Uncertainty intervals

SIADS 542: Presenting uncertainty – Week 2, Lecture 2

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Today

We've seen how to formally describe
continuous uncertainty encodings

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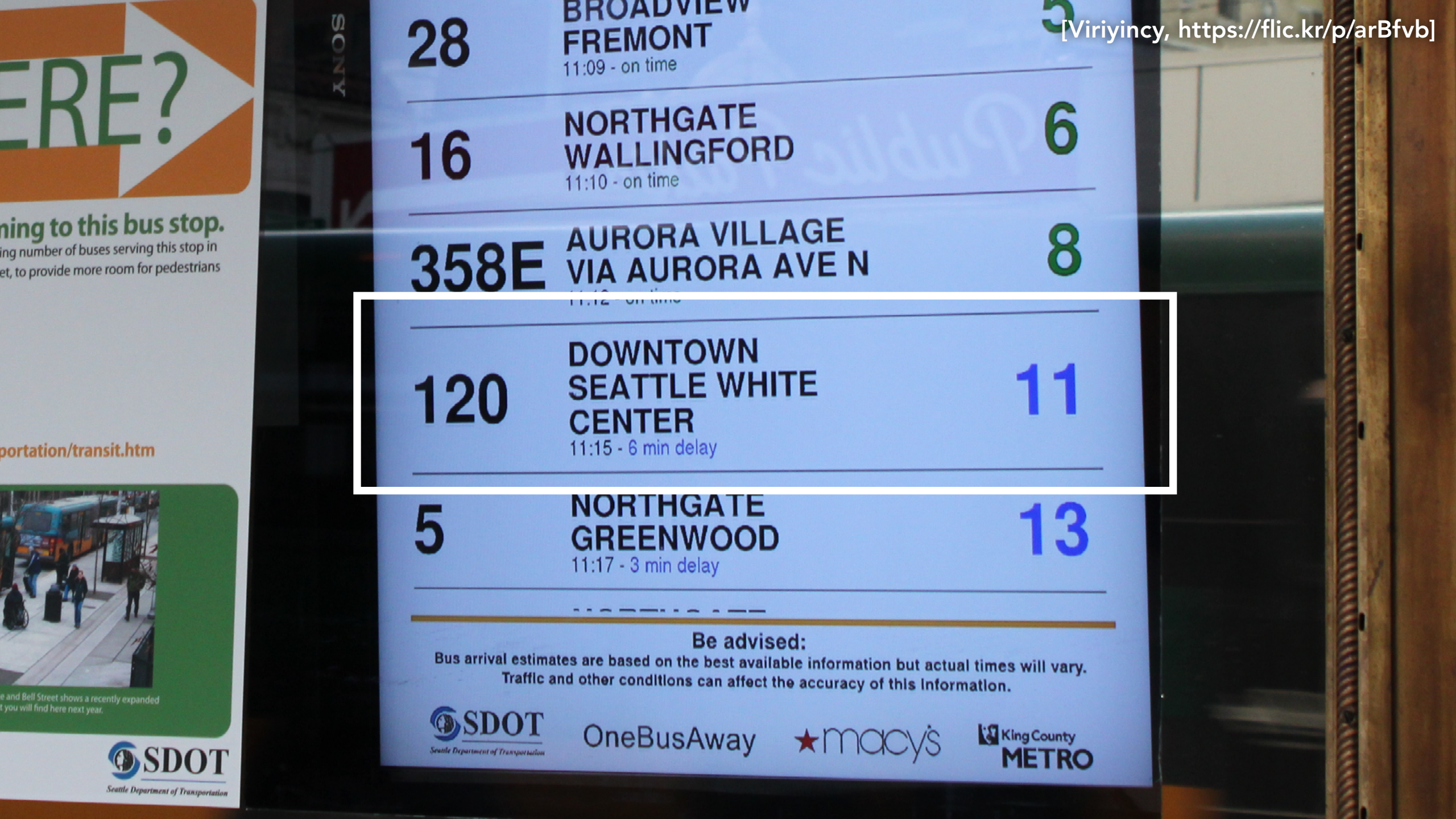
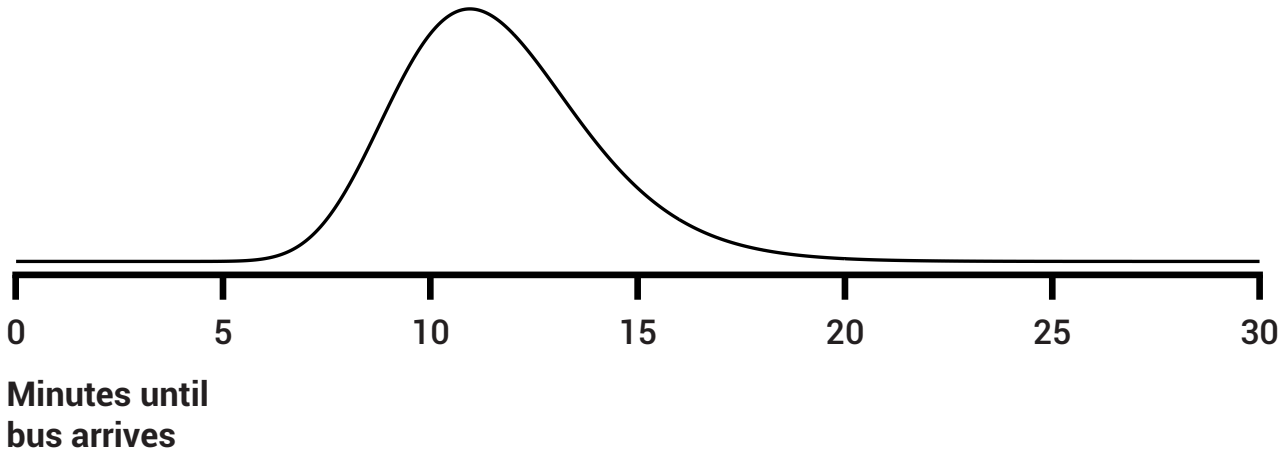
We've seen how to formally describe
continuous uncertainty encodings

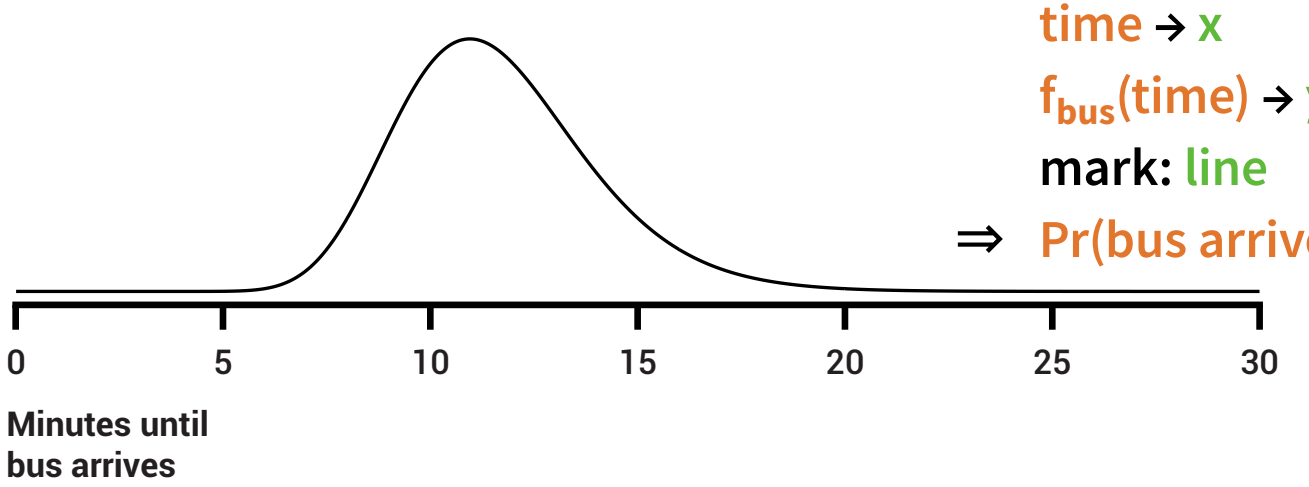
Let's look at uncertainty intervals

Let's return to the bus tracking scenario



[Viriyincy, https://flic.kr/p/arBfvb]


