Circuit Theory and Electronics Fundamentals

David Cristino (96518), João Marques (96542), Rodrigo Pereira (96563)

Masters in Physics and Technological Engineering, Técnico, University of Lisbon

Simple Circuit Analysis Report

March 25, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing resistors that follow Ohm's law, independent and linearly dependent voltage and current sources. With this analysis, we want to determine the potential at each of the eight nodes and the current flowing through all of the ten branches. With this purpose, we'll use both node method, that is, the usage of KVL theorem at all the nodes, considering the voltage of the nodes, and mesh method, that is, applying KCL theorem in all of the four meshes, each one with a current defined by us.

The circuit explained, as well as all numbered nodes and mesh currents, can be seen in Figure 1

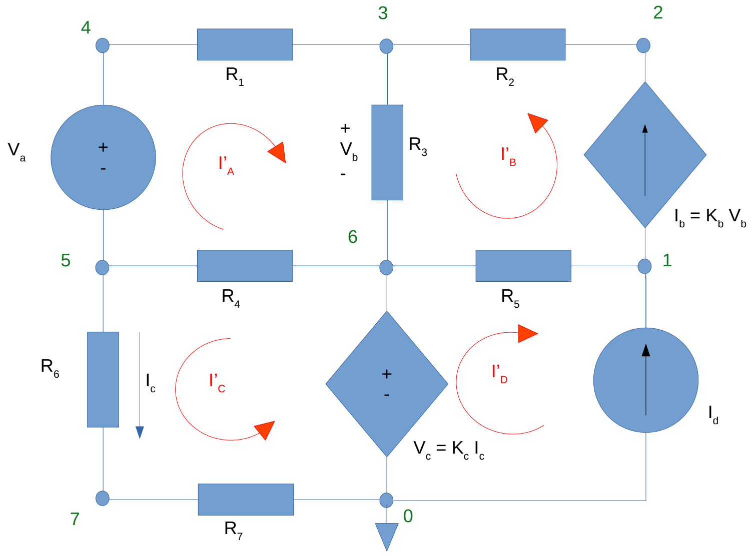

Figure 1: Analyzed circuit.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown earlier in Figure 1 is analyzed, using both node and mesh methods, as explained in section 1. It is important to notice that node 0 corresponds to ground, meaning we assign it has having null potential.

A table with all the given values can be seen in table 1

Name	Values
R_1	1.04005394176 $k\Omega$
R_2	2.07146823978 $k\Omega$
R_3	3.06015694112 $k\Omega$
R_4	4.13750728298 $k\Omega$
R_5	3.13205467735 $k\Omega$
R_6	2.01065636997 $k\Omega$
R_7	1.00318758033 $k\Omega$
V_a	5.14514577871 V
I_d	1.03830911265 mA
K_b	7.04881622155 mS
K_c	8.3495605781 $k\Omega$

Table 1: Given values for the circuit components.

2.1 Node method

Using Kirchhoff Current Law (KCL) at all nodes, we end up with these equations:

$$\begin{cases}(1)I_d = \frac{V_1-V_6}{R_5} + I_b \\ (2)I_b = \frac{V_2-V_3}{R_2} \\ (3)\frac{V_2-V_3}{R_2} = \frac{V_3-V_4}{R_1} + \frac{V_b}{R_3} \\ (4)V_4 - V_5 = V_a \\ (5)\frac{V_6-V_5}{R_4} = \frac{V_5-V_7}{R_6} + \frac{V_4-V_3}{R_1} \\ (6)V_6 = V_c \\ (7)\frac{V_5-V_7}{R_6} = \frac{V_7}{R_7} \\ I_b = K_bV_b = K_b(V_3 - V_6) \\ V_c = K_cI_c = K_c\frac{V_5-V_7}{R_6}\end{cases} \tag{1}$$

Where V_i represents the voltage at node i. It is important to notice that, because nodes 4 and 5 were connected to a voltage source, V_a, we use the source equation on equation 1.4, and for equation 1.5, we consider the super-node 4-5. We used a similar though process for equation 1.6.

simplifying these expressions and substituting variables by their numeric number given in table 1, and solving this system of linear equations using *Octave*, we end up with:

Node number	**Voltage (V)**
V_1	1.183121031110900e+01
V_2	7.046930516437613e+00
V_3	7.640710157945843e+00
V_4	7.917802934993277e+00
V_5	2.772657156283278e+00
V_6	7.681376099908987e+00
V_7	9.229061854445412e-01

Table 2: Node voltage obtained by the theorethical analysis.

We can get the current flowing through each branch using Ohm's law.

