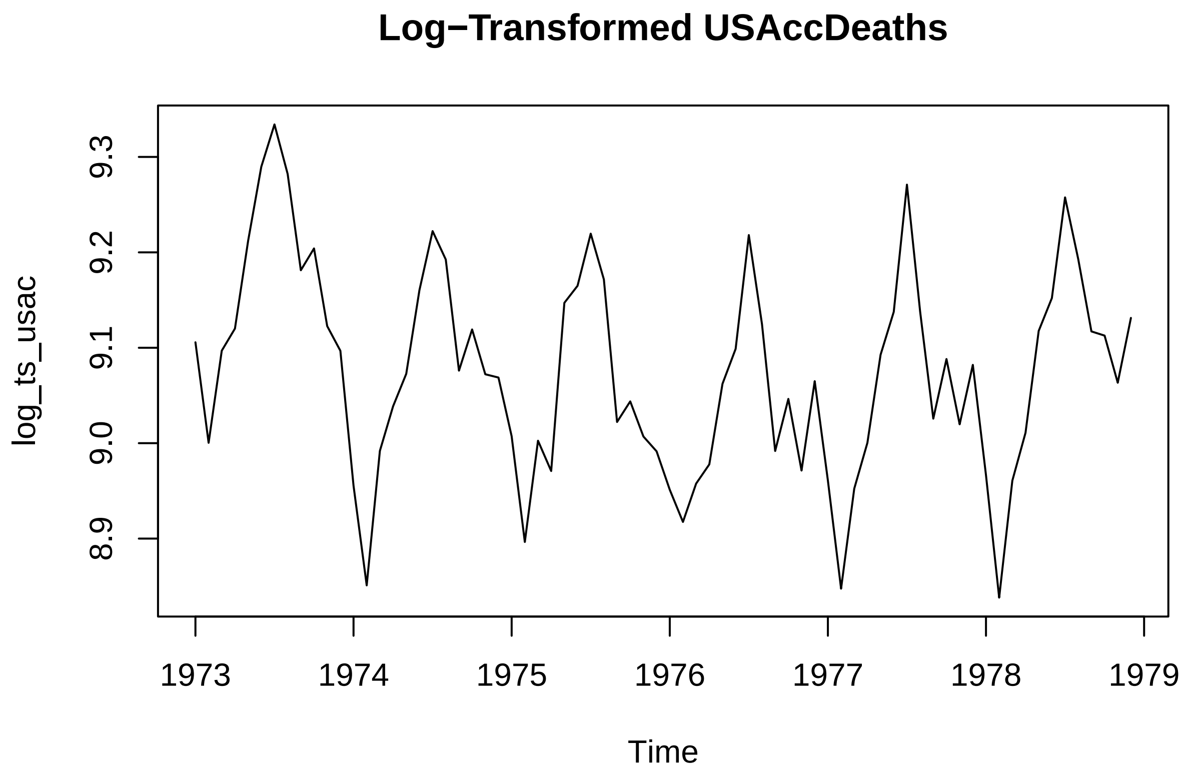
```r
# Decomposition (monthly data has a frequency of 12)
decomp_usac <- decompose(USAccDeaths, type = "additive")
plot(decomp_usac)
```