0
5
10
15
20
25
30
Minutes until
bus arrives

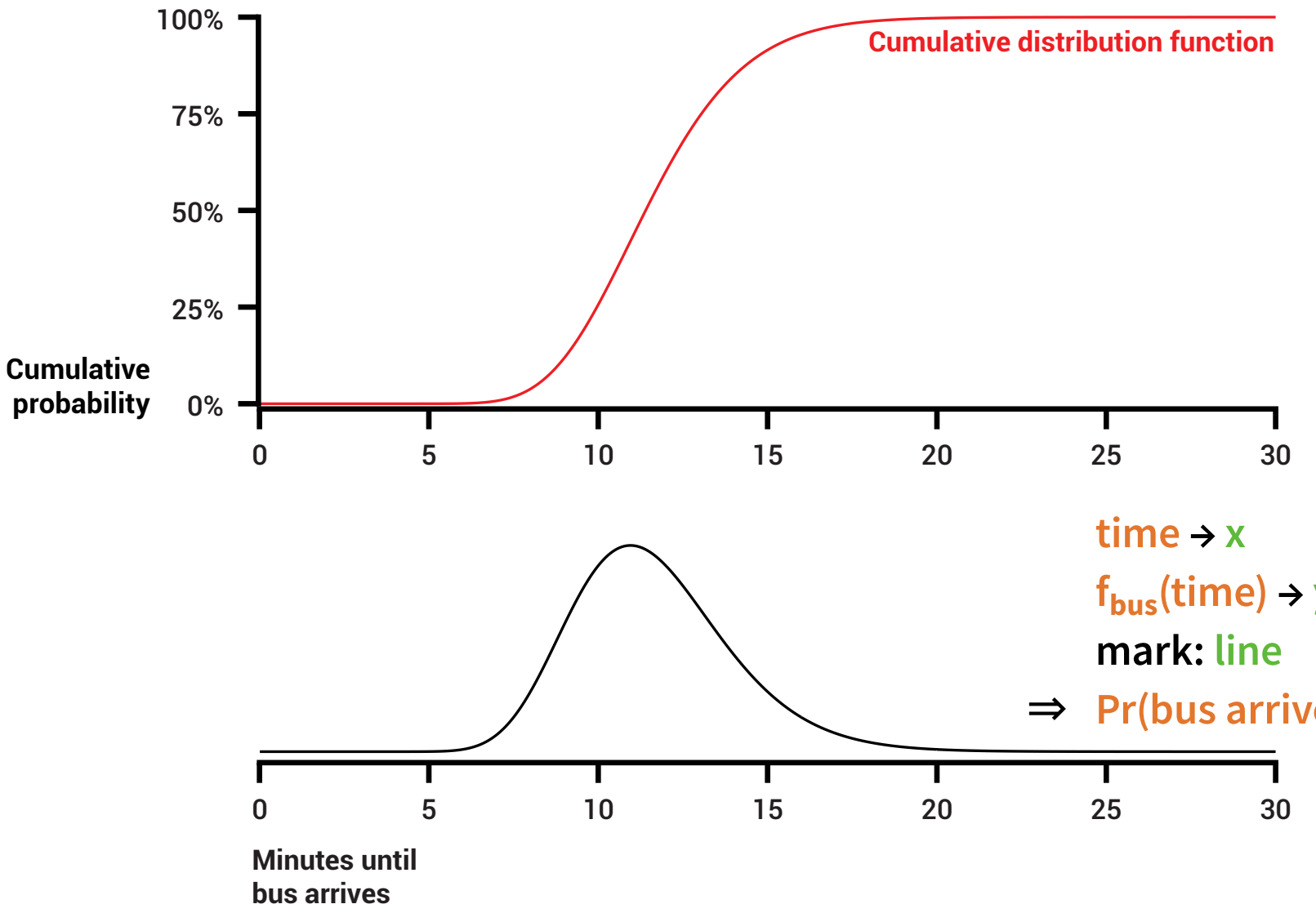


time → x

f_{bus}(time) → y (*density*)

mark: line

⇒ Pr(bus arrives ≤ time) → area

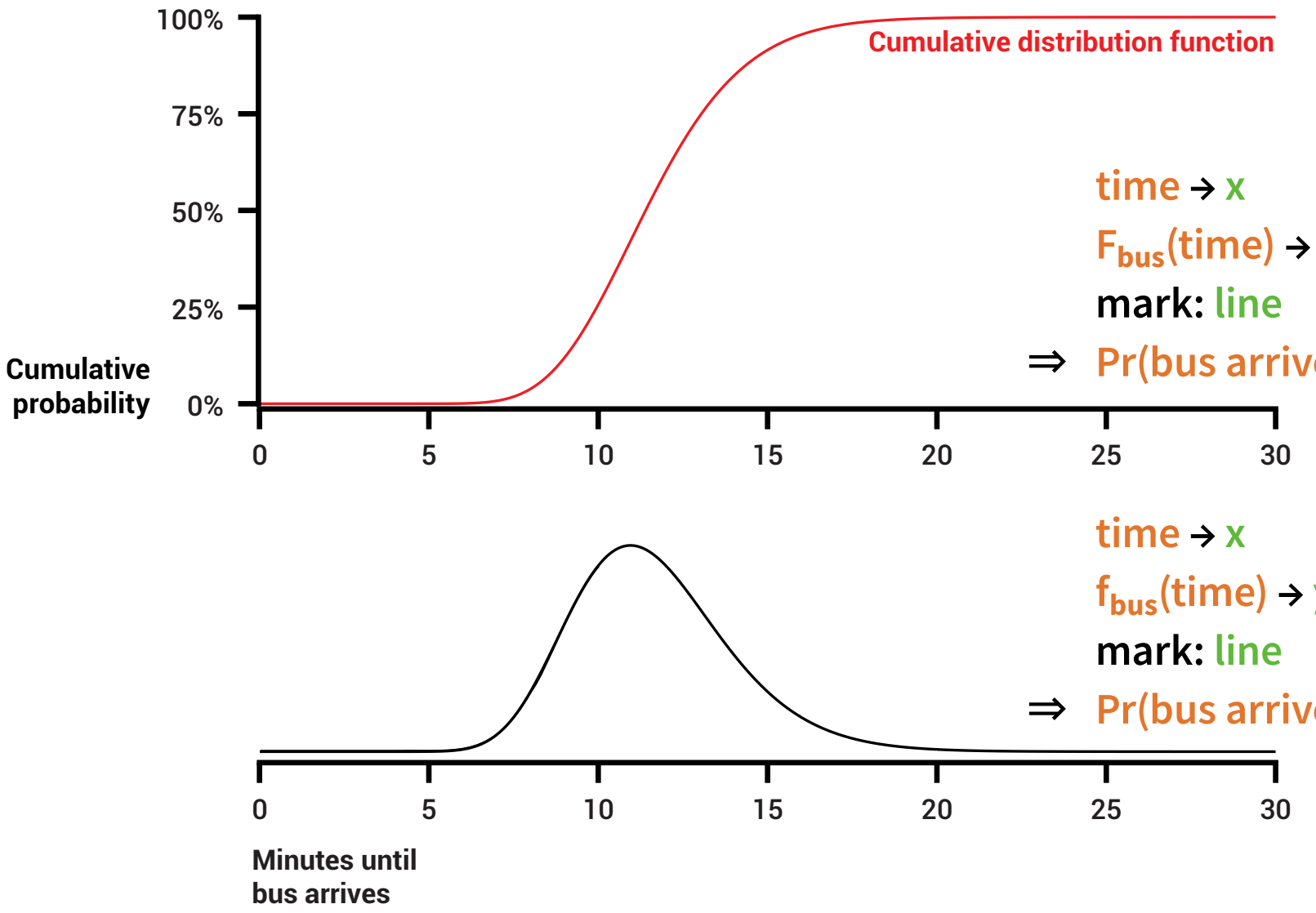


time → x

f_{bus}(time) → y (*density*)

mark: line

⇒ Pr(bus arrives ≤ time) → area



time → x

F_{bus}(time) → y (*CDF*)

