Final project

Part 1

Paper title: Randomized Social Choice Functions Under Metric Preferences
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Main claim: randomized choice Mechanisms significantly decrease “*worst-case distortion”* over deterministic mechanisms. In this paper *“distortion*” measures the performance of choice mechanism on the worst-case scenarios by the ratio between the worst-case social cost and the optimal choice social cost.
“worst-case distortion” refers to the distortion in the worst-case **profile**.

Main results:

Definitions

notations

M – the set of possible alternatives.
N – the set of all agents/voters.
S – the set of ordinal preference profiles
$d(i, X)$ – the cost of agent i for the X alternative to win

The Sum objective

The social cost of an alternative $X \in M$ is the sum of agent costs for that X.
Formally: $SC(X, d) = \sum_{i \in N} d(i, X)$

The Median objective

The quality of an alternative $X \in M$ is the median agent’s cost for that alternative.
Formally: $med(X, d) = med_{i \in N}\big(d(i, X)\big)$

The Proportional to squares voting rule.

the odds of the alternative $X \in M$ are proportional to the amount of people (notated as $|Y^*|$) that prefer that alternative, formally:

$$p(X) = \frac{|X^*|^2}{\sum_{z \in M} |Z^*|^2}$$

Probabilistic outcome:

Notation: Probabilistic social choice function $f: S^n \to \Delta(M)$ is a mapping from the set of preference profiles to the space of all probability distributions over the alternatives
$p(X)$ – is the probability of alternative X being selected according to randomized $f(\sigma)$
For randomized social choice function we will use $SC(f(\sigma), d) = \mathbb{E}[SC(X, d)] = \sum_{X \in M} p(X) SC(X, d)$ and $med(f(\sigma), d) = \mathbb{E}[med(X, d)] = \sum_{(X \in M)} p(X)\, med(X, d)$.

Distortion

The distortion of social choice mechanism f on a profile (ordinal ordering) $\sigma \in S$ is the worst-case ratio between the social cost of f and the social cost of the true

optimal alternative.
Formally:

$$dist_{\Sigma}(f,\sigma) = \underset{d \in \rho^{-1}(\sigma)}{sup} = \frac{SC(f(\sigma),d)}{\min_{X\in M} SC(X,d)}$$

$$dist_{med}(f,\sigma) = \underset{d \in \rho^{-1}(\sigma)}{sup} = \frac{med(f(\sigma),d)}{med_{X\in M}\, med(X,d)}$$

The notation of $\rho^{-1}(\sigma)$ used to signify all the metrics d which may have induced σ under the assumption that the social cost to an agent determines its ordering (i.e. an agent won't vote for an outcome it's not interested at.)
Note that the worst-case is taken over all metrics d which may have induced σ since the social choice function don't have access to the metrics that actually determines agents ordering (only the ordering itself).
"worst-case distortion" takes the maximum over σ as well as d

α -decisive metric space

A metric space is $\alpha - decisive$ iff for every agent the cost of its first choice is less than α times the cost of its second choice for some $\alpha \in [0,1]$
This constraint how indifferent the agents can be between their first and second choice
Note that every metric space is $1 - decisive$

Results

Notations:
$|X^*| =$ the amount of people voting for alternative $X \in M$

1) **Theorem 3:** "*if a metric space is α-decisive, then the distortion of randomized dictatorship is $\leq 2+\alpha-\frac{2|W^*|}{|N|}$ where $W = arg \min_{Y\in M:|Y^*|>0)} |Y^*|$ and this bound is tight*".
   Note: in this case "tight bound" mean that it is possible to get the upper bound by choosing appropriate profile and social costs.
   The authors prove that for arbitrary m alternatives randomized dictatorship achieves strictly less than 3 distortion (and is at most $3-\frac{2}{|N|}$ for $\alpha = 1, |W^*| = 1$ in the theorem
   a. This simple randomized mechanism has better distortion than any deterministic mechanism, since no deterministic mechanism can have distortion strictly better than 3 in the worst case, worst case here means the worst profile. (proven in another paper)
   b. This is both surprising and useful because randomized dictatorship operates only on the first preferences of every agent: the full preference ranking is not required.
2) for $|M| = 2$ alternatives the authors designed a generalization for the proportional to squares voting rule using the α value of the metric space and proved that it is optimal.
   The rule is as follows:

$$p(Y) = \frac{(1+\alpha)|Y^*|^2-(1-\alpha)|X^*||Y^*|}{(1+\alpha)(|X^*|^2+|Y^*|^2)-2(1-\alpha)|X^*||Y^*|}$$

Where X is the second alternative.

a. The rule get distortion of $1 + \alpha$ which achieve the bound for the case of $m = 2$ & α-decisive space.
b. They go farther and suggest a mechanism (based on this social choice function) for α-decisive, 1- Euclidean space for arbitrary m that by reducing the possible optimal choices to 2 able to get $1 + \alpha$ distortion which again is the best possible worst-case distortion.