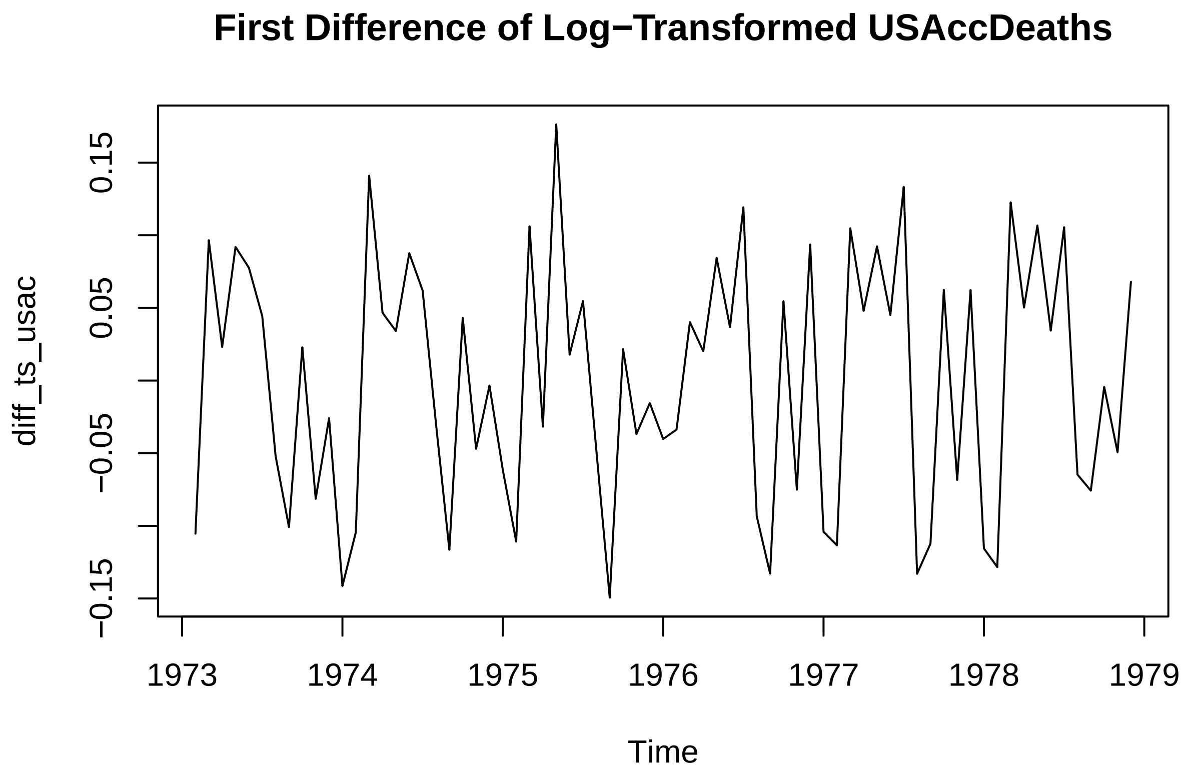

```r
# Data Preprocessing
# Check for missing values (none)
sum(is.na(USAccDeaths))
```

```
[1] 0
```

```r
# Apply log transformation
log_ts_usac <- log(USAccDeaths)
plot(log_ts_usac, main = "Log-Transformed USAccDeaths")
```




```r
# Apply first differencing
diff_ts_usac <- diff(log_ts_usac, differences = 1)
plot(diff_ts_usac, main = "First Difference of Log-Transformed USAccDeaths")
```




```r
# Stationarity Testing
# Original series tests
adf_original_usac <- adf.test(USAccDeaths)
print(adf_original_usac)
```

```
	Augmented Dickey-Fuller Test

data:  USAccDeaths
Dickey-Fuller = -3.8221, Lag order = 4, p-value = 0.02268
alternative hypothesis: stationary
```

