Hierarchical spatial modelling for applied population and community ecology

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24-27 June 2024

Spatially-varying coefficient occupancy models

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Spatial modelling

- So far, our discussion of spatial models has focused on accounting for residual spatial autocorrelation.
- As we've seen, this has a lot of benefits, e.g., more accurate predictions, better uncertainty estimates
- All of the models we have talked about have done so via some form of **spatially-varying intercept**

Recall our spatial SSOM

Occupancy (ecological) sub-model

$$z_j \sim \mathrm{Bernoulli}(\psi_j)$$
$$\mathrm{logit}(\psi_j) = \beta_1 + \beta_2 \cdot X_{2,j} + \cdots + \beta_r \cdot X_{r,j} + \mathrm{w}_j$$
$$\mathbf{w} \sim \mathrm{Normal}(\mathbf{0}, \tilde{\boldsymbol{C}}(d, \phi, \sigma^2))$$

Detection (observation) sub-model

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Non-spatial component

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There is nothing restricting us from doing this only with the intercept. We can make any coefficient spatial!

Why might species-environment relationships vary spatially?

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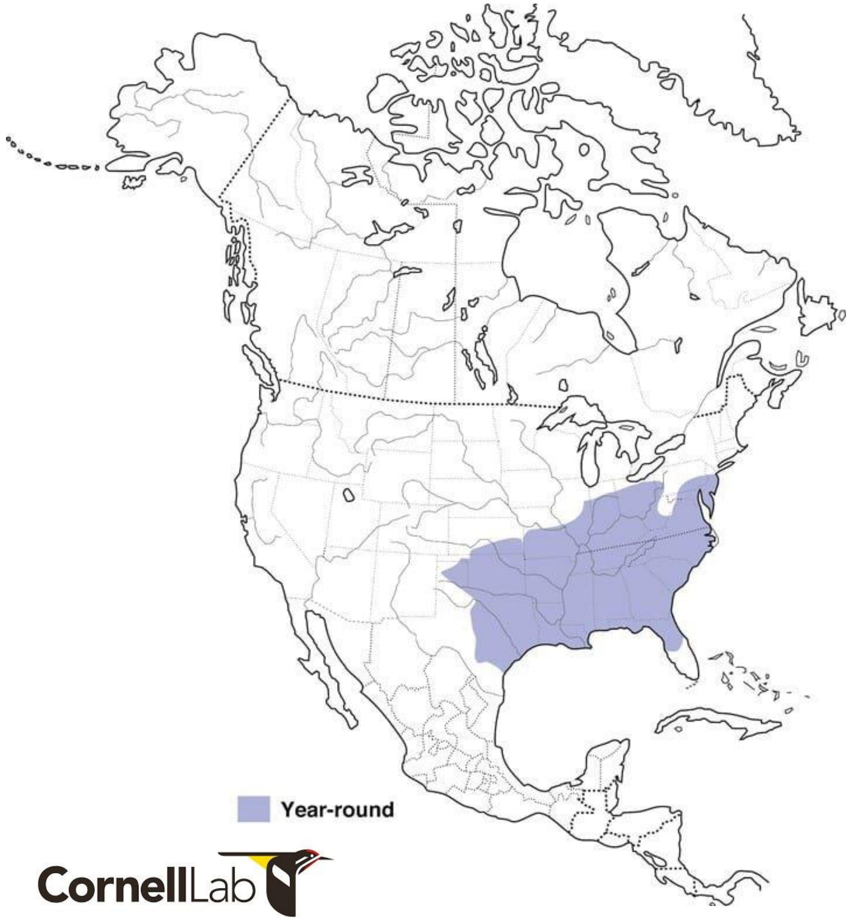
- Interactions with abiotic factors such as:
  - o Historical disturbance regimes
  - o Fine-scale habitat characteristics (e.g., vegetation quality)
  - o Local site conditions (e.g., soil content)
  - o Spatial variation in resource availability

Why might species-environment relationships vary spatially?