The paper proves bounds for worst-case scenarios. However, I was interested to examine how various methods fare in more typical or probable situations. To accomplish this, I randomly generated 10000 profiles with 10 voters and 2 candidates each, by uniformly selecting values for agents' cost vectors. Then, I computed the distortion for each profile.
Figure 1 illustrates both the worst-case scenario bounds for each social choice function and the actual performance of the profiles. Additionally, I did the same process for 10000 profiles that are 0.5-decisive shown in

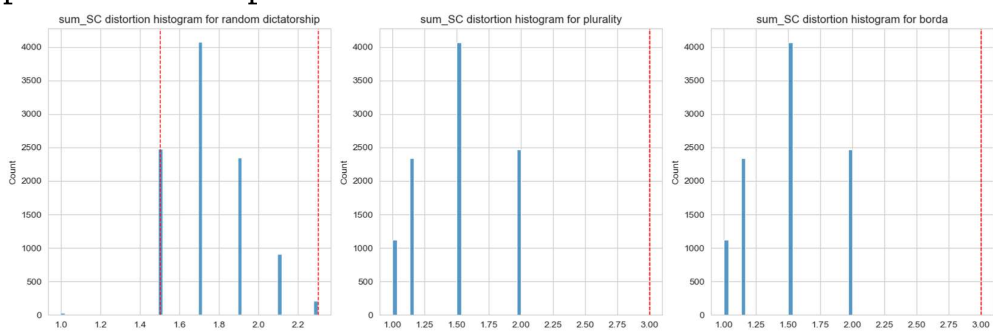

Figure 2

There are few things worth to note.

1. The bounds in red are for worst-possible profile and as such the lower bound does not limit the results on other profile (the case where all voter are unanimous gets distortion of 1 in all cases presented)
2. The large gap between the actual results for the deterministic functions and their bound in

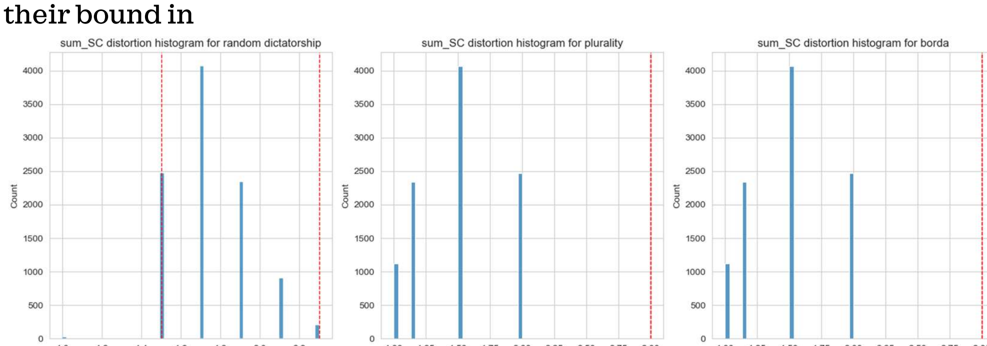

Figure 2 is due to the effect alpha decisiveness has on the bound.
3. It seems that the worst-case scenario is much more rare for the random dictatorship setting (after accounting for the effect mentioned in 2) than it is for the deterministic cases.