```r
kpss_original_usac <- kpss.test(USAccDeaths)
```

```
Warning in kpss.test(USAccDeaths): p-value greater than printed p-value
```

```r
print(kpss_original_usac)
```

```
	KPSS Test for Level Stationarity

data:  USAccDeaths
KPSS Level = 0.19799, Truncation lag parameter = 3, p-value = 0.1
```

```r
# Differenced series tests
adf_diff_usac <- adf.test(diff_ts_usac)
```

```
Warning in adf.test(diff_ts_usac): p-value smaller than printed p-value
```

```r
print(adf_diff_usac)
```

```

	Augmented Dickey-Fuller Test

data:  diff_ts_usac
Dickey-Fuller = -4.4292, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

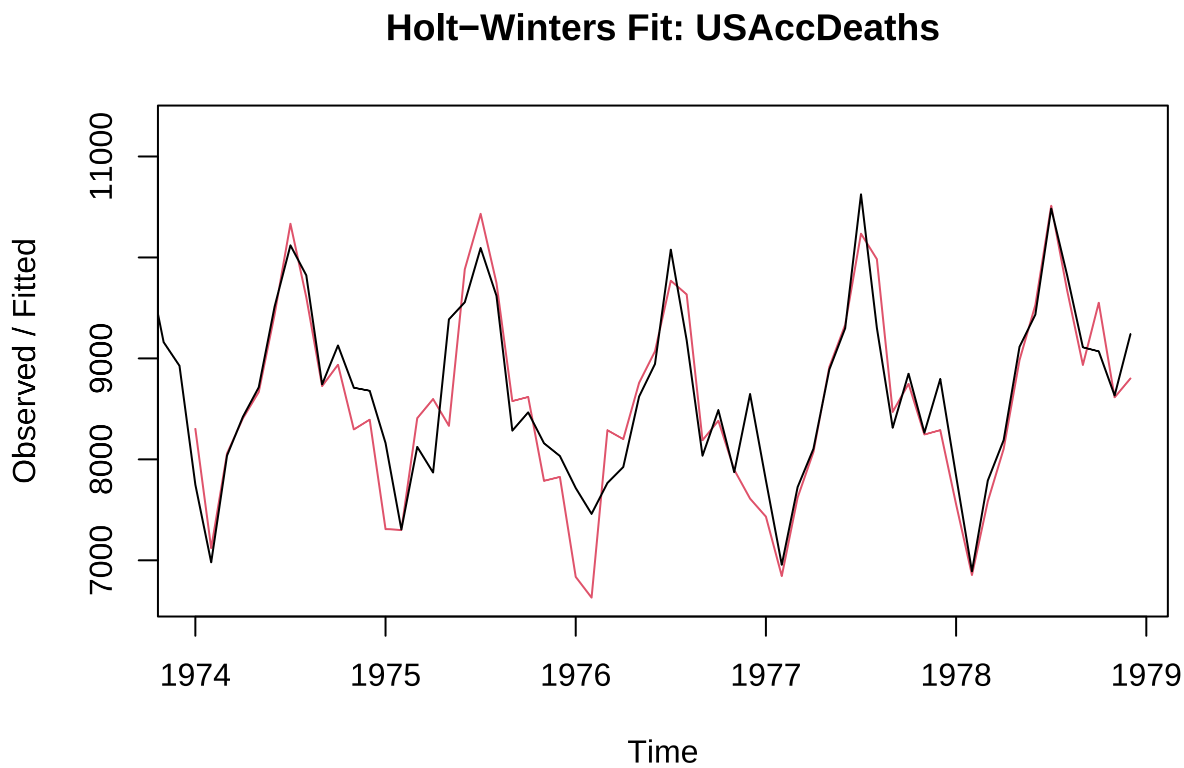
```r
kpss_diff_usac <- kpss.test(diff_ts_usac)
```

