Hierarchical spatial modelling for applied population and community ecology

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Spatial multi-species occupancy models

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Spatial autocorrelation in MSOMs

- Spatial autocorrelation may be more prevalent in MSOMs since different species are driven by different habitat requirements.

- Likely won't be able to include all covariates relevant to all species in the model.

- Can extend the MSOM just like we did with the SSOM to account for spatial autocorrelation in occupancy probability for each species.

- Two "flavors" of spatial MSOMs included in `spOccupancy`

Type 1: Spatial MSOMs

- Estimate a separate spatial random effect **w** for each species
- Each **w** is estimated with a NNGP as before
- Nothing new here, now just have N (the number of species) NNGP spatial random effects
- Would very rarely want to use a full GP instead of an NNGP, since we now need to estimate N spatial random intercepts
- `spMsPGOcc()` function

Type 1 Spatial MSOMs

$$z_{i,j} \sim \text{Bernoulli}(\psi_{i,j})$$

$$\text{logit}(\psi_{i,j}) = \beta_{1,i} + \beta_{2,i} \cdot X_{2,j} + \cdots + \beta_{r,i} \cdot X_{r,j} + \text{w}_{i,j}$$

$$\beta_{r,i} \sim \text{Normal}(\mu_{\beta_r}, \tau^2_{\beta_r})$$

$$\mathbf{w}_i \sim \text{Normal}(\mathbf{0}, \tilde{\boldsymbol{C}}(d, \phi_i, \sigma^2_i)$$

$$\mu_{\beta_r} \sim \text{Normal}(\mu 0_r, \sigma^2_{\beta_r})$$

$$\tau^2_{\beta_r} \sim \text{Inverse gamma}(a_{r,\beta}, b_{r,\beta})$$

$$\phi_i \sim \text{Uniform}(a_{\phi,i}, b_{\phi,i})$$

$$\sigma^2_i \sim \text{Inverse gamma}(a_{\sigma,i}, b_{\sigma,i})$$

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$$\boxed{\begin{array}{l} \phi_i \sim \text{Uniform}(a_{\phi,i}, b_{\phi,i}) \\ \sigma_i^2 \sim \text{Inverse gamma}(a_{\sigma,i}, b_{\sigma,i}) \end{array}}$$

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The detection sub-model is exactly the same as the nonspatial MSOM

Problems with Type 1 spatial MSOMs

- Can become quite slow when the number of species in the community is greater than 10 or so
- Species-specific spatial random effects can be very difficult to estimate for rare species
- Most useful when working with a modest number of spatial locations (e.g. 100-500) and a modest number of species (e.g., <10) that are not extremely rare.

Type 2: Spatial factor MSOMs

- More powerful and efficient than the previously discussed spatial MSOMs

- A multi-species modeling approach that simultaneously accounts for:
  - Imperfect detection
  - Spatial autocorrelation
  - Species correlations

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Joint species distribution models with imperfect detection for high-dimensional spatial data

Jeffrey W. Doser[1,2] | Andrew O. Finley[2,3] | Sudipto Banerjee[4]

Spatial factor MSOMs

- An extension of the latent factor MSOM we discussed previously
- Identical to the latent factor MSOM, but now the factors are estimated as spatial NNGPs!
- **Foundational concept**: spatial autocorrelation in the distributions of individual species may arise from a set of shared, common factors that have a spatial pattern
- Only difference from the previous spatial MSOM is in how the spatial random effects are modeled

Spatial factor MSOMs

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Spatial factor MSOMs

Species-specific spatial random effect

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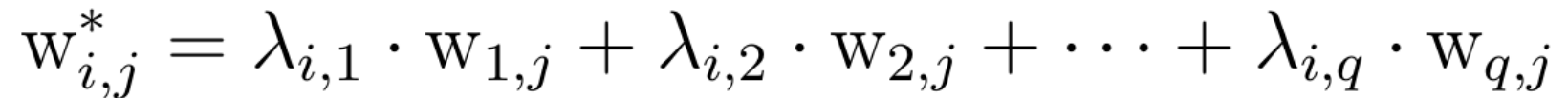
Spatial factor MSOMs

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$$\text{w}_{i,j}^* = \lambda_{i,1} \cdot \text{w}_{1,j} + \lambda_{i,2} \cdot \text{w}_{2,j} + \cdots + \lambda_{i,q} \cdot \text{w}_{q,j}$$

Species specific spatial effect is the sum of a set of q spatial factors and their species-specific coefficients (loadings)

Spatial factor MSOMs

Species-specific spatial random effect

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"Missing covariates" that account for residual spatial autocorrelation

Spatial factor MSOMs

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Species specific spatial effect is the sum of a set of q spatial factors and their species-specific coefficients (loadings)

Effects of the missing covariates

"Missing covariates" that account for residual spatial autocorrelation

Spatial factor MSOMs

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Identical to latent factor MSOMs, except we now model each w with a NNGP instead of from a Normal(0, 1).

