GPT-4 Self-Graded Responses (With Markscheme)

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This document compiles the GPT-4 responses when prompted to grade the solutions which have been generated. The markscheme is given to GPT-4 in the prompt. This is the grading for the Quantum Mechanics (QM) questions.

1 QM

Question 1:

Response 1:
"Solution 1:
Grade:
Part a: 3 marks
Part b: 2 marks
Total: 5 marks
Feedback: Solution 1 comprehensively addresses both parts of the problem with accurate definitions and explanations. The student correctly defines the commutator and explains the concept of compatibility between operators, meeting the criteria for full marks in part a. In part b, the explanation correctly outlines the implications of non-compatibility on measurement outcomes, aligning with the mark scheme's requirements for a full score. The response is clear, directly addresses the questions posed, and provides a thorough understanding of the quantum mechanical principles involved.
Solution 2:
Grade:
Part a: 3 marks
Part b: 2 marks
Total: 5 marks
Feedback: Solution 2 effectively addresses all aspects of the problem, offering precise definitions and logical explanations. The student accurately defines the commutator and the concept of operator compatibility, earning full marks in part a. The response in part b correctly explains the effects of measuring non-compatible operators on a quantum system, consistent with the mark scheme. This solution demonstrates a solid understanding of quantum mechanics, particularly the significance of operator compatibility and measurement sequence.

Solution 3:
Grade:
Part a: 3 marks
Part b: 2 marks
Total: 5 marks
Feedback: Solution 3 provides a detailed and accurate response to the problem, correctly defining the commutator and explaining compatibility between operators for full marks in part a. The explanation of the effects of measuring non-compatible operators in part b is also accurate, adhering to the mark scheme for full marks. This solution demonstrates a comprehensive understanding of the principles of quantum mechanics, particularly in the context of operator compatibility and the impact of measurement sequence on system states."

Response 2:

"Solution 1 Feedback
Grade: 5/5
Part a: 3/3. The student accurately defines the commutator and explains the concept of compatibility between operators, including the implication of zero commutator for compatible operators.
Part b: 2/2. The explanation of the consequences of measuring non-compatible operators in sequence is correct and aligns with the fundamental principles of quantum mechanics, emphasizing the uncertainty in the outcome due to the collapse of the wavefunction into different eigenstates.
General Feedback: Excellent answer. The student provided a clear and accurate explanation for both parts of the problem. The explanation of compatibility and its implications for measurements in quantum mechanics is particularly well articulated. The understanding of the effect of non-compatible operator measurements on the state of the system is correctly described and shows a deep understanding of the principles of quantum mechanics.
Solution 2 Feedback
Grade: 5/5
Part a: 3/3. The student's definition of the commutator and explanation of operator compatibility is correct and succinct, correctly linking compatibility to zero commutator and common eigenstates.
Part b: 2/2. The description accurately reflects the quantum mechanical behavior of measurements with non-compatible operators, including the uncertainty in repeated measurements due to state collapse.
General Feedback: This solution accurately addresses all parts of the problem with precise definitions and clear explanations. The student effectively communicates the relationship between operator compatibility, commutators, and the implications for quantum measurements. The explanation of how measurements of non-compatible operators affect the system's state is well done, demonstrating a good grasp of quantum measurement principles.
Solution 3 Feedback
Grade: 5/5
Part a: 3/3. The student correctly defines the commutator and the condition for operator compatibility, emphasizing the significance of zero commutator for compatible operators and their shared

eigenstates.

Part b: 2/2. The explanation of the measurement process for non-compatible operators correctly describes the quantum mechanical principle of state alteration due to measurement, leading to uncertainty in subsequent measurements of the initially measured operator.

General Feedback: An excellent response that concisely addresses the requirements of the question. The student has provided a thorough explanation of operator commutation, compatibility, and the impact of these concepts on quantum measurements. The clear articulation of how the measurement process affects the state of a quantum system, especially after measuring non-compatible operators, demonstrates a solid understanding of quantum mechanics."

Response 3:

"Solution 1:

Grade: 5/5

Feedback: Solution 1 accurately defines the commutator of two operators and explains the concept of compatibility between operators, meeting the criteria for full marks in part a). It correctly identifies that compatible operators have a commutator equal to zero and share a common set of eigenstates, allowing for simultaneous precise measurements. The explanation provided for part b) is also comprehensive, correctly stating the effect of measuring incompatible operators on the system and the uncertainty of subsequent measurements. This response fully addresses the problem statement and adheres to the mark scheme, earning a perfect score.

Solution 2:

Grade: 5/5

Feedback: Solution 2 presents a clear and correct explanation of the commutator and compatibility of operators, justifying full marks for part a). It properly discusses the implications of compatibility, including the sharing of a common set of eigenstates and the ability to make simultaneous measurements. For part b), the response accurately describes the impact of the sequence of measurements on a system with incompatible operators, highlighting the non-commutative nature of quantum mechanics and the fundamental uncertainty in measurement outcomes. This solution meets all the requirements of the mark scheme, thus achieving a full score.

Solution 3:

Grade: 5/5

Feedback: Solution 3 provides a precise definition of the commutator and a correct explanation of operator compatibility, warranting full marks in part a). The response effectively communicates the meaning of compatibility, emphasizing the shared eigenstates and the potential for precise simultaneous measurements. In part b), the explanation of the effects of measuring incompatible operators is accurate and aligns with quantum mechanical principles, properly explaining the uncertainty in repeated measurements of $\hat{P}$ after $\hat{Q}$ has been measured. The student has successfully met the criteria laid out in the mark scheme, earning the maximum score."

Question 2:

Response 1:

Solution 1 Evaluation

Part a)

Correct identification of the eigenvalues as real: 1 mark

Accurate physical interpretation linking eigenvalues to observable physical quantities and emphasizing real, measurable results: 1 mark

Part b) Clearly explains both orthogonality and normalization in context of quantum mechanics, with detailed mathematical description and significance: 2 marks

Total: 4 marks

Feedback: Solution 1 provides a comprehensive and correct explanation for both parts of the question. It successfully links the mathematical properties of Hermitian operators to their physical interpretations, and it explains the orthonormality of eigenfunctions with clear mathematical and physical reasoning.

Solution 2 Evaluation

Part a) Correctly states that the eigenvalues of a Hermitian operator are real: 1 mark

Provides a relevant physical interpretation of the real eigenvalues in terms of observable quantities: 1 mark

Part b) Correctly defines orthonormality in terms of the Kronecker delta and explains its significance in quantum mechanics: 2 marks

Total: 4 marks

Feedback: Solution 2 accurately addresses both parts of the problem, providing correct definitions and physical insights. The explanation of orthonormality is succinct and directly relates to the properties required by the mark scheme.

Solution 3 Evaluation

Part a) Accurate statement regarding the reality of eigenvalues for Hermitian operators: 1 mark

Sufficient physical interpretation connecting real eigenvalues to the measurement of observable quantities: 1 mark

Part b) Precisely defines orthonormality and its implications for quantum mechanics, with clear mathematical notation: 2 marks Total: 4 marks

Feedback: Solution 3 offers a thorough explanation of the mathematical and physical aspects of Hermitian operators and their eigenfunctions. The response is well-aligned with the mark scheme, providing both accurate definitions and clear interpretations of orthonormality."

Response 2:

"Solution 1

a) This solution correctly identifies that the eigenvalues of a Hermitian operator are always real and provides an appropriate physical interpretation of these eigenvalues. It goes beyond the basic requirement by explaining the relevance of real eigenvalues in measurements of physical quantities, aligning perfectly with the markscheme. [2 marks]

b) The explanation of orthonormality is detailed, correctly explaining both orthogonality and normalization in the context of quantum mechanics, with a clear demonstration of how this property is fundamental for representing quantum states. It goes into sufficient depth, matching the criteria set out in the markscheme. [2 marks]
Feedback: Your solution is comprehensive and accurately addresses both the mathematical properties and the physical significance of Hermitian operators in quantum mechanics. The explanations are detailed and well-structured, providing a clear understanding of the concepts.
Solution 2
a) This solution also correctly states that the eigenvalues of a Hermitian operator are real and explains the significance of this property in the context of observable physical quantities. It effectively links the mathematical property to its physical interpretation, in line with the markscheme. [2 marks]
b) The explanation of the orthonormality of eigenfunctions is concise and accurate, correctly defining the mathematical expression of the orthonormality property and its significance in quantum mechanics. It meets the requirements of the markscheme by explaining both orthogonality and normalization. [2 marks]
Feedback: Your explanation is clear and to the point, effectively covering both the mathematical and physical aspects of the question. Your solution demonstrates a good understanding of the principles of quantum mechanics related to Hermitian operators.
Solution 3
a) This solution correctly explains that the eigenvalues of a Hermitian operator are real, including a mathematical demonstration of why this is the case, which matches the requirement of the markscheme. It also correctly links this property to the physical interpretation of observable quantities. [2 marks]
b) The description of orthonormality is accurate, providing both the mathematical definition and its physical significance in quantum mechanics. It meets the criteria for full marks by correctly explaining the concepts of orthogonality and normalization. [2 marks]
Feedback: Your solution is well-articulated, providing both the mathematical proof and physical interpretation clearly. It demonstrates a deep understanding of the role of Hermitian operators in quantum mechanics, effectively explaining the significance of their eigenvalues and eigenfunctions."

Response 3:

"Solution 1 Grading
Part a)
The eigenvalues are real: 1 mark Physical interpretation provided: 1 mark Total for part a): 2 marks
Part b)
Explanation of orthogonality and normalization with mathematical representation: 2 marks Total for part b): 2 marks
Overall Marks: 4/4
Feedback: The solution provides a comprehensive explanation of the mathematical properties and

physical interpretations of the eigenvalues of a Hermitian operator, as well as a clear description of the orthonormality of its eigenfunctions. The mathematical expressions and physical significance are both well-articulated, demonstrating a strong understanding of quantum mechanics principles.
Solution 2 Grading
Part a)
The eigenvalues are real: 1 mark Physical interpretation provided: 1 mark Total for part a): 2 marks
Part b)
Explanation of orthonormality with mathematical representation: 2 marks Total for part b): 2 marks
Overall Marks: 4/4
Feedback: This solution accurately explains the real nature of eigenvalues for Hermitian operators and their physical implications in quantum mechanics. It correctly describes the orthonormality of the eigenfunctions of a Hermitian operator, providing the essential mathematical expression. The connection to quantum mechanics and the importance of these concepts for forming a complete basis for the space of states are well presented.
Solution 3 Grading
Part a)
The eigenvalues are real: 1 mark Physical interpretation provided, but slightly verbose: 1 mark Total for part a): 2 marks
Part b)
Explanation of orthonormality with mathematical representation: 2 marks Total for part b): 2 marks
Overall Marks: 4/4
Feedback: The student gives a thorough explanation of the realness of eigenvalues for Hermitian operators, including a mathematical justification and its physical significance. The orthonormality of the eigenfunctions is correctly explained with appropriate mathematical notation, emphasizing its importance in quantum mechanics. The response could be more concise but is comprehensive and demonstrates a solid understanding of the topic."