mark: line

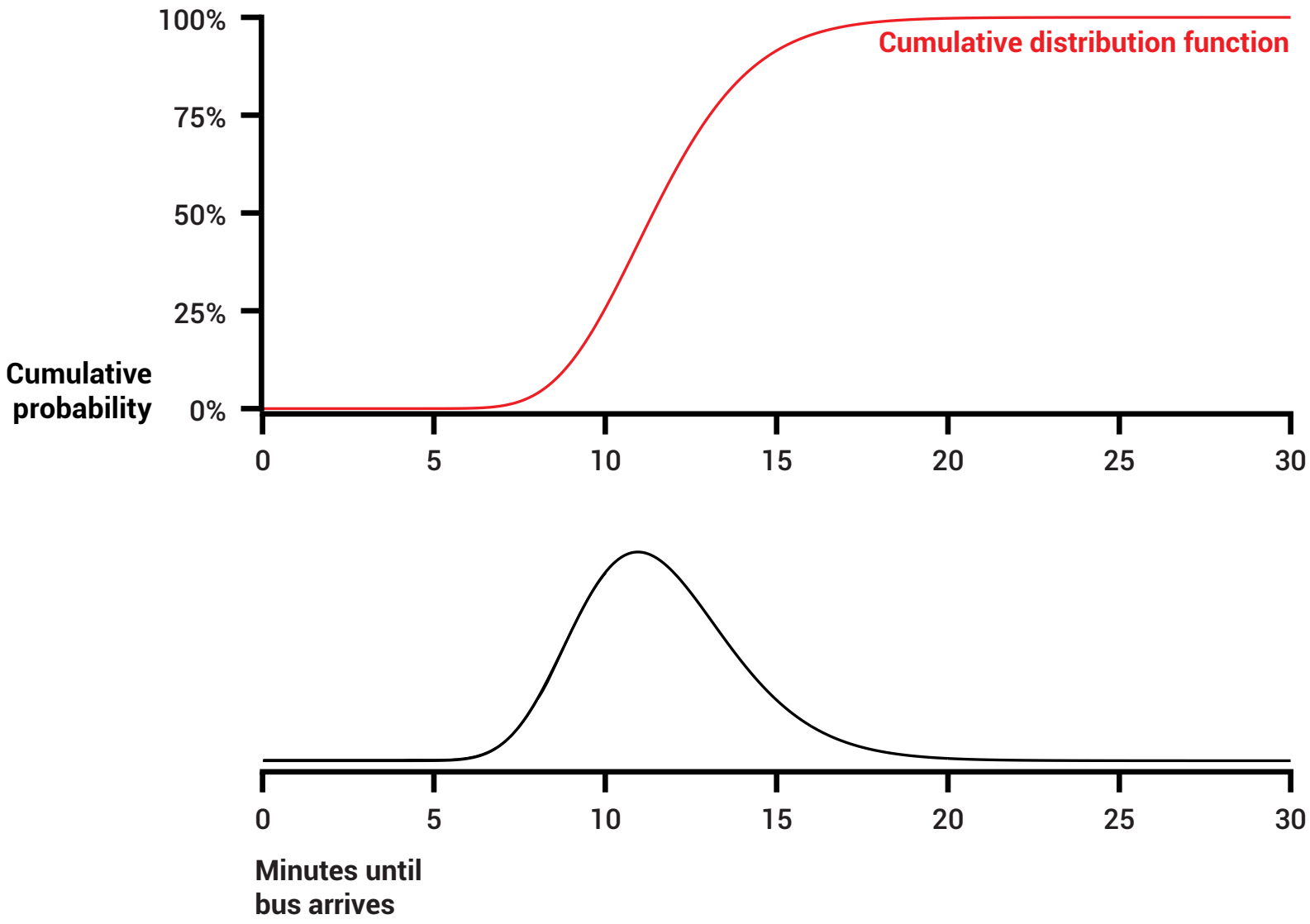
⇒ Pr(bus arrives ≤ time) → y

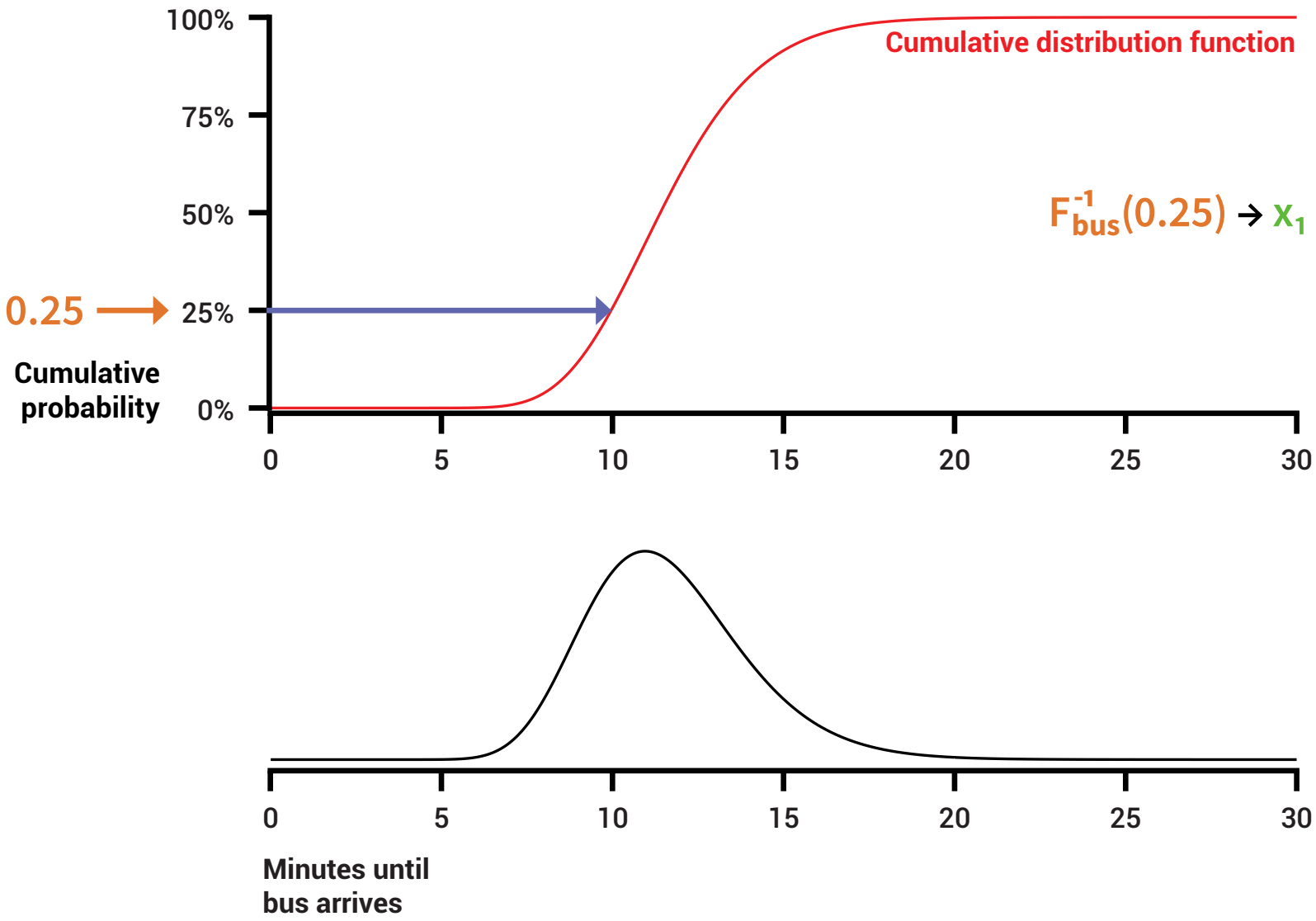
time → x

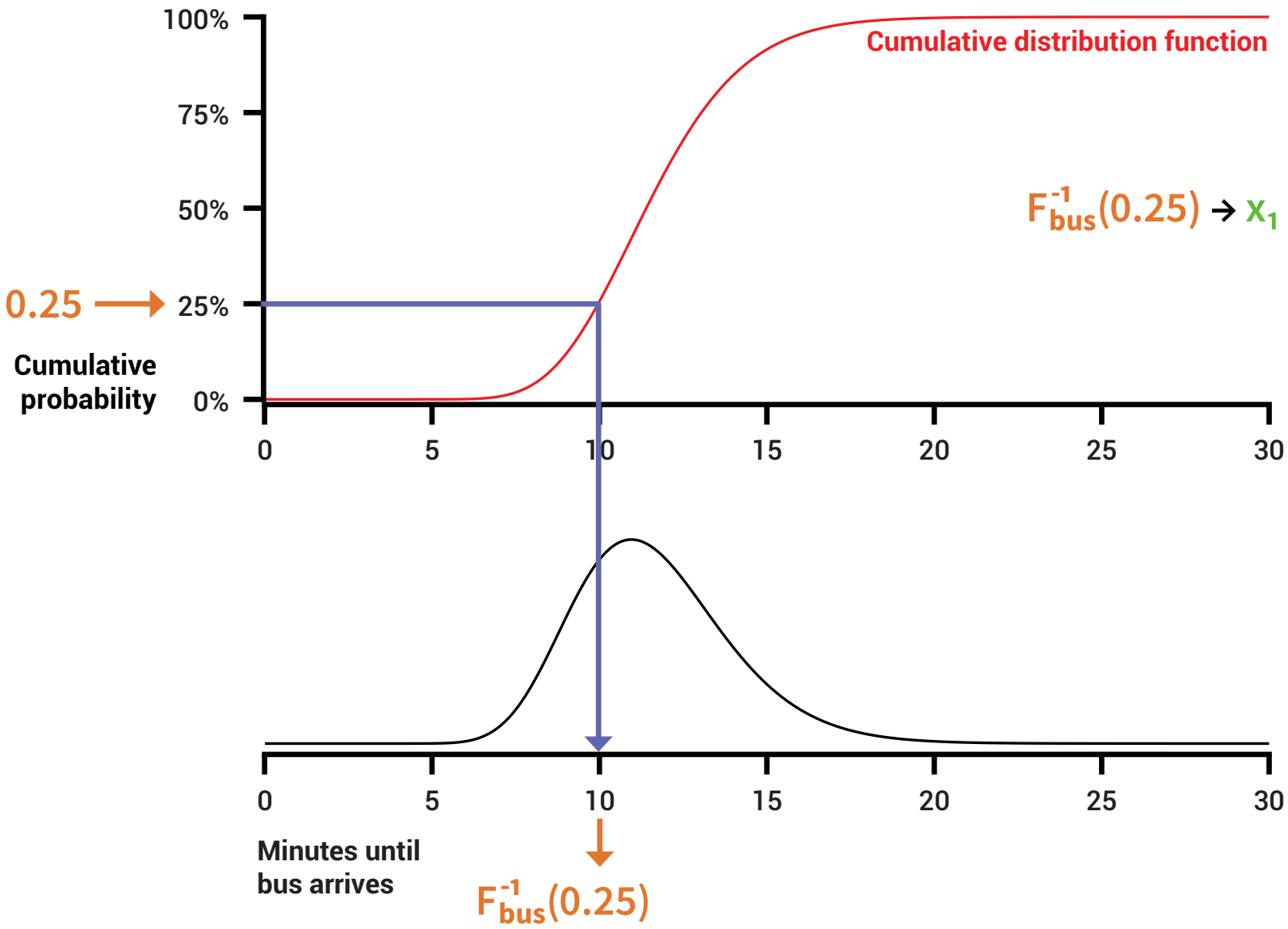
f_{bus}(time) → y (*density*)

