自测题1答案

一、选择题

B C C C A C

二、填空题

1. $\frac{6}{\sqrt{61}}$ 2. $(x+y)e^{xy}f_1+(\frac{1}{y}-\frac{x}{y^2})f_2$ 3. $dx-dy$ 4. $(1,1,2)$

5. $\int_{-1}^{0}dx\int_{x^2}^{\sqrt{-x}}f(x,y)dy$ 6. $\frac{4\pi}{5}$

三、计算下列各题

1. 解 $\frac{dy}{dx}=\frac{1-\frac{y}{x}}{1+2\frac{y}{x}}$，令 $\frac{y}{x}=u$，则

$$u+x\frac{du}{dx}=\frac{1-u}{1+2u},\quad \frac{1+2u}{1-2u-2u^2}du=\frac{1}{x}dx$$

两边积分，$\int\frac{1+2u}{1-2u-2u^2}du=\int\frac{1}{x}dx$，

$$-\frac{1}{2}\int\frac{1}{1-2u-2u^2}d(1-2u-2u^2)=\int\frac{1}{x}dx$$

$$-\frac{1}{2}\ln|1-2u-2u^2|=\ln|x|\underbrace{-\frac{1}{2}\ln|C|}$$

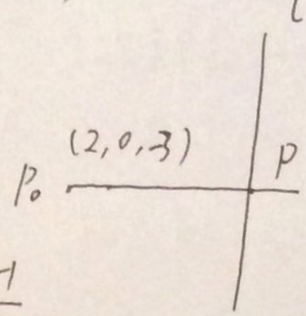
（反正是任意常数）

$$x^2(1-2\frac{y}{x}-2\frac{y^2}{x^2})=C$$

$$x^2-2xy-2y^2=C$$

$y|_{x=1}=1$，得 $C=-3$． 即 $x^2-2xy-2y^2=-3$

2. 解：在直线 $\begin{cases} x+y-1=0 \\ x+z-2=0 \end{cases}$ 上任取一点 $A(1,0,1)$



$$\vec{s}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix}=(1,-1,-1)$$

直线的对称式方程为：$\dfrac{x-1}{1}=\dfrac{y}{-1}=\dfrac{z-1}{-1}$

参数方程为 $\begin{cases} x=1+t \\ y=-t \\ z=1-t \end{cases}$

设点 P 的坐标为 $(1+t,-t,1-t)$，则

$\overrightarrow{P_0P}=(t-1,-t,-t+4)$.

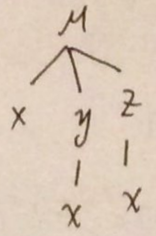
由于 $\vec{s}\perp\overrightarrow{P_0P}$，得 $t-1+t+t-4=0$，$t=\dfrac{5}{3}$，

$\overrightarrow{P_0P}=\left(\dfrac{2}{3},-\dfrac{5}{3},\dfrac{7}{3}\right)\,//\,(2,-5,7)$

所求直线方程为 $\dfrac{x-2}{2}=\dfrac{y}{-5}=\dfrac{z+3}{7}$

四.

1.



$$\frac{du}{dx}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\cdot\frac{dy}{dx}+\frac{\partial u}{\partial z}\cdot\frac{dz}{dx}$$

$$=\frac{e^{ax}\cdot a(y+z)}{a^2+b^2}+\frac{e^{ax}}{a^2+b^2}\cdot a\cos x+\frac{e^{ax}}{a^2+b^2}\cdot(-b\sin x)$$

$$=\frac{e^{ax}}{a^2+b^2}\left[(a^2-b)\sin x+(ab+a)\cos x\right]$$

2. 设 $F(x,y,z)=x^3+y^3+z^3+xyz-4$

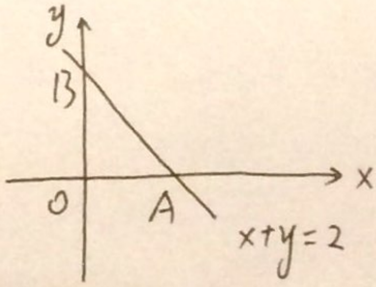
$F_x=3x^2+yz$，$F_y=3y^2+xz$，$F_z=3z^2+xy$.