Question 3:

Response 1:
"Grading and Feedback for Each Solution
Solution 1
Part a): 4 marks

2 marks Correctly substitutes $\hat{p}$ into the definition and uses integration by parts, effectively handling the vanishing terms at infinity.

2 marks Accurately simplifies the integral, correctly arranges into the form showing LHS equals RHS, and states $\hat{p}$ is Hermitian.

Part b): 4 marks

2 marks Correctly substitutes the operator into the integral and applies integration by parts, handling the vanishing terms due to limits and well-behaved functions. However, does not explicitly show the application of integration by parts twice as required for full marks.

2 marks Correctly arranges into the correct form and states $\hat{Q}$ is Hermitian. The explanation could be clearer in showing the second application of integration by parts.

Feedback: The student demonstrates a strong understanding of Hermitian operators and correctly applies integration by parts. For part b, more explicit steps showing the application of integration by parts twice would enhance the clarity and completeness of the solution.

Solution 2

Part a): 3 marks

2 marks Attempts to substitute $\hat{p}$ and simplifies correctly but lacks a clear, detailed explanation of integration by parts and its implications for the vanishing terms.

1 mark The final comparison is made, but the explanation lacks the detailed step-by-step simplification seen in Solution 1.

Part b): 3 marks

1 mark Substitutes the operator and outlines the approach but lacks detailed execution of integration by parts, crucial for full marks.

2 marks Correctly identifies that the operator is Hermitian but does not provide detailed step-by-step proof, particularly the double application of integration by parts and handling of boundary conditions.

Feedback: The student has a basic understanding of how to check for Hermitian operators but needs to provide more detailed mathematical steps and explanations. Specifically, more explicit demonstration of integration by parts and handling of boundary terms is needed.

Solution 3

Part a): 4 marks

2 marks Correctly substitutes $\hat{p}$ and uses integration by parts effectively, with a clear explanation of the vanishing boundary terms.

2 marks Successfully simplifies the integrals, showing that LHS equals RHS, and clearly states that $\hat{p}$ is Hermitian.

Part b): 5 marks

2 marks Appropriately substitutes the operator into the integral and applies integration by parts twice, with a correct approach to removing vanishing terms due to well-behaved functions at limits.

1 mark Correctly applies integration by parts again, clearly handling the vanishing terms after evaluating limits.

2 marks Successfully arranges into the correct form and states that $\hat{Q}$ is Hermitian, with a clear and thorough explanation.

Feedback: The student has demonstrated a thorough understanding of the Hermitian operator concept and skillfully applies integration by parts, including handling vanishing terms. The explanations are clear, and the mathematical steps are well-presented."

Response 2:

"Grading and Feedback for Each Solution Solution 1 Part a)

Substitution and integration by parts: 2 marks Simplification and conclusion: 2 marks Total for part a): 4 marks

Part b)

Substitution and first integration by parts: 2 marks Second integration by parts and conclusion: 3 marks Total for part b): 5 marks

General Feedback: Solution 1 effectively demonstrates the Hermitian property for both operators through detailed mathematical steps and logical reasoning. The approach for each part is clear, and the solution follows the definition accurately. Both parts a) and b) are thoroughly explained, leading to the correct conclusion. Well done.

Overall Grade: 9/9

Solution 2

Part a)

Substitution but incomplete integration by parts explanation: 1 mark

Missing detailed simplification or explicit integration by parts steps: 0 marks

Total for part a): 1 mark

Part b)

Substitution correct but lacks detailed integration by parts steps: 1 mark

Missing clear second application of integration by parts and vague conclusion: 1 mark

Total for part b): 2 marks

General Feedback: Solution 2 outlines the correct approach but lacks detailed mathematical derivations, especially in the application of integration by parts and the simplification of terms. This makes it difficult to follow the logic and verify that the operators meet the Hermitian condition accurately. More detailed steps and explicit demonstrations of integration by parts are needed for both parts.

Overall Grade: 3/9

Solution 3

Part a)

Substitution and detailed integration by parts: 2 marks

Correct simplification but somewhat redundant explanation: 2 marks

Total for part a): 4 marks

Part b)

Correct substitution and integration by parts application: 2 marks

Proper second application of integration by parts and clear conclusion: 3 marks

Total for part b): 5 marks

General Feedback: Solution 3 provides a comprehensive and detailed explanation for both operators, accurately demonstrating their Hermitian properties through clear mathematical steps. The solution successfully applies integration by parts and logically arrives at the correct conclusions, albeit with some redundancy in explanations that could be streamlined for clarity.

Overall Grade: 9/9"

Response 3:

"Solution 1 Grading

Part a)

- Substitution of $\hat{p}$ into the LHS or RHS of definition, Integrate by parts, removing vanishing terms since functions vanish at infinity: 2 marks
- Simplifying the remaining integral, arranging into correct form showing LHS equals RHS, stating $\hat{p}$ is Hermitian: 2 marks

Total for part a): 4 marks

Part b)

- Substituting the operator into the integral and applying integration by parts, removing vanishing terms due to limits and well-behaved functions: 2 marks
- Applying integration by parts again, again removing vanishing terms after evaluating limits: 1 mark
- Arranging into the correct form, stating $\hat{Q}$ is Hermitian: 2 marks

Total for part b): 5 marks

Overall feedback: The student has provided a comprehensive and accurate analysis for both parts of the question, demonstrating a clear understanding of how to verify whether an operator is Hermitian. The solution correctly uses integration by parts and addresses the boundary conditions to argue the Hermitian nature of both operators. Full marks awarded.

Solution 2 Grading

Part a)

- Integration by parts and the elimination of boundary terms is mentioned, but the steps are not clearly outlined or justified as in the mark scheme: 1 mark
- The final conclusion that $\hat{p}$ is Hermitian is correct but lacks detailed explanation of simplifying the integral and properly showing that LHS equals RHS: 2 marks

Total for part a): 3 marks
Part b)

- The solution hints at using integration by parts twice and removing vanishing terms, but the explanation lacks clarity and detail, making it hard to justify full marks: 1 mark
- No clear step-by-step application of integration by parts again and removing vanishing terms explicitly: 0 marks
- The conclusion that $\hat{Q}$ is Hermitian is correct, but the justification is weak due to lack of detail: 1 mark

Total for part b): 2 marks
Overall feedback: The student has the right idea but fails to provide detailed explanations and step-by-step processes as required by the mark scheme. The answers lack the necessary depth in explaining how integration by parts is applied and how the terms are simplified. More detail is needed to achieve full marks.
Solution 3 Grading
Part a)

- Detailed substitution and integration by parts with correct elimination of boundary terms: 2 marks
- Correctly simplifies the integral and clearly shows that LHS equals RHS, with a clear statement that $\hat{p}$ is Hermitian: 2 marks

Total for part a): 4 marks
Part b)

- Correctly substitutes the operator and applies integration by parts twice, with explicit removal of vanishing terms: 2 marks
- Correctly applies integration by parts again and clearly shows the elimination of vanishing terms, which is assumed but not explicitly shown in the solution: 1 mark
- The arrangement into the correct form and stating $\hat{Q}$ is Hermitian is correct, but the explicit steps of the second integration by parts and its direct impact on the final form could be clearer: 2 marks

Total for part b): 5 marks
Overall feedback: The student presents a well-structured and accurate analysis for both operators, showing a good understanding of the Hermitian property and how to apply integration by parts to prove it. The explanation is clear, and the mathematical manipulations are correctly performed. Full marks awarded."

Question 4:

Response 1:

"To grade each solution according to the provided markscheme, let's evaluate how well each student's solution meets the criteria for each part of the problem.

Solution 1

a) 3 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly states that since $\hat{X}$ commutes with the relevant operators, the terms cancel out. - Correctly concludes the commutator equals zero.

b) 3 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly states that since $\hat{P}_x$ commutes with the relevant operators, the terms cancel out. - Correctly concludes the commutator equals zero.

c) 4 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly changes the order of commuting terms. - Correctly simplifies to $i\hbar\hat{Z}$ using the commutation relationship between $\hat{Y}$ and $\hat{P}_y$.

d) 4 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly changes the order of commuting terms. - Correctly simplifies to $i\hbar\hat{P}_z$ using the commutation relationship between $\hat{Y}$ and $\hat{P}_y$.

e) 6 marks - Correctly rewrites the commutation as a sum of three commutators. - Correctly applies the identity for the commutation with squared terms. - Correctly identifies that the terms cancel out, leading to a zero commutator.

Total: 20 marks

Feedback: Excellent work. The solution accurately follows quantum mechanics principles and the commutation relations. The explanations are clear, and the mathematical derivations are correctly applied to reach the required conclusions.

Solution 2

a) 2 marks - Correctly expands the commutation relation but does not explicitly show the steps for substitution as required by the mark scheme. - Correct conclusion that the commutator equals zero, but lacks the detailed reasoning awarded by the markscheme.

b) 2 marks - Similar to (a), correctly identifies the commutator equals zero but lacks detailed substitution steps. - Does not explicitly justify the zero result with detailed reasoning as per the markscheme.

c) 3 marks - Correctly identifies the commutator and the outcome. - Lacks the detailed step-by-step process (changing order, factorizing) explicitly noted in the markscheme.

d) 3 marks - Correctly identifies the outcome. - Misses explicit demonstration of steps required by the markscheme for full marks.

e) 4 marks - Identifies the correct approach for dealing with the squared momentum operator. - Lacks the detailed justification and explicit step-by-step reasoning for each term as noted in the markscheme.

Total: 14 marks

Feedback: The solutions are on the right track, but the work lacks the detailed explanations and step-by-step process required by the markscheme. More explicit derivation showing how each term is handled would improve clarity and completeness.
Solution 3
a) 3 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly states the terms cancel out, with a clear explanation. - Correctly concludes the commutator equals zero.
b) 3 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly states the terms cancel out, with a clear explanation. - Correctly concludes the commutator equals zero.
c) 4 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly changes the order of commuting terms. - Correctly simplifies to $i\hbar\hat{Z}$, following the markscheme precisely.
d) 4 marks - Correctly expands the commutation relation and substitutes $\hat{L_x}$ in. - Correctly changes the order of commuting terms. - Correctly simplifies to $i\hbar\hat{P}_z$, following the markscheme precisely.
e) 6 marks - Correctly rewrites the commutation as a sum of three commutators. - Correctly applies the identity for the commutation with squared terms. - Correctly concludes the terms cancel out, leading to a zero commutator.
Total: 20 marks
Feedback: Excellent work. The solution correctly and clearly follows the markscheme's requirements for each part of the problem. Each step of the derivation is well-explained and aligns with the principles of quantum mechanics."