Priors and identifiability considerations

- Same priors as we've seen before
- Normal priors on community-level means
- Inverse-gamma priors on community-level variances
- Uniform priors on spatial decay parameters
- Same restrictions on the factor loadings matrix as with latent factor models
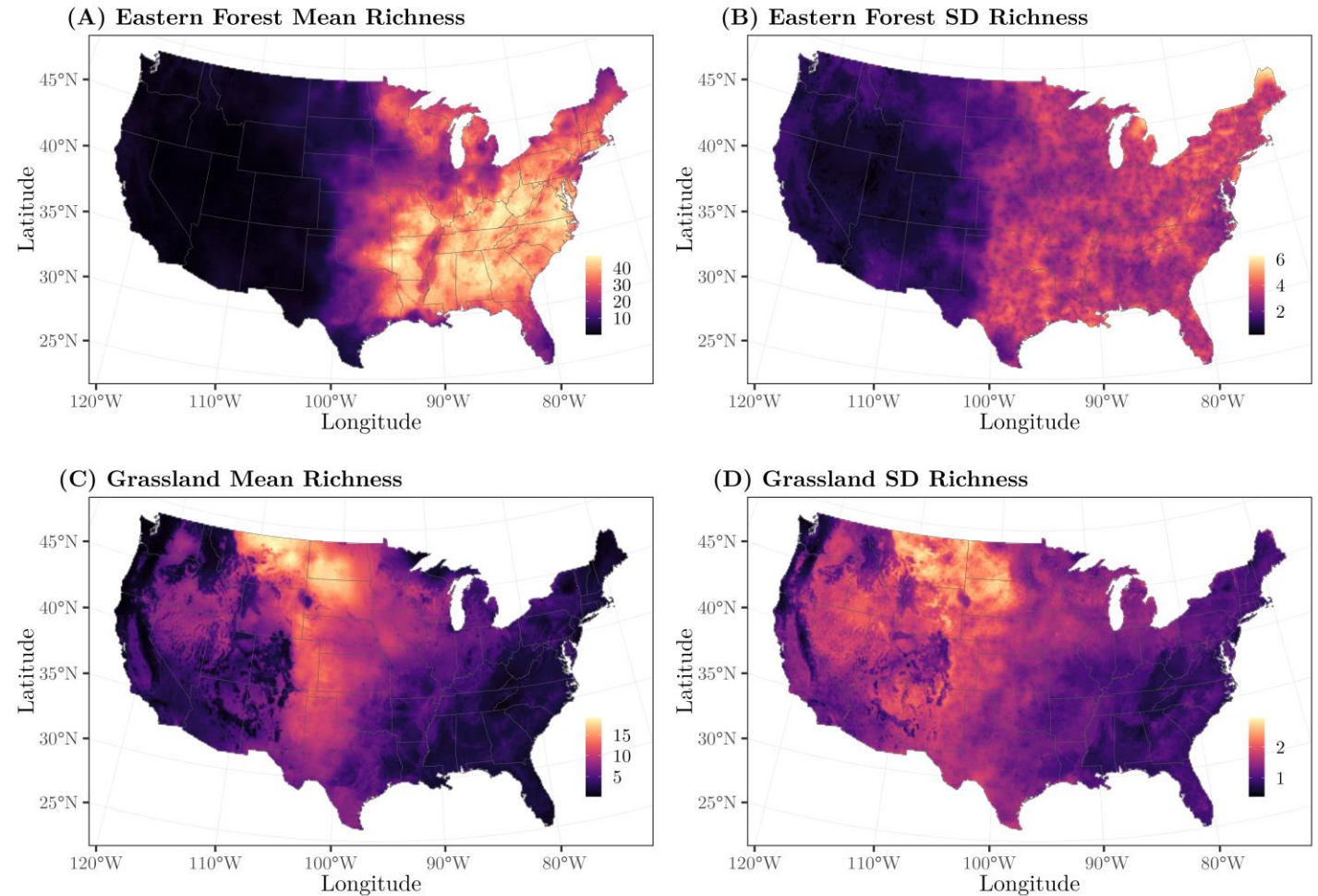
Why use the spatial factor MSOM?

Performs well in a variety of scenarios while being more computationally efficient than alternative spatial MSOMs.

TABLE 2 Estimated coverage rates of simulated species-specific occurrence probabilities and covariate effects for six different simulation scenarios and six models of varying complexity, as well as average run time.