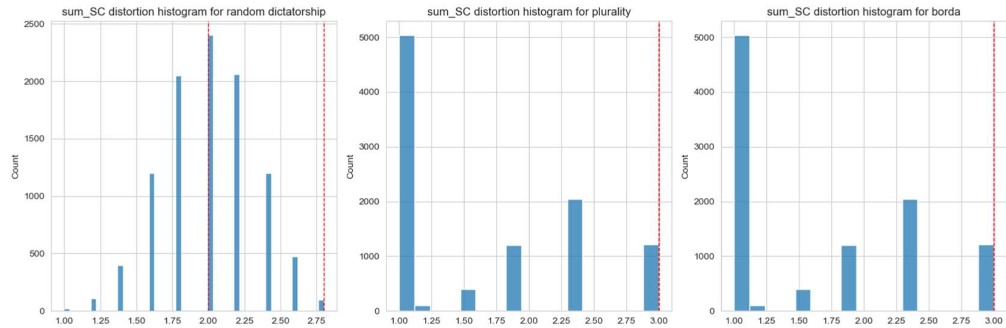

Figure 1

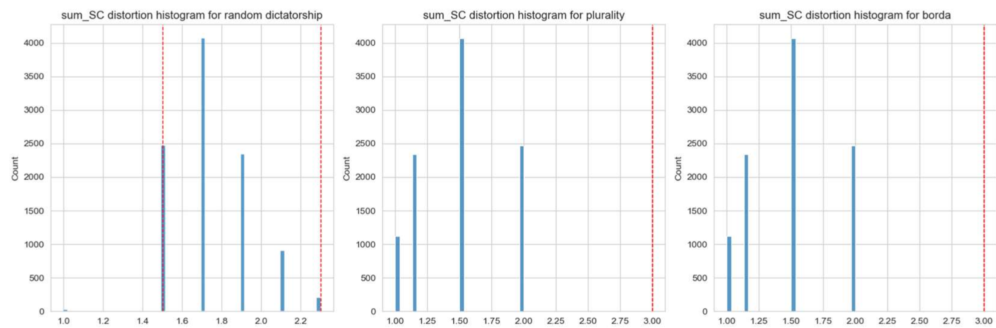

Figure 2

Important paper cited by this paper:

Edith Elkind and Piotr Faliszewski. Recognizing 1-euclidean preferences: An alternative approach. In Proc. 7th Int. Symp. Algorithmic Game Theory, pages 146–157. Springer, 2014.

The paper was being cited multiple times:

1) As an example of how well studied the special case of the 1-Euclidean preferences are.
2) When designing an optimal random social choice function for the 1-Euclidean preferences the paper relies on the properties described in the Edith Elkind and Piotr Faliszewski paper (more specifically, how by using only the preference profile its possible to determine the ordering of the agents & agents on the line)

Paper citing this paper:

Distortion in Social Choice Problems: The First 15 Years and Beyond by Elliot Anshelevich, Aris Filos-Ratsikas, Nisarg Shah, and Alexandros A. Voudouris

The paper was being cited several times for different reasons:

1) for breaking the barrier of distortion of 3 proven for deterministic voting rules, as mentioned above by using Random Dictatorship to get distortion of $3 - \frac{2}{n}$
2) For finding lower bound for the distortion namely 2, for any randomized voting rule. although still leaving open the question of “what is the best possible distortion for randomized social choice rules in the metric setting?”
3) For introducing the notion of α-decisiveness to measure preference strength and for showing lower and upper bound parametrized by this quantity

Review

Some definitions and explanations for notations are missing and relay heavily on familiarity with the subject and other papers, an example will be the use of $\rho^{-1}(d)$ which is explained in depth in “Approximating optimal social choice under metric preferences”, it also sometimes fail do distinguish between distortion and “worst-case distortion” which is confusing at times, and misleading at others.

The paper proves new bounds for random social function worst-case distortion, and explains algorithm (algorithm I in the paper) that can achieve these bounds. However, I think that the usefulness of “worst-case distortion” as a metric is limited due to these cases relative rarity, even in the paper the authors mentioned that in most cases people tend to have strong preference for their first candidate over their second and mentioned that high distortion for many social choice functions comes from cases where voters are almost indifferent between first and second place (thus introducing alpha-decisiveness). Nonetheless I think it can be interesting to investigate in the context of adversarial planning.

While reading the paper I came across a problem, in the paper the authors discuss the α-Generalized Proportional to Squares social choice function (explained in result 2)

However, for some values (such as $|Y^*| = 2, |X^*| = 8, \alpha = 0.5$) the result is not a portability at all! ($p(Y) = -\frac{1}{43}, p(X) = \frac{44}{43}$. I couldn’t find the likely true intent of the authors. And I’m unsure what to make of it.

They later use develop this function to the more general case (|M|>2, in 1-Eculiden) but the problem remains.

Plan for the project

I plan to do an implementation project, I already started implementing the objectives and the distortion metric as well as some simulation capabilities, I'm interested to expand on that and implement the Optimal randomized mechanism for the α-decisive, 1-Euclidean space that appears in part 3.1 of the article. As well as generators for elections that are 1-Euclaidean and election that are (m-1) simplex

I'm interested to see how the random social function perform in the mean and meading cases(in addition to showing that the performance on the worst case is as proven in the article)