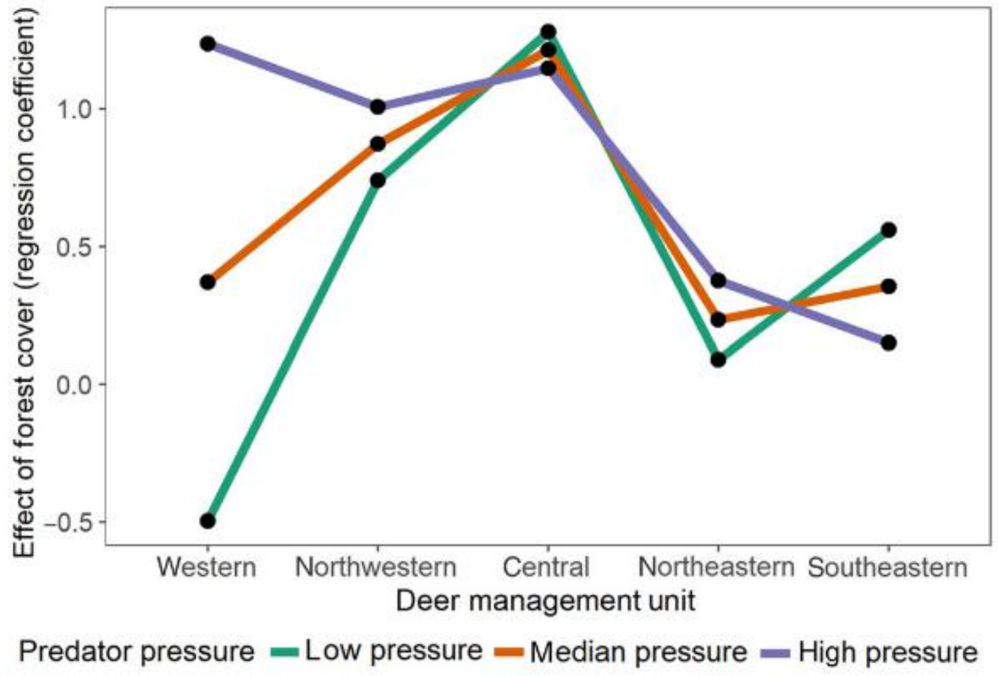
- Impacts of climate may differ between range core and range boundaries

Carolina chickadee



Why might species-environment relationships vary spatially?

- Biotic processes
  - Local genetic adaptations
  - Spatial variation in species interactions



Pease, Pacifici, Kays (2022) Ecosphere

Spatially-varying species-environment relationships

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  1. Test and generate hypotheses regarding what is driving the relationship.
  2. Better understand the relative importance of different drivers across a species range.
  3. Inform conservation and management at both local and broad scales.

Estimating spatial variability in species-environment relationships

- Consider a general model for a spatial occupancy model with one covariate

$$\text{logit}(\psi_j) = (\beta_1 + \text{w}_{1,j}) + f(X_{2,j}, \boldsymbol{\beta})$$

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Let's consider 5 forms of this functional relationship

1. Linear

$$f(X_{2,j}, \boldsymbol{\beta}) = \beta_2 \cdot X_{2,j}$$

- The simplest form.
- Does not account for spatial variation in species-environment relationships.

2. Quadratic

$$f(X_{2,j}, \boldsymbol{\beta}) = \beta_2 \cdot X_{2,j} + \beta_3 \cdot X_{2,j}^2$$

- Allows occupancy probability to peak at some optimum level or peak at the extremes
- Useful if the species-environment relationship is non-linear

3. Stratum

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- Overall linear effect of the environmental predictor as well as stratum-specific adjustments in the effect across a set of strata (e.g., management units)
- Stratum-specific effects could be fixed or random
- Accounts for nonstationarity, but...
  - Spatial variation is limited to pre-defined strata.
  - Lots of uncertainty if some strata have small sample sizes.

4. Interaction

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- Allows for spatial variation in species-environment relationship through an interaction with another variable.
- Very useful for testing explicit hypotheses.
- Spatial variation is limited to the interaction with the additional variable.
- May not always have the interacting variable available (or may not know what the variable is).

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- Can be viewed as an extension of the interaction model where we now estimate the interacting variable.