Response 2:

"Grading each solution according to the provided mark scheme:
Solution 1:

- **a)** Correctly expands the commutator relation and states the reason for the commutator being zero. **3/3 marks**
- **b)** Correctly expands the commutator relation and states the reason for the commutator being zero. **3/3 marks**
- **c)** Correctly expands the commutator relation and identifies the non-zero term, arriving at the correct result. **4/4 marks**
- **d)** Correctly identifies the non-zero term and arrives at the correct result, though the explanation is slightly brief. **3/4 marks** (Deducting 1 mark for not fully demonstrating the change of order of commuting terms and the factorization process.)
- **e)** Correctly identifies that the commutator of $\hat{L_x}$ with $\hat{P}^2$ is zero based on the previous parts but does not fully detail the application of the identity for each term. **5/6 marks** (Deducting 1 mark for not explicitly showing the application of the identity for each term of $\hat{P}^2$.)

Total: 18/20

Feedback: The solution correctly addresses most parts of the problem with adequate explanations. However, a more detailed step-by-step factorization and application of commutation relations in

part d) and a more explicit application of the identity in part e) would enhance clarity and completeness.
Solution 2:

- **a)** Correctly expands the commutator relation and concludes the result. **3/3 marks**
- **b)** Correctly concludes the result, though without explicitly expanding the commutator relation. **2/3 marks** (Deducting 1 mark for not explicitly showing the expansion of the commutation relation.)
- **c)** Correctly arrives at the final result but lacks a detailed explanation of the factorization and simplification steps. **3/4 marks** (Deducting 1 mark for not fully explaining the factorization and simplification process.)
- **d)** Correctly concludes the result, but the explanation lacks detail on the commutation and factorization process. **3/4 marks** (Deducting 1 mark for the same reason as in c).)
- **e)** Correctly identifies that the total commutator is zero but lacks detailed explanation for each term's commutation. **4/6 marks** (Deducting 2 marks for not explicitly showing the application of the identity for each term and the factorization process.)

Total: 15/20
Feedback: The solution provides correct answers but lacks detailed explanations and explicit expansions in parts b), c), d), and e). More thorough derivations and explanations of the steps would improve the clarity and completeness of the solution.
Solution 3:

- **a)** Correctly expands and concludes the result. **3/3 marks**
- **b)** Correctly expands and concludes the result. **3/3 marks**
- **c)** Correctly expands and concludes the result, with a slight oversight in the detailed explanation. **4/4 marks**
- **d)** Correctly expands and concludes the result. **4/4 marks**
- **e)** Correctly identifies that the total commutator is zero, with a good explanation but lacks explicit detail on the use of the identity for each momentum component squared. **5/6 marks** (Deducting 1 mark for not explicitly showing the detailed application of the identity for each term.)

Total: 19/20
Feedback: The solution effectively addresses all parts of the problem, providing correct results with adequate explanations. To further enhance the solution, a more detailed explanation in part

e), specifically on the application of the identity for each momentum component squared, would provide additional clarity."

Response 3:

"To grade each solution according to the provided mark scheme, we will evaluate the responses for each part of the problem and allocate marks accordingly.

Solution 1

- **Part a) [3 marks]:** Correctly expands the commutator and substitutes $\hat{L_x}$ in. Identifies that $\hat{X}$ commutes with all other operators involved and concludes the commutator is zero. Full marks awarded.

- **Part b) [3 marks]:** Correctly expands the commutator and substitutes $\hat{L_x}$ in. Identifies that $\hat{P}_x$ commutes with all other operators involved and concludes the commutator is zero. Full marks awarded.

- **Part c) [4 marks]:** Correctly expands the commutator and identifies the non-zero term due to $\hat{Z}\hat{P}_y$ acting on $\hat{Y}$, which gives $i\hbar\hat{Z}$. Full marks awarded.

- **Part d) [4 marks]:** Correctly identifies the non-zero term from $\hat{Y}\hat{P}_z$ acting on $\hat{P}_y$, yielding $i\hbar\hat{P}_z$. Full marks awarded.

- **Part e) [6 marks]:** Correctly applies the logic from previous parts and concludes that the commutator $\left[\hat{L_x}, \hat{P}^2\right]$ is zero due to the properties of angular momentum and momentum operators. However, this solution lacks the detailed explanation based on the identity for $[\hat{A}, \hat{B}^2]$ and the specific commutation relations needed for a full derivation. 4 marks awarded for the correct conclusion but without the full derivation detail as per the mark scheme.

Total: 20 marks

General Feedback: This solution effectively addresses each part of the problem, reaching the correct conclusions with clear reasoning. However, for part e), a more detailed derivation using the provided identity and commutation relations would enhance the completeness of the answer.

Solution 2

- **Part a) [3 marks]:** Correctly expands and simplifies the commutator, concluding it is zero. Full marks awarded.

- **Part b) [3 marks]:** Correctly identifies that the commutator is zero due to the non-interaction of $\hat{P}_x$ with components of $\hat{L_x}$. Full marks awarded.

- **Part c) [2 marks]:** Provides a general explanation but lacks the detailed step-by-step factorization and the use of the commutation relation $[\hat{Y}, \hat{P}_y] = i\hbar$ explicitly for full marks. Partial marks awarded.

- **Part d) [2 marks]:** Simplifies the commutator but does not explicitly use the commutation relation $[\hat{Y}, \hat{P}_y] = i\hbar$ in the derivation. Partial marks awarded.

- **Part e) [4 marks]:** Correctly concludes that the commutator with $\hat{P}^2$ is zero but does not fully utilize the given identity $[\hat{A}, \hat{B}^2]$ for each term in the sum explicitly or the detailed steps involving $[\hat{L_x}, \hat{P}_y]$ and $[\hat{L_x}, \hat{P}_z]$. Partial marks awarded for correct conclusion but lacking detail.

Total: 14 marks
General Feedback: The solution correctly identifies the outcome of each commutation relation but lacks the explicit step-by-step calculations and detailed derivations expected in the mark scheme, especially for parts c), d), and e).
Solution 3

- **Part a) [3 marks]:** Correctly expands and concludes the commutator is zero. Full marks awarded.
- **Part b) [3 marks]:** Correctly identifies the commutator is zero with appropriate justification. Full marks awarded.
- **Part c) [4 marks]:** Adequately expands the commutator and uses the commutation relations to conclude $i\hbar\hat{Z}$. Full marks awarded.
- **Part d) [4 marks]:** Correctly derives $i\hbar\hat{P}_z$ from the commutator. Full marks awarded.
- **Part e) [6 marks]:** Correctly states the outcome of the commutator with $\hat{P}^2$ as zero and references the results from part b). However, similar to Solution 1, it lacks the detailed explanation for each term based on the identity for $[\hat{A}, \hat{B}^2]$. 4 marks awarded for the correct conclusion but without the full derivation detail as per the mark scheme.

Total: 20 marks
General Feedback: This solution provides a strong approach to each part, reaching the correct conclusions with logical reasoning. For part e), including a more detailed derivation based on the identity for $[\hat{A}, \hat{B}^2]$ would complete the answer according to the mark scheme."

Question 5:

Response 1:
"To grade each solution according to the mark scheme provided, we'll evaluate them against the criteria listed for parts a) and b).
Solution 1 Evaluation Part a)
Stating the equation for energy relevant to the ground state energy: 1 mark.
Substituting $\hat{H}' = V_0 \sin(\frac{2\pi x}{L})$ into the integral: 1 mark.
Solving the integral and showing the perturbation equals 0: 2 marks.
Total for part a): 4 marks.
Part b)

Applying perturbation theory correctly: 1 mark.
Correctly solving the integral and simplifying to $2\alpha\sin^2(\frac{n\pi}{2})$: 1 mark
Stating 2α for all odd values of n, and 0 for all even values of n: 1 mark.
Total for part b): 3 marks.
Overall: 7 marks.
Feedback: Solution 1 thoroughly follows the mark scheme for both parts a) and b), providing a detailed step-by-step approach that accurately calculates the first-order corrections. The solution correctly applies quantum mechanics principles and perturbation theory, resulting in a clear and comprehensive answer.
Solution 2 Evaluation Part a)
Stating the equation for energy relevant to the ground state energy: 1 mark (implicit in the solution, but not explicitly related to the ground state energy as required).
Substituting $\hat{H}' = V_0\sin(\frac{2\pi x}{L})$ into the integral with the limits [0,L] and simplifying: 0 marks (the solution makes an attempt but does not simplify to the form required by the mark scheme). Solving the integral and showing the perturbation equals 0: 2 marks (indirectly, as the solution does not explicitly show this step but arrives at the correct conclusion).
Total for part a): 3 marks.
Part b)
Applying perturbation theory correctly: 1 mark.
Correctly solving the integral and simplifying to $2\alpha\sin^2(\frac{n\pi}{2})$: 1 mark
Stating 2α for all odd values of n, and 0 for all even values of n: 1 mark.
Total for part b): 3 marks.
Overall: 6 marks.
Feedback: Solution 2 provides a solid understanding of the perturbation theory and correctly addresses the problem. However, there is a slight discrepancy in part a) regarding the explicit demonstration of the integral's simplification to meet the mark scheme's specific requirements. The solution effectively covers the key concepts, but could improve clarity in demonstrating the integral simplification in part a).
Solution 3 Evaluation Part a)
Stating the equation for energy relevant to the ground state energy: 1 mark.
Substituting $\hat{H}' = V_0\sin(\frac{2\pi x}{L})$ into the integral: 1 mark.
Solving the integral and showing the perturbation equals 0: 2 marks.
Total for part a): 4 marks.
Part b)
Applying perturbation theory correctly: 1 mark.
Correctly solving the integral and simplifying to $2\alpha\sin^2(\frac{n\pi}{2})$: 1 mark
Stating 2α for all odd values of n, and 0 for all even values of n: 1 mark.
Total for part b): 3 marks.
Overall: 7 marks.

Feedback: Solution 3 meticulously adheres to the mark scheme, providing an accurate and detailed explanation of both parts a) and b). The solution demonstrates a strong grasp of quantum mechanics and perturbation theory, leading to a comprehensive and correct answer."

Response 2:

"Grading Solutions

Solution 1 Grading

Part a)

- Stating the equation for energy and making it relevant to the ground state energy: 1 mark.
- Substituting $\hat{H}'$ into the integral for energy with the limits [0,L] and simplifying: The solution incorrectly combines the sine functions without directly stating the required $\sin^3$ term but proceeds with the correct method. 0.5 marks (rounded to 0 marks for whole number grading).
- Solving the integral and showing the perturbation equals 0: The solution does not explicitly solve the integral as per the mark scheme but implies the correct outcome. 1 mark.

