自测题2-答案

一. B A B C B D

二. 1. $y=e^{2x}(C_1\cos x+C_2\sin x)$　2. $z=5(x^2+y^2)$　3. $\frac{\pi}{3}$　4. 2

5. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta \int_0^{\frac{2}{\cos\theta}} r f(r^2)\,dr$　6. $-\frac{2\pi}{3}$

三. 1. $y'+\frac{1}{x}y=\frac{\sin x}{x}$

$$y=e^{-\int\frac{1}{x}dx}\left[\int\frac{\sin x}{x}e^{\int\frac{1}{x}dx}dx+C\right]$$

$$=\frac{1}{x}[-\cos x+C]$$

2. $\vec{n_1}=(1,2,3)$　$\vec{n_2}=(6,-1,5)$

$$\vec{n}=\vec{n_1}\times\vec{n_2}=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\1&2&3\\6&-1&5\end{vmatrix}=(13,13,-13)\,//\,(1,1,-1)$$

所求平面方程:

$$x+y-z=0$$

四. 1. $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}=\cdots$

$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}=\cdots$　最后 u, v 代入.

2. 令 $F=e^z-z+xy-3$

$F_x=y$, $F_z=e^z-1$, $\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{y}{e^z-1}$

$$\frac{\partial^2 z}{\partial x^2}=\frac{ye^z\cdot\frac{\partial z}{\partial x}}{(e^z-1)^2}=-\frac{y^2e^z}{(e^z-1)^3}$$

3. 解: $f_x = 3x^2 - 3y = 0$
$f_y = y - 3x = 0$，得驻点 $(0,0)$，$(3,9)$

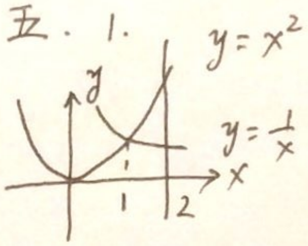
$A = f_{xx} = 6x$，$B = f_{xy} = -3$，$C = f_{yy} = 1$

$\Delta = AC - B^2 = 6x - 9$

对于点 $(0,0)$，$\Delta = -9 < 0$，$f(0,0)$ 不是极值.

对于点 $(3,9)$，$\Delta = 9 > 0$，$A > 0$，极小值 $f(3,9) = -\frac{27}{2}$.

五. 1.



解: $\iint\limits_D xy^2\,dx\,dy = \int_1^2 dx \int_{\frac{1}{x}}^{x^2} xy^2\,dy$

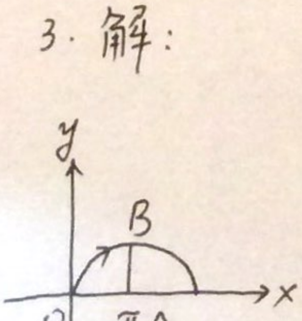
$= \frac{1}{3}\int_1^2 \left(x^7 - \frac{1}{x^2}\right)dx = \frac{1}{3}\left[\frac{1}{8}x^8 + \frac{1}{x}\right]\Big|_1^2$

$= \frac{251}{24}$

2. 解 $\iiint\limits_V xy\,dx\,dy\,dz = \int_0^{\frac{\pi}{2}} d\theta \int_0^1 dr \int_0^1 r^3 \sin\theta\cos\theta\,dz$

$= \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\,d\theta \int_0^1 r^3\,dr = \frac{1}{8}\sin^2\theta\Big|_0^{\frac{\pi}{2}} = \frac{1}{8}$

3. 解: $P = x^4 + 4xy^3$，$Q = 6x^2y^2 - 5y^4$



$\frac{\partial Q}{\partial x} = 12xy^2$，$\frac{\partial P}{\partial y} = 12xy^2$. $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

可知积分与路径无关.

原式 $= \int_{\overline{OA} + \overline{AB}} (x^4 + 4xy^3)\,dx + (6x^2y^2 - 5y^4)\,dy$

$$\overline{OA}:\begin{cases} x=x \\ y=0 \end{cases},\quad x:0\to\frac{\pi}{2}$$

$$=\int_0^{\frac{\pi}{2}} x^4 dx+\int_0^1\left(\frac{3\pi^2}{2}y^2-5y^4\right)dy$$

$$\overline{AB}:\begin{cases} x=\frac{\pi}{2} \\ y=y \end{cases},\quad y:0\to 1$$

$$=\cdots$$

六、1. 解：先考虑级数 $\sum\limits_{n=1}^{\infty}\left|(-1)^{n-1}\frac{\sqrt{n}}{2^n}\right|=\sum\limits_{n=1}^{\infty}\frac{\sqrt{n}}{2^n}$

$$u_n=\frac{\sqrt{n}}{2^n},\quad \lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}\frac{\sqrt{n+1}}{2^{n+1}}\cdot\frac{2^n}{\sqrt{n}}=\frac{1}{2}<1,$$

可知级数 $\sum\limits_{n=1}^{\infty}\frac{\sqrt{n}}{2^n}$ 收敛，故级数 $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{2^n}$ 绝对收敛，

因此级数 $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{\sqrt{n}}{2^n}$ 收敛.

2. 解：$u_n=(-1)^{n+1}\frac{x^{2n+1}}{2n+1}$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=\lim_{n\to\infty}\left|\frac{x^{2(n+1)+1}}{2(n+1)+1}\cdot\frac{2n+1}{x^{2n+1}}\right|=x^2$$

当 $x^2<1$ 时，$-1<x<1$，级数收敛

当 $x^2>1$ 时，$|x|>1$，级数发散.

收敛半径 $R=1$，收敛区间 $(-1,1)$

$x=1$，$\sum\limits_{n=0}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ 收敛，

$x=-1$，$\sum\limits_{n=0}^{\infty}(-1)^{n+1}\frac{(-1)^{2n+1}}{2n+1}=\sum\limits_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$ 收敛. 收敛域为 $[-1,1]$.

设 $S(x)=\sum\limits_{n=0}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{2n+1}$，$x\in[-1,1]$

$$S'(x)=\sum_{n=0}^{\infty}(-1)^{n+1}x^{2n}=-\sum_{n=0}^{\infty}(-x^2)^n=-\frac{1}{1+x^2}$$

$$\int_0^x S(x)dx=\int_0^x-\frac{1}{1+x^2}dx,\quad S(x)-S(0)=-\arctan x$$

$$S(0)=0,\quad S(x)=-\arctan x.$$