Simulation experiment

- Simulated data under six different species environment relationships:
    1. Linear
    2. Quadratic
    3. Stratum
    4. Interaction
    5. Interaction with an unknown "missing" covariate
    6. The sum of all the above
- Compared the performance of the 5 models

Linear

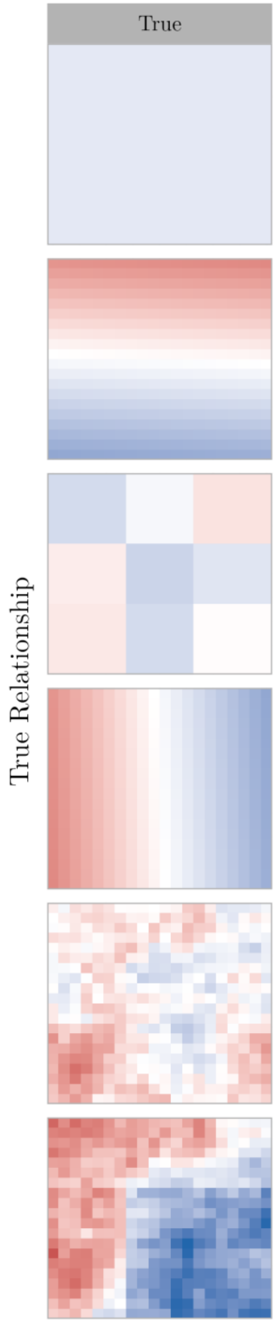
Quadratic

Stratum

Interaction

Missing interaction

Full



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SVC occupancy models

- A simple extension of spatial occupancy models, but we now allow the regression coefficients to vary spatially as well!
- Example SVC ocupancy model with an SVI and 1 SVC

$$\text{logit}(\psi_j) = \tilde{\beta}_{1,j} + \tilde{\beta}_{2,j} \cdot X_{2,j}$$
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Each w is modelled with an NNGP with its own set of spatial parameters

Fitting SVC Occupancy models

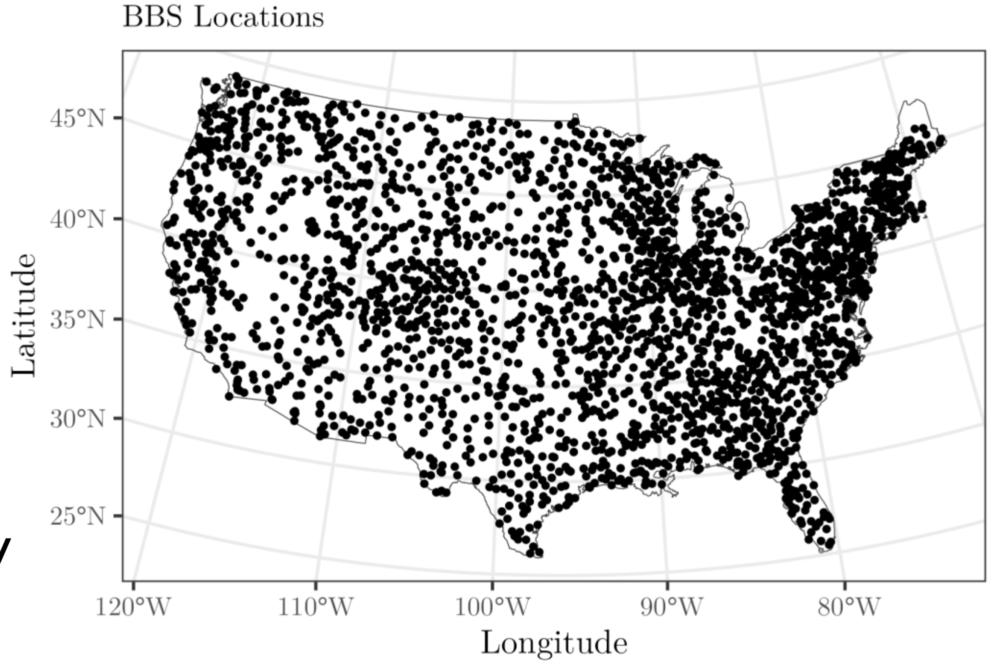
- `svcPGOcc()` function (spatially-varying coefficient Pólya-Gamma occupancy model)
- Each SVC (and SVI) have a different set of spatial parameters (spatial decay and spatial variance) that control their form. Can specify different priors on the different SVCs
- Often useful to set a more informative prior for the spatial decay parameter on SVCs to prevent extremely small effective spatial ranges
- Using `predict()` we can generate maps of the SVCs!!

