THE MECHANICS OF BINARY INTERACTIONS

Our system comprises an infinite number of mechanically identical, spherical particles known as monomers. These particles have masses m_1 and radii R_1. Although their masses are quite small, they can interact with each other gravitationally. When two monomers collide, they lose a certain amount of impact energy and rebound with a coefficient of restitution ε. If the impact energy is below a specific threshold value $E_{\mathrm{imp}} \leqslant E_{\mathrm{agg}}$, the monomers stick together due to surface forces like van der Waals forces, forming a larger aggregate particle with mass m_2 and radius R_2. This process is known as *aggregation* and allows the creation of larger particles from individual monomers.

Conversely, there is another mechanism called *fragmentation* that decreases the sizes of aggregates. If the impact energy is higher than a certain threshold value $E_{\mathrm{imp}} \geqslant E_{\mathrm{frag}}$, the colliding aggregates break into smaller pieces.

Distribution function

The statistical description of the system is given by

Collision mechanics

We consider a collision of two particles of masses m_i, m_j, and velocities $\boldsymbol{v}_i$, $\boldsymbol{v}_j$. If the particles did not exert gravitational influence, the collision geometry would have been a linear problem. However, the gravitational interaction of the particles result in a deflection of the trajectories of motion, aka *gravitational focusing.* To analyze the collision mechanics of two gravitationally interacting bodies, let us reformulate the problem into a mathematically equivalent one. Consider a point size particle with mass $\mu = m_i m_j / M$ and velocity $\boldsymbol{g}_0 = \boldsymbol{v}_i - \boldsymbol{v}_j$, that moves in a gravitational field of a stationary body of mass $M = m_i + m_j$ and radius $R = R_i + R_j$. The collision happens if the point mass falls onto the surface of the stationary body. Let us introduce a parameter b, which is a distance from the center of the stationary mass to the trajectory asymptote of the moving particle. The vector $\boldsymbol{b}$ is then orthogonal to the velocity $\boldsymbol{g}$. There is a certain value of this parameter b_{max}, which defines the collision threshold, e.g. when the impact parameter b is smaller than this threshold value, the moving particle falls onto the surface of the stationary body. The value of this threshold value is

$$b_{\mathrm{max}}(g_0) = R\sqrt{1 + \frac{2GM}{Rg_0^2}}, \tag{1}$$

where G is the gravitational constant. The value of the relative velocity g_0 is considered at infinity, hence at the point of impact, the relative speed, or in other words, the impact speed g is

$$g = g_0\sqrt{1 + \frac{2GM}{Rg_0^2}}, \tag{2}$$

slightly larger than g_0. The point of impact on the surface of the stationary body can be represented by a unit vector $\hat{\boldsymbol{e}}$, pointing from the center of the body. However, the actual impact speed does not depend on $\hat{\boldsymbol{e}}$, i.e. as long as the impact parameter of the point mass particle is less than the threshold value b_{max}, it falls onto the surface of the stationary body with the same speed g for any value b of the impact parameter, due to energy conservation and spherical shape of the body.

Aggregation mechanics