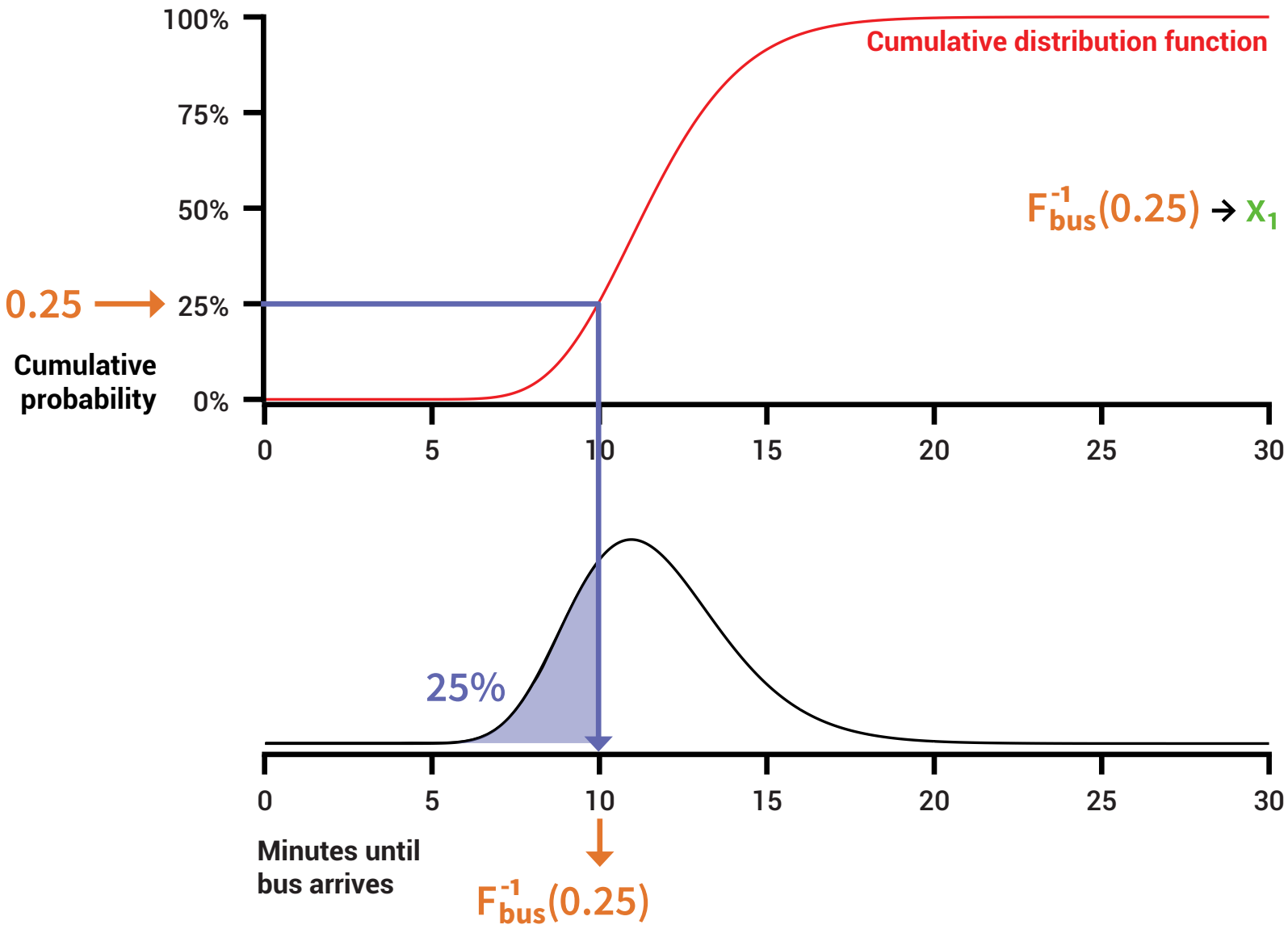
mark: line

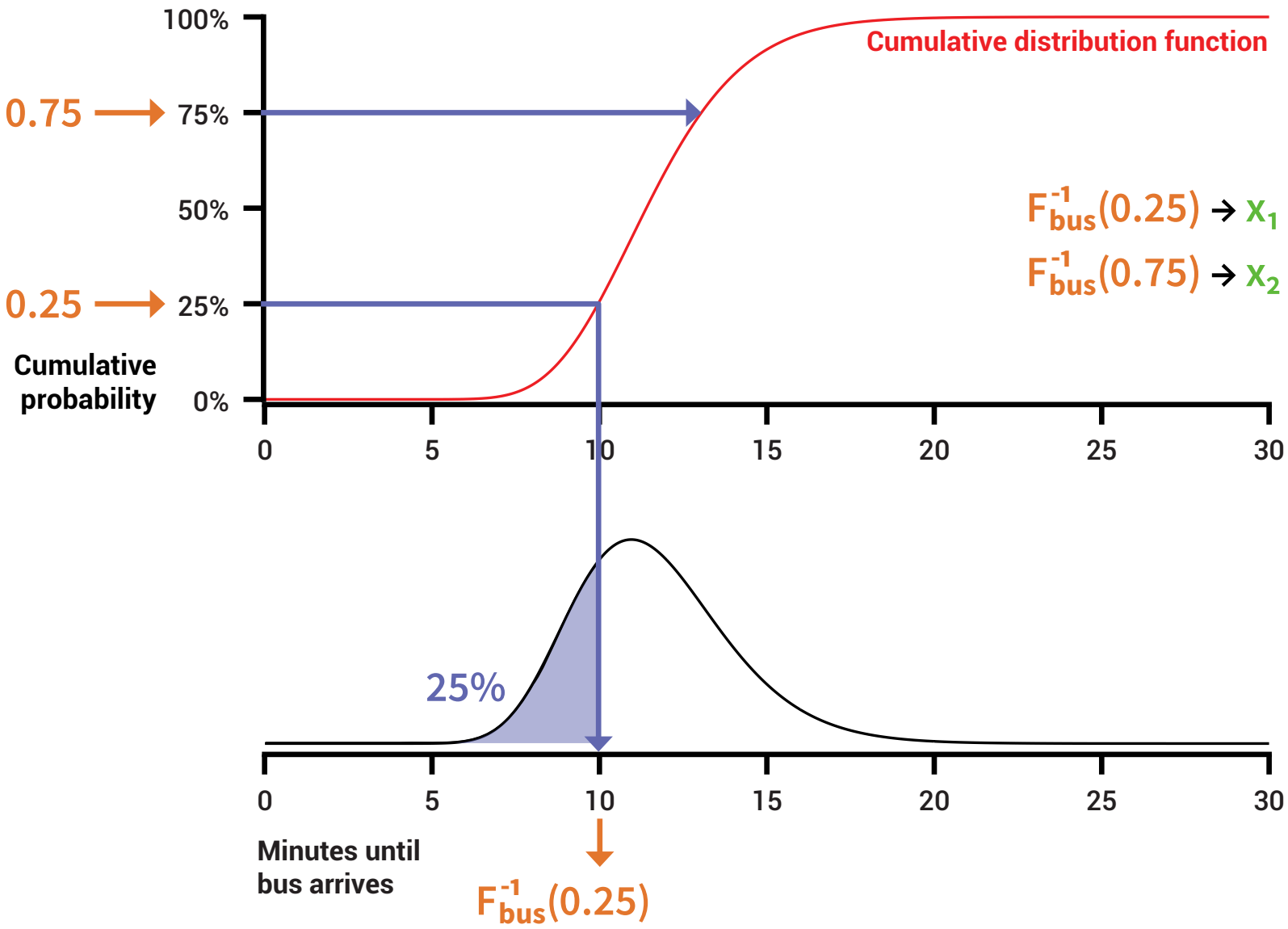
⇒ Pr(bus arrives ≤ time) → area

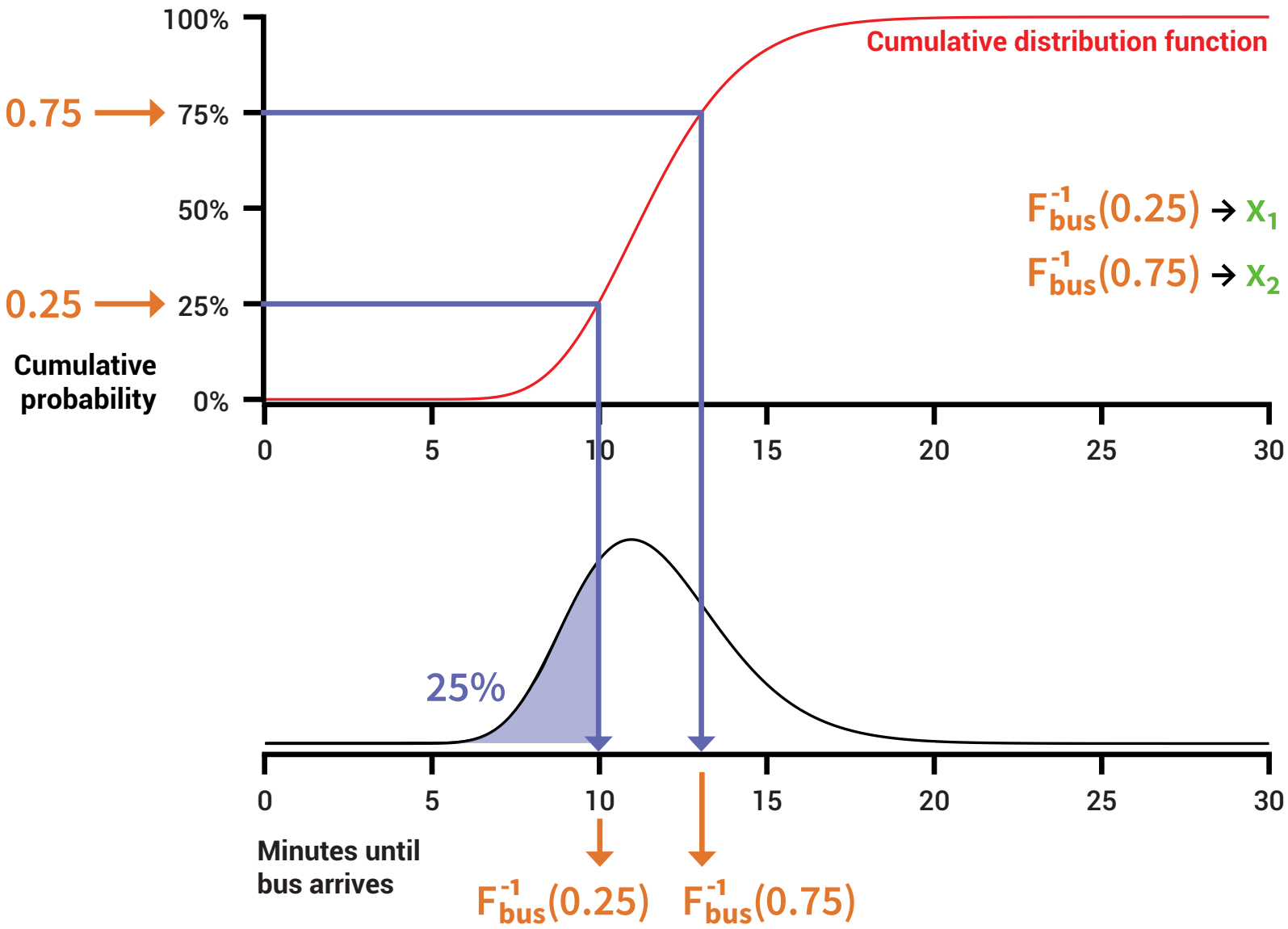

100%
75%
50%
25%
0%
Cumulative probability
Cumulative distribution function
