

P54. 2.10. 解: (1). 由牛顿第二定律:

$$F_{\text{合}} = -kv = ma = m \frac{dv}{dt}$$

分离变量: $-\frac{k}{m} dt = \frac{dv}{v}$

$$\int_{v_0}^v \frac{dv}{v} = \int_0^t -\frac{k}{m} dt$$

$$\ln \frac{v}{v_0} = -\frac{k}{m} t$$

$$v = v_0 e^{-\frac{k}{m} t}$$

$$\begin{aligned} (2) \quad x &= \int_0^t v dt = \int_0^t v_0 e^{-\frac{k}{m} t} dt \\ &= \frac{mv_0}{k} (1 - e^{-\frac{k}{m} t}) \end{aligned}$$

$$(3) \quad \text{当 } t \rightarrow \infty \quad x = \frac{mv_0}{k}$$

$$(4) \quad \text{当 } t = \frac{m}{k}, \quad v = v_0 e^{-\frac{k}{m} \cdot \frac{m}{k}} = v_0 e^{-1}$$

$$\therefore \frac{v}{v_0} = \frac{1}{e}$$

P77. 3.12 解: 设左侧绳子拉力为 T_1 , 右侧绳子拉力为 T_2 ,

由牛顿定律:
$$\begin{cases} m_2 g - T_2 = m_2 a_2 \\ T_1 - m_1 g = m_1 a_1 \\ T_2 R - T_1 r = (\frac{1}{2} m r^2 + \frac{1}{2} M R^2) \alpha \\ a_1 = \alpha r \\ a_2 = \alpha R \end{cases}$$

代入数据:
$$\begin{cases} 2 \times 9.8 - T_2 = 2 a_2 \\ T_1 - 2 \times 9.8 = 2 a_1 \\ T_2 \times 0.2 - T_1 \times 0.1 = (\frac{1}{2} \times 4 \times 0.1^2 + \frac{1}{2} \times 10 \times 0.2^2) \alpha \\ a_1 = 0.1 \times \alpha \\ a_2 = 0.2 \times \alpha \end{cases}$$

解得:
$$\begin{cases} \alpha = 6.125 \text{ rad/s}^2 \\ T_1 = 20.825 \text{ N} \\ T_2 = 17.15 \text{ N} \end{cases}$$

3.14 解: (1) $mg \frac{l}{2} = J \alpha = \frac{1}{3} m l^2 \alpha$

$$\alpha = \frac{3g}{2l}$$

(2) 由机械能守恒可得:

$$mg \frac{l}{2} \sin \theta = \frac{1}{2} J \omega^2 = \frac{1}{2} \cdot \frac{1}{3} m l^2 \omega^2$$

$$\omega = \sqrt{\frac{3g}{l} \sin \theta}$$

3.15 解: (1) 碰撞瞬间, 角动量守恒.

可得: $m v_0 l = m v l + \frac{1}{3} M l^2 \omega$

由于是弹性碰撞, 动能守恒:

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{1}{3} M l^2 \omega^2$$

杆子上摆过程, 机械能守恒.

$$\frac{1}{2} \cdot \frac{1}{3} M l^2 \omega^2 = M g \frac{l}{2} (1 - \cos 30^\circ)$$

解得: $v_0 = \frac{\sqrt{6(2-\sqrt{3})} g l}{12} \cdot \frac{3m+M}{m}$

(2) 小球受到一冲量为其动量之改变.

$$\begin{aligned} \Delta p &= m v - m v_0 \\ &= -\frac{\sqrt{6(2-\sqrt{3})} g l}{6} M \end{aligned}$$

3.16 解: (1) 飞出去一瞬间, 碎片二角速度为 ω .

$$\therefore \omega = \omega_0 R$$

$$\therefore h = \frac{v^2}{2g} = \frac{\omega^2 R^2}{2g}$$

(2) 由于碎片飞出瞬间, 角动量守恒.

故飞后二角速度不变, 仍为 ω .