Total for Part a): 2 marks

Part b)

- Applying perturbation theory and substituting the correct terms: 1 mark.
- Correctly solving the integral and simplifying: 1 mark.
- Stating 2α for all odd values of n, and 0 for all even values of n: The solution correctly derives the expression for the correction but does not explicitly state the conclusion about odd and even values of n. 0.5 marks (rounded to 1 mark for whole number grading).

Total for Part b): 3 marks

Overall: 5 marks

Feedback: The solution correctly identifies the key concepts and applies perturbation theory accurately. However, it lacks explicit clarity in part a) regarding the simplification step for the integral. Part b) is well-handled but could improve by directly stating the implications for odd and even values of n.

Solution 2 Grading

Part a)

- Stating the equation for energy and making it relevant to the ground state energy: 1 mark.
- Substituting $\hat{H}'$ into the integral for energy with the limits [0,L] and simplifying: The solution transitions directly to the integral with correct limits but skips the explicit mention of $\sin^3$ term. 0.5 marks (rounded to 0 marks for whole number grading).
- Solving the integral and showing the perturbation equals 0: The solution implies the outcome without showing explicit steps. 1 mark.

Total for Part a): 2 marks
Part b)

- Applying perturbation theory and substituting the correct terms: 1 mark.
- Correctly solving the integral and simplifying: 1 mark.
- Stating 2α for all odd values of n, and 0 for all even values of n: The solution correctly derives the expression for the correction but does not explicitly mention the variance between odd and even n. 0.5 marks (rounded to 1 mark for whole number grading).

Total for Part b): 3 marks
Overall: 5 marks
Feedback: The approach is methodical, with clear application of perturbation theory. However, the solution would benefit from more explicit steps in solving integrals in part a) and a clearer statement regarding the behavior for odd and even n in part b).
Solution 3 Grading
Part a)

- Stating the equation for energy and making it relevant to the ground state energy: 1 mark.
- Substituting $\hat{H}'$ into the integral for energy with the limits [0,L] and simplifying: The solution correctly approaches the substitution but does not explicitly simplify to the $\sin^3$ form as required. 0.5 marks (rounded to 0 marks for whole number grading).
- Solving the integral and showing the perturbation equals 0: The solution implies correct outcome through the use of trigonometric identities but does not show the actual integral solution. 1 mark.

Total for Part a): 2 marks
Part b)

- Applying perturbation theory and substituting the correct terms: 1 mark.
- Correctly solving the integral and simplifying: 1 mark
- Stating 2α for all odd values of n, and 0 for all even values of n: The solution correctly derives the expression for the correction and indirectly implies the behavior for odd and even n but could be more explicit. 0.5 marks (rounded to 1 mark for whole number grading).

Total for Part b): 3 marks
Overall: 5 marks
Feedback: The solution demonstrates a solid understanding of perturbation theory and its application. In part a), more explicit steps in the integral solution would improve clarity. Part b) is well executed, though direct commentary on odd and even n would further enhance the explanation."
Response 3:

"Grading and Feedback
Solution 1:
Part a)

- **[1 mark]** for stating the equation for energy and making it relevant to the ground state energy correctly.
- **[1 mark]** for substituting $\hat{H}' = V_0 \sin\left(\frac{2\pi x}{L}\right)$ into the integral for energy correctly.
- **[0 marks]** The solution incorrectly uses trigonometric identities and does not directly solve the integral as required by the markscheme.
- **Total for Part a):** 2 marks

Part b)

- **[1 mark]** for correctly applying perturbation theory with the correct integral expression.
- **[1 mark]** for correctly solving the integral and simplifying to $2\alpha \sin^2\left(\frac{n\pi}{2}\right)$.
- **[1 mark]** for correctly stating 2α for all odd values of n, and 0 for all even values of n.
- **Total for Part b):** 3 marks

Overall: 5/7 marks
Feedback: Your approach to part a) correctly outlines the process but lacks the correct final step in solving the integral as per the markscheme. In part b), you correctly applied perturbation theory and derived the correct expression for the first-order correction to all energy eigenvalues. Ensure to closely follow the requirements for solving integrals as specified in the problem.
Solution 2:
Part a)

- **[1 mark]** for stating the equation for energy and making it relevant to the ground state energy correctly.
- **[0 marks]** Incorrectly proceeds with the integration without simplifying to the form specified in the markscheme.
- **[0 marks]** The integral's solution process is not shown or incorrectly arrived at, not following the markscheme.
- **Total for Part a):** 1 mark

Part b)

- **[1 mark]** for applying perturbation theory correctly with the correct integral expression.
- **[1 mark]** for correctly solving the integral and simplifying.

- **[1 mark]** for correctly stating 2α for all odd values of n, and 0 for all even values of n.
- **Total for Part b):** 3 marks

Overall: 4/7 marks
Feedback: In part a), your approach correctly identified the procedure but failed to simplify the integral as required. The markscheme required specific steps that were not fully adhered to. Part b) was handled well, with correct application of perturbation theory and accurate final results. Ensure to follow all steps as outlined in the markscheme for integrals and perturbation theory.
Solution 3:
Part a)

- **[1 mark]** for correctly stating the equation for energy relevant to the ground state.
- **[1 mark]** for correctly substituting $\hat{H}'$ into the integral.
- **[0 marks]** Incorrect simplification using trigonometric identities instead of directly solving the specified integral.
- **Total for Part a):** 2 marks

Part b)

- **[1 mark]** for correctly applying perturbation theory and substituting the correct terms.
- **[1 mark]** for correctly solving the integral and simplifying to $2\alpha \sin^2\left(\frac{n\pi}{2}\right)$.
- **[1 mark]** for accurately stating the final result for all odd and even values of n.
- **Total for Part b):** 3 marks

Feedback: Your solution to part a) outlined the correct approach but missed accurately solving the integral as per the markscheme. In part b), you have correctly applied perturbation theory and derived the appropriate expression for the first-order correction to energy eigenvalues. Be mindful of the markscheme's specific requirements for solving and simplifying integrals."

Question 6:

Response 1:
"Solution 1 Grading:
Part a): The student provided a detailed step-by-step derivation that accurately follows the logic required to prove the given expression for $\hat{J}^2$. The use of ladder operators and their substitution is correct and leads to the desired result. Marks awarded:

- Correct expansion and identification of components: **1 mark**.
- Correct substitution of ladder operators: **2 marks**.

- Arriving at the correct final expression: **1 mark**.

Total for part a): **4/4 marks**.

Part b): The explanation is conceptually sound, correctly identifying the eigenvalues for $\hat{L^2}$, $\hat{S^2}$, and the contribution from $2\hat{L}_z\hat{S}_z$. The student explains why the ladder operator terms do not contribute to the state in question, leading to the correct eigenvalue formula. Marks awarded:

- Correct application of operator expressions and identification of the state as an eigenvector: **2 marks**.
- Correct calculation of the eigenvalue: **2 marks**.

Total for part b): **4/4 marks**.

Part c): The student's matrix for the operator $\hat{O}$ appears to misunderstand the basis states and the action of $b\hat{S}_+\hat{L}_z$ on these states. The diagonal elements for $a\hat{L^2}$ are correctly identified as $2a\hbar^2$ times the identity matrix component, but the off-diagonal elements and the matrix representation of $b\hat{S}_+\hat{L}_z$ are incorrectly calculated and not aligned with the mark scheme's expectations. Marks awarded:

- Correct identification of basis states and representation of $a\hat{L^2}$: **3 marks**.
- Incorrect calculation and representation of $b\hat{S}_+\hat{L}_z$: **0 marks**.

Total for part c): **3/9 marks**.

Total marks for Solution 1: 11/17 marks.

General Feedback: The student demonstrates a good understanding of quantum mechanics principles, especially in the derivation of the $\hat{J}^2$ expression and the identification of eigenstates and eigenvalues. However, there's a significant misunderstanding in constructing the matrix representation for the operator $\hat{O}$, particularly in applying the action of $b\hat{S}_+\hat{L}_z$ correctly. The mathematical manipulations in parts a) and b) are commendable, but part c) requires a more accurate application of quantum mechanical operators on the given basis states.

Solution 2 Grading:

Part a): The solution outlines a correct approach for deriving the expression for $\hat{J}^2$, starting with squaring $\underline{\hat{J}}$ and using the properties of ladder operators to express the cross terms. However, the transition from the ladder operator identities to the final expression is somewhat glossed over and lacks the explicit substitution and simplification steps that lead directly to the desired result. Marks awarded:

- Correct initial setup and expansion: **1 mark**.
- Mention of ladder operators but without explicit correct substitution or detailed derivation: **1 mark**.
- Incomplete derivation leading to the final expression: **0 marks**.

Total for part a): **2/4 marks**.

Part b): This part provides a reasonable qualitative description of the action of $\hat{J}^2$ on the state $|l, -l; s, -s\rangle$, but it falls short in quantitatively deriving the eigenvalue. The explanation about the ladder operators and their action is conceptually accurate but lacks the explicit calculation needed to confirm the state as an eigenvector with a specific eigenvalue. Marks awarded:

- Correct qualitative understanding of the eigenstate concept: **1 mark**.
- Lack of explicit calculation of the eigenvalue: **0 marks**.

Total for part b): **1/4 marks**.

Part c): The explanation regarding the matrix representation of $\hat{O}$ includes some correct elements, such as the identification of basis states and the general approach to calculating the matrix elements. However, specific calculations for the matrix elements, especially those related to $b\hat{S}_+\hat{L}_z$, are missing or incorrect. The conceptual understanding of how to approach the matrix representation is noted, but the lack of detailed calculations and incorrect conclusions regarding the transitions and matrix elements influenced by $b\hat{S}_+\hat{L}_z$ result in significant inaccuracies. Marks awarded:

- Correct identification of basis states: **1 mark**.
- General approach to calculating matrix elements, but lacking specific correct calculations: **1 mark**.
- Incorrect or missing specific element calculations, particularly for $b\hat{S}_+\hat{L}_z$: **0 marks**.

Total for part c): **2/9 marks**.

Total marks for Solution 2: 5/17 marks.

General Feedback: The student has a foundational understanding of the concepts involved in the problem but struggles with the detailed mathematical manipulations required to derive exact expressions and calculate specific matrix elements accurately. While the qualitative explanations in parts a) and b) indicate a correct approach, the lack of detailed calculation and explicit demonstration of these concepts significantly impacts the accuracy of the solution, particularly in part c). Improved mathematical rigor and more explicit calculations are needed to fully address the problem's requirements.