0
5
10
15
20
25
30
0
5
10
15
20
25
30
Minutes until bus arrives

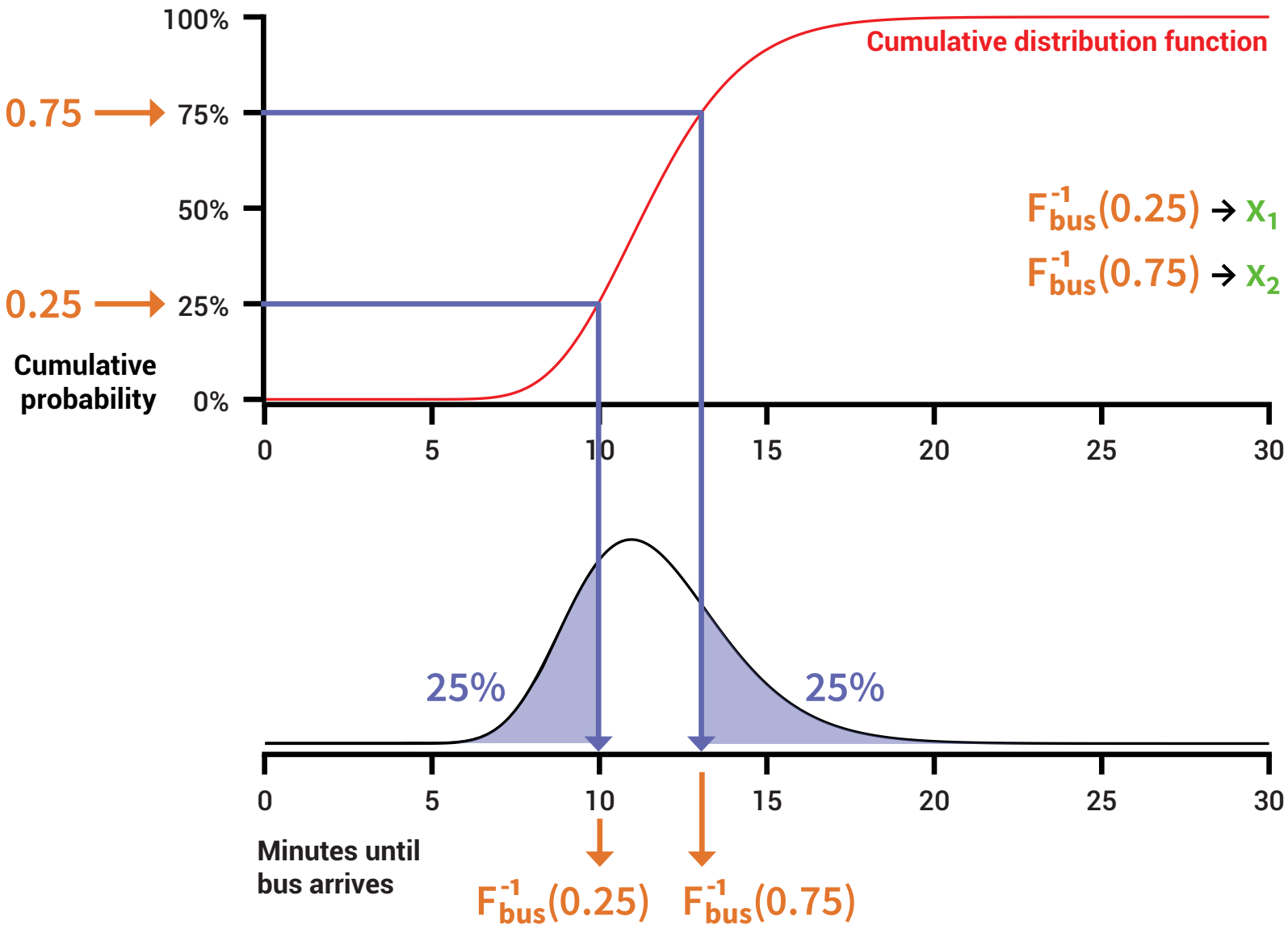

100%
75%
50%
25%
0%
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
0
5
10
15
20
25
30
Minutes until bus arrives