2.2 Mesh method

Using Kirchhoff Voltage Law (KVL) in all meshes, we can write these equations:

$$\begin{cases}(A)R_1I'_A + R_3(I'_A + I'_B) + R_4(I'_A + I'_C) = V_a \\ (B)I'_B = I_b = K_bV_b = K_bR_3(I'_A + I'_B) \\ (C)R_4(I'_A + I'_C) + R_6I'_C + R_7I'_C = K_cI_c = K_cI'_C \\ (D)I'_D = -I_d\end{cases} \tag{2}$$

Because mesh B was connected to a current source, we can write the equation 2.B for this mesh. Similarly, we can write equation 2.D, for mesh D. As done in the previous subsection, we can rearrange terms and replace the variables by their numeric number given in table 3. Solving this system of linear equations using *Octave*, we end up with:

Mesh name	Current (A)
I'_A	2.663681225830452e-04
I'_B	-2.793171676344597e-04
I'_C	9.197892270708518e-04
I'_D	-1.038309112650000e-03

Table 3: Theorethical Current flowing in each mesh.

Using this information, we can now solve for the current flowing in each component, as seen in table 4

Component	Current (A)
I_b	-2.793171676344597e-04
$R_1[i]$	2.663681225830452e-04
$R_2[i]$	-2.793171676344597e-04
$R_3[i]$	-1.294904505141448e-05
$R_4[i]$	1.186157349653897e-03
$R_5[i]$	-1.317626280284460e-03
$R_6[i]$	9.197892270708518e-04
$R_7[i]$	9.197892270708518e-04
$V_c[i]$	1.185198855791483e-04
$V_a[i]$	-2.663681225830452e-04

Table 4: Theorethical Current flowing through each component.

3 Simulation Analysis

For the circuit simulation we used the *Ngspice* software. It is important to point that, for this analysis we needed to add an independent voltage source with value 0V to compute the current I_c, needed on the dependent voltage source, V_c. Figure 2 represents the simulated circuit

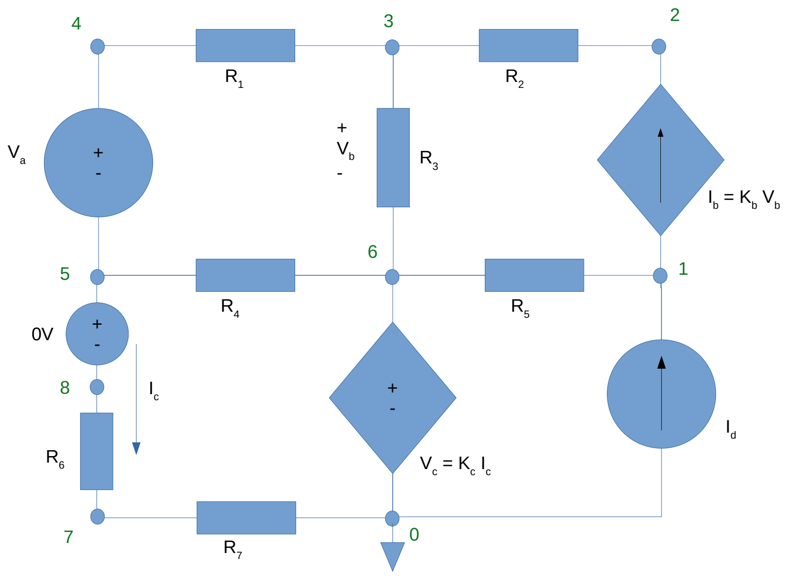

Figure 2: Simulated circuit.

3.1 Node Analysis

Table 5 shows the node voltages obtained using the simulated operating point results for the circuit under analysis

Node number	Voltage (V)
v(1)	1.180671e+01
v(2)	7.061613e+00
v(3)	7.640210e+00
v(4)	7.917247e+00
v(5)	2.772101e+00
v(6)	7.679836e+00
v(7)	9.227211e-01
v(8)	2.772101e+00

Table 5: Node voltage obtained by the simulation analysis.

Compared to the theoretical analysis results, we notice that both are similar, having small numeric differences between them ($\sim$ 0.2%). The most likely cause of this is due to the fact that *Octave* and *Ngspice* have different numerical precision.

3.2 Mesh Analysis

Table 6 shows the current flowing in each component using the simulated operating point results for the circuit under analysis

Component	Current (A)
@gb[i]	-2.79317e-04
@r1[i]	2.663681e-04
@r2[i]	-2.79317e-04
@r3[i]	-1.29490e-05
@r4[i]	1.186157e-03
@r5[i]	-1.31763e-03
@r6[i]	9.197892e-04
@r7[i]	9.197892e-04
hc-branch	1.185199e-04
va-branch	-2.66368e-04

Table 6: Simulated Current flowing through each component.

Compared to the theoretical analysis results, we notice that, like in subsection 3.1, both are similar, having differences $\sim 1 \times 10^{-4}$%. We think the causes are the same that we described earlier.

4 Conclusion

In this laboratory assignment the objective of analysing a simple circuit using both node and mesh method has been achieved. The theoretical equations have been solved using *Octave*, and the circuit simulation was obtained using *Ngspice* software. The simulation results don't match precisely with the theoretical ones. This is due to the fact that these tools use different numerical precision in their calculations. However, the differences were negligible. The reason for this small errors is the fact that this circuit only contains linear components, so the theorethical and simulation results cannot differ very much. In conclusion, even though the two analysis don't match completely, given the small differences we consider the results to be highly satisfying.