Extensions to multiple seasons and multiple species

- `svcMsPGOcc()` fits single-season multi-species SVC occupancy models using a spatial factor approach.
- `svcTPGOcc()` fits multi-season single-species SVC occupancy models
- `svcTMsPGOcc()` fits multi-season, multi-species SVC occupancy models.

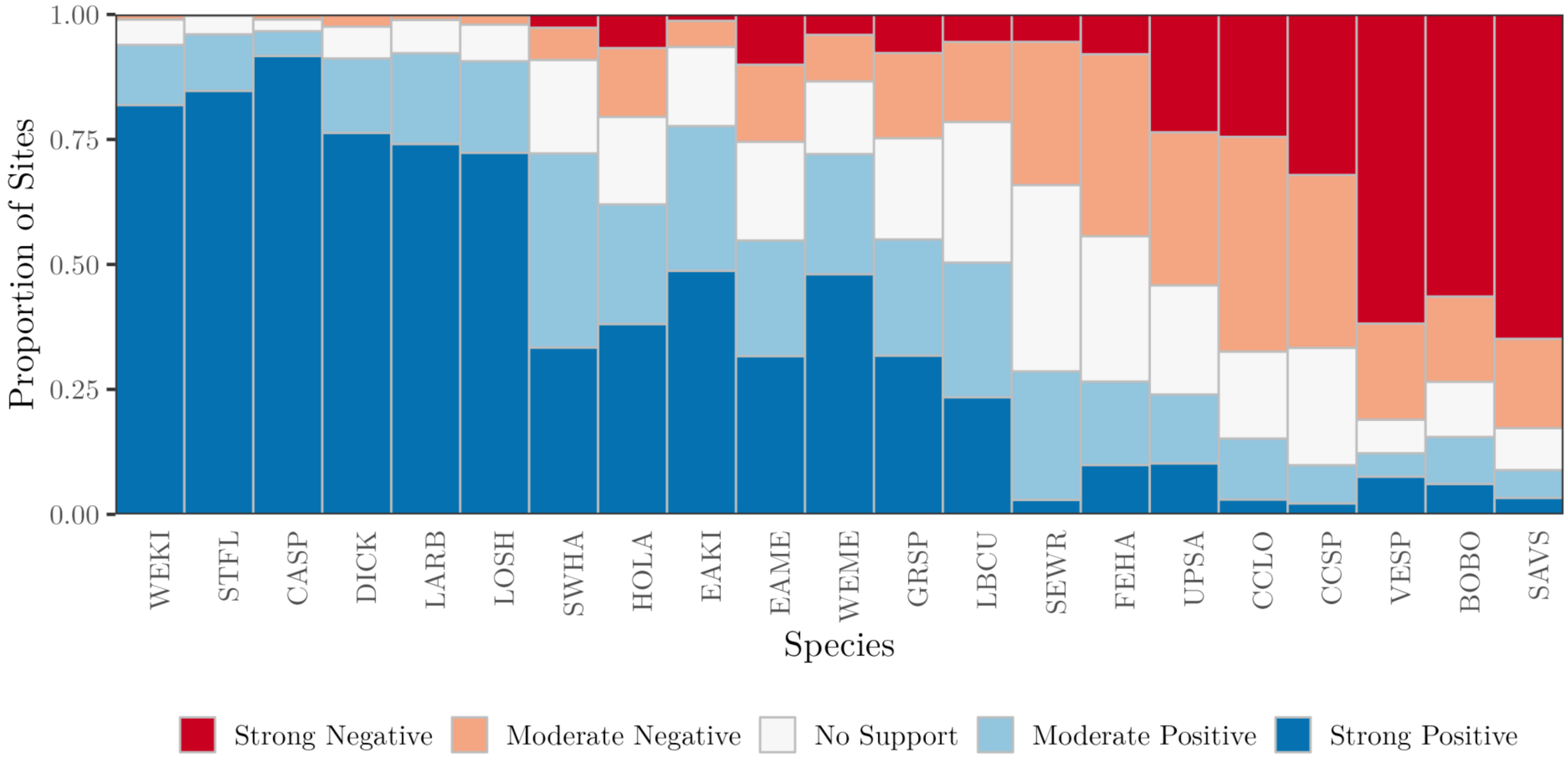
Example: Grassland birds in the US

- North American Breeding Bird Survey Data in 2019
- 21 grassland bird species across 2,486 BBS routes
- Does the effect of maximum temperature vary spatially?
- Fit a multi-species SVC occupancy model