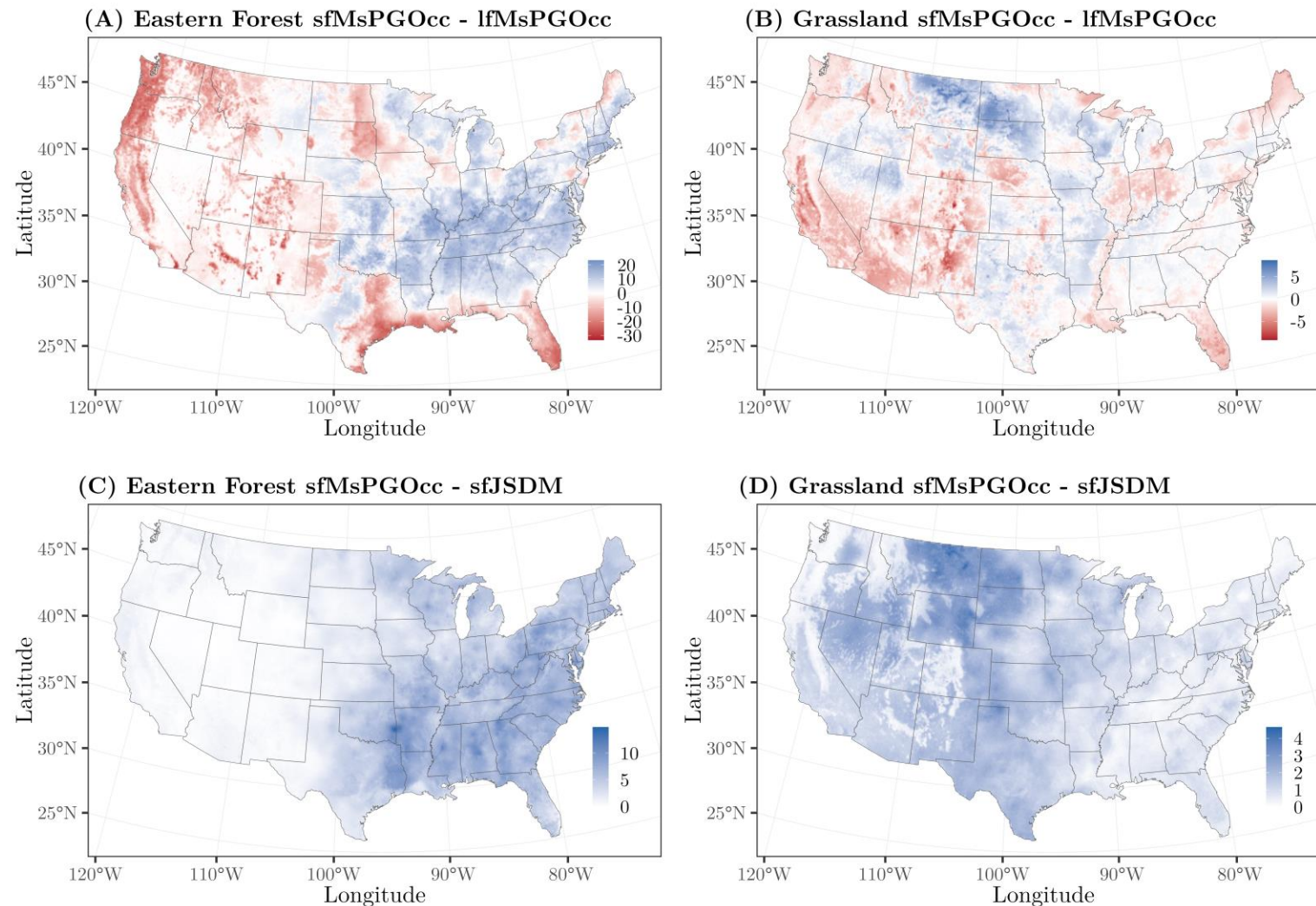| Parameter	Scenario	Model					
		lfJSDM	sfJSDM	msPGOcc	spMsPGOcc	lfMsPGOcc	sfMsPGOcc
$\psi_i(s_j)$	1	91.5	90.8	68.9	88.1	95.6	95.3
	2	91.6	91.0	69.1	89.1	95.5	95.4
	3	85.6	84.8	95.0	96.4	95.5	95.5
	4	77.5	76.4	80.2	93.1	95.7	95.5
	5	75.3	74.2	71.3	88.5	95.5	95.3
	6	76.0	75.0	72.2	89.6	95.3	95.2
β_i	1	88.7	88.2	82.0	91.1	95.2	95.1
	2	88.8	88.2	82.2	91.7	94.9	94.9
	3	73.8	73.1	95.1	94.4	90.4	90.8
	4	65.9	65.0	89.1	94.0	94.7	94.7
	5	64.2	63.6	83.6	91.7	95.2	95.0
	6	65.7	64.6	85.1	92.7	94.9	94.9

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Why use the spatial factor MSOM?