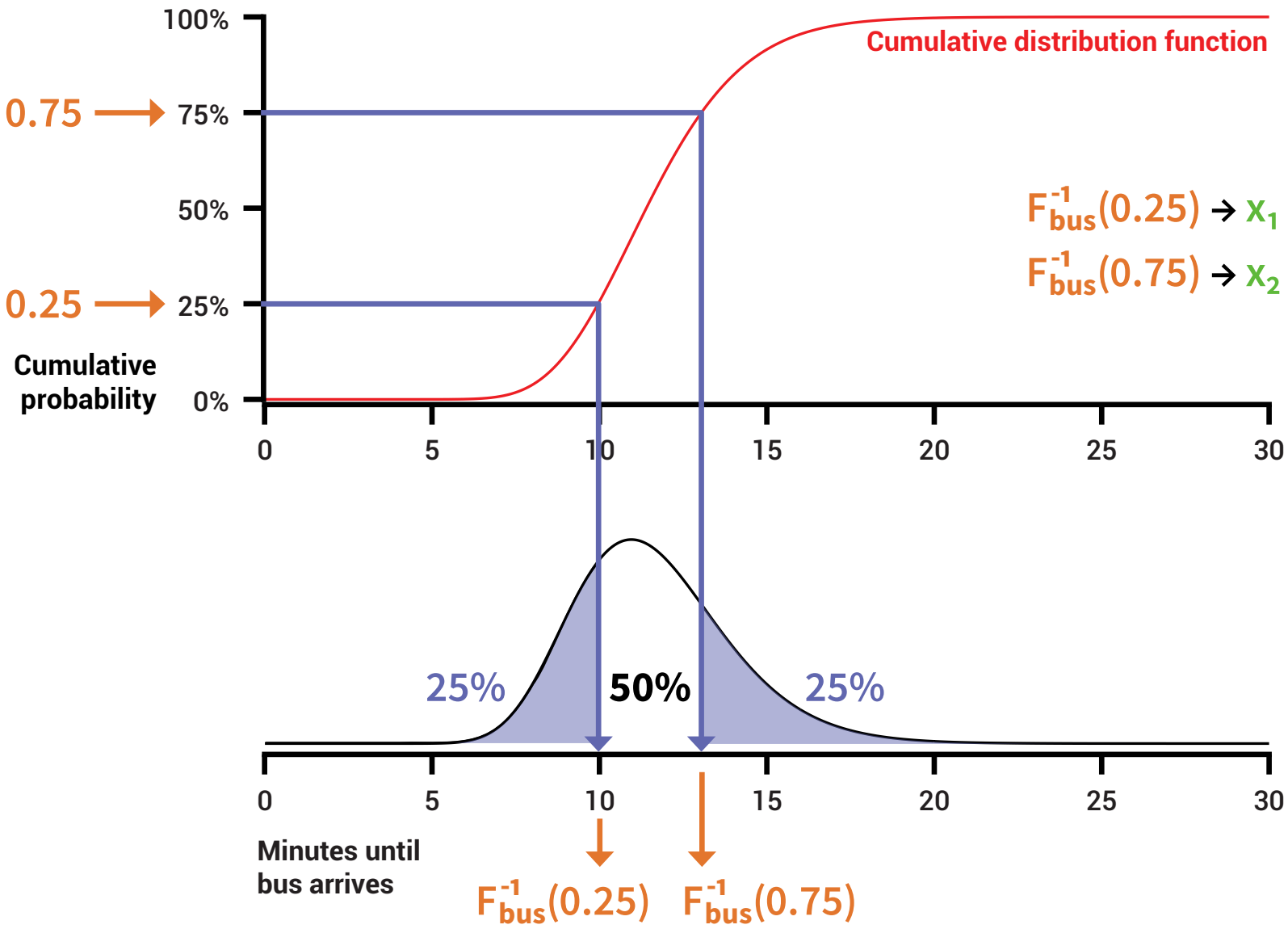

100%
75%
50%
25%
0%
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
0
5
10
15
20
25
30
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$

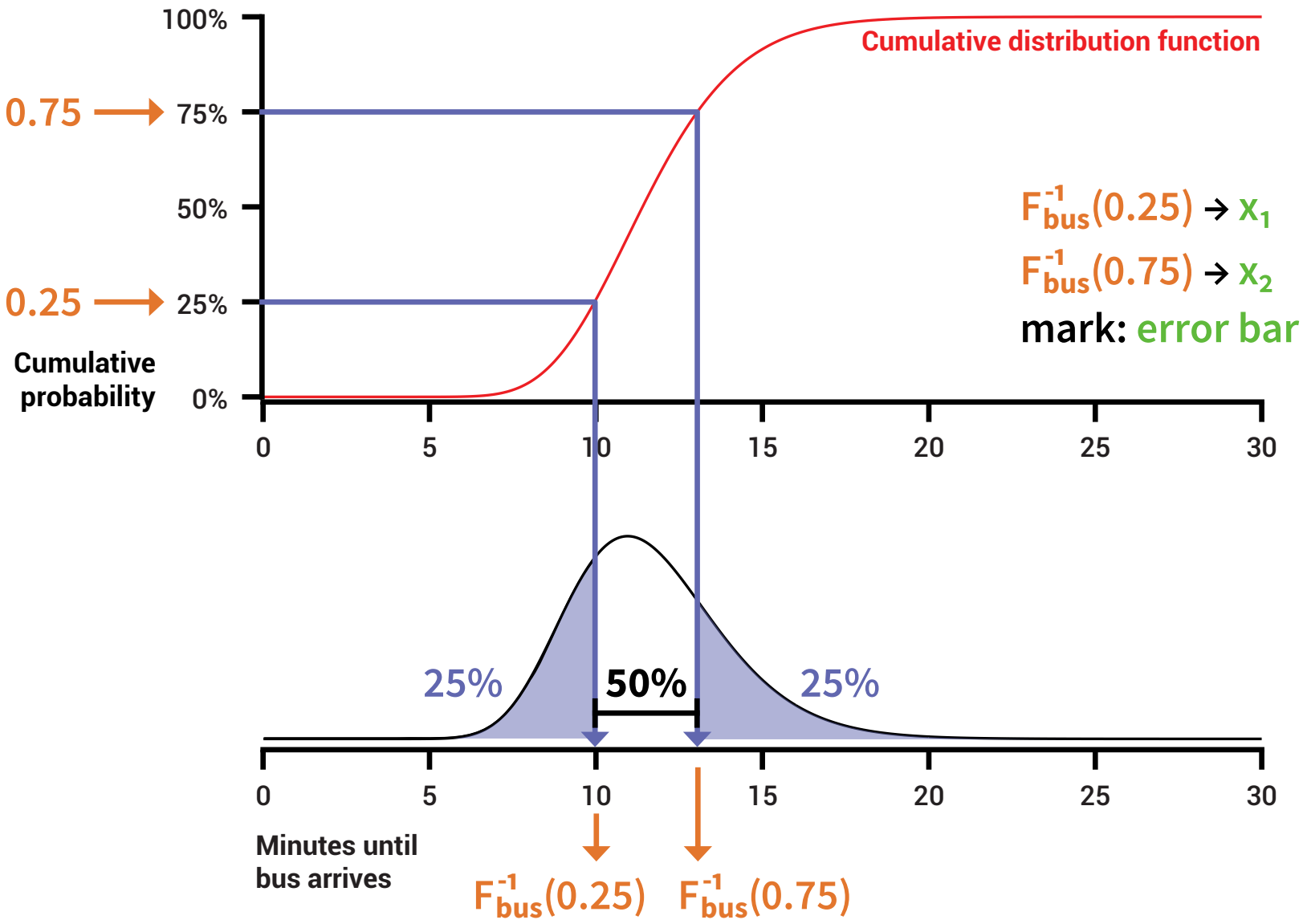

100%
75%
50%
25%
0%
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
0
5
10
15
20
25
30
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$


100%
75%
50%
25%
0%
0.75
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
$F^{-1}_{bus}(0.75) \rightarrow x_2$
0
5
10
15
20
25
30
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$


100%
75%
50%
25%
0%
0.75
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
$F^{-1}_{bus}(0.75) \rightarrow x_2$
0
5
10
15
20
25
30
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$
$F^{-1}_{bus}(0.75)$


100%
75%
50%
25%
0%
0.75
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
$F^{-1}_{bus}(0.75) \rightarrow x_2$
0
5
10
15
20
25
30
25%
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$
$F^{-1}_{bus}(0.75)$

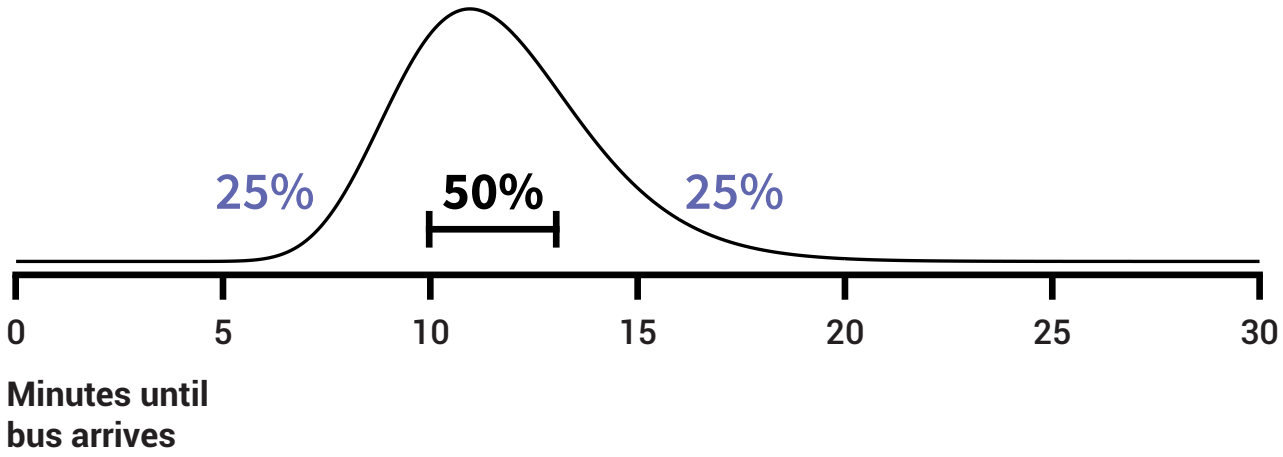

100%
75%
50%
25%
0%
0.75
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
$F^{-1}_{bus}(0.75) \rightarrow x_2$
0
5
10
15
20
25
30
25%
50%
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$
$F^{-1}_{bus}(0.75)$