Doser et al. (2024) *Journal of Agricultural, Biological, and Environmental Statistics*

Large spatial variation in maximum temperature effect



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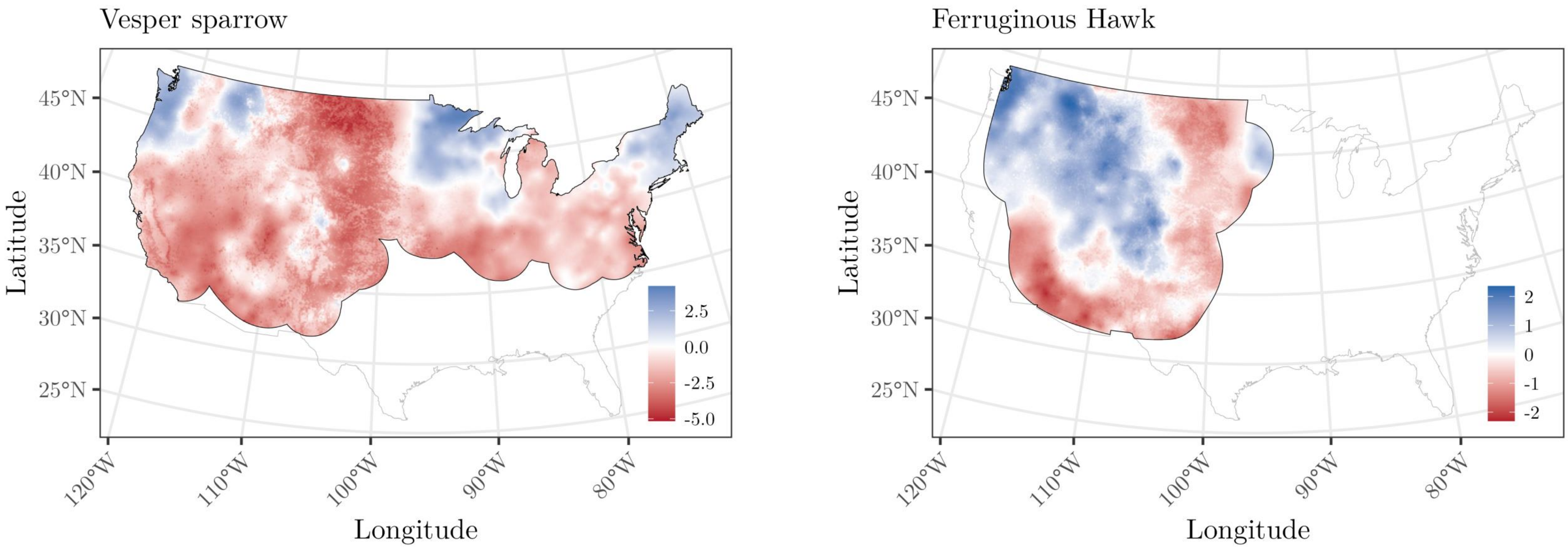
Effects of maximum temperature



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Estimating spatially-varying trends

- One important application of SVC models is estimating spatially-varying trends
- Can provide fine-scale insight on population/distribution changes

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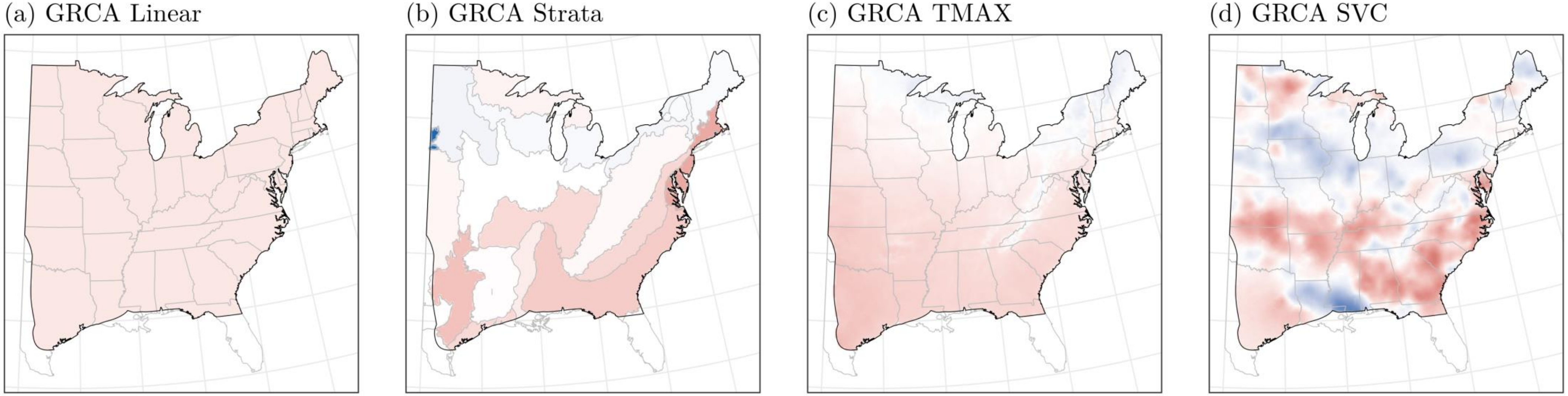
Could also model spatially-varying nonlinear trends by using SVCs with temporal spline covariates