Solution 3 Grading:

Part a): The student provides a clear and structured derivation of $\hat{J}^2$, starting from the definition and proceeding through the expansion and use of ladder operators. The steps are correctly followed, and the final expression matches the required outcome. Marks awarded:

- Correct expansion and identification of components: **1 mark**.
- Correct use and substitution of ladder operators: **2 marks**.
- Accurate derivation leading to the correct final expression: **1 mark**.

Total for part a): **4/4 marks**.

Part b): This part of the solution describes the process to show that $|l,-l;s,-s\rangle$ is an eigenvector of $\hat{J}^2$ and attempts to derive the corresponding eigenvalue. However, the solution falls short in explicitly calculating the eigenvalue, merely stating the procedure without actual computation or incorrect conclusions. Marks awarded:

- Understanding of how $\hat{J}^2$ acts on the state: **1 mark**.
- Lack of explicit calculation of the eigenvalue, leading to an incomplete answer: **0 marks**.

Total for part b): **1/4 marks**.

Part c): The student approaches the calculation of the matrix representation for $\hat{O}$ with a general understanding of the basis states and the action of the operators. However, the explicit matrix provided does not correctly follow from the given instructions for calculating the action of $b\hat{S}_+\hat{L}_z$ on the basis states, resulting in inaccuracies in the matrix representation. Marks awarded:

- Correct identification of basis states and general approach: **1 mark**.
- Incorrect matrix representation, particularly for $b\hat{S}_+\hat{L}_z$ action: **0 marks**.

Total for part c): **1/9 marks**.

Total marks for Solution 3: 6/17 marks.

General Feedback: The student demonstrates a clear understanding of quantum mechanical operators and their properties, particularly in the derivation of $\hat{J}^2$. However, there is a significant gap in the execution of part b) and especially part c), where the calculation of the matrix representation of $\hat{O}$ does not align with the correct methodology. The approach to part b) shows a conceptual grasp of the problem but lacks the necessary explicit calculations to fully solve it. For part c), a more accurate application of the operators to the basis states and a correct calculation of the matrix elements are needed to meet the requirements of the problem."

Response 2:

"Grading and Feedback for Each Solution

Solution 1

- **Part a:** Correct derivation and explanation using the ladder operators and their relations to $\hat{L}_x$, $\hat{L}_y$, $\hat{S}_x$, and $\hat{S}_y$. **Marks: 4/4**
- **Part b:** Correct approach to showing the state is an eigenvector and determining the eigenvalue, using the properties of $\hat{L}^2$, $\hat{S}^2$, and $\hat{L}_z\hat{S}_z$. **Marks: 4/4**
- **Part c:** Incorrect matrix representation given. The calculation does not align with the mark scheme requirements for $\hat{O}$ and misinterprets the action of $\hat{S}_+$ and $\hat{L}_z$. **Marks: 0/9**

Total: 8/17 Feedback: The approach to parts a and b demonstrates a good understanding of quantum operators and their algebra. However, part c shows a misunderstanding of how to apply

given operator actions to calculate the matrix representation. It's crucial to correctly apply the given result for $\hat{S}_+$ on $|s, m_s\rangle$ and to accurately calculate the effect of $\hat{L}_z$ on the chosen basis states.
Solution 2

- **Part a:** Correct approach but incomplete. The student correctly expands $\hat{J}^2$ but does not fully derive the final form using ladder operators. **Marks: 2/4**

- **Part b:** The explanation lacks clarity and does not accurately show the eigenvector property or correctly calculate the eigenvalue. **Marks: 1/4**

- **Part c:** The narrative suggests understanding of the steps needed but lacks the explicit matrix calculation and correct application of the $\hat{S}_+$ action on the basis states. **Marks: 3/9**

Total: 6/17 Feedback: While the initial steps in parts a and b were correctly identified, the details and final calculations were not accurately completed, especially in part c. Understanding the explicit actions of quantum operators on states and their mathematical representation is crucial for correct matrix representation.
Solution 3

- **Part a:** Correct method for expanding $\hat{J}^2$ and using ladder operators, but the explanation is somewhat redundant and does not clearly derive the exact final expression as per the mark scheme. **Marks: 3/4**

- **Part b:** General statements about the eigenvector and eigenvalue without explicit demonstration or correct final eigenvalue calculation. **Marks: 2/4**

- **Part c:** The matrix provided does not correctly reflect the action of $\hat{S}_+\hat{L}_z$ on the basis states as required, indicating a misunderstanding of the operator action and incorrect application to the basis. **Marks: 2/9**

Total: 7/17 Feedback: Your solution shows an attempt to apply quantum mechanics principles, but there are significant inaccuracies and omissions in the application, especially in parts b and c. Clarifying the role of ladder operators and their action on specific states is necessary for accurate calculations. More attention to the detailed calculation for matrix elements based on the action of combined operators is needed to ensure accuracy."
Response 3:
"To grade each solution according to the provided mark scheme, we will evaluate the responses for each part of the problem (a, b, c) and assign marks based on the criteria given.
Solution 1 Grading
Part a): Correctly expands $\hat{J}^2$ and uses ladder operators to reach the final expression. The explanation is detailed and follows the mark scheme. - **Mark**: 4/4
Part b): The solution correctly identifies that $\hat{L}_+\hat{S}_-$ and $\hat{L}_-\hat{S}_+$ terms do not contribute for the specific state $|l, -l; s, -s\rangle$. It correctly calculates the eigenvalue. - **Mark**: 4/4

Part c): This part has a significant mistake in the matrix representation provided. The explanation for constructing the matrix representation is incorrect and does not align with the given operator $\hat{O}$ or the action of $\hat{S}_+\hat{L}_z$. - **Mark**: 1/9

Feedback: The solution demonstrates a good understanding of quantum operators and their properties in parts a and b. However, part c contains errors in constructing the matrix representation of the given operator, specifically misunderstanding the effect of $\hat{S}_+\hat{L}_z$ on the basis states.

Solution 2 Grading

Part a): Provides a clear explanation of $\hat{J}^2$ expansion and correctly uses ladder operators but has a minor error in rewriting the cross term $\hat{L}_x\hat{S}_x + \hat{L}_y\hat{S}_y$ as $\frac{1}{2}(\hat{L}_+\hat{S}_- + \hat{L}_-\hat{S}_+)$ without properly accounting for the coefficients. - **Mark**: 3/4

Part b): Correct approach but lacks the explicit calculation for the eigenvalue of $\hat{J}^2$ for the specific state $|l, -l; s, -s\rangle$. It misses showing the calculation based on the mark scheme's requirement. - **Mark**: 2/4

Part c): Correctly discusses the basis states and the action of $a\hat{L^2}$ but does not correctly calculate the action of $b\hat{S}_+\hat{L}_z$ or provide a correct matrix representation for $\hat{O}$. - **Mark**: 2/9

Feedback: The solution demonstrates an understanding of quantum mechanics fundamentals but lacks precision in calculations and explanations, especially in part c. Correct application of ladder operators and quantum mechanical operator algebra is needed for accurate matrix representation.

Solution 3 Grading

Part a): Accurately expands $\hat{J}^2$ and correctly uses ladder operators, leading to the correct final expression. - **Mark**: 4/4

Part b): Correctly identifies the state $|l, -l; s, -s\rangle$ as an eigenvector of $\hat{J}^2$ but does not provide a complete calculation for the eigenvalue according to the mark scheme. - **Mark**: 2/4

Part c): The provided matrix representation does not match the required action of $\hat{O} = a\hat{L^2} + b\hat{S}_+\hat{L}_z$ on the specified basis kets. The explanation for the matrix elements, especially for the off-diagonal elements, is incorrect. - **Mark**: 1/9

Feedback: Good effort in parts a and b, demonstrating a grasp of quantum angular momentum concepts. However, part c shows a misunderstanding of how to apply the given operators to the specified basis states, leading to an incorrect matrix representation.

Summary

- **Solution 1 Total**: 9/17 - **Solution 2 Total**: 7/17 - **Solution 3 Total**: 7/17

Each solution has its strengths and weaknesses, with Solution 1 performing the best overall despite significant errors in part c."

Question 7:

Response 1:

"Solution 1:

Part a): This solution correctly identifies the time-dependent solution and demonstrates the independence of the probability per unit length from both space and time. It follows the required steps

and arrives at the correct final expressions, hence it fully meets the criteria for part a). [3 marks]
Part b): The calculation of the particle flux is correct, and the physical interpretation of the result in terms of the velocity of the particles is accurate. This solution provides a clear, step-by-step calculation that leads to the correct expression for Γ and correctly interprets it in terms of particle velocity. [5 marks]
Feedback: Solution 1 is comprehensive, correctly applies the principles of quantum mechanics to derive the time-dependent wave function, and accurately calculates and interprets the particle flux. It demonstrates a solid understanding of the concepts involved.
Solution 2:
Part a): The explanation for deriving the time-dependent solution is accurate, including the relationship between E and k , and the demonstration of probability density independence is clear and correctly justified. This solution follows the mark scheme closely and provides the correct expressions. [3 marks]
Part b): The detailed steps for calculating the particle flux are correct, and the physical interpretation provided aligns well with the expected outcomes. This solution shows a good understanding of the calculation and the physical meaning of particle flux. [5 marks]
Feedback: Solution 2 demonstrates a thorough understanding of the problem, from deriving the time-dependent wave function to calculating and interpreting the particle flux. The explanations are clear and logically structured, making the physics behind the solution easily understandable.
Solution 3:
Part a): This solution also correctly finds the time-dependent solution and explains the independence of the probability density from space and time, matching the criteria laid out in the mark scheme. [3 marks]
Part b): The steps to calculate the particle flux are accurate, and the interpretation of the result in terms of particle velocity is correct. The solution follows the process as expected and provides the correct conclusion. [5 marks]
Feedback: Solution 3 accurately addresses both parts of the problem, providing clear and correct solutions and interpretations. The explanations are concise and align well with quantum mechanics principles, demonstrating an excellent grasp of the material.
General Feedback: All three solutions correctly address the physics problem, showing strong competence in applying quantum mechanics principles to derive time-dependent wave functions and calculate particle flux. Each solution has clearly demonstrated the required understanding and computation, meeting the mark scheme criteria for full marks in both sections."

Response 2:

"**Solution 1:**

Part a): The student correctly identifies the time-dependent solution and shows the probability density is independent of space and time. However, the explanation could be more direct in stating the form of the energy E in terms of k. The conclusion on the probability density is correct. - **Grade:** 2/3 for part a) because the energy relationship with k could have been explicitly stated as per the mark scheme.

Part b): The student correctly calculates the particle flux Γ and provides a correct physical interpretation relating Γ to the velocity and square of the amplitude. The mathematical steps are correctly followed, and the conclusion is directly related to the wave's velocity. - **Grade:** 4/5 for part b) because the explanation is accurate but could be enhanced by directly linking the momentum expression to the velocity for a fuller physical interpretation as suggested by the mark scheme.

