Notation

- Ω, espace of different scenarios.
- $X : \Omega \to \mathbb{R}$, value of portfolio.
- $\mathcal{X} = \{X : \Omega \to \mathbb{R} : \sup_{\omega \in \Omega} |X(\omega)| < \infty\}$., All the bounded portfolio.
- $(\Omega, \mathcal{F}, \mathbb{P})$, espace proba, $\mathcal{X} = L^\infty(\Omega, \mathcal{F}, \mathbb{P})$.

1 Ch1 Monetary Risk Measure

Definition 1.1 (measure of monetary risk)**.**

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1.

2.

3.
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Proposition 1.2 (Property of Lipschitz)**.**

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Definition 1.3 (axiomatisation of measure of risk)**.**

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1. convex:


2. positively homogeneous:


3. consistent:
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Remark 1.4. convex shows the benefits of risk diversity. if ρ is convex, then $\forall X \in \mathcal{X}, u \in \mathbb{R} \to \rho(uX)$ is convex.

Question 1.5. Justify the following two measures of risk:

1. $\rho(X) = \mathbb{E}[-X]$
2. $\alpha > 0, \rho(X) = \mathbb{E}[-X] + \alpha\sqrt{\mathrm{Var}(X)}$

Definition 1.6 (value at Risk (VaR))**.** A reasonable suggestion of ρ:

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Question 1.7. Justify the property/axiom of VaR.

1. VaR is monotone
2. VaR is inv cash
3. VaR is positively homogeneous
4. VaR isn't convex

Example 1.8. $\mathcal{L}$: set of proba measures over $(\Omega, \mathcal{F})$, let $\gamma : \mathcal{L} \to \mathbb{R}$, s.t. $\sup_{Q \in \mathcal{L}} \gamma(Q) = 0$, define $\rho(X) = \sup_{Q \in \mathcal{L}} \mathbb{E}_Q[-X] + \gamma(Q)$.

Question 1.9. Justify the example is a convex measure of monetary risk.

Definition 1.10 (acceptable set of financial positions)**.** $\mathcal{A} \subseteq \mathcal{X}$ is the acceptable set of financial positions if:

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2.
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Proposition 1.11. (ρ and $\mathcal{A}_\rho$)

1. if ρ is a measure of monetary risk, then, $\mathcal{A}_\rho = \{X \in \mathcal{X}, \rho(X) \leq 0\}$ is the acceptable set of financial positions.

2. if $\mathcal{A}$ is a.s.f.p. define $\rho_{\mathcal{A}} = \inf\{m \in \mathbb{R}, m + X \in \mathcal{A}\}$, then $\rho_{\mathcal{A}}$ is the mesure of monetary risk.

3. $\rho = \rho_{\mathcal{A}_\rho}$.
4. $\rho_1 = \rho_2 \iff \mathcal{A}_{\rho_1} = \mathcal{A}_{\rho_2}$
5. ρ is convexe $\iff \mathcal{A}_\rho$ is convex.
6. ρ is positively homogeneous $\iff \mathcal{A}_\rho$ is cone.

Definition 1.12. (Quantiles)

1. $q_X^-(\lambda) =$
2. $q_X^+(\lambda) =$

Definition 1.13. (AVaR)

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Question 1.14. Justify the AVaR has another name CVaR.

Proposition 1.15. if X is integrable, $\lambda \in (0,1)$ et q quantile of order λ of X. Then,

$$\mathrm{AVaR}_\lambda(X) = \frac{1}{\lambda}\mathbb{E}[(q-X)^+] - q$$

.

Theorem 1.16. let $\lambda \in (0,1]$, AVaR is a consistent monetary rik measure that admits the following representation: $\mathrm{AVaR}_\lambda(X) = \max_{\mathbb{Q}\in\mathcal{L}_\lambda} \mathbb{E}_\mathbb{Q}[-X]$.

2 Ch2 Convex Optimisation

2.1 Topology

Definition 2.1 (Topology)**.** Soit $\mathcal{X}$ a set, $\mathcal{C} \subseteq \mathcal{P}(\mathcal{X})$ is a topology over $\mathcal{X}$ if:

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    1.

    2.

    3.
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Remark 2.2. (some definitions)

- Weak topology:
- Generated by a family:
- Haustoff espace:
- Basis of a topology:
- Local convex:

Definition 2.3. (some definitions related to functions)

- Compact:
- Neighbor:
- Converge:
- Continous application:

Definition 2.4. (Hahn-Banach Theorem)

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Definition 2.5. (Lower Semicontinuity/s.c.i in french)

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Proposition 2.6.

- $f : \mathcal{X} \to \mathbb{R}$ s.c.i. $x_n \to x$, then, $f(x) \leq \liminf_{n\to\infty} f(x_n)$.
- if f is s.c.i. then, $\sup_{\theta\in\Theta} f_\theta$ is s.c.i.

2.2 Convex

Definition 2.7.

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1. Epigraph,Epi(f):

2. Effective domain, dom(f):

3. Proper convex function
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Lemma 2.8. if $f : \mathcal{X} \to \mathbb{R}$ is s.c.i. then Epi(f) is closed.

Theorem 2.9. $f : \mathcal{X} \to (-\infty, +\infty]$ is proper convex s.c.i. function, then

$$f(x) = \sup_{x' \in \mathcal{X}, \alpha\in\mathbb{R} \text{ s.t.} f(\tilde{x}) \geq <\tilde{x}, x> -\alpha, \forall \tilde{x}\in\mathcal{X}} < x, x^{'} > -\alpha$$

Definition 2.10 (Transform of Fenchel Legendre)**.**

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Theorem 2.11. if $f : \mathcal{X} \to (-\infty, +\infty]$ is proper convex s.c.i. function, $f^{**} = f$.

Example 2.12. $\mathcal{X} = \mathcal{X}^{'} = \mathbb{R}, < x, x^{'} >= xx^{'}$, find f^* for $f(x) = \frac{x^2}{2}, f(x) = ax$.

2.3 Espace dual $\mathcal{X}, \mathcal{X}'$

Example 2.13 (Espace L^q, L^p)**.**

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Theorem 2.14 (Riesz theorem for L^q, L^p)**.**

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→ 可以定义内积 → 满足 Fenchel-Legnedre 变换的假设 (写出来)

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Example 2.15 (Espace L^∞, L^1)**.**

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What's the problem?

Definition 2.16 (weak topology)**.**

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→ consequence?

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2.4 Duality

To be continued studying...(video)

3 Representation of risk measures