```
Warning in kpss.test(diff_ts_usac): p-value greater than printed p-value
```

```r
print(kpss_diff_usac)
```

```

	KPSS Test for Level Stationarity

data:  diff_ts_usac
KPSS Level = 0.028077, Truncation lag parameter = 3, p-value = 0.1
```

```r
# Modelling and Forecasting
# Fit Holt-Winters model
hw_model_usac <- HoltWinters(USAccDeaths, seasonal = "additive")
plot(hw_model_usac, main = "Holt-Winters Fit: USAccDeaths")
```




```
summary(hw_model_usac)
```

```
             Length Class  Mode
fitted       240    mts    numeric
x             72    ts     numeric
alpha          1    -none- numeric
beta           1    -none- numeric
gamma          1    -none- numeric
coefficients  14    -none- numeric
seasonal       1    -none- character
SSE            1    -none- numeric
call           3    -none- call
```

```
# Fit AR(1) and MA(1) models on the differenced series
ar_model_usac <- arima(diff_ts_usac, order = c(1, 0, 0))
summary(ar_model_usac)
```

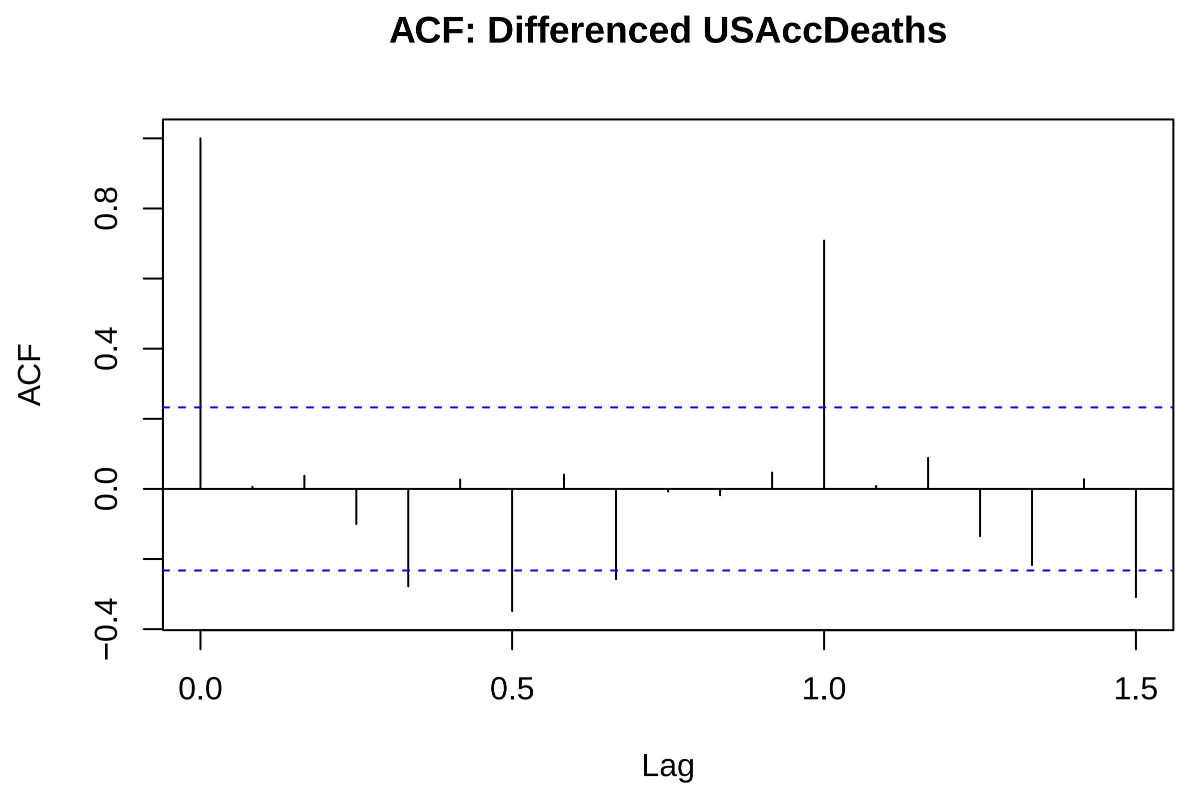
```
Call:
arima(x = diff_ts_usac, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.0062      4e-04
```