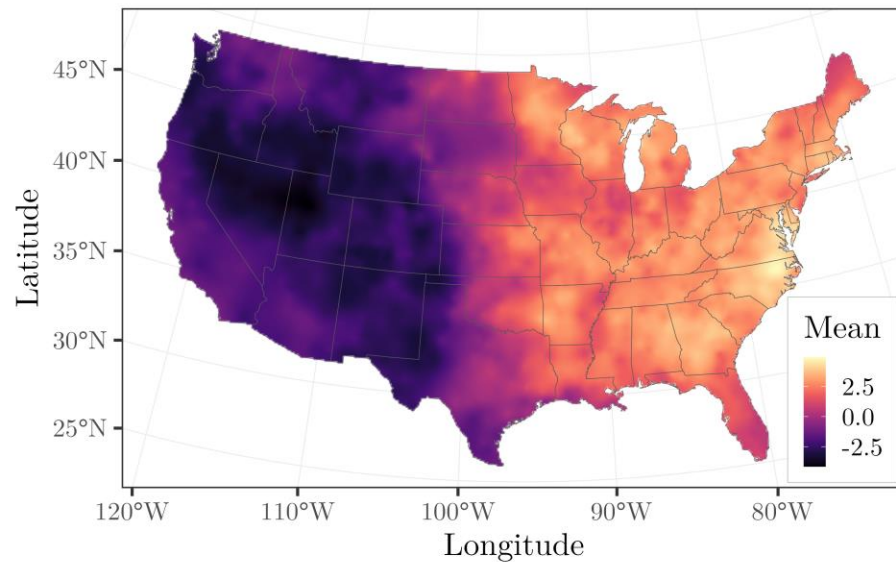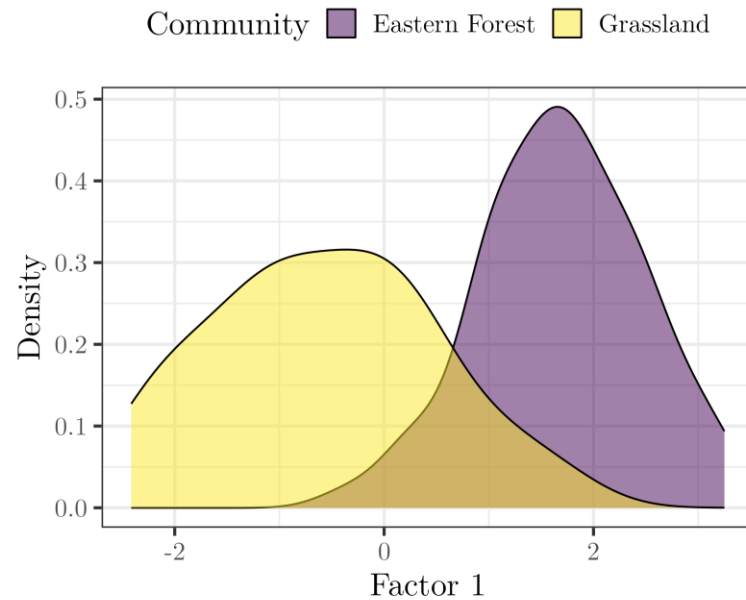Estimate distributions and biodiversity metrics with big multi-species data sets (ex: 2,619 sites and 98 species)

(A) Eastern Forest Mean Richness

(B) Eastern Forest SD Richness

(C) Grassland Mean Richness

(D) Grassland SD Richness

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Why use the spatial factor MSOM?

Improved estimates
of biodiversity metrics

(A) Eastern Forest sfMsPGOcc - lfMsPGOcc

(B) Grassland sfMsPGOcc - lfMsPGOcc

(C) Eastern Forest sfMsPGOcc - sfJSDM

(D) Grassland sfMsPGOcc - sfJSDM

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Why use the spatial factor MSOM?

Model-based ordination

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Fitting spatial factor MSOMs in `spOccupancy`

- `sfMsPGOcc()`

- As with all spatial models, we break up the MCMC iterations into a set of batches, where each batch has a set number of iterations.

- Need to specify number of spatial factors to use

Multi-species, single-season models

spOccupancy function	Species correlations	Spatial autocorrelation	Imperfect detection
lfJSDM	✓		
sfJSDM	✓	✓	
msPGOcc			✓
spMsPGOcc		✓	✓
lfMsPGOcc	✓		✓
sfMsPGOcc	✓	✓	✓

Exercise: Niche partitioning in a simulated plant community

6-plant-spatial-multi-species-occ.R