Example: Spatial heterogeneity in forest bird trends in eastern USA



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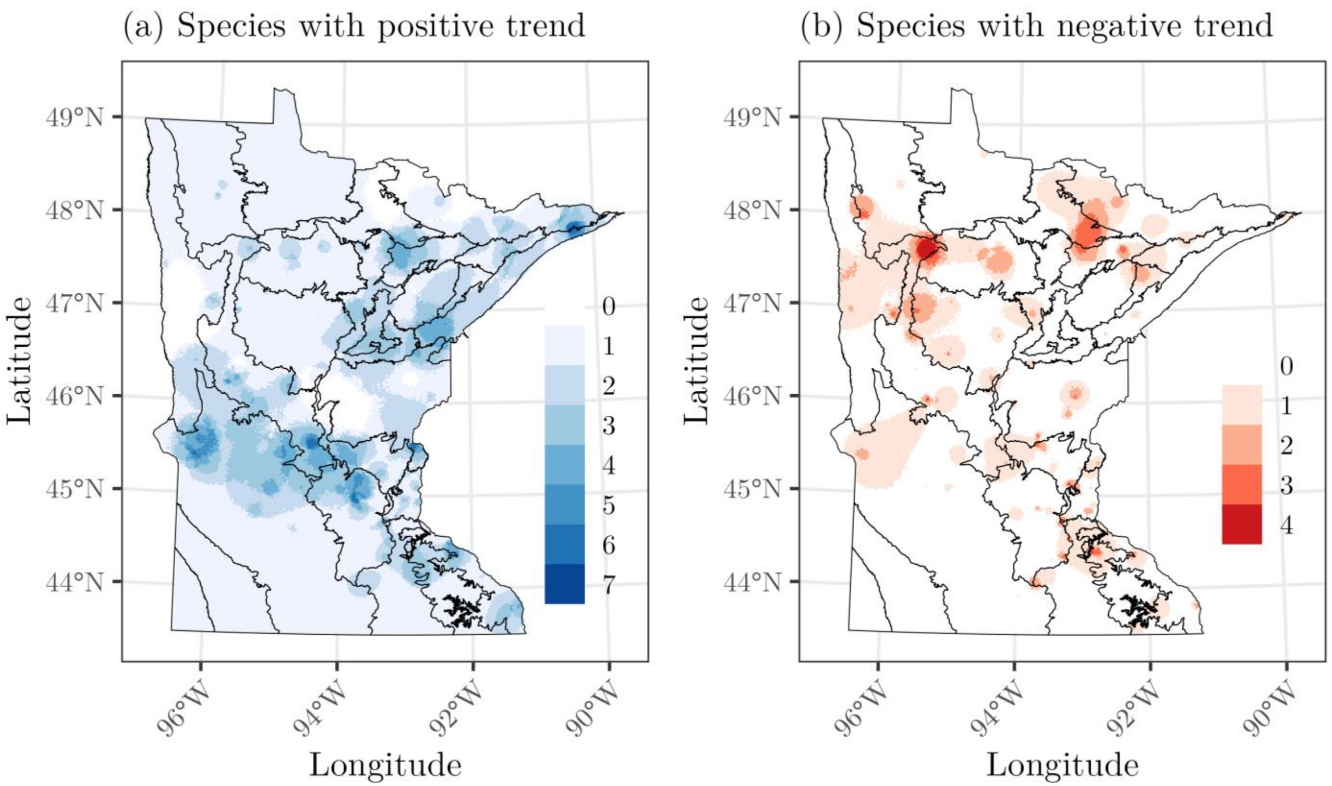
Example: Identify hotspots of decline for targeted monitoring/management