```
s.e.  0.1197      1e-02

sigma^2 estimated as 0.007029:  log likelihood = 75.25,  aic = -144.5

Training set error measures:
                       ME       RMSE        MAE      MPE     MAPE      MASE
Training set 9.236496e-06 0.08384114 0.07374253 99.78922 99.78922 0.7626056
                      ACF1
Training set -0.0002881081
```

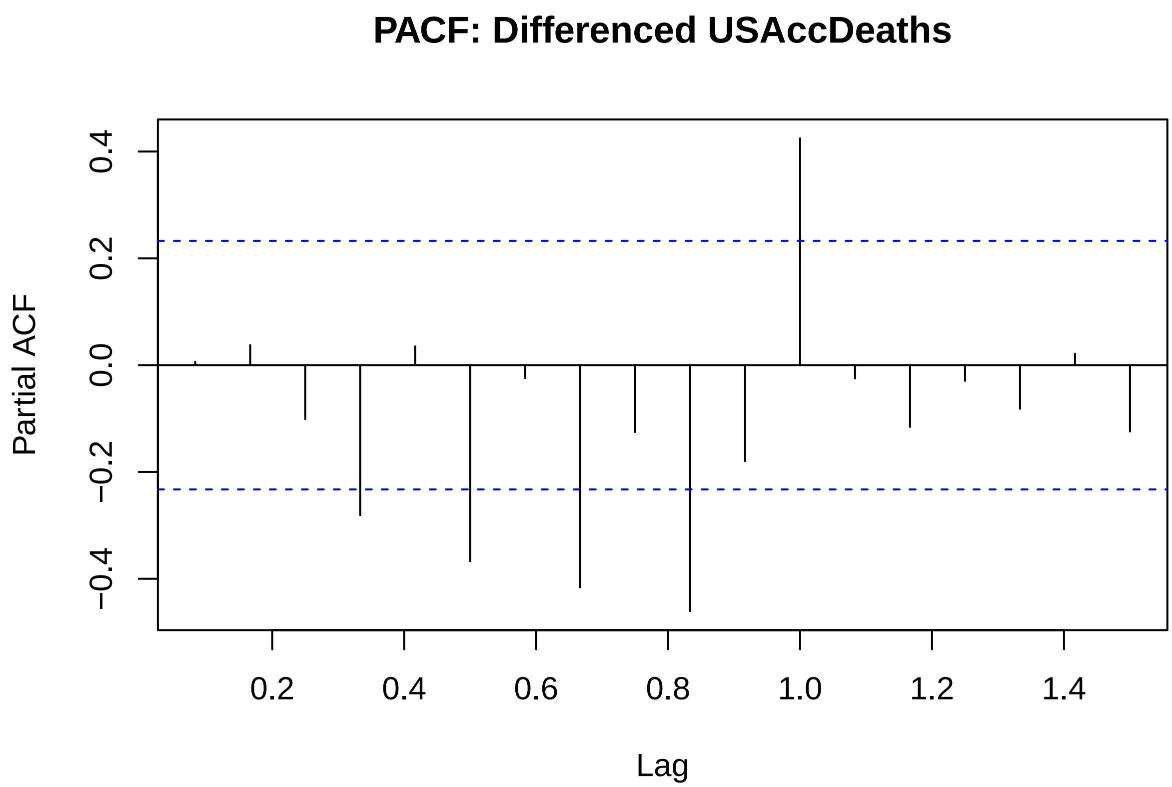
```r
ma_model_usac <- arima(diff_ts_usac, order = c(0, 0, 1))
summary(ma_model_usac)
```

```
Call:
arima(x = diff_ts_usac, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.0058      4e-04
s.e.  0.1153      1e-02