100%
75%
50%
25%
0%
0.75
0.25
Cumulative probability
Cumulative distribution function
$F^{-1}_{bus}(0.25) \rightarrow x_1$
$F^{-1}_{bus}(0.75) \rightarrow x_2$
mark: error bar
0
5
10
15
20
25
30
25%
50%
25%
Minutes until bus arrives
$F^{-1}_{bus}(0.25)$
$F^{-1}_{bus}(0.75)$

$F^{-1}_{bus}(0.25) \rightarrow x_1$

$F^{-1}_{bus}(0.75) \rightarrow x_2$

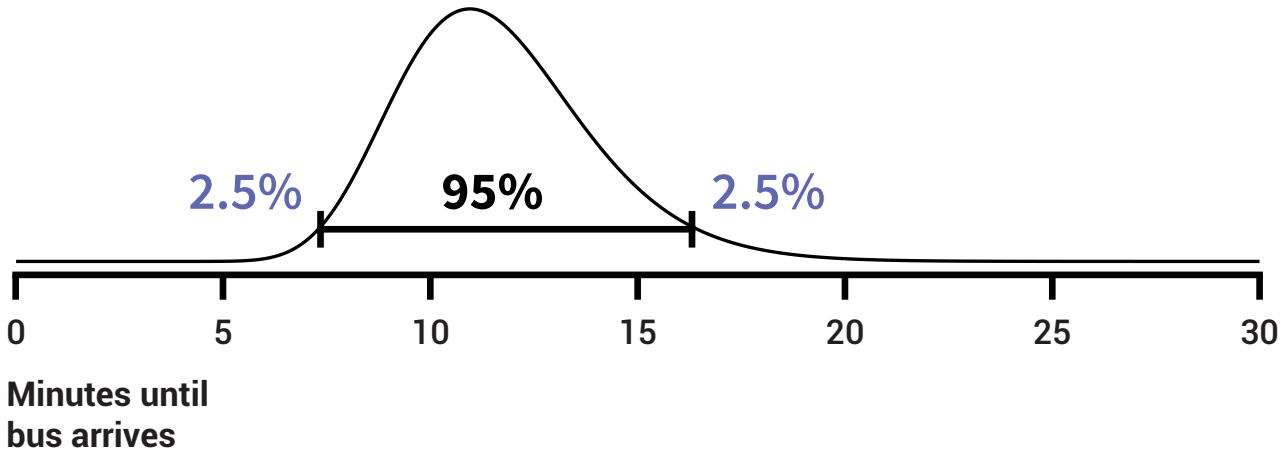
mark: error bar



$F^{-1}_{bus}(0.025) \rightarrow x_1$

$F^{-1}_{bus}(0.975) \rightarrow x_2$

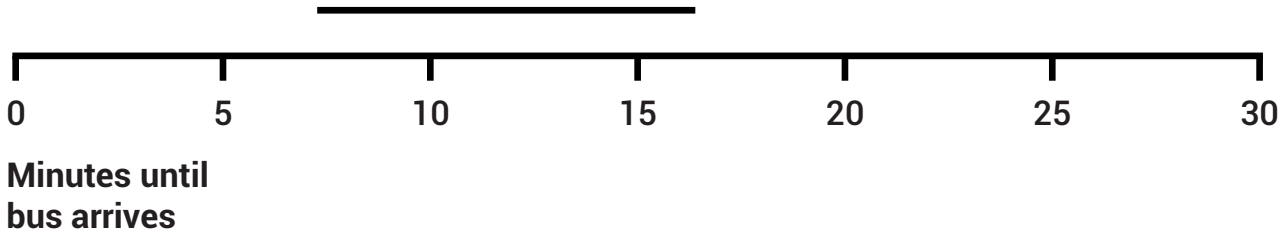
mark: error bar



$F^{-1}_{bus}(0.025) \rightarrow x_1$

$F^{-1}_{bus}(0.975) \rightarrow x_2$

mark: error bar



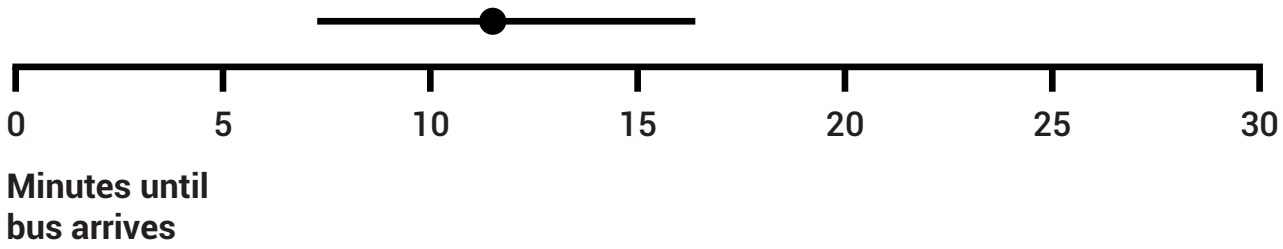
$F^{-1}_{bus}(0.025) \rightarrow x_1$

$F^{-1}_{bus}(0.975) \rightarrow x_2$

mark: error bar

$F^{-1}_{bus}(0.50) \rightarrow x$

mark: point



$F^{-1}_{bus}(0.025) \rightarrow x_1$

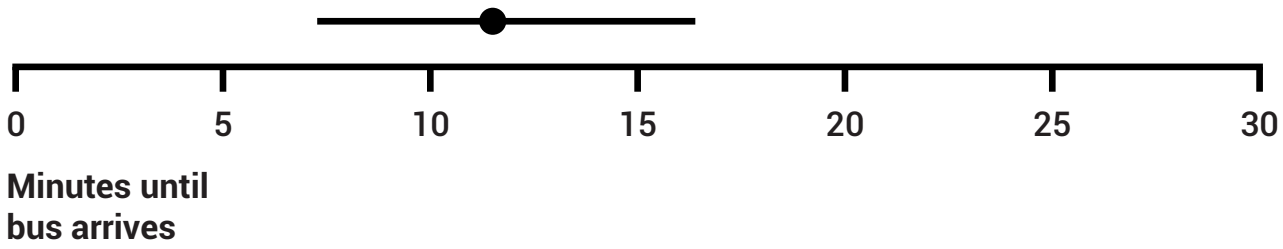
$F^{-1}_{bus}(0.975) \rightarrow x_2$

mark: error bar

$F^{-1}_{bus}(0.50) \rightarrow x$

mark: point

$\Rightarrow$ ~SD(bus arrival time) → length $|x_2 - x_1|$



$F^{-1}_{bus}(0.025) \rightarrow x_1$