Doser, Edwards, Zipkin, Jennelle (*In prep*)

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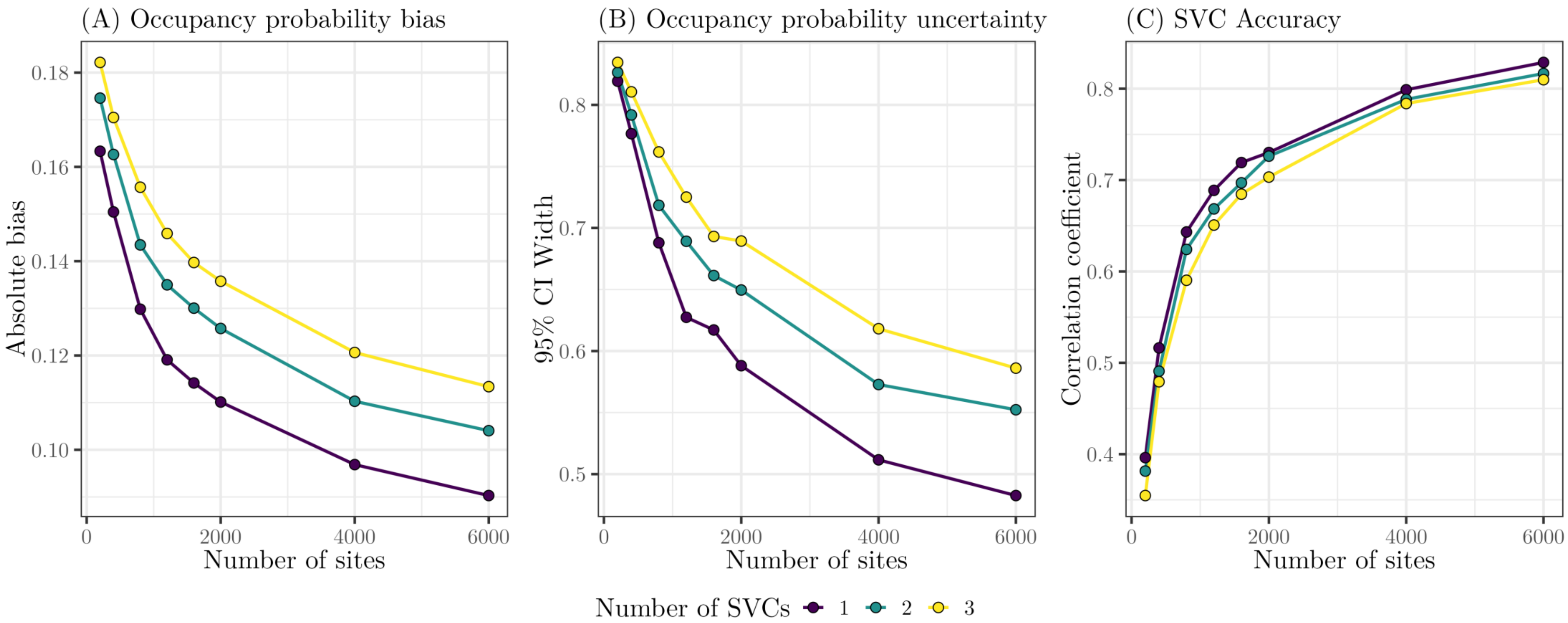
- Compare SVC models to a spatial occupancy model
- More here

Priors for spatial decay on SVCs

- Default `spOccupancy` prior for spatial decay parameter ϕ is Uniform(3/max, 3/min).
- Recall the effective spatial range is $3 / \phi$
- This can lead to overfitting in SVC models.
- Can often be useful to specify a more informative prior to restrict the effective spatial range.
- Best option is to set the bounds based on ecology of the species or the spatial scale of interest.
- Setting upper bound of ϕ to $3 / q_{0.25}$ can help minimize the potential for misleading inferences (where $q_{0.25}$ is the 25% quantile of the intersite distance matrix)