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{3x^2+yz}{3z^2+xy}，\quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{3y^2+xz}{3z^2+xy}$$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,1,1)}=-1，\quad \left.\frac{\partial z}{\partial y}\right|_{(1,1,1)}=-1$$

$$\left.dz\right|_{(1,1,1)}=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-(dx+dy)$$

3.



$f(x,y)=y^2-xy+\frac{1}{2}x$

$f_x=-y+\frac{1}{2}$，$f_y=2y-x$，得驻点 $\overset{(1,\frac{1}{2})}{\cancel{(\frac{1}{2},1)}}\in D$.

$f(1,\frac{1}{2})=\frac{1}{4}$

在~~直线~~线段 OA上：$\begin{cases}0\le x\le 2\\ y=0\end{cases}$，$f(x,y)=\frac{1}{2}x$，$f(0,0)=0$，$f(2,0)=1$

在~~直线~~线段 OB上：$\begin{cases}x=0\\ 0\le y\le 2\end{cases}$，$f(x,y)=0$

在~~直段~~线段 AB上：$\begin{cases}0\le x\le 2\\ y=2-x\end{cases}$，$\varphi=f(x,y)=(2-x)^2-x(2-x)+\frac{1}{2}x$

$$=2x^2-\frac{11}{2}x+4$$

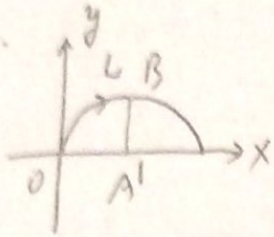
$$\varphi'=4x-\frac{11}{2}=0，\quad x=\frac{11}{8}$$

$$f(\frac{11}{8},\frac{5}{8})=\frac{7}{32}$$

所以 最大值为1，最小值为0

五. 1. $\iint\limits_D\sqrt{1+x^2+y^2}\,dxdy=\int_0^{2\pi}d\theta\int_1^2 r\sqrt{1+r^2}\,dr=\cdots$

2. $\iiint\limits_V(x+y)\,dxdydz=\int_0^1dx\int_0^{1-x}dy\int_0^{1-x-y}(x+y)\,dz=\cdots$

3. 

$P = e^x \sin y - x + 1$, $Q = e^x \cos y - 1$

$\dfrac{\partial Q}{\partial x} = e^x \cos y$, $\dfrac{\partial P}{\partial y} = e^x \cos y$ $\quad \dfrac{\partial Q}{\partial x} = \dfrac{\partial P}{\partial y}$

可知积分与路径无关.

$\overline{OA}: \begin{cases} x = x \\ y = 0 \end{cases} \quad x: 0 \to 1$

原式 $= \displaystyle\int_{\overline{OA}+\overline{AB}} (e^x \sin y - x + 1)dx + (e^x \cos y - 1)dy$

$= \displaystyle\int_0^1 (-x+1)dx + \int_0^1 (e\cos y - 1)dy$

$\overline{AB}: \begin{cases} x = 1 \\ y = y \end{cases} \quad y: 0 \to 1$

$= \cdots$

六 1. 解: $u_n = \dfrac{2^n n!}{n^n}$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{2^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n n!} = \lim_{n\to\infty} \frac{2}{(1+\frac{1}{n})^n} = \frac{2}{e} < 1$$

可知级数 $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ 收敛.

2. 解: 令 $x - 1 = t$, $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} \frac{t^n}{n \cdot 2^n}$

$a_n = (-1)^{n-1} \dfrac{1}{n \cdot 2^n}$, $R = \displaystyle\lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| = \lim_{n\to\infty} \frac{(n+1) \cdot 2^{n+1}}{n \cdot 2^n} = 2$

$t = 2$, $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} \frac{t^n}{n \cdot 2^n} = \sum_{n=0}^{\infty} (-1)^{n-1} \cdot \frac{2^n}{n \cdot 2^n} = \sum_{n=0}^{\infty} (-1)^{n-1} \frac{1}{n}$ 收敛.

$t = -2$, $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} \frac{t^n}{n \cdot 2^n} = \sum_{n=0}^{\infty} \frac{-1}{n}$, 发散.

$-2 < t \le 2$, $-2 < x - 1 \le 2$, $-1 < x \le 3$

幂级数 $\displaystyle\sum_{n=0}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n \cdot 2^n}$ 的收敛域为 $(-1, 3]$.