Notation

- Ω, espace of different scenarios.
- $X : \Omega \to \mathbb{R}$, value of portfolio.
- $\mathcal{X} = \{X : \Omega \to \mathbb{R} : \sup_{\omega \in \Omega} |X(\omega)| < \infty\}$., All the bounded portfolio.
- $(\Omega, \mathcal{F}, \mathbb{P})$, espace proba, $\mathcal{X} = L^{\infty}(\Omega, \mathcal{F}, \mathbb{P})$.

1 Ch1

Definition 1.1 (measure of monetary risk)**.**

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Proposition 1.2 (Property of Lipschitz)**.**

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Definition 1.3 (axiomatisation of measure of risk)**.**

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1. convex:


2. positively homogeneous:


3. consistent:
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Remark 1.4. convex shows the benefits of risk diversity. if ρ is convex, then $\forall X \in \mathcal{X}, u \in \mathbb{R} \to \rho(uX)$ is convex.

Question 1.5. Justify the following two measures of risk:

1. $\rho(X) = \mathbb{E}[-X]$
2. $\alpha > 0, \rho(X) = \mathbb{E}[-X] + \alpha\sqrt{\mathrm{Var}(X)}$

Definition 1.6 (value at Risk (VaR))**.** A reasonable suggestion of ρ:

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Question 1.7. Justify the property/axiom of VaR.

1. VaR is monotone
2. VaR is inv cash
3. VaR is positively homogeneous
4. VaR isn't convex

Example 1.8. $\mathcal{L}$: set of proba measures over $(\Omega, \mathcal{F})$, let $\gamma : \mathcal{L} \to \mathbb{R}$, s.t. $\sup_{Q \in \mathcal{L}} \gamma(Q) = 0$, define $\rho(X) = \sup_{Q \in \mathcal{L}} \mathbb{E}_Q[-X] + \gamma(Q)$.

Question 1.9. Justify the example is a convex measure of monetary risk.

Definition 1.10 (acceptable set of financial positions)**.** $\mathcal{A} \subseteq \mathcal{X}$ is the acceptable set of financial positions if:

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Proposition 1.11. (ρ and $\mathcal{A}_\rho$)

1. if ρ is a measure of monetary risk, then, $\mathcal{A}_\rho = \{X \in \mathcal{X}, \rho(X) \leq 0\}$ is the acceptable set of financial positions.

2. if $\mathcal{A}$ is a.s.f.p. define $\rho_\mathcal{A} = \inf\{m \in \mathbb{R}, m + X \in \mathcal{A}\}$, then $\rho_\mathcal{A}$ is the mesure of monetary risk.

3. $\rho = \rho_{\mathcal{A}_\rho}$.
4. $\rho_1 = \rho_2 \iff \mathcal{A}_{\rho_1} = \mathcal{A}_{\rho_2}$
5. ρ is convexe $\iff \mathcal{A}_\rho$ is convex.
6. ρ is positively homogeneous $\iff \mathcal{A}_\rho$ is cone.