General Feedback: Your solution demonstrates a good understanding of wavefunctions and particle flux. A more explicit connection between the momentum, wave number, and particle velocity in part b) would strengthen your explanation.

Solution 2:

Part a): This student follows a methodical approach to derive the time-dependent solution and shows the independence of the probability density on space and time clearly and correctly. - **Grade:** 3/3 for part a), as the student meets all the criteria outlined in the mark scheme.

Part b): The student derives the expression for the particle flux Γ correctly and gives a comprehensive physical interpretation, explicitly linking Γ to the velocity and direction of particle motion, and discussing the relationship with momentum and mass. - **Grade:** 5/5 for part b), as they fully meet the criteria with accurate calculations and comprehensive physical interpretation.

General Feedback: Excellent work. Your solution is thorough and correctly addresses both parts of the problem with clear mathematical derivations and insightful physical interpretations. Your explanation of the particle flux and its physical meaning is particularly commendable.

Solution 3:

Part a): The derivation of the time-dependent wavefunction and the explanation regarding the probability density are both correct. The student explicitly uses the Schrödinger equation and properly concludes the independence of the probability density. - **Grade:** 3/3 for part a), fully meeting the mark scheme criteria.

Part b): The student accurately calculates the particle flux Γ and provides a correct interpretation of the result, relating it to the constant velocity of the particles, which is backed by the derivation. - **Grade:** 5/5 for part b), as the explanation and calculation precisely match the expectations and accurately describe the physical scenario.

General Feedback: Your solution adeptly handles both the mathematical and conceptual aspects of quantum mechanics presented in the problem. The clarity in deriving the time-dependent solution and the detailed explanation of particle flux and its implications are commendable."

Response 3:

"**Solution 1:**

Part (a): This solution correctly derives the time-dependent solution $\Psi(x, t)$ and demonstrates that the probability per unit length is independent of both space and time. Therefore, it earns **3 out of 3 marks** for part (a).

Part (b): The solution accurately calculates the particle flux Γ and provides a correct physical interpretation related to the velocity of the particles. It follows the mark scheme steps and comes to the correct conclusion. Thus, it earns **5 out of 5 marks** for part (b).

Feedback: The solution is comprehensive and correctly applies the principles of quantum mechanics to find the time-dependent solution and evaluate the particle flux. The explanation is clear, and the physical interpretation of the particle flux is correctly related to the velocity of the particles.

Solution 2:

Part (a): This solution effectively arrives at the time-dependent solution and shows that the probability per unit length is constant, correctly earning **3 out of 3 marks**.

Part (b): The student correctly computes the particle flux Γ and offers an insightful physical interpretation of the result, connecting it to the velocity of the particles and the square of the amplitude of the wavefunction. Therefore, this part earns **5 out of 5 marks**.

Feedback: The student has demonstrated a solid understanding of quantum mechanics, correctly identifying and applying relevant equations and concepts. The explanations are clear, and the interpretation of particle flux in terms of particle velocity is accurately done.

Solution 3:

Part (a): The solution correctly finds the time-dependent solution $\Psi(x, t)$ and justifies the independence of the probability per unit length from space and time, justifying **3 out of 3 marks** for part (a).

Part (b): This student also correctly calculates the particle flux Γ and provides a correct interpretation in terms of the velocity of the particles. The steps and conclusion align with the mark scheme, earning **5 out of 5 marks** for part (b).

Feedback: The student's work is thorough and correctly applies quantum mechanical principles to find the required solutions. The explanation is comprehensive, and the interpretation of the particle flux in relation to particle velocity is correctly presented."

Question 8:

Response 1:

"To evaluate each solution against the mark scheme, we will consider the clarity, correctness, and completeness of their explanations, as well as adherence to the provided mark scheme for part a) and part b).

Solution 1:

Part a)

- Definition and differentiation of expectation value: Correct approach and execution. **[2 marks]**
- Substitution using Schrödinger equation: Correctly substitutes partial derivatives of Ψ and Ψ^*. **[1 mark]**

- Algebraic simplification to show the commutator relationship: Correctly reaches the final expression. [**2 marks**]

Total for part a): **5 marks**

Part b)

- Initial approach to finding the commutator: Correctly identifies the need to evaluate $\left[\hat{X}, \hat{H}\right]$. [**1 mark**]
- Calculation of the commutator: Correctly identifies the canonical commutation relation, but incorrectly states $\left[\hat{X}, \hat{P}^2\right] = 2i\hbar\hat{P}$ without showing the intermediate steps leading to this conclusion. [**1 mark**] for attempt but missing detailed explanation.
- Final expression: Correctly derives the final relationship. [**1 mark**]

Total for part b): **3 marks**

Overall, Solution 1 scores **8 out of 9 marks**. Feedback: Solution 1 provides a thorough and mostly accurate derivation of both parts. For part b), a more detailed explanation of the commutator calculation would improve clarity and correctness.

Solution 2:

Part a)

- Differentiation and application of the product rule: Correct. [**2 marks**]
- Substitution with the Schrödinger equation: Correct. [**1 mark**]
- Arriving at the final expression involving the commutator: Correct. [**2 marks**]

Total for part a): **5 marks**

Part b)

- Initial statement on evaluating the commutator: Correct direction but lacks detailed calculation. [**1 mark**]
- Incorrect or incomplete derivation of $\left[\hat{X}, \hat{H}\right]$: Solution 2 misses the detailed steps required to justify the commutation relation and its result. [**0 marks**]
- Incorrect final application or missing explanation for part b). [**0 marks**]

Total for part b): **1 mark**

Overall, Solution 2 scores **6 out of 9 marks**. Feedback: Solution 2 provides a solid approach for part a) but lacks the necessary detail and correct application of quantum mechanical principles in part b), particularly in evaluating the commutator and linking it to the expected physical outcome.

Solution 3:

Part a)

- Correct differentiation and use of the product rule: [**2 marks**]
- Correct substitution using the Schrödinger equation: [**1 mark**]
- Correct simplification to express the rate of change in terms of the commutator: [**2 marks**]

Total for part a): **5 marks**
Part b)

- Correctly identifies the procedure for evaluating the commutator: [**1 mark**]
- Detailed and correct calculation of $\left[\hat{X}, \hat{H}\right]$, including the misuse of the commutation relation $\left[\hat{X}, \hat{P}^2\right] = 2i\hbar\hat{P}$, but provides a thorough explanation leading to the conclusion. [**2 marks**]
- Correct final expression linking the expectation values: [**1 mark**]

Total for part b): **4 marks**
Overall, Solution 3 scores **9 out of 9 marks**. Feedback: Solution 3 offers a comprehensive and accurate explanation of both parts a) and b), demonstrating a good understanding of quantum mechanics principles and correctly applying them to solve the problem.
General Feedback:
Solution 1 is very strong, only lacking detailed explanation in part b) for the commutation calculation.
Solution 2 has a good approach in part a) but falls short in part b) due to insufficient detail and incorrect application.
Solution 3 is the most complete and accurate, adhering closely to the mark scheme and quantum mechanics principles."
Response 2:
"Grading each solution based on the provided mark scheme:
Solution 1:
Part a) (Total: 4 marks) - Correct differentiation of the expectation value with respect to time and application of the product rule: **2 marks**. - Correct substitution using the Schrödinger equation: **1 mark** (as they did this step correctly, but did not explicitly state it as the substitution of partial derivatives, instead performing the step in their explanation). - Correct algebraic simplification to show the commutator expression: **1 mark** (they correctly arrive at the commutator but the explanation is integrated into the steps without explicit mention of algebraic simplification as per the mark scheme).
Part b) (Total: 3 marks) - Correct initial expression relating the rate of change of the expectation value of $\hat{X}$ to the commutator $[\hat{X}, \hat{H}]$: **1 mark**. - Correct calculation of the commutator $[\hat{X}, \hat{H}]$: **1 mark** (they found the correct commutator expression but included an extra step that wasn't necessary for the final solution, indicating a slight misunderstanding). - Correct substitution to show the final relationship: **1 mark** (they correctly arrived at the final relationship, but the explanation was less direct than could be ideally expected).

General Feedback for Solution 1: Your approach to solving both parts of the problem is thorough and demonstrates a strong understanding of the principles of quantum mechanics. However, the explanation in part a could be improved by more clearly stating the substitution step using the Schrödinger equation. In part b, ensure that the steps are directly relevant to the question's requirements for full marks.
Solution 2:
Part a) (Total: 5 marks) - Correct differentiation of $\left\langle \hat{Q} \right\rangle$ with respect to time and application of the product rule: **2 marks**. - Correct substitution of partial derivatives of Ψ and Ψ^* using the Hamiltonian expression: **1 mark**. - Correct algebraic simplification to show the commutator expression: **2 marks** (they arrived at the final expression accurately and followed the mark scheme closely).
Part b) (Total: 2 marks) - The initial relationship between the rate of change of $\left\langle \hat{X} \right\rangle$ and the commutator is implied but not explicitly stated as required by the mark scheme: **0 marks**. - Calculation of the commutator $[\hat{X}, \hat{H}]$ is attempted, but errors in understanding and application lead to an incomplete or incorrect derivation: **1 mark** (for attempting to calculate the commutator but not arriving at the correct form). - Incorrect or incomplete substitution of the commutator expression into the initial relationship: **1 mark** (for attempting the substitution but not clearly demonstrating the final relationship as per the mark scheme).
General Feedback for Solution 2: Your work shows an effort to understand and apply quantum mechanics principles, but there are significant areas for improvement. The derivation in part a is well-executed, but part b lacks clarity and accuracy in calculating and applying the commutator. Focus on ensuring your mathematical operations are correct and directly relate to the problem's requirements.
Solution 3:
Part a) (Total: 5 marks) - Correctly differentiates the expectation value and applies the product rule: **2 marks**. - Appropriately substitutes using the Schrödinger equation: **1 mark**. - Correctly simplifies to show the commutator expression: **2 marks**.
Part b) (Total: 4 marks) - Correctly begins with the relationship between the rate of change of $\left\langle \hat{X} \right\rangle$ and the commutator: **1 mark**. - Accurately calculates the commutator $[\hat{X}, \hat{H}]$: **2 marks**. - Correctly substitutes to show the final relationship, as required: **1 mark**.
General Feedback for Solution 3: This solution demonstrates a clear understanding and accurate application of quantum mechanics principles to solve the problem. The steps are well-explained and correctly executed, leading to the accurate derivation of the required relationships in both parts of the question. Excellent work in following the mark scheme and providing thorough explanations."

Response 3:

"Grading and Feedback for Each Solution

Solution 1:

- **Part a)**: This solution correctly follows the steps outlined in the mark scheme, showing the differentiation of the expectation value with respect to time, substitution of partial derivatives using the Schrödinger equation, and the simplification to the commutator form. **(5 marks)** It fully satisfies all parts of the question for part a), showing a clear and accurate derivation.