SVC occupancy models are data hungry

Single-season occupancy models



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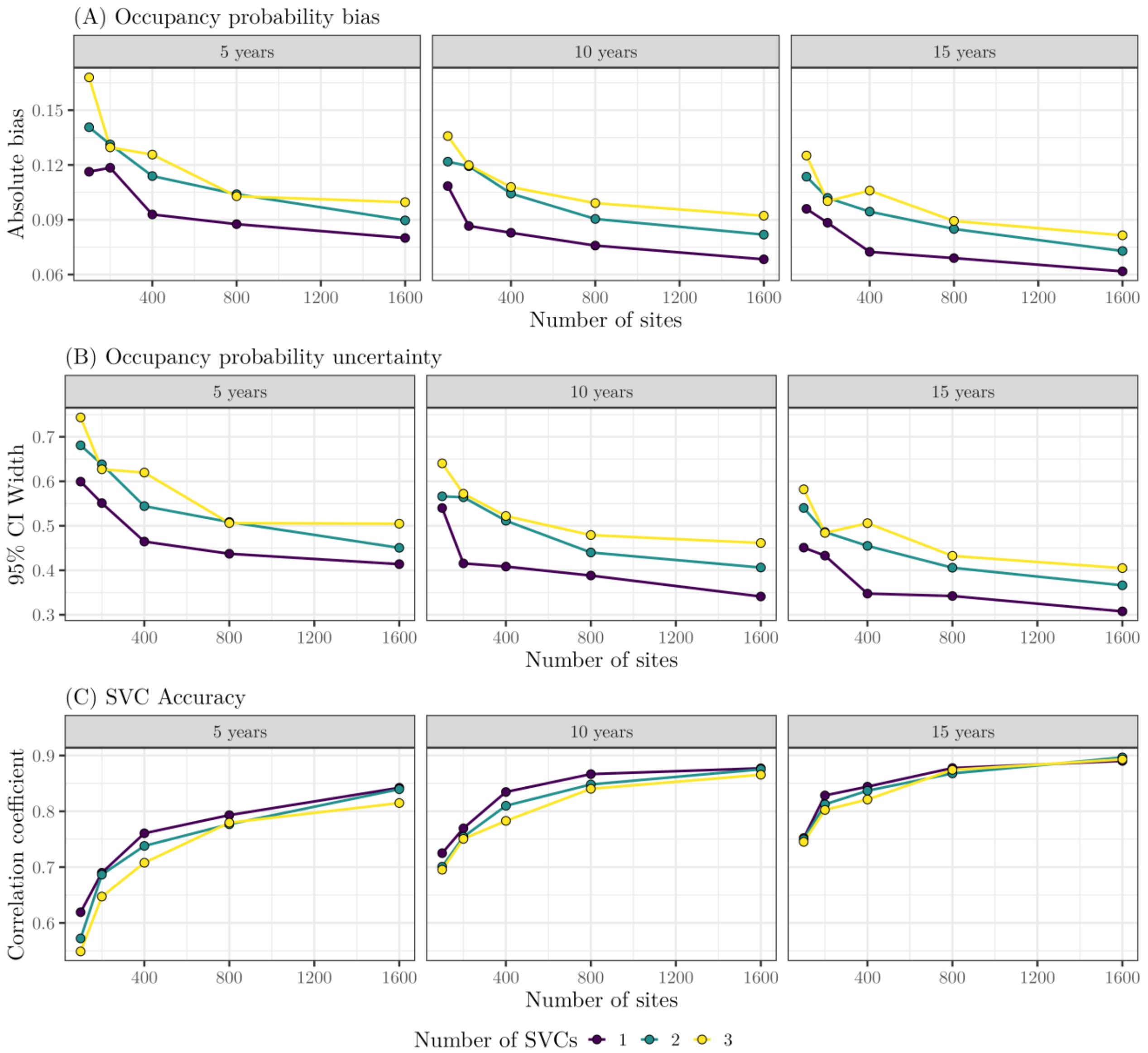
SVC occupancy models are data hungry

Multi-season SVC occupancy models are somewhat less data hungry



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Exercise: Spatially-varying trend in wood thrush occupancy in the eastern US

8-wood-thrush-spatial-trend-occ.R