$F^{-1}_{bus}(0.975) \rightarrow x_2$

mark: error bar

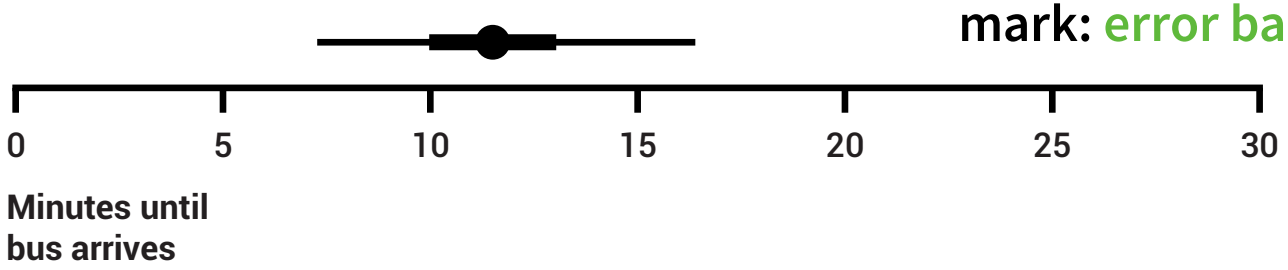
$F^{-1}_{bus}(0.50) \rightarrow x$

mark: point

$F^{-1}_{bus}(0.25) \rightarrow x_1$

$F^{-1}_{bus}(0.75) \rightarrow x_2$

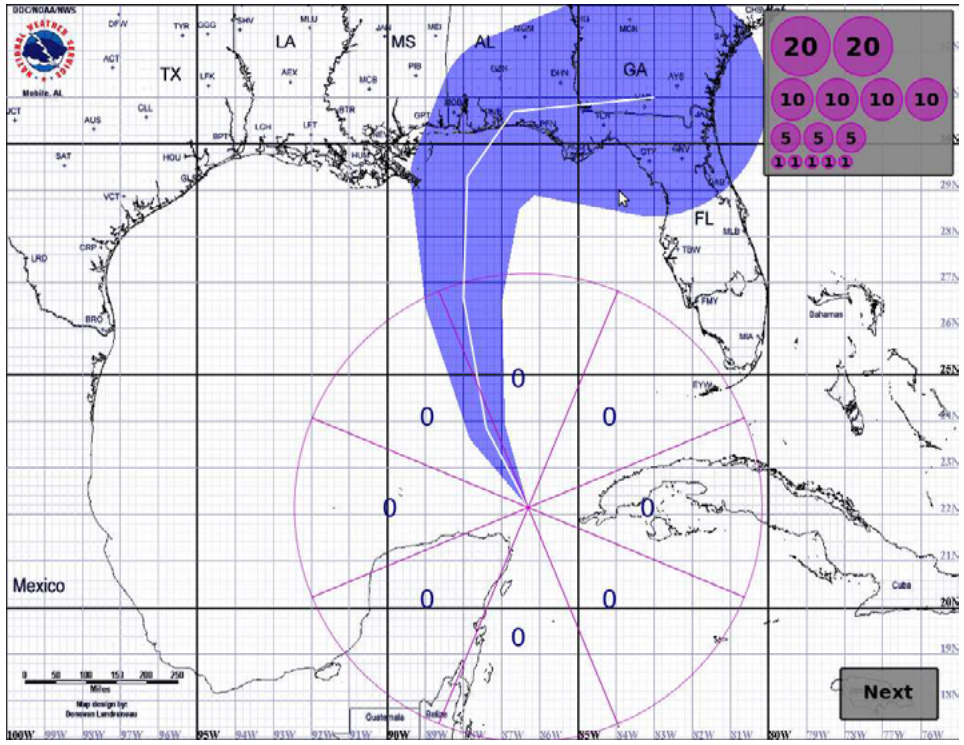
mark: error bar



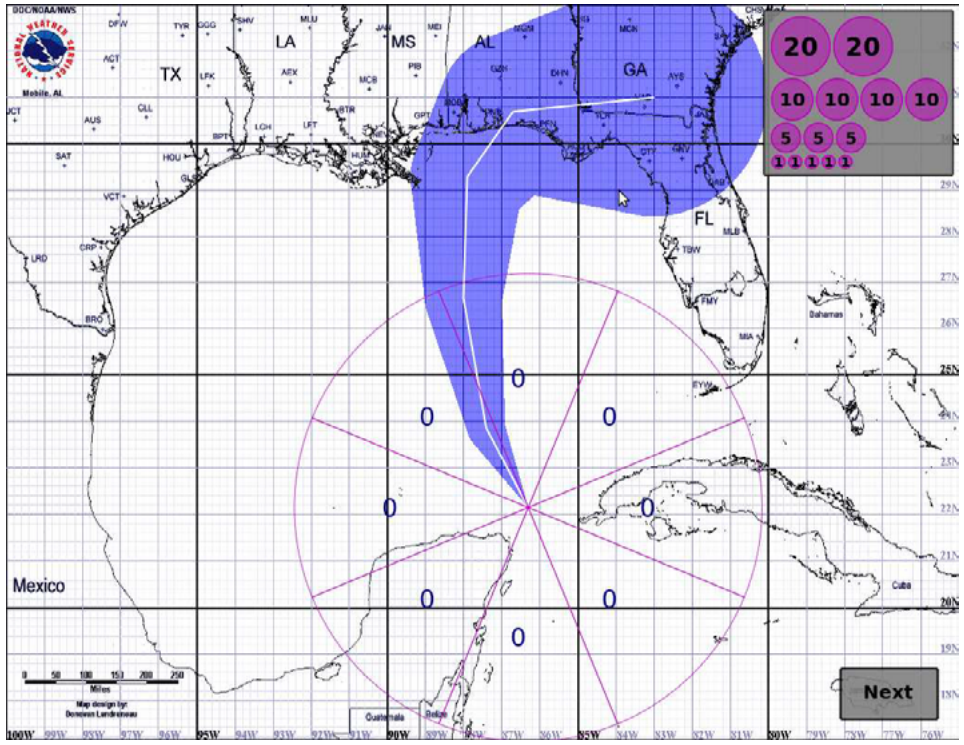
More examples...

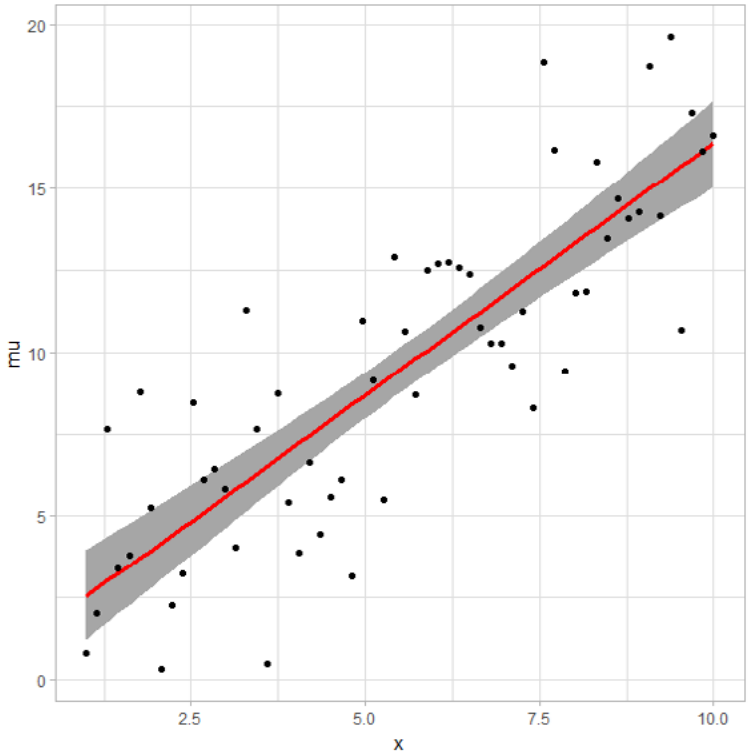
Hurricane error cones

[Cox, House, Lindell. Visualizing Uncertainty in Predicted Hurricane Tracks. International Journal for Uncertainty Quantification, 3(2), 143–156, 2013]



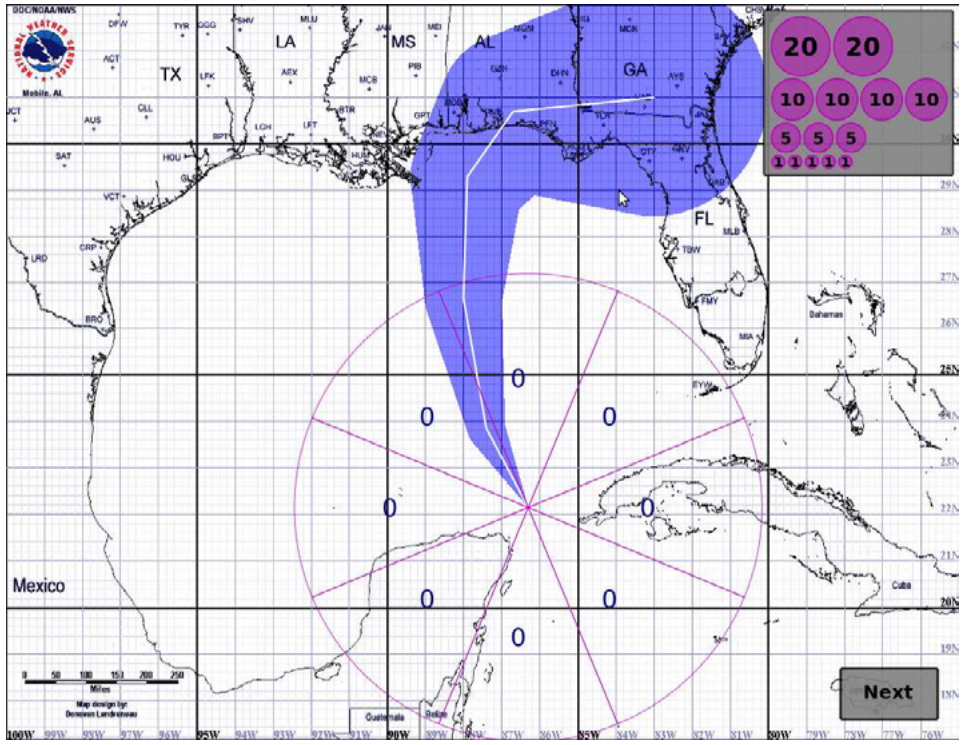
Fit line uncertainty band

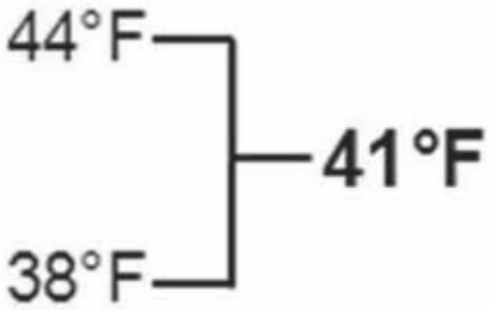




Deterministic construal errors

[Joslyn & LeClerc. Decisions With Uncertainty: The Glass Half Full. Current Directions in Psych. Science, 22(4), 2013]





Summing up

Uncertainty intervals

Can be thought of as mapping quantiles of a distribution onto bounds of an error bar or band

Implied encoding roughly corresponds to variance

Beware deterministic construal errors

Uncertainty intervals

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