sigma^2 estimated as 0.007029:  log likelihood = 75.25,  aic = -144.5

Training set error measures:
                       ME       RMSE        MAE      MPE     MAPE     MASE
Training set 8.807277e-06 0.08384126 0.07374478 99.81644 99.81644 0.762629
                     ACF1
Training set 0.0001709799
```

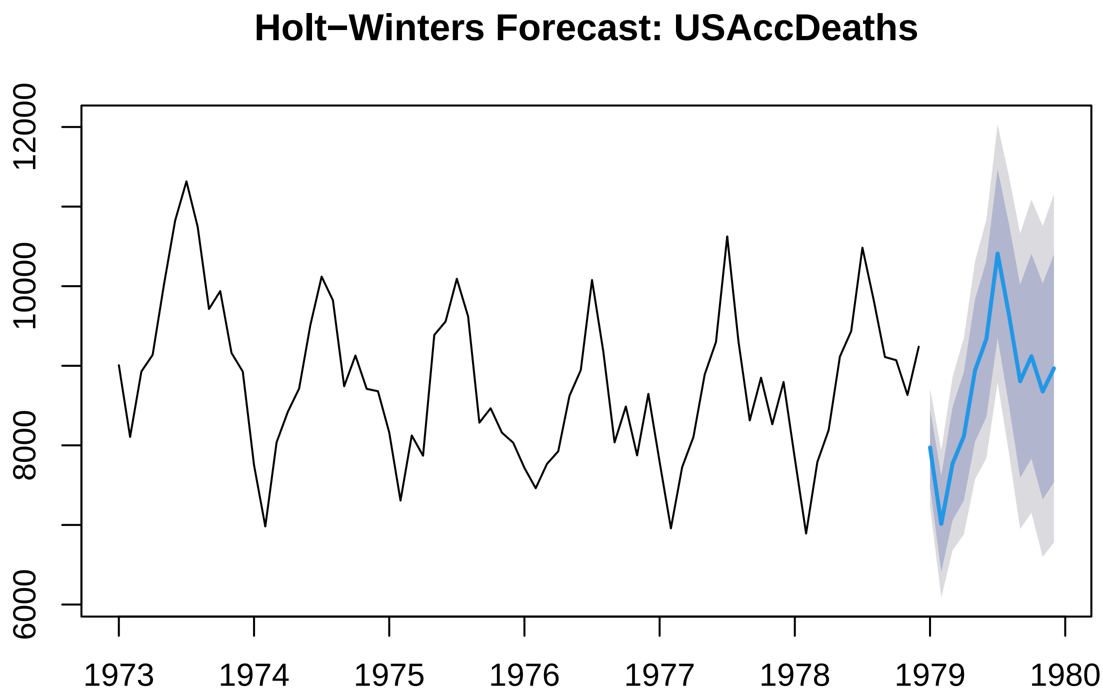
```r
# ACF/PACF plots
acf(diff_ts_usac, main = "ACF: Differenced USAccDeaths")
```




```r
pacf(diff_ts_usac, main = "PACF: Differenced USAccDeaths")
```




```r
# Final Forecast
# Forecast using Holt-Winters
hw_forecast_usac <- forecast(hw_model_usac, h = 12) # 1 year
plot(hw_forecast_usac, main = "Holt-Winters Forecast: USAccDeaths")
```




```r
print(hw_forecast_usac)
```

```
         Point Forecast    Lo 80     Hi 80    Lo 95     Hi 95
Jan 1979       7973.432 7489.493  8457.371 7233.311  8713.553
Feb 1979       7013.363 6407.222  7619.504 6086.350  7940.376
Mar 1979       7770.193 7058.507  8481.878 6681.763  8858.622
Apr 1979       8113.432 7306.220  8920.645 6878.907  9347.958
May 1979       8944.468 8048.465  9840.472 7574.149 10314.788
Jun 1979       9336.042 8356.086 10315.997 7837.329 10834.755
Jul 1979      10410.583 9350.304 11470.862 8789.027 12032.140
Aug 1979       9648.107 8510.309 10785.905 7907.995 11388.219
Sep 1979       8805.344 7592.241 10018.447 6950.063 10660.625
Oct 1979       9120.328 7833.696 10406.959 7152.594 11088.061
Nov 1979       8676.792 7318.072 10035.512 6598.809 10754.774
Dec 1979       8967.858 7538.228 10397.488 6781.428 11154.288
```