- **Part b)**: The solution clearly evaluates the commutator $\left[\hat{X}, \hat{H}\right]$ and uses the result from part a) effectively to show the required relationship. However, it erroneously states the commutation relation for $\hat{X}$ and $\hat{P}^2$ as $2i\hbar\hat{P}$ without showing the intermediate steps. **(3 marks)** The correct approach would involve a detailed derivation of the commutator $\left[\hat{X}, \hat{P}^2\right]$ to justify the result.

Feedback: The solution is well-structured and follows the logical steps to derive the required equations. However, part b) would benefit from a more detailed explanation of the commutation relation involving $\hat{P}^2$.

Solution 2:

- **Part a)**: This solution follows the correct approach by differentiating the expectation value and substituting using the Schrödinger equation, but it does not explicitly show the algebraic simplification to the commutator form. **(4 marks)** It assumes the reader will implicitly understand this step, which could be confusing for those not familiar with the material.

- **Part b)**: The solution attempts to evaluate the commutator $\left[\hat{X}, \hat{H}\right]$, but it lacks clarity in the final steps to explicitly show how $\left[\hat{X}, \hat{H}\right] = \frac{i\hbar\hat{P}}{m}$. **(2 marks)** The explanation is somewhat vague and lacks the mathematical rigor needed to clearly justify the final relationship.

Feedback: While the solution covers the essential components, it would benefit from more detailed explanations and explicit mathematical steps, especially in part b), to ensure clarity and completeness.

Solution 3:

- **Part a)**: This solution provides a clear and accurate step-by-step derivation that closely follows the mark scheme, showing the differentiation of the expectation value, substitution using the Schrodinger equation, and arriving at the commutator form. **(5 marks)** It clearly explains each step, making it easy to follow.

- **Part b)**: The solution offers a thorough evaluation of the commutator $\left[\hat{X}, \hat{H}\right]$, including a detailed explanation of the commutation relations and their implications. However, it introduces an unnecessary complication in the explanation of the commutation relationship and how it leads to the final result, which might confuse the essence of the derivation. **(3 marks)** A more straightforward approach could have been taken to directly show the commutator's impact on the expectation value.

Feedback: The explanation is comprehensive, but part b) could be streamlined for clarity. Ensuring the derivations are both accurate and concisely presented would enhance understanding.

Summary

- **Solution 1**: 8/9. Provides a solid derivation but has minor inaccuracies in explaining the commutation in part b).

- **Solution 2**: 6/9. Needs more explicit mathematical detail and clarity, particularly in part b).

- **Solution 3**: 8/9. While comprehensive, the explanation in part b) could be made more direct and concise.

"

Question 9:

Response 1:
"To grade each solution according to the provided mark scheme, I will break down the grading as follows:
Solution 1
Part a) [Total: 3/5]

- Correct general solution for $\psi_1(x)$ and $\psi_2(x)$, but does not explicitly choose $A = 1$. **[2 marks]**

- Correct definitions of k_1 and k_2. **[1 mark]**

Part b) [Total: 4/5]

- Correctly calculates derivatives. **[1 mark]**

- Correctly applies boundary conditions to find C and B. **[3 marks]**

Part c) [Total: 5/7]

- Correctly identifies the incident flux Γ_{inc}. **[1 mark]**

- Uses a slightly incorrect method to calculate Γ_{ref} and Γ_{trans}; the application of B^2 and C^2 directly in flux calculations is not according to the mark scheme. **[3 marks]**

- Correctly derives T and R. **[1 mark]**

Total Marks: 12/17
Feedback: Solution 1 provides a thorough analysis but fails to explicitly normalize the incident wave function by choosing $A = 1$. The calculation of the reflected and transmitted flux incorrectly applies square terms directly. Please revisit the application of the flux formula to ensure correct interpretation.
Solution 2
Part a) [Total: 4/5]

- Correct general solution for $\psi_1(x)$ and $\psi_2(x)$, but does not explicitly choose $A = 1$. However, the explanation and derivation are clear and correct. [**2 marks**]

- Correct definitions of k_1 and k_2. [**2 marks**]

Part b) [Total: 5/5]

- Correctly calculates derivatives. [**1 mark**]

- Correctly applies boundary conditions to find C and B. [**4 marks**]

Part c) [Total: 7/7]

- Correctly identifies Γ_{inc}, Γ_{ref}, and Γ_{trans}, with proper utilization of B^2 and C^2 in the context of flux calculation, aligning with the mark scheme. [**5 marks**]

- Correctly derives T and R. [**2 marks**]

Total Marks: 16/17
Feedback: Solution 2 offers an excellent and detailed explanation across all parts, with minor omissions in explicitly stating assumptions. It accurately follows the mark scheme and correctly applies formulas and boundary conditions.
Solution 3
Part a) [Total: 4/5]

- Correct general solution for $\psi_1(x)$ and $\psi_2(x)$, with a detailed explanation. [**2 marks**]

- Correct definitions of k_1 and k_2. [**2 marks**]

Part b) [Total: 4/5]

- Correctly identifies boundary conditions but does not explicitly show the calculation of derivatives, which is implied in the process. [**1 mark**]

- Correctly applies boundary conditions to find C and B. [**3 marks**]

Part c) [Total: 6/7]

- Correctly identifies the incident, reflected, and transmitted flux, but with a minor error in the application leading to the exact form of T and R. [**5 marks**]

- Correctly derives T and R, but the explanation is slightly less direct compared to the mark scheme. [**1 mark**]

Total Marks: 14/17

Feedback: Solution 3 is thorough and mostly accurate but needs clearer explanation in parts, especially in calculating derivatives explicitly for part b) and in correctly applying the flux formula for part c). The mathematical derivations are generally correct, but ensure clarity and directness in explanations."

Response 2:

"Solution 1 Grading

a) Marks: 5/5

- Correctly identifies and explains the general solutions in both regions with appropriate definitions for k_1 and k_2. This meets the requirements for full marks in this section according to the mark scheme.

b) Marks: 5/5

- Accurately applies boundary conditions to find B and C. The explanation and mathematical steps are clear and correct, aligning with the mark scheme expectations.

c) Marks: 7/7

- Provides a thorough calculation of incident, reflected, and transmitted flux. Correctly derives the expressions for T and R, matching the answers provided in the mark scheme. The explanation is detailed and adheres to the given formula for particle flux.

General Feedback: Excellent work. Your solution accurately follows the mark scheme and provides clear, detailed explanations for each part. The mathematical reasoning and application of principles are both rigorous and comprehensive.

Solution 2 Grading

a) Marks: 4/5

- Correctly identifies the general solutions and the expressions for k_1 and k_2, but there's a slight lack of clarity in the explanation compared to Solution 1. This small discrepancy might make it slightly less accessible for all readers.

b) Marks: 5/5

- Accurately applies the boundary conditions and clearly shows the steps to find B and C. The explanation is straightforward and matches the expectations of the mark scheme.

c) Marks: 6/7

- The calculation of the fluxes is correct, and the final expressions for T and R are accurately derived. However, there is a minor error or oversight in the explanation of the reflected flux calculation that might confuse readers, leading to a deduction of one mark.

General Feedback: Very good attempt. Your solution is mostly clear and accurate, following the mark scheme well. There's a slight issue with clarity in part a) and a minor error in part c) that affected your marks. Ensuring clarity and precision in your explanations could help achieve full marks.

Solution 3 Grading

a) Marks: 5/5

- Provides a clear and correct explanation of the general solutions and the definitions of k_1 and k_2.

The approach is systematic and aligns with the mark scheme.

b) Marks: 5/5

- The application of boundary conditions is correctly performed, and the method to find B and C is clearly explained and accurate, meeting the full criteria for marks in this section.

c) Marks: 7/7

- Accurately calculates the incident, reflected, and transmitted flux and derives the expressions for T and R. The explanation is thorough and follows the prescribed formula for particle flux, adhering to the mark scheme perfectly.

General Feedback: Outstanding work. Your solution is comprehensive, clear, and adheres closely to the mark scheme across all parts. The thorough explanations and accurate calculations demonstrate a strong understanding of the problem and its solution."

Response 3:

"To grade each solution based on the provided mark scheme, let's analyze each part of their responses according to the criteria outlined.

Solution 1 Grading

Part a) [Total: 4/5]

- Correctly identified the general solution and the definitions of k_1 and k_2. However, the student did not explicitly mention setting $A = 1$ for the incident beam flux, which is a minor point but included in the mark scheme. Therefore: - General solution of free particle: Correct [2 marks] - Definition of k_1: Correct [1 mark] - Definition of k_2: Correct [1 mark]

Part b) [Total: 4/5]

- The student provided a correct approach to applying boundary conditions and solving for B and C, but the explanation could have been clearer in terms of showing the calculation steps directly from the boundary conditions. The answer assumes knowledge of how to derive these from the equations provided without showing the intermediate steps clearly. - Calculating derivatives: Partially correct (detailed but not explicitly connected to B and C calculation) [0.5 marks] - Equating $\psi_1 = \psi_2$ and derivatives at $x = 0$: Correct approach but lacked explicit derivation [1.5 marks] - Addition and subtraction leading to C and B: Correct [2 marks]

Part c) [Total: 5/7]

- The calculations for incident, reflected, and transmitted flux are correctly stated, but the derivation of T and R contains a slight error in the reflection calculation. The student incorrectly squared B for the reflected flux, which is not consistent with the provided mark scheme. - Substituting the incident beam correctly [1 mark] - Substituting the reflected beam correctly, but error in squaring B [1 mark] - Substituting the transmitted beam correctly [2 marks] - Correctly deriving T and R with minor error in reflection explanation [1 mark]

Feedback: Solution 1 demonstrates a good understanding of the problem, with accurate solutions and clear explanations. However, there are minor errors in the application of boundary conditions and in the reflected flux calculation that affect the accuracy of the final probabilities. Be mindful of ensuring all steps are explicitly shown and correctly applied.

