1. $M = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\text{Determinant} = (-0)\left(\begin{vmatrix} r_{12} & r_{13} & t_1 \\ r_{22} & r_{32} & t_2 \\ r_{32} & r_{33} & t_3 \end{vmatrix}\right) + 0\left[\begin{vmatrix} r_{11} & r_{13} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{33} & t_3 \end{vmatrix}\right]$$

$$+ 0\left(\begin{vmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{21} & r_{32} & t_3 \end{vmatrix}\right) + 1\begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{32} \\ r_{31} & r_{32} & r_{33} \end{vmatrix}$$

$$= \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix}$$

$$= r_{11}\begin{vmatrix} r_{22} & r_{23} \\ r_{32} & r_{33} \end{vmatrix} - r_{21}\begin{vmatrix} r_{12} & r_{13} \\ r_{32} & r_{33} \end{vmatrix} + r_{31}\begin{vmatrix} r_{12} & r_{13} \\ r_{22} & r_{23} \end{vmatrix}$$

$$= r_{11}\left(r_{22}r_{33} - r_{32}r_{23}\right) - r_{21}\left(r_{12}r_{33} - r_{13}r_{32}\right) + r_{31}\left(r_{12}r_{23} - r_{13}r_{22}\right.$$

$$M^T = \begin{bmatrix} r_{11} & r_{21} & r_{31} & 0 \\ r_{12} & r_{22} & r_{32} & 0 \\ r_{13} & r_{23} & r_{33} & 0 \\ t_1 & t_2 & t_3 & 1 \end{bmatrix}$$

$$M^{-1} = \left[\frac{1}{r_{11}(r_{22}r_{33} - r_{32}r_{23}) + r_{21}(r_{12}r_{33} - r_{13}r_{22}) + r_{31}(r_{12}r_{23} - r_{13}r_{22})}\right]$$

$$\times$$

$$\begin{bmatrix} r_{11} & r_{21} & r_{31} & 0 \\ r_{12} & r_{22} & r_{32} & 0 \\ r_{13} & r_{23} & r_{33} & 0 \\ t_1 & t_2 & t_3 & 1 \end{bmatrix}$$

2. $\phi(P) = Mp$ 3D Transformation $P = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$

$$\Phi(P_2 - P_1) = \Phi(P_2) - \Phi(P_1)$$
$$M(P_2 - P_1) = M(P_2) - M(P_1)$$

Yes, Φ is linear in 3D,

linear must satisfy $\Phi(aP) = a\phi(P)$

$$\phi(P_1 + P_2) = \phi(P_1) + \phi(P_2)$$

$$\Phi(P_2 - P_1) = \Phi(P_2) + \phi(-P_1)$$
$$= \phi(P_2) - \Phi(P_1)$$