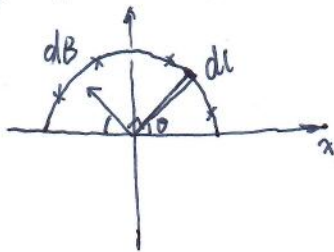
$$\therefore \text{角动量大小: } L = J \omega$$

$$= (\frac{1}{2} M R^2 + m R^2) \omega$$

转动动能

$$\begin{aligned} E_k &= \frac{1}{2} J \omega^2 \\ &= \frac{1}{2} (\frac{1}{2} M R^2 + m R^2) \omega^2 \end{aligned}$$

10.11 解: 利用无限长通电直导线之理论: $B = \frac{\mu_0 I}{2\pi r}$



单位长度之电流 $j = \frac{I}{\pi R}$

$$dI = \frac{I}{\pi R} \cdot dl = \frac{I}{\pi} d\theta$$

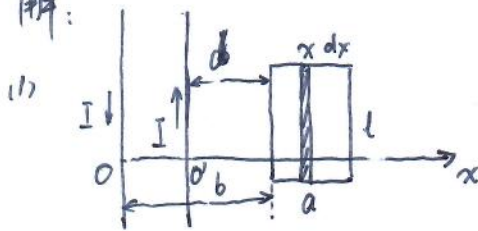
$$dB = \frac{\mu_0}{2\pi R} \cdot dI$$

$$dB_x = dB \cdot \cos\left(\frac{\pi}{2} - \theta\right) = dB \cdot \sin\theta$$

$$B_x = \int dB_x = \int_0^\pi \frac{\mu_0}{2\pi R} \cdot \frac{I}{\pi} d\theta \cdot \sin\theta$$

$$= \frac{\mu_0 I}{\pi^2 R} = \frac{4\pi \times 10^{-7} \times 5}{\pi^2 \times 0.01} = 6.37 \times 10^{-5} \text{ T}$$

11.6 解:



解: 如图建立坐标系:

对于左边电流 I , $B = \frac{\mu_0 I}{2\pi x}$

$$\phi = \int_b^{b+a} \frac{\mu_0 I}{2\pi x} \cdot l \cdot dx = \frac{\mu_0 I l}{2\pi} \ln \frac{b+a}{b}$$

对于右边电流 I

$$\phi' = \int_d^{d+a} \frac{\mu_0 I}{2\pi x} \cdot l \cdot dx = \frac{\mu_0 I l}{2\pi} \ln \frac{d+a}{d}$$

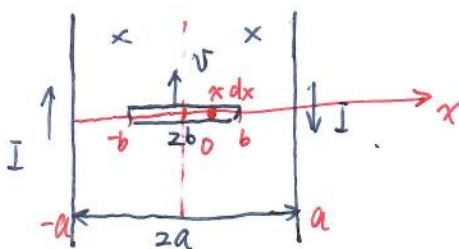
电流方向相反.

$$\begin{aligned} \phi_{\Sigma} &= \phi' - \phi = \frac{\mu_0 I l}{2\pi} \left(\ln \frac{d+a}{d} - \ln \frac{b+a}{b} \right) \\ &= \frac{\mu_0 I l}{2\pi} \ln \frac{(d+a)b}{d(b+a)} \end{aligned}$$

$$(2) \quad \mathcal{E} = -\frac{d\phi}{dt} = -\left[\frac{\mu_0 l}{2\pi} \ln \frac{(d+a)b}{d(b+a)} \right] \frac{dI}{dt}$$

11.12 解:

如图建立坐标系:



$$B = \frac{\mu_0 I}{2\pi(a+x)} + \frac{\mu_0 I}{2\pi(a-x)}$$

$$d\mathcal{E} = B \cdot v \cdot dx$$

$$\mathcal{E} = \int_{-b}^b \left[\frac{\mu_0 I}{2\pi(a+x)} + \frac{\mu_0 I}{2\pi(a-x)} \right] \cdot v \cdot dx$$

$$\mathcal{E} = \int_{-b}^b \left[\frac{\mu_0 I}{2\pi(a+x)} + \frac{\mu_0 I}{2\pi(a-x)} \right] \cdot v \cdot dx$$

$$= \frac{\mu_0 I v}{\pi} \ln \frac{a+b}{a-b}$$

左端为正极.

右端为负极.