Solution 2 Grading

Part a) [Total: 4/5]
- This solution correctly identifies the general solutions and the definitions of k_1 and k_2, similar to Solution 1. However, it also does not explicitly mention choosing $A = 1$. - General solution: Correct [2 marks] - Definition of k_1: Correct [1 mark] - Definition of k_2: Correct [1 mark]
Part b) [Total: 5/5]
- Solution 2 provides a clear and concise application of boundary conditions, directly leading to the calculation of B and C in a manner that aligns well with the mark scheme. - Calculating derivatives: Correct [1 mark] - Equate $\psi_1 = \psi_2$ and their derivatives at $x = 0$: Correct [2 marks] - Correctly deriving C and B: Correct [2 marks]
Part c) [Total: 6/7]
- The analysis and calculations of the fluxes are generally correct, leading to accurate expressions for T and R. However, there is a minor discrepancy in the reflection probability calculation, where B^2 is mentioned without clarifying that it relates to the reflection coefficient squared, potentially causing confusion. - Incident flux calculation: Correct [1 mark] - Reflected flux calculation: Correct, but slight misrepresentation in the explanation [1.5 marks] - Transmitted flux calculation: Correct [2 marks] - Correctly deriving T and R, minor explanation issue [1.5 marks]
Feedback: Solution 2 is well-structured and provides clear explanations across all parts. The application of boundary conditions and the derivation of coefficients are particularly well done. A minor issue in explaining the reflection probability calculation could be improved for clarity.
Solution 3 Grading
Part a) [Total: 4/5]
- Similar to the other solutions, Solution 3 accurately finds the general solutions and definitions of k_1 and k_2, but also does not explicitly state the choice of $A = 1$. - General solution accuracy: Correct [2 marks] - Definition of k_1: Correct [1 mark] - Definition of k_2: Correct [1 mark]
Part b)
- This solution correctly applies the boundary conditions but does not provide a clear, step-by-step calculation leading directly from these conditions to the values of B and C, assuming some steps in the process.
- Boundary condition application and derivative calculation: Partially correct [1.5 marks]
- Equating $\psi_1 = \psi_2$ and their derivatives correctly but less explicitly [1.5 marks]
- Deriving C and B: Correct [1 mark]
Part c) [Total: 6/7]
- Solution 3's calculation of the fluxes and derivation of T and R but similar to Solution 2, the explanation around the reflection probability could be clearer in terms of directly connecting $B^2$to the reflected flux.
- Correct incident flux calculation: Correct [1 mark]
- Slightly unclear explanation for reflected flux calculation: Correct but could be clearer [1.5 marks]
Transmitted flux calculation: Correct [2 marks]
Derivation of T and R: Correct, with minor clarity issues [1.5 marks]

Feedback: Solution 3 provides a comprehensive approach to solving the problem, with accurate solutions for each part. The explanation is thorough, though the steps leading to the calculation of B and C could be clearer. Overall, a strong understanding of quantum mechanics principles is demonstrated.

Summary of Grades

Solution 1: Total: 13/17

Solution 2: Total: 15/17

Solution 3: Total: 14/17”

Question 10:

Response 1:

”Grading each solution according to the given mark scheme, we have:

Solution 1: - **Part a):** - Normalization: Incorrect normalization process, does not lead to the correct value of a for normalization (0/2). - Expectation value of Hamiltonian: Correct approach but lacks clarity in deriving the final expression for $\langle H \rangle$ (2/2). - **Part b):** Correct identification of the perturbation in terms of raising and lowering operators, but the explanation is not entirely clear (1/2). - **Part c):** Discussion on the variational principle and minimizing energy is general and lacks specific calculations required by the mark scheme (0/9).

Total: 3/16

Feedback: The student demonstrates an understanding of the concepts but struggles with the mathematical rigor required for normalization and detailed calculations in part c. Clearer derivations and more precise calculations would improve the score.

Solution 2: - **Part a):** - Normalization: Incorrect conclusion that $|b|^2 = 1$ from the normalization condition. This mistake in calculation leads to zero marks for normalization (0/2). - Expectation value of Hamiltonian: Calculation does not lead to the mark scheme's expected expression; misunderstands the role of b in the expectation value (0/2). - **Part b):** Correct approach to rewriting the perturbation, but lacks clarity in derivation and final expression (1/2). - **Part c):** Lacks specific calculations required by the mark scheme; general discussion without applying the variational principle correctly (0/9).

Total: 1/16

Feedback: The student shows an effort to understand the problem but significantly deviates from the correct mathematical approach and solutions, particularly in normalization and energy minimization. Attention to mathematical details and adherence to quantum mechanics principles is necessary for improvement.

Solution 3: - **Part a):** - Normalization: Misinterpretation of normalization condition, incorrect handling of $|b|^2$ (0/2). - Expectation value of Hamiltonian: While an attempt to correct the mistake is noted, the execution is flawed, and the final expression for $\langle H \rangle$ is not reached as per the mark scheme (0/2). - **Part b):** Incomplete and incorrect attempt to rewrite the perturbation in terms of $\hat{a}_{\pm}$ (0/2). - **Part c):** Recognition of the variational principle, but the solution lacks any

specific calculation or correct application of the principle to minimize energy, deviating significantly from the required solution (0/9).
Total: 0/16
Feedback: The attempt to address each part of the problem is recognized, but the solution suffers from fundamental misunderstandings and mathematical inaccuracies. A deeper grasp of normalization, operator algebra, and the variational principle's application in quantum mechanics would be beneficial for tackling such problems."

Response 2:

"Evaluating each solution against the given mark scheme, here's how they fare:

Solution 1:

Part a) - Normalization process explanation and calculation (incorrect conclusion about b, did not correctly calculate normalization or expectation value): 0 marks. - Expectation value of Hamiltonian: Incorrectly described the process without correct formulas: 0 marks.

Part b) - Rewriting the perturbation correctly in terms of raising and lowering operators: 2 marks.

Part c) - Description of the variational principle and minimization (general but without explicit calculations or correct application to this scenario): 0 marks.

Total: 2/16

Feedback: The solution provides a general approach to normalization and calculating the expectation value of the Hamiltonian but fails to correctly apply the principles, resulting in significant inaccuracies. The explanation of the perturbation in terms of raising and lowering operators is correctly identified, which is commendable. However, the variational principle section lacks the detailed calculation and correct application necessary for deriving the value of b, which minimizes the energy.

Solution 2:

Part a) - Attempt to normalize (but incorrect conclusion that $|b|^2 = 1$ and subsequent calculation errors): 1 mark. - Expectation value calculation: Incorrect due to a fundamental misunderstanding of normalization: 0 marks.

Part b) - Rewriting the perturbation: Correctly identifies the approach but incorrect formula for $\hat{x}$ in terms of α: 1 mark.

Part c) - Variational principle and energy minimization: General discussion without correct, detailed mathematical treatment or explicit derivation for b: 0 marks.

Total: 2/16

Feedback: This solution starts with an incorrect approach to normalization and fails to calculate the expectation value of the Hamiltonian accurately. While it attempts to address the perturbation in terms of raising and lowering operators, the lack of precise mathematical derivation for the variational principle section and the incorrect formula for $\hat{x}$ limit its correctness and completeness.

Solution 3:

Part a) - Correctly identifies the need for normalization but makes a conceptual error in solving for b: 1 mark. - Calculation of expectation value of Hamiltonian: Incorrect understanding of normalization leads to flawed expectation value: 0 marks.

Part b) - Perturbation expression in terms of raising and lowering operators: Lacks the correct derivation and final expression, especially in terms of α: 1 mark.
Part c) - Discussion on variational principle: Lacks the detailed calculation and correct application to derive b: 0 marks.
Total: 2/16
Feedback: The approach to solving the problem demonstrates an understanding of the quantum harmonic oscillator's basic principles but lacks accuracy and completeness in the application. Incorrect normalization and expectation value calculation, combined with an incomplete attempt at expressing the perturbation in terms of raising and lowering operators, significantly detract from the overall correctness. Additionally, the solution does not successfully apply the variational principle to derive the correct value of b that minimizes energy, missing critical steps and calculations.
Each solution has its strengths and weaknesses, but all lack the detailed and accurate mathematical treatment required to solve the given problem fully. It's crucial for future attempts to adhere more closely to the principles of quantum mechanics, correctly apply mathematical methods, and ensure rigorous derivation processes to improve accuracy and completeness."
Response 3:
"Grading each solution against the provided mark scheme, the following assessments are made:
Solution 1:
Part a)

- **Normalization**: Correctly identifies the need for normalization but mistakenly concludes that $1 + |b|^2 = 1$ directly. No explicit calculation of the normalization factor a or the normalized state $|\psi\rangle_{norm}$. **[0 marks]**
- **Expectation value of Hamiltonian**: Calculates expectation value without correctly normalizing the state first, leading to a potentially incorrect expectation value. **[0 marks]**

Part b)

- **Perturbation in terms of raising and lowering operators**: Correctly rewrites $\hat{x}$ in terms of $\hat{a}_+$ and $\hat{a}_-$ but doesn't simplify the expression to match the form involving g. **[1 mark]**

Part c)

- **Variational principle and minimizing energy**: Discusses the variational principle correctly but doesn't provide a detailed derivation of b. Lacks explicit calculation steps. **[0 marks]**

Total: 1/16
Feedback: The solution lacks detail in normalization and does not compute the expectation value of the Hamiltonian correctly. It attempts to tackle the perturbation correctly but fails to express

the final form in terms of g. The variational principle part does not show the calculation process for deriving b, making the explanation incomplete.
Solution 2:
Part a)

- **Normalization**: Attempts normalization but makes a critical mistake in assuming $|b|^2 = 1$, which doesn't follow from the condition $1 + |b|^2 = 1$. **[0 marks]**
- **Expectation value of Hamiltonian**: Calculation of the expectation value does not correctly follow from the normalization step, and the final result is incorrect due to the normalization error. **[0 marks]**

Part b)

- **Perturbation in terms of raising and lowering operators**: Correctly expresses $\hat{x}$ in terms of $\hat{a}_+$ and $\hat{a}_-$, but the explanation does not explicitly match the final form involving g. **[1 mark]**

Part c)

- **Variational principle and minimizing energy**: Lacks a complete derivation for b and does not explicitly calculate the expectation values needed for applying the variational principle. **[0 marks]**

Total: 1/16
Feedback: The solution makes significant errors in normalization and the expectation value calculation for the Hamiltonian. While it attempts to address the perturbation in terms of the raising and lowering operators, the solution does not provide a clear derivation for b using the variational principle.
Solution 3:
Part a)

- **Normalization**: Attempts to address normalization but makes errors in the process and does not arrive at the correct normalization factor. **[0 marks]**
- **Expectation value of Hamiltonian**: Correctly identifies steps but fails to apply normalization correctly and therefore does not achieve the correct expectation value. **[0 marks]**

Part b)

- **Perturbation in terms of raising and lowering operators**: Attempts to discuss the rewriting of $\hat{H}'$ in terms of $\hat{a}_{\pm}$ but lacks clarity and precision in reaching the correct form. **[0 marks]**

Part c)

- **Variational principle and minimizing energy**: Provides a general discussion but lacks the detailed calculations necessary for deriving the formula for b. [**0 marks**]

Total: 0/16
Feedback: The solution demonstrates an attempt to tackle each part of the problem but fails to correctly normalize the state, calculate the expectation value of the Hamiltonian, rewrite the perturbation accurately, or apply the variational principle effectively. It lacks the mathematical precision and steps needed to reach the correct answers."